I Introduction

In a traditional search market setting, buyers and sellers expend time and money in order to find each other. This process is traditionally viewed as expensive and costly to society. In this paper I make the opposite argument that in some cases, the existence of search friction produces rational and efficient outcomes. I discuss the costs and time spent searching for a match between a buyer and a seller. A portion of one group is willing to bear the costs and this reveals information about their unobserved quality. I apply this idea to the Angel Capital market, where Angel Capitalists attempt to screen entrepreneurs by requiring the entrepreneur to hunt for them. Only good entrepreneurs will bear the costs of searching for an Angel, because a successful match is worth more the better the entrepreneur.

Angel Capitalists make small scale, high risk investments with high expected returns. Typically in this market, one "Angel" matches with one Entrepreneur and funds their project. A survey of New England Angels completed by Wetzel (1983) discusses the “continuing perception that gaps exist in the capital markets for small firms.” He means that while Venture Capital, formal companies that fund projects and advertise their services, exist; it does not account for much the funding of small young firms. Many small firms are funded by Angels, who are not easily observed and do not advertise their willingness to fund projects. Angels are much harder to find than Venture Capitalists. This paper shows how the costly search process can lead to better outcomes for all parties. Angels “hide,” Entrepreneurs “seek” and reveal the quality of their projects

The Angel has capital, but lacks an investment project. The entrepreneur lacks capital but has an investment project. This paper assumes that the entrepreneur has exhausted her own financial resources as well as any family or friend's willingness to contribute.[1] The project is at an infant stage and it is unclear whether it will be a success or failure. This precludes the entrepreneur from traditional capital sources such as a bank. Angels offer small amounts of capital to give a project the chance to improve; the eventual goal of both the groups is either to sell the project to an existing company or to sell it to the public. Because the potential profits are so large the angel is willing to accept a long period with little to no return. This further differentiates the investor from the traditional capital markets, especially bank loans.[2]

This paper makes an important assumption that the entrepreneur has better information about the probability of success than the Angel; this is reasonable insofar that the entrepreneur is not delusional and irrationally believes in her abilities and ideas. I assume the projects that Entrepreneurs wish to find funding have a range of profitability. These are expected profits as the outcome of the project is unknown. The Angel attempts to screen the entrepreneurs by traditional means. These screens include examining job and education history or IQ and other ability means tests. Each of these will be correlated with the success of the project; however, at every level of observed ability there remains a large variation in the probability of success. Additionally the correlation may disappear above a certain level of observed ability. A survey by Prowse (1998) of Angels in the Dallas-Fort Worth area discusses these search costs as if they were inefficient, "The search process does appear to be very time

[1] This type of investment is known as Love Capital. I make a distinction from rational Angel Capital, and possibly irrational Love Capital.
[2] The fragility of the liabilities of a bank makes long term uncollateralized contracts unattractive.

consuming and awkward." The screening by imposing search costs occurs before candidates are screened in more traditional ways. I argue that the search process is both purposeful and consistent with rational Angels. At least for some Angels it screens the pool of potential applicants and therefore increases their expected profit. That the search for the Entrepreneur is "time consuming and awkward" serves the Angel well. This paper will proceed as if no other screening process can gain information about the quality of the entrepreneur.

Imagine that the Angel remains in her office working on other projects waiting to be found. The Angel accepts the possibility that no Entrepreneur will find them in exchange for an increase in the quality of the pool of applicants. The Entrepreneur must exert considerable effort, in the form of shoe leather, brow sweat, and opportunity cost of other activities, in order to search the business world for an Angel to fund their project. The search may take place while the entrepreneur is on the job; however, she will likely give up some chance at advancement and success when using all her effort to search for an angel. The Entrepreneur bears this cost in order to have an increased probability of a match with an Angel. For the better Entrepreneurs matches with Angels are worth more.

After the agents and their respective problems are formally stated, I discuss the matching technology. All entrepreneurs are assumed to have the same "ability" to search. A natural extension is to consider the possibility that the entrepreneurs with good projects are intrinsically better at search. The search and bargaining models build upon Rogerson et al**.** (2005) and Pissarides (2000), which apply the search framework to the labour market.

Equilibrium is established if every agent is satisfied with their choice of search intensity.. If I can show that deviation from their choice yields lower expected profit. In any equilibrium each marginal agent is at best indifferent between changing her choices. Entrepreneurs face a crowding externality; other Entrepreneurs that are searching for the same Angel Capital reduce the likely hood of a successful match. Each Angel faces a choice in how difficult to make herself to find by potential Entrepreneurs. The higher the cost the better the expected profit of the firm, but the lower the probability of a match for the Angel. Angels will choose a search function from a continuum, only Entrepreneurs that attempt to search using that method will find those Angels.

Section 2 reviews the relevant economic and business literature regarding the Angel Capital market. Section 3 summarizes the search theory literature as it directly pertains to this paper. Section 4 declares the notation used in this paper and begins the model by considering the search market with entry decisions of both agents. Section 5 provides a proof of a Nash Equilibrium where every Angel has the same search function and derives the optimal level for the financial market. Section 6 discusses social welfare and government policy. It will also discuss some of the public policy implications Section 7 briefly concludes.

2. Angel Literature

Wetzel (1983) describes the market for the seed capital or start up financing for "small technology based firms." The paper that argues many claim there is a substantial funding gap in the banking and financial industry for these small scale projects. However, Wetzel remarks that such observers likely consider only traditional capital sources and fail to recognize "business

angels," which he claims are the largest source of funding for small risky ventures in the technology industry. For Entrepreneurs attempting to find the initial capital that comes from outside friends and family is the main barrier to the continuation of projects. Most Entrepreneurs found their funding through "business associations, friends and personal search" with substantially fewer finding their funding with the aid of investment bankers. He believes that the invisibility of Angel Capitalists is commonly misinterpreted to imply that they do not exist.

Wetzel surveyed 133 Angels from New England across many of the industries that are funded using Angel Capital. Among those with experience funding projects as an Angel he found remarkably high returns for those lucky enough to have successful projects. 50% per annum over 5 years is a staggeringly high return. The problem of survivor bias is not addressed; it is likely that his survey only includes Angels that were successful; he does not have any data on the numbers of discouraged Angels. Besides these omissions the sample offers interesting insight into how the Angels view their market. Many were dissatisfied with the difficulty of finding potential projects; however, the privacy of their information was very important to them. In most markets the participants wish to announce their willingness to engage in trade as loudly as possible, contrary to the apparent findings among Angel investors.

Prowse (1998) performs the same sort of survey as Wetzel except on Angels in the Dallas-Fort Worth region. He confirms many of the conclusions that Wetzel reached about the risk and potential rewards of funding such projects as well as the difficulty and costs of the search process. He claims that the firms ultimately wish to become publicly traded firms in an

IPO —few end so successfully. The Texas Capital Network attempted to reduce the difficulties of the search process by attempting to match projects with financing directly. However, according to Prowse the effort was successful in matching firms and funding, but very few deals were signed. Apparently Angels are not satisfied with the projects they are matched with in a less costly environment for Entrepreneurs.

Lerner (1998) reviews the public policy in the United States to encourage Angel Capital. The author argues that private Angels offer a socially sub optimal level of funding to small young technology firms. He finds a case for government intervention to fund these projects. The moral hazard issues discussed by Lerner deal with the ex post actions of the Entrepreneurs; after a firm is capitalized the Entrepreneur have the incentive to shirk and to spend the firms funds personal rather than firm interests. This paper will assume that the principal agent problem is eliminated by enforceable contracts or efficient monitoring. Lerner focuses on this problem which is a famous application of information asymmetries. However, he ignores the greater information asymmetry that the Entrepreneur may be better ex ante informed about the probability of success of the project.

Lerner finds venture capital firms unable to efficiently fund the small technology based firms and recommends that public policy be formed to encourage Angels. He wishes to explore ways to increase the value added by an investor to a firm beyond the capital provided. This is an issue that this paper will not address.

Vo (2007) imposes a search model on the problem of which public policy will be the most effective to encourage Angels to invest in the projects that Entrepreneurs offer. He finds

policy to reduce the search costs will have a “mild impact” on the amount of funding. Instead he suggests that Angels be better educated in the workings of the market as well as the skills that will add value to any potential firm. Additionally decreasing the capital gains tax on Angel type investments will increase the value of a match for the Angel and encourage activity as the Angel will internalize a greater percentage of the project return.

3. Search Literature

In his 2000 book, Christopher Pissarides presents the theoretical backing behind modeling the labour market with the problem of matching treated directly. Unemployed workers search for jobs that are randomly assigned with some frequency from a pool of vacant positions. The below model borrows heavily from the methodology and language of the basic model described in the first sections of the book.

The unemployment search theory has several applications to economic issues. Firstly, the simple neo-classical model of labour supply provides no compelling explanation of why workers would accept periods of unemployment. In the neo-classical world every worker is assumed to be efficiently allocated and all unemployed workers cannot be employed profitably. In contrast the Pissarides matching model argues that workers face a number of possible employers with different wage offers. Offers are randomly assigned to potential workers they will wait for a period to receive an optimal number of offers or an offer above their reservation wage. This period of unemployment is costly to the searching worker; however, the longer they search the better the match in terms of employer.

In addition to explaining the micro foundations for an individual's acceptance of unemployment the model is tractable to macro unemployment forecasting and modelling. So termed "frictional" unemployment is an important statistical reality plagued business cycle authors until Pissarides offered a micro foundational model for unemployment. This paper uses the same essential idea that the process of searching for a match is a less than perfect science, applies it to the market for Angel Capital and shows why Angels will not wish to destroy these frictions.

4 Notation and Assumptions

4.1 The Entrepreneur

Entrepreneurs begin with a potential project but little wealth. Entrepreneurs are homogenous expect for one aspect the probability of success of their project. All projects either succeed or fail. Success will result in a payoff of one, failure in a payoff of zero. The probability of success denoted for each entrepreneur*: P* and the probability of failure: *1-P*. Probabilities of success are uniformly distributed on the unit line: $P \in [0,1]$, uniformly across the unit of entrepreneurs. This parameter is known to the entrepreneur herself but is not public knowledge; no direct screening will reveal this information. The project costs a total of k whether it fails or succeeds. The firm has an expected value of $P - k$ the entrepreneur does not have k to spend on the project; they have spent all the resources they have on the project and the entrepreneurs have used up all the "Love Capital" from friends and family. Due to the variance of the return of the project the traditional method of financing through banks is not available to many entrepreneurs. These entrepreneurs must therefore find an Angel to fund

their project. The potential Entrepreneurs that could search have a unit mass; the Entrepreneurs that decide to search at a given cost C have mass e.

The Entrepreneur must exert a cost C when the searching for the Angel. The cost of this search can come in several ways. Firstly the entrepreneur has a job, the hours spent searching are not spent in advancement in their career. Secondly the search may draw time away from leisure, and as long as search is not a pleasurable activity for the entrepreneur this must be considered a cost. Lastly the search process may be explicitly expensive; the production of business plans, the transportation costs that may be required for search, the entrance fees to trade shows, are all examples of direct financial costs that an entrepreneur will expend in the search process. The parameter C abstracts all of these costs, and varies in direct portion to some notion of the expected search length.

$$C\epsilon[0,1]$$

No entrepreneurs will search for capital where the cost dominates the expected return; therefore, a further restriction on the cost that Angels will make the entrepreneur exert:

$$C < P(1-k)$$

The continuum of search costs corresponds to different methods of search, with different probabilities of success. Therefore only Entrepreneurs that pay the cost C will search for the Angels to be found at C. The Entrepreneurs can only search using one method at a time. The more difficult the process selected by the Angel the less likely they are to be found, and the greater the cost to the entrepreneur. Additionally it must be noted that the search costs are not

"paid" to anyone, they are a loss to the entrepreneur and therefore a loss to society. The availability of the information regarding each Angel's choice of search function will be discussed in Section 7 below.

4.2. The Angels

All Angels are assumed to be identical. Imagine the Angel at work at their desk on some other project with a pile of capital sitting next to them earning a low but stable return. Their door however is open; waiting for the diligent but poor entrepreneur to walk through with the brilliant idea that will make both rich. The multitudes of possible project span almost every industry and encompass a wide range of skill sets that Angels have. These skills when perfectly matched with the skills and idea of an Entrepreneur might lead to a wildly more successful project than when the match in industry is less than perfect. However, for simplicities sake I will not consider heterogeneity of project beyond the probability of success. The below model applies to only one industry where all matches with industry outsiders are rejected.

There are a Angels who are willing to fund the projects of the entrepreneurs find them; it is assumed that this quantity is much smaller than e. The Angels face an outside investment opportunity for the k cost of the project with net return assumed to be zero. I assume that financers are risk neutral and consider only the expected value when making the decisions to enter, and when choosing a search function.

4.3. The Search Technology

The matching process between the Angels and Entrepreneurs is modeled as a simple isomorphism to the matching process described in the first chapter of Pissarides (2000). The expected number of firms created which I denote M corresponds to mL which is the number of matches between job searchers and vacancies. Entrepreneurs e take the place of uL, the unemployed workers; and the Angels a replace vL, which denoted the number of vacancies. The number of firms or matches is increasing and concave in both a and e, with constant returns to scale as imposed by Pissarides. Common sense makes it impossible for there to be more matches than either the number of Angels or Entrepreneurs. The number of matches in any period:

$$M(e,a) = \max(Ra^{\beta}e^{1-\beta}, a, e)$$

$$0 < \beta < 1$$

I assume that each Entrepreneur will find an Angel with equal probability; dividing the number of matches by the amount a particular type of agent yields the probability of that agent's success in finding a match. Therefore the probability for the Entrepreneur and Angel are respectively:

$$\frac{M}{e} = \max\left(Ra^{\beta}e^{-\beta}, 1\right) = \max\left(R\left(\frac{a}{e}\right)^{\beta}, 1\right)$$

$$\frac{M}{a} = \max\left(Ra^{\beta-1}e^{1-\beta}, 1\right) = \max\left(R\left(\frac{e}{a}\right)^{1-\beta}, 1\right)$$

The concavities of the matching function in both f and e have an important implication; an additional Entrepreneur or Financer reduces the probability of a successful match for all the agents of the type added, but increase the probability of a successful match for the other type:

$$\frac{\partial\left(\frac{M}{e}\right)}{\partial e} < 0, \qquad \frac{\partial\left(\frac{M}{a}\right)}{\partial e} > 0, \quad \frac{\partial\left(\frac{M}{a}\right)}{\partial a} < 0, \qquad \frac{\partial\left(\frac{M}{e}\right)}{\partial a} > 0$$

In a significant departure from Pissarides, the multiplicative coefficient R on the function depends on the choice by the Angel. The "technology" of the search process depends on the expected effort that the entrepreneur must exert at the cost *C*. The variable C can be chosen by the Angel in an attempt to maximize the expected profits of the firm. R is therefore a decreasing function of *C*; here assumed linear:

$$R = B(1 - C)$$

B is an exogenous parameter. When $C = 0$ the required search costs are at their minimum the Angel has chosen the technology with the highest probability of a successful match for any given amount of Angels or Entrepreneurs. When $C = 1$ the required search cost is maximized, there are no matches as $R = 0$, additionally as shown above no entrepreneurs would search at that high a cost. The harder the search process for the Entrepreneur the more costly it is to undertake and less likely a successful match for either an agent.

4.4 The Firm

When an Entrepreneur finds an Angel they form a firm. Should the firm fail it will be a total loss. The Angel will bear the loss k and the Entrepreneur will lose nothing. The expected profit of the firm is $P - k$. Some portion of this equity in the firm ϑ is paid upon success to the Entrepreneur and the residual $(1 - \vartheta)$ is retained by the Angel. I have little ability to make claims about the value of ϑ; I only purpose that $0 < \vartheta < 1$ so that each party is paid a positive amount. Limited liability implies that the Angel cannot impose a penalty upon the Entrepreneur after the discovery of the project's success or failure. The binary nature of the payoffs where only success or failure is observed implies that the Angel cannot ex post infer the type of the Entrepreneur. The potential negative consequences for a poor Entrepreneur make agreement unlikely to an unlimited liability or state contingent contract. Therefore the payoff to the Entrepreneur upon the success of the firm is; this will be denoted F_P:

$$F_P = \vartheta(1 - k)$$

Therefore the expected profit after a match for the Entrepreneur is simply the probability of success multiplied by F_P:

$$E(F_P) = P\vartheta(1 - k)$$

The Angel upon project success will have payoff denoted $F_C = 1 - \vartheta(1 - k) - k$, should the project fail they will have payoff $-k$ The Angel when discovered by an Entrepreneur does not know what probability success the matched Entrepreneur has. The can infer that the probability is above the probability of success of the Entrepreneur that is indifferent between

searching and not. The Angel finds the average of the distribution of Entrepreneurs that are searching at that level. This expected probability of success will be denoted $E(P|C)$. The expected firm payoff for the Angel who has chosen search costs of C:

$$E(P|C)\big(1-\vartheta(1-k)\big)-k$$

The angel will make a profit when:

$$E(P|C) \geq \frac{k}{1-\vartheta(1-k)}$$

Some firms must have positive value for the Angel, a project that is successful with certainty must have a positive profit. Therefore, the following condition on k must be satisfied:

$$k < \frac{1-\vartheta}{1+\vartheta}$$

The expected payoff for the Entrepreneur who has chosen to search with cost C:

$$E(\Pi_e) = B(1-C)\left(\frac{a}{e}\right)^{\beta} F_P - C$$

As the Entrepreneur knows their probability of success, they will be profitable given:

$$E(\Pi_P) = B(1-C)\left(\frac{a}{e}\right)^{\beta} \vartheta P(1-k) - C \geq 0$$

P must be no less than the threshold probability of success $\boldsymbol{P}$:

$$P \geq \boldsymbol{P} \equiv \frac{C}{B(1-C)\left(\frac{a}{e}\right)^{\beta} \vartheta(1-k)}$$

If $\mathrm{C} = 0$ the lower bound is $\boldsymbol{P} = 0$, *ceteris paribus.* Entrepreneurs of any quality will search. Conversely when $\mathrm{C} \to 1$ an upper bound $\boldsymbol{P}$>1 is reached and no Entrepreneur has a probability sufficiently high. The comparative statics for the required level of $\boldsymbol{P}$:

$$\frac{\partial \boldsymbol{P}}{\partial B} < 0, \qquad \frac{\partial \boldsymbol{P}}{\partial a} < 0, \qquad \frac{\partial \boldsymbol{P}}{\partial e} > 0, \qquad \frac{\partial \boldsymbol{P}}{\partial \vartheta} < 0, \qquad \frac{\partial \boldsymbol{P}}{\partial k} > 0$$

The required probability of success for an Entrepreneur to profitable search at a level C increases in the number of Entrepreneurs "crowding" the market, and the cost of the project. $\boldsymbol{P}$ decreases in the exogenous search efficiency, the number of Angels to be found at a given level, and the proportion of the profits paid to the Entrepreneur

The lower bound $\boldsymbol{P}$ allows us to identify the set of entrepreneurs who will search at C. Without better opportunities, all Entrepreneurs who do not lie below this bound will participate, $e = 1 - \boldsymbol{P}$. Substituting this relationship into the RHS yields a relationship between participation e and search C, *ceteris paribus*. Under the above assumption, $a = A$, there are no other endogenous variables and we have the following lemma.

Lemma. The cost of search is related to the number of entrepreneurs as follows:

$$C(e) = \frac{1}{1 + \frac{e^{\beta}}{1-e}\frac{1}{BA^{\beta}[\vartheta(1-k)]}}$$

$C(e)$ is decreasing in e. The first derivative with respect to e is:

$$\frac{\partial C(e)}{\partial e} = -\frac{\left[e^{\beta-1}(\beta(1-e)+e\right]}{\left(1 + \frac{e^{\beta}}{1-e}\frac{1}{BA^{\beta}[\vartheta(1-k)]}\right)^2}\frac{1}{BA^{\beta}[\vartheta(1-k)]}$$

This is negative for all $e\epsilon[0,1]$. $C(e)$ is bounded between zero and one. $\lim_{e\to 0}\frac{\partial C(e)}{\partial e}\to -\infty$ and $\lim_{e\to 1}\frac{\partial C(e)}{\partial e}\to 0$ as $\frac{e^{\beta}}{1-e}\to\infty$. Over the portion of the range that is invertible there exists a function $e^{*}(C)$ that again monotonically maps a given search technology to a number of Entrepreneurs. Unfortunately, there is no closed form solution for $e(C)$. However some comparative statics of this function result:

$$\frac{\partial e^{*}}{\partial C}<0,\qquad \frac{\partial e^{*}}{\partial f}>0,\qquad \frac{\partial e^{*}}{\partial \theta}>0\,,\qquad \frac{\partial e^{*}}{\partial B}>0,\qquad \frac{\partial e^{*}}{\partial k}<0,$$

The number of Entrepreneurs decreases with the cost of the search; this guarantees a screen in so far that only the e^{*} best Entrepreneurs will search. It is not profitable on expectation for the remaining *1-e** Entrepreneurs to engage in the Angel Capital market. More Angels attract more Entrepreneurs; better terms for the Entrepreneur and greater efficiency of search do as well. That the decision of the Entrepreneur to seek funding now depends on the cost of the project and its expected profit is an improvement; if there were no search cost the Entrepreneurs own solely the upside of the project. In the absence of the cost of finding capital Entrepreneurs with any project with any potential upside, $P>0$ was looking for capital; the quality of the project shapes their entry decision.

5. Market Efficient Outcome

Equilibrium

In the following analysis, we consider the Nash equilibrium of the game where Entrepreneur's and Angels move simultaneously. In general, a Nash equilibrium is a set of

decisions by Angels to set search costs and decisions by Entrepreneurs of how much to search where all agents are optimizing given the behaviour of other agents. In this game, as in many games, there are many Nash equilibria. This is the case in this model. The following examines equilibria corresponding to unanimous decisions on C.

Proposition. There exist parameter values such that Nash equilibria exist where all angels are active, $a = A$ and choose the same search costs $C > 0$. The number of Entrepreneurs search at C is $e > 0$ and consists of the set of most able Entrepreneurs, $P\epsilon(1-e, 1]$.

In order to prove this result, first suppose that parameters are such that all agents expected payoffs are non-negative. Now consider a single deviant Entrepreneur, then a single deviant Angel. If an Entrepreneur that was searching at the potential equilibrium level was to stop exerting any search cost then there payoff would be zero, which is weakly dominated by searching at C^{EQM}. If the deviant was to search at any other level of C other than C^{EQM}, the Entrepreneur would exert C but have no potential return. They exert effort for no return; therefore, searching at any other level is strictly dominated.

Any Entrepreneur that was not searching faces negative expected profits while searching at C^{EQM}, and for the same reason as the other Entrepreneurs will not search at any other C. Therefore not exerting any search effort dominates exerting any. This is why the Angel forces the Entrepreneurs to find them, so that the presumably lower quality projects do not muddy the pool of higher quality projects.

If all the Angels will choose the same search function, they can collectively determine the outcome that will maximize their profits. Given that the Angels know how high a cost to force a given number of Entrepreneurs to search for them. Two opposite effects must be balanced by the Angel; firstly an increase in *C* decreases the probability of an Entrepreneur finding the Angel. Secondly, an increase in *C* increases the quality of the applicants increasing the profitability of the firm. The Financer now considers their expected profit given only the top e^* will search. The marginal Entrepreneur has a probability of success of $1 - e^*$ therefore the probability of success of any firm will be uniformly distributed between of $1 - e^*$ and 1. The probability of success of the expected firm for a given level of *e**:

$$E(P|C) = \frac{2 - e^*}{2}$$

The problem for the individual Angel is well formed:

$$\max_e E(\Pi_C(e)) = \mathrm{B}\big(1 - C(e)\big)\left(\frac{e}{a}\right)^{1-\beta} F_C(e)$$

The individual Angel's problem is the same as the collective Angels' problem provided that there is no deviation. This implies that the profit maximizing levels of C and e are the same for each individual Angel and all Angels together. When considering the profits of all the Angels $\sum \Pi_C(e)$ together the expected number of number of matches is multiplied by the expected firm quality:

$$\max_e E\left(\sum \Pi_C(e)\right) = \mathrm{B}\big(1 - C(e)\big)a^{\beta} e^{1-\beta} F_C(e)$$

This is simply the above individual's problem multiplied by a.

Now the problem can be solved either for the individual Angel or all Angels. The Angel can take the first derivative with respect to their choice variable, e:

$$\frac{\partial E(\Pi_C(e))}{\partial e} = -B\frac{\partial C(e)}{\partial e}\left(\frac{e}{a}\right)^{1-\beta} F_C(e) + B(1-C(e))(1-\beta)\left(\frac{e}{a}\right)^{-\beta} e^{1-2\beta} F_C(e)$$

$$+B(1-C(e))\left(\frac{e}{a}\right)^{1-\beta}\frac{\partial F_C(e)}{\partial e}$$

The first term is a direct effect, the choice of C directly influences the probability of a successful match through the technology parameter of the chosen function. The second term shows the effect of increasing the number of Entrepreneurs searching has on the probability of a successful match. The last term is the quality effect, when the number of Entrepreneurs decreases the expected quality of the match increases.

Proposition : The profit maximizing number of Entrepreneurs for the Angel is interior, $e^* \in (0,1)$, this implies that the Market Efficient level of $C \in (0,1)$

To show this result without expanding the expression which again has no closed form, one can evaluate the derivative at the boundaries. If $C = 0$ then all the Entrepreneurs are willing to search $e = 1$. Therefore there are many more Entrepreneurs than Angels, and the ratio $\frac{e}{a}$ becomes very large, therefore the first and last terms dominate. As the quality effect is negative, $\frac{\partial F_C(e)}{\partial e} < 0$, the derivative is negative as well, $\frac{\partial E(\Pi_a)}{\partial e} < 0$ for $e \to 1$.

Conversely, when $C \to 1$, the number of Entrepreneurs searching goes to zero, $e \to 0$. Therefore the ratio $\frac{e}{a}$ becomes very small and the second term will dominate. All the factors are positive scalars, and as the expected firm $F_C(e)$ is near its maximum value, it must be

positive[3]. As the only term that does not vanish is positive then the derivative is positive, $\frac{\partial E(\Pi_a)}{\partial e} > 0$, for $e \to 0$.

Angels who have almost no Entrepreneurs searching for them should decrease C to encourage more Entrepreneurs. Angels who have all the Entrepreneurs searching for them should increase C in order to ensure a better mix of projects. These two results coupled with the intermediate value theorem imply that the profit maximizing levels of C and e are interior for financial industry optimum. These levels are denoted $C^*$and $e^*$and are referred to as the financial industry optimum.

It can be assumed that the cost of project that is under consideration is sufficiently small so that the Angel will earn a positive profit given a successful match: $E(P|C) \geq \frac{k}{1-\vartheta(1-k)}$. The expected profit of an Angel is the expected value of a match multiplied by the probability of a match. As the probability of a match is greater than zero, and as assumed above the firm will have positive expected profit for the Angel, search for each Angel will have a positive expected value. Therefore, $a = A$, all the Angels are willing to fund the projects they receive. The group of Entrepreneurs each positive expected profit as the worst member has $P = 1 - e$ and is indifferent between searching and not. At least for some set of parameter values the players in this model have non-negative expected payoffs prior to search.

[3] For $k < 1$

6. Welfare and Policy Implications

Welfare

Social welfare can be loosely defined as the expected undivided surplus of the firm multiplied by the number of expected firms. There are many factors that this simplification does not include. Consider the financial market optimal level of C. The positive externalities from an Entrepreneurial society and the learning by doing or failing are not taken into account. The social planner's problem is to maximize:

$$\max_{e} E(W(e)) = \mathrm{B}\big(1 - C(e)\big)a^{\beta} e^{1-\beta} F_{total}(e) - eC(e)$$

Where F_{total} is equal to the firms expected profit $P - k$ not the profit for the Angel. Letting the two can be related multiplicatively: $F_{total} = \frac{F_C}{1-\vartheta} > 1 \; \forall \; \vartheta > 0$. Recall F_C is the value of the firm for the Angel and $E(\Pi_C(e))$ is the before search expected payoff for the Angel. Taking the first order condition with respect to e:

$$\frac{\partial E(W(e))}{\partial e} = \frac{1}{1-\vartheta}\frac{\partial E(\Pi_C(e))}{\partial e} - \left(C(e) + e\frac{\partial C(e)}{\partial e}\right)$$

Proposition: The socially optimal level of C may be above or below the capital market efficient outcome.

Angels fail to internalize the direct costs to the society of the Entrepreneurs engaging in search. The final term in the welfare first order condition is the direct cost effect on welfare. Evaluating the expression at the financial industry optimum (where $\frac{\partial E(W(e))}{\partial e} = 0$), only the direct cost effect remains. The total search costs $eC(e)$ are approximately a parabolic function.

$\frac{\partial eC(e)}{\partial e} = C(e) + e\frac{\partial C(e)}{\partial e}$. When $C = 0$ the costs are zero, however, as $C \to 1$ the number of Entrepreneurs falls to zero as illustrated above and the total search cost is again zero. As both e and $C(e)$ are continuous positive functions, the mean value theorem implies an interior maximum. Recall that $\frac{\partial C(e)}{\partial e} < 0$ and $\lim_{e\to 0} \frac{\partial C(e)}{\partial e} \to -\infty$ If the capital market efficient outcome attracts a number of Entrepreneurs that is below $\operatorname{argmax}_e(eC(e))$, the maximum level of the total search cost then $\frac{\partial eC(e)}{\partial e} > 0$ and $\frac{\partial E(W(e))}{\partial e} < 0$ at the capital market efficient outcome. The cost effect of adding an additional Entrepreneur dominates the cost decrease that is required for an additional Entrepreneur. The social planner would choose fewer Entrepreneurs and a higher level of C when the capital market optimum has a large C and a small e.

If the capital market optimum were to attract a number of Entrepreneurs that was above $\operatorname{argmax}_e\big(eC(e)\big)$ then $\frac{\partial eC(e)}{\partial e} < 0$ and $\frac{\partial E(W(e))}{\partial e} > 0$ at the capital market efficient outcome. The cost of one additional Entrepreneur is more than offset by the decrease in the cost that each Entrepreneur pays. The social planner would choose more Entrepreneurs and a lower level of C when the capital market optimum has a small C and a large e. This effect may be sufficient to ensure a socially optimal level of e as high as 1, and the socially efficient level of $C = 0$. The welfare maximizing number of Entrepreneurs may then be smaller or greater than the capital market efficient outcome.

Additionally at the endpoints of the interval for from the Capital Market efficient outcome the total cost effect has the opposite sign the first order condition for the of the

Angels $\frac{\partial E(\Pi_C(e))}{\partial e}$. When $e = 0$ and $C = 1$, $\frac{\partial E(\Pi_C(e))}{\partial e} > 0$ but $\left(C(e) + e\frac{\partial C(e)}{\partial e}\right) > 0$ as $\lim_{e\to 0} \frac{\partial C(e)}{\partial e} \to -\infty$. Therefore $\lim_{e\to 0} \frac{\partial E(W(e))}{\partial e}$ can be positive or negative. However, at the other limit where $e = 1$ and $C = 0$, $\frac{\partial E(\Pi_C(e))}{\partial e} < 0$ but $\left(C(e) + e\frac{\partial C(e)}{\partial e}\right) < 0$ as $\lim_{e\to 1} \frac{\partial C(e)}{\partial e} \to 0$. Recalling that the direct cost effect is subtracted from the welfare first order condition. At each limit the marginal direct cost effect has the opposite sign as the

Policy

These welfare implications allow some discussion of various government policies. A government may attempt to increase the number of Entrepreneurs. A simple government policy would require the Angels to make them very easy to find ($C = 0$). The effect of this policy would be to destroy the information gained from the choices of the Entrepreneurs. If the expected firm at $C = 0$ is not profitable, there will be no projects funded by private Angels.

This model puts considerable scrutiny on a policy that would directly fund the projects that might be otherwise turn to Angels for capital. The government cannot "hide," as all other methods of screening are inefficient the government will not be able to gain information about the firms they are funding. The government will end up funding projects drawn from the entire distribution. A better policy would fund projects after they have found Angels or to reduce the tax burden on the firm, which would increase the incentive for the higher quality Entrepreneurs.

Another potential policy would be to bear some part of the cost to the Entrepreneurs. This policy could take the form of setting up networks to facilitate an easier search process

(increasing B the constant augmenting the matching function) or by directly paying some part of the financial costs (tax credits or direct subsidies). The Angels would respond to this policy by simply choosing a more costly search method; the Angels learn information from the costs the Entrepreneurs must bear. Thus the policy will either be nullified as Angels become harder to find or the efficiency of the search process will be reduced. If this policy were to move the level of C closer to the efficient level it will be welfare improving. However, the difficulty in the empirical estimation of the structural parameters makes the proper public policy difficult to form. However, this theoretical model should act as a starting point for consideration of any proposed policy.

7. Unproved Suppositions: Speculation for Future Research

The Diamond Paradox of Unobserved Action and the Financial Industry Optimum

The issue of whether the choices of search function by the Angels are directly observable by the Entrepreneurs has been left untreated to this point. That the actions by the Angels are hidden follows directly from the fact that the Angels are hidden; the ability to advertise their choice of search technology will invariably compromise their ability to hide. This additional hidden information is analogous to the "Diamond Paradox," Diamond (1971).

The original Diamond model was roughly "patterned after a retail consumer durable market," but the argument was designed with "great weight has been given to mathematical tractability rather than trying to reflect some specific market." Belleflamme (2010) explains the

argument by considering two identical firms in a goods market, the firms set price as their strategic variable. The classic Bertrand model predicts that each firm will undercut each other and end up supplying the entire market. This will drive the price of the good down to the marginal cost. This result depends crucially on a downward deviation in price resulting in consumers switching products. If the change in price is not observed and there is a positive cost for the consumer to switch to or to search for the other product, then there is no profit from reducing price in an attempt to poach consumers. Each firm will charge the monopoly price. Every consumer has beliefs consistent with this equilibrium and will not choose to switch products given its positive cost. A firm that charges a lower than monopoly price given the demand of their existing consumers will see a decrease in profits but no poaching of the consumers from the other firm. As the monopoly price maximizes the profit from current consumers any other price is not the optimal.

If consumers are the Entrepreneurs and the firms are the Angels, the price charged by the firms is similar to the choice of search method. If one Angel were to choose a Search technology that had a lower imposed cost than the potential equilibrium, they would hope to attract the best Entrepreneurs. A further discussion below will illustrate the same Bertrand augment holds in the Angel Capital market when the actions of the Angels are observed. However, proceeding under the assumption that the choice by any Angel is private information; an analogy drawn from the goods market to the market for Angel Capital:

Conjecture: If the choice of search method is unobserved by the Entrepreneurs, all the Angels will choose the same method of search, a subset of Entrepreneurs, all of whom have

P greater than some P^*, will search for them using that method. This will maximize both individual Angel's expected profit as well as the sum of all Angel's expected profit given that all Angels use the same method.

In order to prove this result it will be shown that no agent can profitably deviate, even with a sequential timing structure where Angels select a search technology before Entrepreneurs act and the profit maximizing solution will be found and discussed. Entrepreneurs cannot observe the choice of search method or function by individual Angels; however they must have a belief about which function the Angels will likely make them use. Angels cannot attract a deviant group Entrepreneurs by selecting search function that is not consistent with the Entrepreneurs' beliefs. Proposition III extends to this case as the lack of information implies that the choices might as well have been made simultaneously. Therefore the same results hold, neither Angels nor Entrepreneurs can profitably deviate from the potential equilibrium. The Angels will choose to be found using the search function consistent with the beliefs of the Entrepreneurs. Entrepreneurs believe, as the consumers did in the Diamond Paradox goods market, that the unobserved actions by the Angels (Firms) will be rational, profit maximizing. The level of search cost that is consistent with this equilibrium is the financial industry optimum.

Just as there is a search cost must be borne by the consumer to discover the price charged by another firm in the Diamond Paradox goods model, the Entrepreneurs must pay a positive cost to discover any deviant. The expected payoff of the financial industry optimal

search process must be forfeited; additionally the cost itself at level other than $C = 0$ is inherently costly.[4]

Observed Action and the Instability of Equilibrium

The previous discussion assumed that the individual choice of a search method and intensity was hidden information, now implications if this information was public must considered.

Conjecture: The financial industry optimal[5] fails to remain consistent with equilibrium when the actions of a measure of deviant Angels λ choose a search method with costs slightly below C^* at C^{dev}.

As the best Entrepreneurs move first they will choose the method with the highest expected profit; therefore, the best ε Entrepreneurs will search at C^{dev}. ε and λ satisfy the following relation rendering the εth best Entrepreneur indifferent between searching at intensity C^* or at C^{dev} :

$$B(1-C^{dev})\left(\frac{\lambda}{\varepsilon}\right)^{\beta}\vartheta[(1-\varepsilon)-k]-C^{dev} = B(1-C)\left(\frac{a-\lambda}{e'}\right)^{\beta}\vartheta\big((1-\varepsilon)-k\big)-C$$

Where e' is the number of Entrepreneurs that search under at C^{dev} given this deviation. The deviation is profitable for the λ:

$$(1-C^{dev})\left(\frac{\varepsilon}{\lambda}\right)^{1-\beta}F_{C^{dev}} > (1-C^*)\left(\frac{e}{a}\right)^{1-\beta}F_{C^*}$$

[4]No Angels would want to be available at $C = 0$, this follows from the below discussion of the optimal level Proposition VI. In essence the Angel does not wish to have their project drawn from the whole distribution.
[5] Or any potential Equilibrium where all the Angels choose the same level of search costs.

This result holds as the multiplicative technology is better $C^{dev} < C^*$ and as the expected firm is better: $E\,(P|C^{dev}) = \frac{2-\varepsilon}{2} > E(P|C^*) = \frac{2-e}{2}$. The ratios $\frac{\varepsilon}{\lambda}$ and $\frac{e}{a}$ are the same as the limit $C^{dev} \to C^*$. Conversely, upon consideration, $E(P|C^{dev})$, the positive firm composition effect is discontinuous. Any deviation will be followed by the best Entrepreneurs. Therefore the firm composition effect dominates for small λ. This result is exactly analogous to allowing the prices of other firm's products become public free information in the Diamond Paradox goods market. The price is driven downwards as price undercutting steals consumers, the classic Bertrand price competition result.

Conjecture: Deviation by a small measure of Angels λ from a potential equilibrium where all the Angels have a single search method to a search method where $C^{dev} > C^*$ will also be profitable as long as the deviation is also small.

The very best Entrepreneurs will search with the method C^*, once e' Entrepreneurs are engaged at that level the return from C^* is sufficiently crowded that a small measure ϵ will search at C^{dev}. These Entrepreneurs will sufficiently crowd the C^{dev} until the C^* method will dominate for all remaining Entrepreneurs. Given a small measure of deviant Angels λ who only deviate to a slightly harder process, the following comparison can be drawn between the return to deviation and remaining with C^*:

$$(1 - C^{dev})\left(\frac{\epsilon}{\lambda}\right)^{1-\beta} F_{C^{dev}} > (1 - C^*)\left(\frac{e}{a}\right)^{1-\beta} F_{C^*}$$

As above the terms that include the direct effect and the probability of matching based on the ratio approach the C^* levels as $C^{dev} \to C^*$. But again the firm composition effect is

discontinuous; some of the best Entrepreneurs will split away from the C^*group and search at intensity C^{dev} as long as the two are not exactly equal. The expected quality of the firm for a deviant Angel is $E(P|C^{dev}) = \frac{2-e^{'}-\epsilon}{2}$ [6]. This is greater than the expected quality using C^* $E(P|C^*) = \frac{2-e}{2}$. Therefore, individual deviation to increase C by a small amount is profitable as long the choice of search technology is observed by the Entrepreneurs.

Corollary: There is no equilibrium such that all Angels can be found at the same intensity of search effort, when the actions of Angels are observed.

This follows directly from the previous two conjectures. An Angel can do better by choosing a smaller C from any potential equilibrium where $C > 0$. In addition an Angel can increase their private profits by choosing a larger C when any equilibrium under consideration is less than least cost at which no Entrepreneurs search. As these results together span the set $C\epsilon[0,1]$ no equilibrium can exist where all the Angels are at the same intensity can be sustained.

The results depart significantly from the goods market analog, under Bertrand competition undercutting drives price down to marginal cost. Price of a good equal to marginal cost is a stable equilibrium; however, when if the undercutting mechanism were to force $C = 0$ then some group of Angels could profitably deviate by selecting a more costly mechanism. There is no stable equilibrium of this type in the Angel Capital market.

[6] Recall ϵ and $e^{'}$ are small.

Conjecture: There exist equilibria that leave every Angel and Entrepreneur indifferent, where Angels choose a range of search methods.

Proving and deriving these equilibria is unfortunately beyond the scope of this paper, future efforts will likely find such consideration fruitful. Allowing the portion of potential return to the Entrepreneur ϑ vary with the information obtained through the search mechanism will allow the Angels to increase the information rents that the very best Entrepreneurs can expect at high levels of C.

8. Conclusion

This paper has argued that the costly search process can act to screen the applicants for Angel Capital. The Angels receive better projects to fund and do not have to deal with superfluous and low quality projects. High quality Entrepreneurs are able to signal their quality by choosing to exert a high level of effort to find the capital they require. As long as the decisions are made simultaneously, a single equilibrium search function where every Angel operates can be sustained. When all the Angels use the same search function, their profits can be maximized by a positive number of Entrepreneurs. Social Welfare is unlikely to be maximized at this level and the private choice of search cost can either be too great or too little. Various departures from simultaneous Nash Equilibriums were discussed. Positive search costs force Entrepreneurs to reveal their private quality of project and will force lower quality projects from the marketplace.

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