The following is a package of Mathematica files, representing various models and concepts that one would encounter in Economics 312, or any similar Urban Land Economics course. These models are intended mainly for comparative static exercises and/or teaching purposes. Screenshots of various scenarios are included with the discussion to highlight the abilities of the models; however, the coding for all models will be sent to you in an email so that you can play with the models yourself. Additional comments have been provided within the files. Thank you to Dr. Martin Farnham for providing the course notes on which these models are based.

Model 1 – Factory Market Diagram

Economic Model:

The existence of comparative advantage and economies of scale in production gives rise to factory cities. Under these conditions, it makes more sense to concentrate production within factories, rather than continuing to produce goods in homes on a much smaller scale. The size of the factory city will be dependent on two main variables: the magnitude of the economies of scale in production, and freight costs. Note that these two variables counteract one another. Namely, increasing the magnitude of the economies of scale (by decreasing factory costs) will increase the factory's market radius, while increasing the freight costs of getting goods from the factory to the residents in their homes, will decrease the market radius.

Computational Model:

The computational model can be seen below as Figure 1(a).

![Figure 1(a)](image)

The line labelled ‘HC’ represents the cost of home production, ‘FC’ represents the cost of factory production, ‘TC’ represents transportation costs, and ‘MFF’ stands for ‘miles from factory.’ Note that transportation costs increase as you move farther away from the factory. The points at which ‘TC’ intersect ‘HC’ give the size of the factory market area, based on the combination of the cost of factory production and the freight costs. This is because once total costs (production plus freight) have passed the cost of home production, no one will demand goods from the factory. They will simply make the goods more cheaply in their own homes. These points are traced down to 'MFF' using the dashed lines to give the mile distance of the factory market area. For example, in Figure One, the factory serves a market area of seven miles (three and a half miles on either side of the factory, which is located in the centre of the dashed lines, at the intersection between the black line (FC) and MFF).
The coding allows you to change the cost of home production: 'Home Cost', the cost of factory production: 'Factory Cost,' and the freight cost: 'Transport Cost.' Examples of such changes are given in the next section.

**Highlights (Experiments):**

I chose five 'experiments' to highlight the abilities of the coding. First, I increased the costs of factory production (as seen in Figure 1(b)). Second, I decreased freight costs (as seen in Figure 1(c)). Third, I decreased the cost of in-home production (Figure 1(d)). Finally, I give two 'special cases' of freight costs: zero freight costs (Figure 1(e)), and infinite freight costs (Figure 1(f)).

Figure 1(b) shows that an increase in factory costs decreases the size of the factory market radius, as previously discussed, while Figure 1(c) shows that a decrease in freight costs increases the size of the market radius, also as previously discussed.

Figure 1(d)
Figure 1(d) shows that a decrease in the cost of home production decreases the size of the factory market radius from its original seven miles, to five miles.
Finally, Figure 1(e) shows that with zero transportation costs, the size of the factory market area is infinitely large, and Figure 1(f) illustrates that with infinitely large freight costs, there is no factory market area at all.

Discussion:

An increase in factory costs decreases the economies of scale from factory production. Thus, except for those living very close to the factory and paying very low freight costs, it is cheaper to simply make products in their own homes. This reduces the size of the factory market area, as no one will buy from the factory when they can make the products themselves for cheaper. A decrease in home production costs has the same effect. A decrease in transportation costs increases the factory market area as it now cheaper for those living farther away to get products from the factory. The extreme of this case is seen with zero freight costs. With zero freight costs, as long as the cost of factory production is less than the cost of home production (which is reasonable to assume with economies of scale), the size of the factory market area is infinite. This is because it is cheaper for everyone, no matter how far away they live from the factory, to buy factory-produced goods. Finally, the opposite case to having zero transportation costs is having infinite transportation costs. With infinite transportation costs, it is prohibitively expensive for anyone to buy factory-produced goods, except for those who live on top of the factory. Thus, there is no factory market area at all.

Model 2 – Filtering and Quality Ladder

Economic Model:
This is a filtering model to show why the poor tend to live in old housing units. The rich have a higher willingness to pay for housing quality than the poor, as a result of higher levels of disposable income, so they will end up in higher quality units than the poor. As a unit of housing ages, the willingness to pay for such a unit decreases, due to increased maintenance costs etc. The rich will hold a unit of housing until it declines to a quality level at which they no longer desire the unit. They can then either upgrade the unit or buy a new unit. However, it is often cheaper to buy a new unit than upgrade old ones. Thus, the rich will buy new housing and the middle-income households will buy the castoffs of the rich. They will then, in turn, hold the units until they decline in quality, at which point the poor will buy them. When the poor no longer desire the units, they will be retired, or finally upgraded. So, as long as it is more costly to upgrade houses than to build new ones, the rich will end up in new homes and the poor will end up in used homes.
Computational Model:

The computational model can be seen below as Figure 2(a):

![Figure 2(a)](image)

In this model, the rich (HH income of 15 in this example) start off consuming housing at point ‘H.’ They hold these units until they decline in quality to point ‘I.’ The middle-income group (HH income of 10) buy the castoffs of the rich (point ‘M’), and hold these units until they decline in quality to point ‘N.’ Finally, the poor (HH income of 5) will buy the castoffs of the middle-income group (point ‘Z’) and hold them until they decline in quality to point ‘L.’

The coding allows you to change the income levels of any individual group, as well as the quality of housing they desire.

Highlights (Experiments):

Two scenarios are displayed here to highlight the features of the model. Figure 2(b) shows the effect of increasing the income differences between the various income groups, and Figure 2(c) shows what would happen if households decided to hold onto a particular unit of housing for a longer period of time. In other words, the various income groups would show a higher tolerance for lower quality homes.

![Figure 2(b)](image) ![Figure 2(c)](image)

Figure 2(b) shows that increasing the income differences between the various income groups widens the steps in the filtering and quality ladder, while Figure 2(c) shows that increasing the tolerance of various groups for lower quality housing increases the length of the steps in the filtering and quality ladder.
**Discussion:**

With an increase in the income differences between groups, the groups still demand the same quality of housing. This makes intuitive sense, because their preferences for quality are not changing at all. As long as the rich begin consumption at the same level, and hold onto that housing for the same period of time, the middle-income group will always consume the same quality of housing, regardless of income. The same is true about the consumption of the middle-income group affecting the quality choices of the poor. However, if preferences towards quality change, so do the consumption patterns. With an increased tolerance for lower quality housing, the rich wind up still consuming the same quality of housing initially, but by holding these units of housing longer, they force the middle-income groups to consume lower quality housing units than previously. Similarly, the middle-income group force the poor to consume lower quality housing units by holding onto their units longer. Thus, the poor wind up in very low quality housing. These quality preferences are fully adjustable with the controls. It is even possible, although not entirely likely, to have all three income groups demand the same quality of housing.

**Model 3 – Costs/Benefits from Clustering**

**Economic Model:**

There are certain benefits that firms may receive from locating near each other. For example, several firms may locate near a common labour pool if they are uncertain about their future labour or skills needs. By clustering in this manner, firms can potentially reduce the costs of hiring and firing workers. For example, a firm in Port Hardy, where there are few available jobs, may have to pay workers higher wages than a firm in Vancouver, where there are many available jobs, to compensate for higher job insecurity. If the worker in Port Hardy is fired or laid off, he/she may have to relocate because there may not be another job in the city. In addition, clustering around a known labour pool reduces hiring costs by ensuring that firms can always quickly find a worker for an open position.

However, the firm in Port Hardy faces a vertical labour supply curve, whereas the firm in Vancouver faces a horizontal labour supply curve, due to the availability of workers. Thus, while the firm in Port Hardy has to charge a higher wage than the Vancouver firm in good times, it may actually be able charge a lower wage in bad times because the workers, having no other job opportunities in the city, will likely stay despite the decrease in wages. If the firm in Vancouver tried to reduce its wages, workers would likely just find a new job offering more money. Clustering thus may reduce wages by reducing job insecurity in good times, but may result in higher wages in bad times.

The coding for this model is done is three parts: Part One analyzes the wage determination and producer surplus for a clustered firm (the Vancouver firm in the above example), Part Two analyzes the wage determination and producer surplus for an isolated firm (the Port Hardy firm), and Part Three brings the producer surplus levels from Part One and Part Two together to determine the gains and losses from clustering.
Part One:
Computational Model:

The computational model for Part One is given below as Figure 3-1(a):

Figure 3-1(a) – Clustered Firm

In Figure 3(a) the curve labelled ‘D-G’ represents the labour demand curve in good times, whereas the curve labelled ‘D-B’ represents the labour demand curve in bad times. For the clustered firm, as mentioned above, the relevant supply curve is horizontal (labelled ‘S-C’). Thus, the intersection of ‘D-G’ with ‘S-C’ gives both the firm’s demand for labour in good times (labelled ‘N\text{CG}’ on the x-axis), and the wages it faces in good times (labelled ‘W\text{C}’, on the y-axis). Similarly, the intersection of ‘D-B’ with ‘S-C’ gives both the firm’s demand for labour in bad times (labelled ‘N\text{CB}’ on the x-axis), and the wages it faces in bad times (labelled ‘W\text{C}’, on the y-axis). It is important to note that due to the horizontal supply curve, the wages in both good and bad times are always the same. The vertical curve (labelled S-I) represents the labour supply curve for the isolated firm, which is included to show comparisons between the wages faced by the clustered firm and the isolated firm in both good times and bad times. The intersection of ‘S-I’ with ‘D-G’ gives both the isolated firm’s demand for labour in good times (labelled ‘N\text{IG}’ on the x-axis), and the wages it faces in good times (labelled ‘W\text{IG}’, on the y-axis). Similarly, the intersection of ‘S-I’ with ‘D-B’ gives both the isolated firm’s demand for labour in bad times (labelled ‘N\text{IB}’ on the x-axis), and the wages it faces in bad times (labelled ‘W\text{IB}’, on the y-axis). It is important here to note that due to the vertical supply curve of the isolated firm, the quantity of labour demanded in both good and bad times is always the same.

Given the demand and supply curves for the clustered firm, the blue shaded area on the graph gives the producer surplus in good times, while the red shaded area gives the producer surplus in bad times.

The controls on the left allow you to change the intercept and/or slope of both demand curves, as well as the positions of either supply curve.

Highlights (Experiments):

Producer surplus is measured as the area between the wage line and the demand curve. Thus, we can see the changes to producer surplus when the demand curve or supply curves are shifted. While many different combinations of changes are possible, Figure 3-1(b) below shows the effect of a decrease in wages (by shifting the supply curve downwards), coupled with an increased slope of ‘D-G.’
The increased slope of ‘D-G’ decreased the amount of producer surplus in good times, by making the area between the wage line and ‘D-G’ smaller. However, the decrease in wages counteracted this effect by increasing both the producer surplus in good times and the producer surplus in bad times.

**Discussion:**

Increasing the slope of either demand curve decreases the level of producer surplus in that state of the world (good times or bad times), while leaving the producer surplus in the other state of the world unchanged. This is due to the nature of a horizontal supply curve. If the demand curve becomes steeper, the firm is demanding less labour, but still paying the same amount in wages. Thus, they are necessarily worse off than before. This makes the area of the producer surplus narrower. The opposite is therefore true with a decrease in the slope of either curve. The decrease in wages, however, makes firms better off, as now the firm can afford to demand more labour. This makes the area of the producer surplus longer. Again, the opposite is true for an increase in wages. In the above example, therefore, we have two effects competing against each other. Thus, we see a narrower, but longer, area of producer surplus.

**Part Two:**

**Computational Model:**

The computational model for Part Two is given below as Figure 3-2(a):

The coding for this model has already been explained. The relevant labour supply curve is vertical (‘S-I’). The intersection of ‘S-I’ with ‘D-G’ gives both the isolated firm’s demand for labour in good times (labelled ‘N’ on the x-axis), and the wages it faces in good times (labelled ‘W_G’, on the y-axis). Similarly, the intersection of ‘S-I’ with ‘D-B’ gives both the isolated firm’s demand for labour in bad times (labelled ‘N’, on
the x-axis), and the wages it faces in bad times (labelled ‘WBR’, on the y-axis). It is important here to note that due to the vertical supply curve of the isolated firm, the quantity of labour demanded in both good and bad times is always the same. Given the demand and supply curves for the clustered firm, the blue shaded area on the graph gives the producer surplus in good times, while the red shaded area gives the producer surplus in bad times.

**Highlights (Experiments):**

In Part One, the effects of changes in slopes were discussed for a horizontal supply curve only. Thus, Figure 3-2(b) shows the effect of a decrease in the slope of ‘D-B’ coupled with a decrease in the supply of labour for a vertical supply curve.

The effect of the decrease in the slope of ‘D-B’ was to decrease the area of the producer surplus in bad times, while leaving the producer surplus in good times unchanged. This effect was magnified by the decrease in the supply of labour, which reduced the producer surplus in both good times and bad times.

**Discussion:**

Decreasing the slope of demand with a vertical supply curve has the opposite effect of decreasing the slope of demand with a horizontal demand curve. Recall, decreasing the slope of demand with a horizontal demand curve increases producer surplus because, as long as the supply curve remains in the same position, the wage must stay the same regardless of changes to the demand curve. However, with a vertical supply curve, that is no longer the case. In this example, a decrease in the slope of demand raises the wage level. This makes the area of producer surplus smaller than before this change as the firm must now hire the same amount of labour, but at a higher price. Thus, they are necessarily worse off than before. The opposite is true for an increase in the slope of demand. A decrease in the supply curve also has the opposite effect as for the clustered firm. Recall, decreasing the supply curve for the clustered firm decreased the wage, allowing firms to hire more labour at the same price, and making them better off. Here, however, a decrease in the supply curve raises wages further, and thus further reduces the area of producer surplus in bad times, but also reduces the area of producer surplus in good times. Firms must once again hire the same amount of labour at a higher price.
Part Three:
Computational Model:

The computational model for Part Three is given below as Figure 3-3(a):

![Figure 3-3(a)](image)

This model, once again, operates very similar to the previous two. However, this time, the blue shaded area represents the gains received from clustering, while the red shaded area represents the losses received from clustering. Thus, given the parameters specified in the original models for Parts One and Two, there are gains associated with clustering in good times, but losses associated with clustering in bad times. This is indeed the typical case. As mentioned in the economic model section, a clustered firm can often charge a lower wage in good times (earning the above gains), but must also often charge a higher wage in bad times (earning the above losses). Given the above parameters, the gains from clustering outweigh the losses from clustering. Thus, if the probability of encountering the good state versus the bad state is always 50%, firms will always choose to cluster. However, as these relative probabilities change, so does the likelihood of firms clustering given the above parameters.

This model was very difficult to produce on one graph (originally it was on three), but it is now fully functional, and should be able to handle any changes.

Highlights (Experiments):

The original computational model gives the 'typical case' for firms choosing whether or not to cluster. This section thus analyzes some of the less typical cases. Figure 3-3(b) looks at a case where there is a zero wage for the clustered firm in both states of the world, and the supply curve for the isolated firm has additionally been shifted inwards. Figure 3-3(c) illustrates the effect of a very high wage for the clustered firm in both states of the world, and an outward shift of the supply curve for the isolated firm. Figure 3-3(d) focuses on a steeper slope for 'D-G,' and finally, Figure 3-3(e) looks at a decreased slope for 'D-G.' These more 'extreme' cases highlight the capabilities of the model, as well as outline the effects of any potential wage scenario.
Figure 3-3(b) shows that when the wages of the clustered firm are equal to zero, there are large gains to be made by clustering both in the good state and the bad states of the world. These gains are further amplified by inward shifts of the supply curve for the isolated firm. However, when the reverse is true, and there are high wages for the clustered firm, there are huge losses to be made in both states of the world. These losses are then further amplified by outward shifts of the supply curve for the isolated firm.

Figure 3-3(d) shows that a steeper slope for 'D-G' eliminates all gains that were previously made in the good state and converts them to losses. Similarly, a decrease in the slope for 'D-G' amplifies the previous gains. In both scenarios, the losses made in the bad state are unaltered.

Discussion:

In this model, gains are made from clustering if the wage from clustering is lower than the wage from being isolated. As previously discussed, in the normal case, the wage from clustering is higher in bad times, but lower in good times, leading to losses in the bad state of the world, but gains in the good state. This same pattern continues with Figures 3-3(b)-3-3(e). In 3-3(b), the wage of the clustered firm is always zero. Thus, unless the wage of the isolated firm is simultaneously zero in at least one state of the world, there will always be gains from clustering. These gains are amplified by inward shifts of the isolated firm's supply curve as these inward shifts increase the wage for the isolated firm and thus increase the wage differential between the two firms. The opposite case is seen in Figure 3-3(c). Here, the wage of the
clustered firm is higher than the wage for the isolated firm in both states of the world, leading to losses. These losses are amplified by outward shifts of the isolated firm's supply curve as these outward shifts decrease the wage for the isolated firm and, once again, increase the wage differential between the two firms. The increased slope in Figure 3-3(d) similarly led to losses by making the wage of the isolated firm lower than the wage of the clustered firm when it was previously higher. Finally, the decreased slope in Figure 3-3(e) amplified gains by making the wage of the isolated firm even higher than the wage of the clustered firm. To summarize, anything that leads the wage of the isolated firm to be higher than the wage of the clustered firm will lead to losses, and the magnitude of these losses will be determined by the magnitude of the wage differential, and vice versa for wages of the isolated firm that are lower than the wages of the clustered firm.

Model 4 – Crime Prevention

Economic Model:

Crime is an important urban issue, as it tends to be concentrated in city centres. High crime rates may deter businesses from locating within that city, which can lead to lower local job opportunities. This may lead to an increase in poverty, which can further increase crime rates, as low wages lower the opportunity cost of crime. Crime may also impact individuals' location decisions and thus may be a significant factor in the determination of land prices. Determining the socially optimal level of crime requires first determining the social costs of crime (victim costs), and the prevention costs. Prevention costs are decreasing in the number of crimes because it's less expensive to allow many crimes to occur than to allow none. Completely eliminating crime would be very expensive. Thus, the socially optimal level of crime occurs where total costs (taking both the aforementioned victim costs and prevention costs into consideration) are minimized.

Computational Model:

The computational model is shown below as Figure 4(a):

![Figure 4(a)](image)

The curves are fully labelled so this model should be fairly easy to understand. Total prevention costs are decreasing in the number of crimes (as discussed above), while total victim costs are increasing in the number of crimes. This makes intuitive sense, as the more crimes that occur, the more victims there are. The total cost curve represents the aggregation of the total prevention costs and the total victim costs. The socially optimal level of crime, as discussed above, occurs at the minimum total cost to society. This is the intersection between the total cost curve and the dashed lines. The dashed line to the y-axis shows what the minimum total cost is in dollar terms, while the dashed line to the x-axis shows the approximate number of crimes that this dollar value corresponds to.

Highlights (Experiments):
Since this model is fairly simple to understand, there was not a lot that could be done to really highlight its potential. Thus, this section returns to the comment made in the economic model discussion about completely eliminating crime. From the original model, it is clear to see that under these conditions, completely eliminating crime is not socially optimal, and would be quite expensive. However, Figures 4(b) and 4(c) present two cases where completely eliminating crime might actually be socially optimal. Figure 4(b) shows the case of very high victim costs, whereas Figure 4(c) shows the case of very low prevention costs.

Discussion:

As discussed previously, it is not typically socially optimal to completely eliminate crime due to the expense of doing so. The costs of such a plan would outweigh the benefits. Yet, there are exceptions. With high victim costs, or low prevention costs, society would prefer to have no crime at all. However, it would be harmful to society to have very high victim costs as then, while the city would try to ensure that there were no crimes, any crimes that did occur, would have very large negative effects to society. This option also comes with the highest cost to the city. With very low prevention costs, zero crime is also socially optimally, yet the cost of maintaining this is very low. However, this is presumably not very feasible, or else it would be done. Thus, unless the city engages in very violent crimes all the time, or it becomes very cheap to prevent crime, it will remain the case that it is not socially optimal to completely eliminate crime.

Model 5 – Majority Voting

Economic Model:

Efficient provision of any good (public or private) occurs at the point where marginal social benefit equals marginal social cost. However, when dealing with public goods, the marginal social benefit curve is very hard to obtain. People have an incentive to lie about how much they want of a particular good. They will overstate their demand for it if they think they won’t have to pay for it, but will understate their demand for it if they think they will have to pay for it. This is because there is a free-rider problem inherent in the provision of any public good. If communities lack information about individual preferences, voting becomes a way to gather such information. However, this does not, in general, lead to an efficient provision of goods. This is because under majority rule, the median voter determines the level of provision.

Computational Model:
The computational model is given below in Figure 5(a). This model looks at the example of a community trying to determine how much park space to allocate to its residents. There are three residents in this community, each with different demands for parks.

**Figure 5(a)**

In this model, 'MB A' is the marginal benefit curve for individual 'A', 'MB B' is the marginal benefit curve for individual 'B', 'MB C' is the marginal benefit curve for individual 'C', and 'SMB' is the social marginal benefit curve, which is the vertical aggregation of the three individual benefit curves. This vertical aggregation presented some problems at first, because when the curves passed zero, they would begin to subtract from the aggregate curve. However, it has been fully solved now, and should work for any combination of marginal benefit curves. The curve labelled 'SMC' is the social marginal cost curve and 'IMC' is the individual marginal cost curve, which is the cost to each individual resident.

The controls allow you to adjust the intercepts and/or slopes of any individual marginal benefit curve, which will also change the marginal social benefit curve, as well as adjust the position of the marginal social cost curve, which will also change the individual marginal cost curve.

The efficient provision occurs at the point where marginal social benefit intersects with marginal social cost. However, recall that the median voter (person B in the above example) is the one who determines the amount actually provided. The demand of the median voter occurs at the intersection between his/her marginal benefit curve and the individual marginal cost curve. Thus, since in this example, the efficient provision is greater than the amount demanded by individual B, the amount of park space will be underprovided.

**Highlights (Experiments):**

It is not necessarily the case that the good will be underprovided by majority voting. It may in fact be overprovided (as seen in Figure 5(b)), or it may in fact lead to efficient provision (as seen in Figure 5(c)). However, this is not the typical case. The only way to constantly achieve the efficient outcome from majority voting would be to somehow ensure that everyone had the same individual marginal benefit curves (as seen in Figure 5(d)).
Figure 5(b) shows the case of overprovision. Here the marginal benefit curve for the median voter intersects the individual marginal cost curve at a level of park provision beyond where the social marginal benefit curve intersects the social marginal cost curve. Thus, too much park space is allocated to residents. Figure 5(c) gives the exception to the claim that majority voting provides an inefficient allocation of goods. If all residents have marginal benefit curves with the same slope and the median voter demands an allocation exactly in the middle of the other two residents, efficient provision will be reached.

Figure 5(d)

As discussed previously, Figure 5(d) represents the only way to consistently get efficient results from majority voting. Here, every individual has the exact same marginal benefit curve. In this situation, the individual marginal benefit curves cross the individual marginal cost curve at the exact same location as the social marginal benefit curve crosses the social marginal cost curve. This will occur no matter what shape the individual marginal benefit curves happen to be.

Discussion:

Majority voting usually produces allocations of public goods that are not efficient. While there is an exception (Figure 5(b)), this is certainly not a typical case. However, through tiebout sorting, communities can get much better results out of majority voting. In a tiebout-sorting model, people sort themselves into communities according to the services they desire. In the above example, therefore, we would see resident A moving into a community that desires a low amount of park space, resident B moving into a community that desires a little more park space, and resident C moving into a community that desires lots of park space. If everyone did this, we could (in theory) get everyone in one community having the exact same demands for services (as seen in Figure 5(c)). Under these conditions, the median voter would choose the
efficient amount. However, this does make the assumption that there is at least one community per type of consumer, so that everyone can find the perfect combination of services for their needs.

**Model 6 – Urban Growth Boundary**

**Economic Model:**

Urban sprawl causes an externality. The total social costs of someone buying a house outside of the city and commuting inside it for work include the private costs of that individual (construction and commuting costs) as well as external costs, such as the loss of nice views and longer commutes for others in the area. As with any externality, since the individual building the house only takes his/her private costs into account, and not the social costs, there tends to be an excess of urban sprawl. One way for a city to limit sprawl is to use urban growth boundaries. These are essentially lines drawn around the city within which all development must occur. These restrictions increase the value of residential land within the limits, and therefore the rents everyone must pay to live within the city. In contrast, the value of the land outside the limits falls in value.

**Computational Model:**

The computational model is given below in Figure 6(a):

![Figure 6(a)](image)

In this model, the centre of the city is located at the y-axis. Under normal conditions, the bid-rent function would consist of the business and residential bid-rent function (Business/Resid BR) until the intersection between the business and residential bid-rent function and the agricultural bid-rent function (Agri BR), at which point it would consist solely of the agricultural bid-rent function. However, with the addition of the boundary, the business and residential bid-rent function is restricted. As mentioned above, this pushes up the value of the land within the boundary, and decreases the value of the land immediately outside of the boundary.

The controls of this model allow you to shift the intercept and/or slope of the business and residential bid-rent function, shift the intercept of the agricultural bid-rent function, and move the boundary.

**Highlights (Experiments):**

Three scenarios are given here to highlight the abilities of this model. Figure 6(b) shows a shift of the boundary closer to the city centre, Figure 6(c) shows a shift of the boundary further away from the city centre, and Figure 6(d) shows the effect of decreasing the slope of the business and residential bid-rent function.
Figure 6(b) shows that when the boundary is moved closer to the city centre, the rents paid within the city get significantly higher. However, when the boundary is moved further away from the city in Figure 6(c), it has no effect whatsoever.

Figure 6(d) demonstrates that when the slope of the business and residential bid-rent function is decreased, the rents paid within the city once again get significantly higher.

**Discussion:**

When the urban growth boundary moves closer to the centre of the city, the amount of land that can be built upon shrinks, and competition for land in the city gets fiercer. Thus, by sheer supply and demand mechanics, the price of land within the city (the rent paid) increases. The closer the boundary gets to the centre of the city, the more rents will rise. In contrast, when the urban growth boundary was moved farther away from the centre of the city, it lost all effect. This is due to the fact that in the original model, the boundary was sitting very close to the intersection between the business and residential bid-rent function and the agricultural bid-rent function, the natural place where the bid-rent function would change. Thus, when the boundary was moved beyond this point, it ceased to have an effect because everyone that far away from the city did not want to be in the city anyways. The natural border of the city would therefore occur before this point, regardless of the existence of some boundary. This is similar to the concept of a non-binding price floor or price ceiling in a supply and demand framework. Finally, the decrease in the slope of the business and residential bid-rent function caused the boundary to have a larger effect because such a change of slope naturally increases the size of the city by forcing the intersection between the business and resident and agricultural bid-rents to occur farther away from the centre of the city. Thus, when the boundary is imposed, it has a larger effect, because it is taking away more land from
Model 7 – Bid-Rents Under Monocentric Assumptions

Economic Model:

The monocentric city dominated approximately 100 years ago. Before the days of truck transport, both manufacturers and office firms wanted to locate at the centre of the city. Office firms wanted to minimize the distance between themselves and their clients, while manufacturers wanted to minimize the distance to the central railroad terminal to minimize shipping costs. Office firms tended to get the very centre of the city as they were better able to make use of small land plots by locating in skyscrapers and thus willing to pay higher rents for a given plot of land at the city centre. In addition, people are more expensive to transport than goods, as they require more space and comfort. Thus, land use was often organized into concentric rings, with office firms at the very centre of the city, followed by a ring of manufacturers, and then a ring of residential users.

Computational Model:

The computational model is given below as Figure 7(a):

In this model, the line 'Office BR' represents the bid-rent function for offices, 'MFG BR' represents the bid-rent function for manufacturers, and 'Resid BR' represents the bid-rent function for residential users. The y-axis represents the centre of the city, so this model should be thought of as symmetric. The above bid-rent functions give the land use patterns from the centre of the city outwards; however, the same patterns would be occurring on the other side of the city centre as well, and this is what would produce the concentric ring pattern mentioned above. The intersection of the bid-rent functions with each other gives the division of its land into its various uses. For example, office firms are located in the centre of the city and are located up to the point where the office bid-rent function intersects the manufacturing bid-rent function, at which point the manufacturers would begin to be located. The manufacturing firms would then locate up to the point where the manufacturing bid-rent function intersects the residential bid-rent function, at which point residents would begin to be located. The green line plots the rents actually paid by these users of land. Thus, we would see a pattern of land usage that looks something like this:
The controls of this model allow you to change the intercepts and/or slope of all three bid-rent functions.

**Highlights (Experiments):**

The first aspect I wish to show is the effect of changes to the slopes of the bid-rent functions (Figure 7(b)). However, the main point to highlight here is that the above pattern of land does not have to be what is observed. That was simply a 'typical case'. Thus, Figures 7(c) and 7(d) illustrate different patterns of land usage.

![Figure 7(b)](image)

Here, all the slopes of the bid-rent functions have been steepened. This has the effect of making the city denser. Office firms now all locate with a mile of the city centre, and the entire city is now only just over 8 miles wide (just over 4 miles on either side of the city centre).

![Figure 7(c)](image) ![Figure 7(d)](image)

Figures 7(c) and 7(d) illustrate the fact that not all users must demand land. In Figure 7(a), there are no manufacturers in the city, while in Figure 7(b), there are no offices in the city. However, there are many other examples of different land patterns that can be shown using this model.

**Discussion:**

Steepening the slope of the bid-rent functions (as in Figure 7(b)) makes the city denser by changing the structure of rents. Now, every additional mile away from the centre of the city is cheaper than before, which causes everyone to demand more land closer to the city. Due to this increase in demand for land at the city centre, everyone must make do with less land. Presumably this means more skyscrapers and smaller land plots. Figures 7(c) and 7(d) illustrated the point that not all cities have all types of land users. In Figure 7(b), for example, there are no manufacturers. This is presumably due to some external force,
such as a lack of adequate inputs or customer base, which decreases their demand for land in the city and causes them to be out bid for land by others. In Figure 7(c), there are no office firms. Potentially these cities have restrictions on building height, or again a small client base, that causes the office firms to have a lower demand in those cities and be outbid by others for land. As mentioned above, it is possible to think of many other patterns of land usage and illustrate them using this model.

**Model 8 – Bid-Rent Under Non-Monocentric Assumptions**

**Economic Model:**

This model looks at the pattern of land usage in a modern city. We still see land use being determined by the highest bidder. However, in a modern city, centralized railroads are no longer as important for shipping manufactured goods. Instead, manufacturers now ship their products by truck, and thus prefer to locate near a highway, rather than the city centre. Therefore, we now see office firms locating in the city centre, manufacturers locating near the highway, and residential users locating elsewhere.

**Computational Model:**

The computational model is given below as Figure 8(a):

In this model, 'OF' represents office firms, 'OW' represents office workers, 'MW' represents manufacturing workers, 'MF' represents manufacturing firms, 'A' represents agricultural users (farmland), and once again, the y-axis represents the centre of the city. Here, unlike in the monocentric model, residential users have been broken up into three groups: office workers, manufacturing workers, and agricultural dwellers. This is due to the fact that previously, all employment was concentrated in the centre of the city, so all residents wanted to be as close to their place of employment as possible. In the modern city, however, employment is more dispersed, and thus, so are the residents. Another important difference in the modern city is that due to this employment dispersal, the rent function is not monotonically decreasing from the centre of the city outwards. Here, the orange line plots the line of the bid-rents actually paid. This line is decreasing from the centre of the city up to the manufacturing sector, and then increasing up to the highway, and then decreasing again to the edge of the city. Also, this model is not necessarily symmetrical, unless there happens to be a highway located in the exact same position on the other side of the city centre. Finally, just as in the monocentric model, the intersections of the various bid-rent functions with themselves determine the allocation of land within the city, and these are marked by the dashed lines.

The controls of this model allow you to change the intercepts and/or slopes of any of the bid-rent functions, as well as move the location of the highway. However, note that the controls for the manufacturing workers and the manufacturing firms change both the slope and the intercept simultaneously in order to force these bid-rents to keep the same pattern while allowing the highway to move.
Highlights (Experiments):

Similar to the monocentric model, the first experiments (shown in Figures 8(b) and 8(c)) illustrate the effects of changing slopes and intercepts on the bid-rent function. However, the main point I want to highlight here is the effect of moving the highway. Thus, in Figure 8(d) the highway has been moved closer to the centre of the city, and in Figure 8(e) the highway is located in the centre of the city.

Figures 8(b) and 8(c) illustrate some of the effects of changing land use patterns by changing the slopes and intercepts of the model. However, once again, these are only examples, and many others can be created using this model. In Figure 8(b), the changes forced fewer manufacturing workers to locate on the left side of the manufacturing firm, and more to locate on the right side. In addition, there are fewer manufacturing firms. In Figure 8(c), there are no manufacturing workers on the left side of the manufacturing firm at all, and manufacturing workers must make do with very little land.

In Figure 8(d), the highway has been moved closer to the centre of the city. This change makes the land between the city centre and the highway more competitive, and as such, manufacturing workers must once again make do with very little land, and manufacturing workers all locate to the right of the manufacturing firm. When the highway is located in the centre of the city (Figure 8(e)), we see much more competition, and therefore much more displacement.

Discussion:

Decreasing the slope of the office workers in Figure 8(b) made every additional mile away from the city centre slightly cheaper for them. Thus, they began to demand more land and displaced some of the manufacturing workers. However, at the same time, the slope of the manufacturing workers was decreased, which led them to demand more land as well. Since the office workers were occupying almost all of the land to the left of the manufacturing firm, the additional land for the manufacturing workers had to be located to the right of the manufacturing firm. Thus, decreasing the slope of any individual bid-rent function leads that group to demand more land, but not necessarily in the same locations as before. In fact, it is possible to
decrease the slope of the office workers to such a point where some office workers may choose to locate beyond the highway and commute into the city. Figure 8(c) makes a similar point, but more dramatically, by having no office workers at all locating to the left of the manufacturing firm. Moving the highway closer to the centre of the city increases the competition for land at the centre of the city, which once again displaces workers. This competition gets even worse as the highway moves even closer to the centre of the city. When the highway is in the centre of the city (Figure 8(e)), we actually get the same results as from the monocentric model. Here, once again, all residents of the city want to locate as closely as possible to the centre of the city. Thus, we get jobs concentrated in the city centre, with office firms locating at the very centre, followed by manufacturing firms, and then residents. Note that, as in the monocentric model, the manufacturing workers and office workers live in the same district under these conditions.
Appendix:

Model 1 – Factory Market Diagram

(*Sandra Argatoff – Econ 353*)
(*Please evaluate notebook before using*)

\textbf{FactoryMarket} = Manipulate[
  
  Show[
    (*Plots cost of home production*)
    Plot[Tooltip[a, "Home Cost"], {x, 0, 15}, PlotStyle \rightarrow \{Thick\},
      PlotRange \rightarrow \{\{0, 15\}, \{0, 15\}\}, AxesOrigin \rightarrow \{0, 0\}, AxesLabel \rightarrow \{"Miles", "Cost"\},
      AspectRatio \rightarrow Automatic, PlotLabel \rightarrow "Factory Market Diagram"],

    (*Plots the per unit factory cost*)
    Plot[Tooltip[b, "Per Unit Factory Cost"], {x, 0, 7.5}, PlotStyle \rightarrow \{Thick\}],

    (*Plots the distance line*)
    Plot[Tooltip[1, "Miles From Factory"], {x, 0, 15}, PlotStyle \rightarrow \{Thick\}],

    (*Plots transport cost*)
    Plot[Tooltip[(b + 7.5 + d) - d \* x, "Transport Cost"], {x, 0, 7.5},
      PlotStyle \rightarrow \{Thick, Blue\}],
    Plot[Tooltip[(b - 7.5 + d) + d \* x, "Transport Cost"], {x, 7.5, 15},
      PlotStyle \rightarrow \{Thick, Blue\}],

    (*Plots factory cost*)
    Graphics[{Thick, Line[[\{\{7.5, 1\}, \{7.5, b\}\}]]}],

    (*Plots lines to determine size of factory*)
    Graphics[
      Thick, Dashed,
      Line[[\{\{7.5 + (b/d) - (a/d), 0\},
                    (7.5 + (b/d) - (a/d), (7.5 + (b/d) - (a/d))\}]],
      Thick, Dashed,
      Line[[\{(a/d) - (b/d) + 7.5, 0\},
                    ((a/d) - (b/d) + 7.5 + 7.5 \* d) + d \*(a/d) - (b/d) + 7.5)\}]],

    (*Adds text to graph*)
    Graphics[{Text["HC", \{2, a + 0.5\}]},
    Graphics[{Text["FC", \{2, b + 0.5\}]},
    Graphics[{Text["MFF", \{13, 1.5\}]},
    Graphics[{Text["TC", \{3.5, (b + 7.5 + d) - d \*(3.5) + 1\}]},
    Graphics[{Text["TC", \{11.5, (b - 7.5 + d) + d \*(11.5) + 1\}]},

    AspectRatio \rightarrow Automatic],

    (*Curves are fully adjustable;
    however, 'd' cannot equal '0' due to a potential division by zero problem*)

    "None" \rightarrow \{\{a, 12.26, "Home Cost"\}, 0, 15, .01, ImageSize \rightarrow Tiny, Appearance \rightarrow "Labeled"
    \{\{b, 4, "Factory Cost"\}, 1, 15, .01, ImageSize \rightarrow Tiny, Appearance \rightarrow "Labeled"},
    \{\{d, 2.38, "Transport Cost"\}, 0.0001, 100, 0.01, ImageSize \rightarrow Tiny,
    Appearance \rightarrow "Labeled"],

    ControlPlacement \rightarrow Left]
Model 2 – Filtering and Quality Ladder

Manipulate[Show[

(*Plots nothing. This was done just to set up the formatting of the plot area.*)
Plot[Tooltip[a - b*x, "nothing"], {x, 0, 16}, AxesOrigin -> {0, 0},
AxesLabel -> {"HH Income", "Quality"}, LabelStyle -> Directive[Bold, 12], PlotRange -> {0, 16},
PlotStyle -> {Thick}, Ticks -> {Automatic, Automatic}, TicksStyle -> Directive[12]],

(*Plots the points in the graph from left to right*)
Graphics[{PointSize[0.02], Tooltip[Point[{r, f}], "point"]}],
Graphics[{PointSize[0.02], Tooltip[Point[{r, s}], "point"]}],
Graphics[{PointSize[0.02], Tooltip[Point[{t, s}], "point"]}],
Graphics[{PointSize[0.02], Tooltip[Point[{t, p}], "point"]}],
Graphics[{PointSize[0.02], Tooltip[Point[{k, p}], "point"]}],
Graphics[{PointSize[0.02], Tooltip[Point[{k, l}], "point"]}],

(*Plots lines connecting points to form step pattern*)
Graphics[{Thick, Line[{{r, f}, {r, s}}]},
Graphics[{Thick, Line[{{r, s}, {t, s}}]},
Graphics[{Thick, Line[{{t, s}, {t, p}}]}],
Graphics[{Thick, Line[{{t, p}, {k, p}}]}],
Graphics[{Thick, Line[{{k, p}, {k, l}}]}],

(*Produces lines from points to axes*)
Graphics[{Thick, Dashed, Blue, Line[{{t, 0}, {t, s}}]}],
Graphics[{Thick, Dashed, Blue, Line[{{k, 0}, {k, p}}]}],
Graphics[{Thick, Dashed, Blue, Line[{{0, s}, {r, s}}]}],
Graphics[{Thick, Dashed, Blue, Line[{{0, p}, {t, p}}]}],
Graphics[{Thick, Dashed, Blue, Line[{{0, l}, {k, l}}]}],

(*Adds text to the graph area to identify dots*)
Graphics[{Text["H", {k + 0.75, 1}]}],
Graphics[{Text["I", {k + 0.75, 0}]}],
Graphics[{Text["M", {t + 0.5, p + 0.5}]}],
Graphics[{Text["N", {t + 0.75, s}]}],
Graphics[{Text["Z", {r + 0.5, s + 0.5}]}],
Graphics[{Text["L", {r + 0.5, f + 0.5}]}],

AspectRatio -> Automatic, PlotLabel -> "Filtering and Quality Ladder", ImageSize -> {400, 300},
ImagePadding -> {{25, 75}, {15, 20}}],

(*The following controls are fully adjustable –
there are no limits to values that can be used. The ranges specified are just an example,
please feel free to change them as you play with the graph.*)

(*Please note the pattern: H U/D moves the dot 'H' up and down,
M/I U/D moves the pair of dots 'M' and I' up and down etc. Similarly,
H/I L/R moves the pair of dots 'H' and I' up and down.*)

{{l, 15, "H U/D"}, p, 15, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{p, 10, "M/I U/D"}, s, 1, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{s, 5, "Z/N U/D"}, f, p, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{f, 0, "L/U/D"}, 0, s, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{k, 15, "H/I L/R"}, t, 15, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{t, 10, "M/N L/R"}, r, k, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{r, 5, "Z/L L/R"}, 0, t, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
ControlPlacement -> Left}
Model 3 – Costs/Benefits from Clustering

Part One:

Clustered = Manipulate[

Show[

(*Equation for Demand in good state*)
Plot[Tooltip[a - b \[ExponentialE]^x, "demand good"], {x, 0, 15}, PlotStyle \[Rule] Thick,
PlotRange \[Rule] {{0, 10}, {0, 10}}, AxesOrigin \[Rule] {0, 0}, AxesLabel \[Rule] {"N", "W"},

(*The following produces labels on the axes*)
Ticks \[Rule] {{{((P - a) / (-b)), "N^G"}, {{(P - c) / (-1), "N^R"}}, {s, "N^I"}},

{{{P, "W^C"}, {(c - s), "W^B"}, {(a - s), "W^I"}}},


(*Equation for demand in bad state*)
Plot[Tooltip[c - d \[ExponentialE]^x, "demand bad"], {x, 0, 15}, PlotStyle \[Rule] {Thick, Red}],

(*Equation for labour supply of clustered firm*)
Plot[Tooltip[P, "price"], {x, 0, 15}, PlotStyle \[Rule] {Thick, Green}],

(*The following Graphics objects produce dashed lines marking equilibrium values*)
Graphics[{Dashed, Line[{{((P - a) / (-b)), 0}, {(P - a) / (-b), P}]}]},

Graphics[{Dashed, Line[{{0, P}, {(P - a) / (-b), P}}]}],

Graphics[{Dashed, Line[{{0, P}, {(P - c) / (-b), P}}]}],

Graphics[{Dashed, Line[{{0, (a - s)}, {s, (a - s)}}]}],

Graphics[{Dashed, Line[{{0, (c - s)}, {s, (c - s)}}]}],

Graphics[{Dashed, Line[{{((P - c) / (-d)), 0}, {(P - c) / (-d), P}}]}],

(*Equation for labour supply of isolated firm*)
Graphics[{Thick, Line[{{s, 0}, {s, 100}}]}],
Part Two:

(*Part Two – Isolated Firm*)

**Isolated** = Manipulate[Show[

(*Equation for demand in good state*)
Plot[Tooltip[a - b x, "demand good"], {x, 0, 15}, PlotStyle -> Thick,
PlotRange -> {{0, 10}, {0, 10}}, AxesOrigin -> {0, 0}, AxesLabel -> {"\(N\)", "\(N\)"},
Ticks -> {{(P - a) / (-b), "\(N^{CG}\)"}, {(P - c) / (-d), "\(N^{CB}\)"}, {s, "\(N^I\)"},
{P, "\(N^C\)"}, {(c - s), "\(N^{IB}\)"}, {(a - s), "\(N^{IG}\)"}},
AspectRatio -> Automatic, PlotLabel -> "Isolated Firm"],}]

Part Two:
(*Equation for demand in bad state*)
Plot[Tooltip[c - d*x, "demand bad"], {x, 0, 15}, PlotStyle -> {Thick, Red}],

(*Equation for labour supply of clustered firm*)
Plot[Tooltip[p, "price"], {x, 0, 15}, PlotStyle -> {Thick, Green}],

Graphics[{Dashed, Line[{((p - a) / (-b)), 0}, {(p - a) / (-b), p}]}],
Graphics[{Dashed, Line[{0, p}, {(p - a) / (-b), p}]}],
Graphics[{Dashed, Line[{0, p}, {(p - c) / (-d), p}]}],
Graphics[{Dashed, Line[{0, (a - s)}, {s / b, (a - s)}]}],
Graphics[{Dashed, Line[{0, (c - s)}, {s / d, (c - s)}]}],
Graphics[{Dashed, Line[{((p - c) / (-d)), 0}, {(p - c) / (-d), p}]}],

(*Equation for labour supply of isolated firm*)
Graphics[{Thick, Line[{s, 0}, {s, 100}]}],

Graphics[{Opacity[0.4], Red, 
    Tooltip[Polygon[{0, (c - s)}, {0, c}, {s / d, (c - s)}],
               "Producer Surplus Bad"]}],
Graphics[{Opacity[0.2], Blue, 
    Tooltip[Polygon[{0, (a - s)}, {0, a}, {s / b, (a - s)}],
               "Producer Surplus Good"]}],

Graphics[{Text["D-G", {2, a - x /. {x -> 2}}]}],
Graphics[{Text["D-B", {1, c - x /. {x -> 1}}]}],
Graphics[{Text["S-I", {{s + 0.5}, 8}]}],
Graphics[{Text["S-C", {8, (p + 0.25)}]}],

AspectRatio -> Automatic],

(*Once again, the controls are fully adjustable*)
"None" -> {{a, 9, "D_G intercept"}, 8, 10, .01, ImageSize -> Tiny, 
    Appearance -> "Labeled"},
{{b, 1, "D_G Slope"}, 0.1, 10, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{c, 6, "D_B intercept"}, 0, 8, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{d, 1, "D_B Slope"}, 0.1, 8, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{p, 3, "Supply_G"}, 0, 10, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
{{s, 4.5, "Supply_I"}, 0, 10, .01, ImageSize -> Tiny, Appearance -> "Labeled"],

ControlPlacement -> Left]
Part
Three:

(*Part Three - Gains/Losses*)

GainsLosses = Manipulate[Show[
    (*Equation for demand in good state*)
    Plot[Tooltip[a - b x, "demand good"], {x, 0, 15}, PlotStyle -> Thick,
    PlotRange -> {{0, 10}, {0, 10}}, AxesOrigin -> {0, 0}, AxesLabel -> {"N", "W"},
    Ticks -> {{(P - a) / (-b), "N^{CG}"}, {(P - c) / (-d), "N^{CH}"}, {s, "N^{I}"},
    {{P, "W^{C}"}, {(c - s), "W^{IB}"}, {(a - s), "W^{IG}"}},
    AspectRatio -> Automatic, PlotLabel -> "Gains/Losses From Clustering"],
    (*Equation for demand in bad state*)
    Plot[Tooltip[c - d x, "demand bad"], {x, 0, 15}, PlotStyle -> {Thick, Red}],
    (*Equation for labour supply of clustered firm*)
    Plot[Tooltip[P, "price"], {x, 0, 15}, PlotStyle -> {Thick, Green}],
    Graphics[{Dashed, Line[{(P - a) / (-b), 0}, {(P - a) / (-b), P}]}],
    Graphics[{Dashed, Line[{0, P}, {(P - a) / (-b), P}]}],
    Graphics[{Dashed, Line[{0, P}, {(P - c) / (-b), P}]}],
    Graphics[{Dashed, Line[{0, (a - s) / b, (a - s)}]}],
    Graphics[{Dashed, Line[{0, (c - s) / d, (c - s)}]}],
    Graphics[{Dashed, Line[{(P - c) / (-d), 0}, {(P - c) / (-d), P}]}],
    (*Equation for labour supply of isolated firm*)
    Graphics[{Thick, Line[{s, 0}, {s, 100}]}],
    Graphics[{Text["D-G", {2, a - x / . x -> 2}]}],
    Graphics[{Text["D-B", {1, c - x / . x -> 1}]}],
    Graphics[{Text["S-I", {(s + 0.5), 8}]}],
    Graphics[{Text["S-C", {8, (P + 0.25)}]}]]}
(*The following is a series of if-else statements that allow the model to calculate gains and losses from clustering when the various supply curves/demand curves are changed*)

If[(c - s) ≤ P ≤ (a - s),
  Graphics[{Opacity[0.4], Red,
    Tooltip[Rectangle[0, (c - s)], ((P - c) / (-d), P)],
    Opacity[0.4], Red,
    Tooltip[Polygon[{{P - c) / (-d), (c - s)}, ((P - c) / (-d), P),
                   {s / d, (c - s)}}, "Losses from Clustering"],
    Opacity[0.2], Blue, Tooltip[Rectangle[0, P], {s / b, (a - s)}],
    Opacity[0.2], Blue,
    Tooltip[Polygon[{{s / b, P}, {s / b, (a - s)}, ((P - a) / (-b), P)}],
    "Gains from Clustering"]}],
If[P < (c - s),
  Graphics[{Opacity[0.2], Blue, Tooltip[Rectangle[0, P], {s / b, (a - s)}],
    Opacity[0.2], Blue,
    Tooltip[Polygon[{{s / b, P}, {s / b, (a - s)}, ((P - a) / (-b), P)}],
    "Gains from Clustering"]}],
Graphics[{Opacity[0.4], Red,
  Tooltip[Rectangle[0, (c - s)], ((P - c) / (-d), P)],
  Opacity[0.4], Red,
  Tooltip[Polygon[{{(P - c) / (-d), (c - s)}, ((P - c) / (-d), P), {s / d, (c - s)}},
                   "Losses from Clustering"],
  Opacity[0.4], Red, Tooltip[Rectangle[0, (a - s)], ((P - a) / (-b), P)],
  Opacity[0.4], Red,
  Tooltip[Polygon[{{(P - a) / (-b), (a - s)}, ((P - a) / (-b), P), {s / b, (a - s)}},
                   "Losses from Clustering"]}],
AspectRatio -> Automatic],

("Once again, the controls are fully adjustable")
"None" -> {{a, 9, "D^c intercept"}, 8, 10, .01, ImageSize -> Tiny,
  Appearance -> "Labeled"},
  {{b, 1, "D^g Slope"}, 0.1, 10, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
  {{c, 6, "D^cb Intercept"}, 0, 8, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
  {{d, 1, "D^g Slope"}, 0.1, 8, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
  {{P, 3, "Supply^c"}, 0, 10, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
  {{s, 4.5, "Supply^t"}, 0, 10, .01, ImageSize -> Tiny, Appearance -> "Labeled"},
  ControlPlacement -> Left]
Model 4 – Crime Prevention

(*Sandra Argatov - Econ 333*)

(*Please evaluate notebook before shifting any curves!*)

```
CrimePrevention = Manipulate[

(*definition of total cost*)

tc = ((a (x + 2)^(-b)) + (c + d*x));

Plot[{a (x + 2)^(-b), (c + d*x), tc},

{x, 0, 20}, PlotStyle -> {Thick},

Epilog ->

(*NArgMin is used to calculate the minimum value of total cost*)

{tc/. x-> NArgMin[tc, x] /. x -> NArgMin[tc, x]}],

(*The following produces text on the graph to identify curves*)

{Text["Total Cost", {10, tc /. x -> 12}]}],

{Text["Total Victim Cost", {15, (c + d*x) /. x -> 12}]}],

{Text["Total Prevention Cost", {17, (a (x + 2)^(-b)) /. x -> 12}]}],

(*Ticks produce writing on axes - tc/*

x = NArgMin returns the value of the minimum cost on the y-axis*)

Ticks -> {{NArgMin[tc, x], "Optimal # Crimes"}, {tc /. x -> NArgMin[tc, x], "Min TC"}},

AxesLabel -> {"# of Crimes", "$"}, LabelStyle -> Directive[Bold, 12], PlotRange -> {0, 15},

AspectRatio -> Automatic, PlotLabel -> "Optimal Number of Crimes", ImageSize -> {500, 400},

ImagePadding -> {{70, 80}, {20, 20}}],

(*Please note: the controls for this particular model are a bit jerky,
which I assume is because the program must calculate new minimum values for each shift. It
works best if you move the bars slowly.*)

(*TPC = Total Prevention Cost, TVC = Total Victim Cost*)

{a, 25, "TPC Intercept"}, 5, 35, .01, ImageSize -> Tiny, Appearance -> "Labeled"},

{b, 0.94, "TPC Slope"}, 0.1, 5, .1, ImageSize -> Tiny, Appearance -> "Labeled"},

{c, 0, "TVC Intercept"}, 0, 5, .1, ImageSize -> Tiny, Appearance -> "Labeled"},

{d, 0.8, "TVC Slope"}, .1, 4, .001, ImageSize -> Tiny, Appearance -> "Labeled"},

ControlPlacement -> Left]
```
Model 5 – Majority Voting

Model 5 – Majority Voting

(plot of the marginal benefit curves for individuals C, B, and A, respectively. The if-else statements plot the social benefit curve -
- the aggregate of the three individual marginal benefit curves. M plots the marginal social cost curve, and 1/M plots the individual marginal cost curve.)

Plot: 

\[ y = \begin{cases} 
2 * x & \text{if} \quad (a-b) > 0, \\
2 * x - (a-b) & \text{if} \quad (a-\overline{b}) > 0, \\
2 * x - (a-b) + (a-\overline{b}) & \text{if} \quad (a-b) + (a-\overline{b}) > 0, \\
-2 * x + (a-b) & \text{if} \quad (a-\overline{b}) > 0, \\
-2 * x + (a-b) + (a-\overline{b}) & \text{if} \quad (a-b) + (a-\overline{b}) + (a-\overline{b}) > 0. 
\end{cases} \]

Ticks: 

\[ (\text{Thick, Dashed, Line}[\{(0,1/3), (0,1/3)\}], \{(0,1/3), (1,1/3)\}]) \]

Epilog: 

\[ (\text{Thick, Dashed, Line}[\{(0,1/3), (1,1/3)\}], \{(0,1/3), (0,1/3)\}]) \]

Efficient: Manipulate[ ]

Once again, the curves are fully adjustable – please feel free to make any changes you wish, including changes to the specified ranges.

{\[ a = 7.98, \quad \text{MB High Intercept}\], 0, 15, .01, ImageSize -> Tiny, Appearance -> "Labeled"}, 
{\[ b = 4.45, \quad \text{MB High Slope}\], 0, 15, .01, ImageSize -> Tiny, Appearance -> "Labeled"}, 
{\[ c = 4.4, \quad \text{MB Medium Intercept}\], 0, 15, .01, ImageSize -> Tiny, Appearance -> "Labeled"}, 
{\[ d = 3.45, \quad \text{MB Low Intercept}\], 0, 15, .01, ImageSize -> Tiny, Appearance -> "Labeled"}, 
{\[ e = 3.3, \quad \text{MB Low Slope}\], 0, 15, .01, ImageSize -> Tiny, Appearance -> "Labeled"}, 
{\[ m = 4.5, \quad \text{Marginal Social Cost}\], 0, 15, .01, ImageSize -> Tiny, Appearance -> "Labeled"}, 
ControlPlacement -> Left]
Model 6 – Urban Growth Boundary

(*Please evaluate notebook before adjusting curves*)

UBB = Manipulate[

(*Plots business/residential BR and constraint for the boundary*)
Plot[
{s*(a+b*x), 0, If[x < -(a/2b), x + (b*(x - (a/2b))/b - (a/b)), If[x > (a+b)/b, If[x < (a+b)/b, (a+b)*x + (b*(x - (a+b)/b))/b, 10], 0]], {x, 0, 10}, AxesOrigin = {0, 0},

AxesLabel = {"Distance from CC", "Bid-Rent Per Hectare"}, LabelStyle = Directive[Bold, 12], PlotRange = {0, 10}, PlotStyle = Directive[Thick],
Ticks = {Automatic, Automatic}, TicksStyle = Directive[12],

Epilog =

(*Plots boundary line*)
{{Thick, Line[{(b, 0), (b, (a+b)*10)}]},

(*The following produces text on the graph to identify curves*)
{Text["Boundary", (a/2, 3)],
Text["Business/Resid BR", (2 - 0.5, (a+b)/2)],
Text["Apri BR", (2, (a+b)/2)],
If[x < (a+b)/b, Text["BR Function", (1, (a+b)*1 + 1)])},
AspectRatio = Automatic, PlotLabel = "Urban Growth Boundary", ImageSize = {800, 600}, ImagePadding = {{25, 55}, {20, 20}}],

(*Curves are fully adjustable; Please feel free to adjust the ranges as well as shift the curves*)

t = 0.5, "Res/Resid BR Intercept"}, 0, 10, .01, ImageSize = Tiny, Appearance = "Labeled"],
t = 0.75, "Bus/Resid BR Slope"}, -2, -0.1, .01, ImageSize = Tiny, Appearance = "Labeled"],
t = 1, "Apri BR"], 0, 10, .01, ImageSize = Tiny, Appearance = "Labeled"],
t = 5, "Boundary"], 0, 10, .01, ImageSize = Tiny, Appearance = "Labeled"],

ControlPlacement = Left]
Model 7 – Bid-Rents Under Monocentric Assumptions

(* Sandra Argastoff - Econ 351 *)

(* Please evaluate notebook before shifting any curves *)

Manipulate[

(* (a>b+s) plots the bid-rent function for offices, (c>d+s) plots the bid-rent function for manufacturers, and (e>f+s) plots the bid-rent for residential users of land *)

(* The if statements here produce a line of the bid-rents actually paid - asks a line that plots the outermost curves at any given point *)

Plot[(a+b+s, c+d+s, e+f+s), If[(c+d+s) > (a+b+s), If[(a+b+s) > (c+d+s), (a+b+s), (c+d+s)], If[(a+b+s) > (e+f+s), (a+b+s), (e+f+s)]], (a, b, c, d, e, f, s)], (a, b, c, d, e, f, s)], AxesOrigin -> (0, 0),

AxesLabel -> {"Distance from UC", "Bid-Rent Per Nectare"}, LabelStyle -> Directive[Bold, 12], PlotRange -> {0, 10}, PlotStyle -> Directive[Thick],

(* Automatic ticks number the axes of the graph *)

Ticks -> {Automatic, Automatic}, TicksStyle -> Directive[12],

(* The epilog section produces dashed lines to the equilibriums as well as identifies the curves with text *)

Epilog -> {Thick, Dashed, Line[{{(a - c) / (d - b), 0}, {(a - c) / (d - b), (a + b) * (a - c) / (d - b))}]},

Thick, Dashed, Line[{{0, (a + b) * (a - c) / (d - b))}, {(a - c) / (d - b), (a + b) * (a - c) / (d - b))}]},

Thick, Dashed, Line[{{(c - d) / (e - f), 0}, {(c - d) / (e - f), (c + d) * (c - d) / (e - f))}]},

Thick, Dashed, Line[{{0, (c + d) * (c - d) / (e - f))}, {(c - d) / (e - f), (c + d) * (c - d) / (e - f))}]},

Text["Office BR", (1.5, a + b + x /. x -> 1.5)],

Text["Mfg BR", (6, (c + d + x) /. x -> 6)],

Text["Resid BR", (12, e + f + x /. x -> 12)]},


(* The curves are fully shiftable, please feel free to make any changes to the specified ranges *)

{(a, 8, "Office BR Intercept"), 2, 10, .01, ImageSize -> Tiny, Appearance -> "Labeled"},

{(b, -1.5, "Office BR Slope"), -2, -.74, .01, ImageSize -> Tiny, Appearance -> "Labeled"},

{(c, 5.5, "Mfg BR Intercept"), 4, 10, .01, ImageSize -> Tiny, Appearance -> "Labeled"},

{(d, -0.5, "Mfg BR Slope"), -0.74, -2, .01, ImageSize -> Tiny, Appearance -> "Labeled"},

{(e, 2.26, "Resid BR Intercept"), 3, 0, .01, ImageSize -> Tiny, Appearance -> "Labeled"},

{(f, -0.15, "Resid BR Slope"), -0.49, -.05, .01, ImageSize -> Tiny, Appearance -> "Labeled"},

ControlPlacement -> Left}
Model 8 – Bid-Rent Under Non-Monocentric Assumptions

Manipulate[

Plot[Tooltip[TraditionalForm[8 x + b, "office firms"), {x, 0, s + 6}, AxesOrigin -> {0, 0}, AxesLabel -> {"Distance from CC", "Bid-Rent Per Hectare"}, LabelStyle -> Directive[Bold, 12], PlotRange -> {0, 10}, PlotStyle -> {Thick, Red}, Ticks -> Automatic, Automatic], TicksStyle -> Directive[12]],

Plot[Tooltip[8 x, "office workers"), {x, 0, s + 6}, PlotStyle -> {Thick, Blue}],

Plot[Tooltip[(5/4) x, "MFg Firms"), {x, s - 4, s}, PlotStyle -> {Thick, Pink}],

Plot[Tooltip[(-1/4) x + (1/4) (s - 4), "MFg Firms"), {x, s + 4, s}, PlotStyle -> {Thick, Pink}],

Plot[Tooltip[((5/4) x) / (5.75) + p - ((p - n) / (5.75)) * (s - 0.25), "MFg Workers"), {x, s - 6, s - 0.25}, PlotStyle -> {Thick, Green}],

Plot[Tooltip[(-1/4) x + (1/4) (s - 4), "MFg Firms"), {x, s + 0.25, s + 6}, PlotStyle -> {Thick, Green}],

Plot[Tooltip["The Highway"], {Line[{{0, 0}, {s, 100}}]},

Plot[Tooltip["AG", {x, 0, 15}, PlotStyle -> {Thick, Purple}],

Graphics[Text["OG", {1, s - 6 + s / x = 1}],

Graphics[Text["AG", {3, c - d + s / x = 3}],

Graphics[Text["NH", {s - 2.5, s}],

Graphics[Text["NM", {s - 1, 0}],

Graphics[Text["NM", {s - 2.5, 5}],

Graphics[Text["NM", {s + 1, 8}],

Graphics[Text["Buy", {s + 0.75, 9.75}],

Graphics[Text["A", {14.5, s + 0.25}]]],

This If-else statement plots a line of the bid-rent actually paid and the layout of the city into sectors:

If[8 x > (c - d) x,

If[((p - n) x) / 5.75 + p - ((p - n) / (5.75)) * (s - 0.25), ((1/4) x) + (1/4) x, (s - 4)],

If[(c - d) x > ((1/4) x) + (1/4) x, (s - 4), (c - d) x, ((1/4) x) + (1/4) x, (s - 4)]],

Plot[Tooltip[If[(c - d) x > (c - d) x,

If[((p - n) x) / 5.75 + m - ((p - n) x) / (5.75)) > ((1/4) x) + (1/4) x / 4, 1 + (1/4) x / 4],

If[b - n x + (1/4) x + (1/4) x, (c - d) x, ((1/4) x) + (1/4) x, (s - 4)],

If[(c - d) x > ((1/4) x) + (1/4) x, (s - 4), (c - d) x, ((1/4) x) + (1/4) x, (s - 4)]],

This statement declares all land left over as agricultural land:

Plot[Tooltip[If[m x > 0, ((p - n) x) / 5.75 + m - ((p - n) x) / (5.75)) + m, ((p - n) x) / 5.75 + m - ((p - n) x) / (5.75))],

s, {s + 6, 15}, PlotStyle -> {Thick, Orange}].