THE APPROACHES AND THE RECOMMENDATIONS FOR FINDING OF PREMIUM FOR CONTROL AND AMENDMENTS TO THE VALUE OF THE EQUITY SHARES OF BUSINESS AND THE STOCKS

Key words: value of equity share, equity stake, amendment to the value, controlling share, premium amendment for the control, blocking share, minority share, business valuation

Abstract

The estimation of value of equity share (equity stake) is based on the preliminary determining of value of equity. Value of equity share depending on its size and on the remaining shares can be characterized by premium for control to the value of share, by the amendment to the value of blocking share, as well as by the discount to the value of minority non–blocking share. The available recommendations on finding of amendments to the value of share generally true, although approximately reflect the intervals of their values, therefore, the refinement of amendments is done, as well as linking the amendments to the size of share in the structure of joint capital. To determine the proportions of the premium amendment to value of controlling share, the amendment to value of blocking share and the discount to value of minority shares the balance assumption is used, which for the value of equity is the sum of all its shares (the equity stocks). Upon availability of controlling and blocking shares the size of discounts to the value of minority non–blocking shares is sustainable at the definite level at which further depreciation of value could lead to loss of interest of minority owners for the participation in the business. The analysis of terms of the discount: it part, corresponded with the amendment to the value of controlling share, and the part corresponded with the amendment to the value of blocking share, allows to find the listed amendments to the values of controlling and blocking shares with the account of the balance assumption of the values of all parts of equity.

Introduction

The value of equity share (equity stake) in business depends on size of the estimated share. With the account of shares liquidity and the degree of control in determining their value, the following types of amendments (premiums and discounts) are usually taken into account: the amendment for controlling or non–controlling (minority) nature, the amendment for lack of liquidity. The premium for controlling nature of stake of equity is used usually for the value of operational complex of company assets. The 51–percent equity stake is characterized by the maximal premium for control to the value [2]. The further increase in size of equity stake (up to absolute control at 76 %) associated with the decrease in the value of share with the account of minority interests.

In addition to these amendments at the determining of shares value the following known recommendations may also apply [8]:

---

**Key words:** value of equity share, equity stake, amendment to the value, controlling share, premium amendment for the control, blocking share, minority share, business valuation

**Abstract**

The estimation of value of equity share (equity stake) is based on the preliminary determining of value of equity. Value of equity share depending on its size and on the remaining shares can be characterized by premium for control to the value of share, by the amendment to the value of blocking share, as well as by the discount to the value of minority non–blocking share. The available recommendations on finding of amendments to the value of share generally true, although approximately reflect the intervals of their values, therefore, the refinement of amendments is done, as well as linking the amendments to the size of share in the structure of joint capital. To determine the proportions of the premium amendment to value of controlling share, the amendment to value of blocking share and the discount to value of minority shares the balance assumption is used, which for the value of equity is the sum of all its shares (the equity stocks). Upon availability of controlling and blocking shares the size of discounts to the value of minority non–blocking shares is sustainable at the definite level at which further depreciation of value could lead to loss of interest of minority owners for the participation in the business. The analysis of terms of the discount: it part, corresponded with the amendment to the value of controlling share, and the part corresponded with the amendment to the value of blocking share, allows to find the listed amendments to the values of controlling and blocking shares with the account of the balance assumption of the values of all parts of equity.

**Introduction**

The value of equity share (equity stake) in business depends on size of the estimated share. With the account of shares liquidity and the degree of control in determining their value, the following types of amendments (premiums and discounts) are usually taken into account: the amendment for controlling or non–controlling (minority) nature, the amendment for lack of liquidity. The premium for controlling nature of stake of equity is used usually for the value of operational complex of company assets. The 51–percent equity stake is characterized by the maximal premium for control to the value [2]. The further increase in size of equity stake (up to absolute control at 76 %) associated with the decrease in the value of share with the account of minority interests.

In addition to these amendments at the determining of shares value the following known recommendations may also apply [8]:

---
the premium to the value for blocking feature of the relatively large minoritary shares (non–controlling), also dependent on the share size;

– the discount on the loss of key figure (for example, inheritance);

– the portfolio discount (for two and more kinds of company’s concurrent activities), etc.

The conditions that restrict the rights of owners of controlling equity stocks [7] are among the factors contributing to the premiums and the discounts: the ownership distribution effect, the voting regime, the contractual restrictions.

It should be noted that the amendments have an impact on the value of equity share in two ways: first, they influence the value of the holistic business equity and the value of each share in it commensurately, the second, they influence the distribution of shares in the value of the entire business previously estimated with the account of the amendments, listed above.

For example, the negative cumulative effect on value of business (the property complex of acting company) may have an unsatisfactory equity structure, which impedes the work of the company and the access of participants of all categories to the distribution of performance results. Without the considering of reasons for indicated circumstances, of the obstacles of the share structure and the sequence of its occurrence, one could give the formal example: two or three blocking shares (each about 25% or more of share or stake of equity). Upon the availability of minoritary non–blocking shares and assuming the absence of controlling share these options of capital structure may indicate the problems in managing of joint business and the excessive competition of the participants acting in their own interests and not having the ability to make the common decision, and the uniting of participants. The emergence of controlling share in this example is not foreseen, since at the stage of such transformation there could be conflicts of interest. These inconsistencies in the valuation could mean the need to make the general amendment, reducing the value of the equity total and the values of the incoming shares in the proportion of their sizes.

Discounts for the insufficient liquidity could also be applied to the shares of non–public companies. The financial conditions of business that reduce the liquidity of shares of participants usually include the following: the unavailability or insufficient dividends for shareowner, the probable difficulties or burdens for sell the share, including the legislative or administrative obstacles. When finding liquidation discounts to the value of shares and premiums for the controlling nature of equity stake the consideration of the federal regulation is essential (the availability of the «golden stock»).

The tasks under decision when finding the amendments to value of shares

At the determining of value of share in business there are the peculiarities of use of the economic liquidity concept and the notion. As it is known the discount for the insufficient liquidity does not apply when finding the market value of objects. This is required when determining the residual value if the object which is to be sold during the exposure period less than the typical period for market conditions, when the seller is forced to make transaction for the alienation of his property. To avoid confusing of terms when determining of value shares of participants the liquidation discount to the value of equity stake is not used below as the notion of the discount on insufficient liquidity for the indicated reasons: the unavailability of dividends, the difficulties in selling of share, the legislative or administrative obstacles, which are not related to finding of the standard liquidation value, and the concept and the notion of the discount for the non–controlling feature is applied to the values of minoritary shares.
When assessing the value of equity, making the amendments to it and evaluating the individual shares of equity it needs to pay attention to the difference of the following tasks:

1) the finding of the amendments used in the valuation of equity, which have the proportional impact on the value of all shares of equity, including the amendments to the value of equity with the account of the quality of the equity composition;

2) the calculation and use of amendments in the distribution of the found value of equity into the shares with the account of each share feature: controlling, blocking and minoritary non-blocking.

In this context, the question of the possibility of changing the value of equity is interesting with variations of the ratios of shares in composition of the equity. At innovation stages of investment projects in the creation of property complexes, that is when investing financial assets in the aggregate of real assets, the additional value may occur, as it is known [1], serving the economic purpose of the investing. This additional value, which corresponds to the difference in the value of the aggregate of real assets and the total value of these assets having separately, can serve as valuation of the group intangible asset – the goodwill, which on the other hand is usually associated with the concepts of business reputation, commitment to customers and etc., because these features arise as result of market response due to creating of real assets complexes. At the same time, it is believed that financial assets, when they are accumulated, do not lead to the additional value, and the amount of money is just added up. This is methodologically correct, since the value of real objects and the additional values in groups of real objects are estimated in financial units, which for the financial assets as the characteristic of value have to be nominally summable. Although it can be assumed that the amount of money along with money accumulation can open qualitatively new investment opportunities can open qualitatively new investment opportunities, in this case the value of financial assets would be disproportionate to their sum. However, this work is based on the standard paradigm, which in the addition of financial assets does not change their valuation as the sum, unlike complexes of real assets, which are to be characterized by the additional value, except the total of values of their parts.

For derivative financial instruments, such as securities, for example equity stocks or shares in the equity rights, apparently, it is better to adhere the balanced judgment regarding the ability of those or others ratios of their amount to influence their total value. The balanced judgment may be that the variation of ratios of the equity shares (the equity stocks) may affect value of each share, further increasing or decreasing of this value to the proportional size of the share, but the total of all shares values with variation of their proportions would not change, as this does not imply the change of the amount and the aggregate of the complex of real assets they control, which is equal in value to this composition of shares.

Those or others special cases are possible, when the equity composition has the weaknesses that hinder the joint activities of the participants and proportionally reducing of their shares values. As the example in the above it indicates the presence in the composition of equity of two or three minoritary blocking shares belonged to independent owners in a competitive relationship with the absence of controlling share, that is common for all participants lack of management and can proportionally decrease the value of each share of the equity, and this factor must be taken into account when performing an evaluation of equity (the specified task 1), but not in the distribution of value found in to the shares (task 2).

The ways to solve the first task are presented in the methodological papers [1–3, 7, 8]. The approaches and recommendations for solving the second task – the value allocation of equity
among the shares, which for the amendments are used to the value of controlling share \( x \), to the value of blocking share \( y \) and the discount \( z \) to the values of minoritary shares having into account their non–controlling feature, are considered as well by number of authors [4–6, 9–11] and are the subject of this work. When deciding the second task, that is, when distributing the estimated value of equity in to shares, the total of all shares of values remains unchanged and equal to its found value.

**The regulatory rules and the expert advices in finding of the amendments**

For the purpose of comprehensive analysis of ways to assess the impact of share of equity (of the amount of shares) on the amendment to the value of this share the graph is done (Fig. 1) of dependence of the amendment \( z \) for the non–controlling feature of equity stake on it size for given values of the control coefficient in accordance with the Decree of the Government of the Russian Federation of February 14, 2006 # 87 «The rules for determining the regulatory price to be privatized state or municipal property», which in the values of the coefficient of control for the intervals of the relative size of equity stake are done: for the stake of equity from 1 stake up to 10 % minus 1 share the coefficient of control is 0.6; for the stake of equity of the size from 10 % up to 25 % of the shares the coefficient of control is 0.7; for the stake of equity from 25 % plus 1 share up to 50 % of the shares the coefficient of control is 0.8; for the stake of equity from 50 % plus 1 share up to 75 % of the shares minus 1 share the coefficient of control is 0.9; for the stake of equity from 75 % up to 100 % of the equity the coefficient of control in accordance with the listed Decree is 1.0. The coefficient of control shows how many times the price of share on stock market is lower in given stake than in the equity stake that gives absolute control over the company.

![Graph](image1.png)

**Fig. 1.** The dependences of: (a) the amendment \( z \) for the non–controlling feature of share evaluated according with Decree of the Government of the Russian Federation of February 14, 2006 # 87 on share size \( a \); (b) the value of the set of all shares in the proportion to the value of equity in the dependence on size of maximal share. The legend:

- The lower limit of the interval
- The upper limit of the interval
In addition to the specified intervals of values for making of the graph it is taken into the account that the value of 100 %–th equity stake is equal to the value in full size with zero amendment (with out the discount) after making in the assessment of equity of all required amendments, relating to the operating and financial activities and to the investment composition of the equity, as well as the amendments for the availability of privileged shares etc., have the impact on business value (the property complex of the acting company), including on the unsatisfactory composition of shares of participants which for there may be obstacles in the work of the company, affecting the value of it equity and thus proportionally affecting the value of it each share, etc.

The graphs show that amendment to the value of share in accordance with the Decree of the Government of the Russian Federation of February 14, 2006 # 87 corresponds to the discount (Fig. 1 a) even for the controlling share. The positive amendment to the value is not provided, and with any composition of equity (joint–stock) capital apart from sole 100 %–th ownership the sum of the value of all shares is less than the value of the equity of the company (Fig. 1 b), that can be considered as methodological contradiction of the dependences of the coefficient of control on the size of equity stake in accordance with the listed Decree.

The recommended ranges of values of the discounts and the allowances described by various authors are summarized in the paper [11] of the Scientific and Practical Center of Professional Evaluation, among them are:

1. The recommendations of Rutgaiser V.M. [8]: the average premium for controlling nature of equity stake is of 40 %, the average discount for non–controlling feature is recommended from 27 up to 29 %, the general range of the allowance for control is from 25 up to 75 %, the general range of the discount for non–controlling feature is from 25 up to 50 %.

2. According to the recommendations specified in the book of Gryaznova A.G. and Fedotova M.A. [7]: the premium amendment to the value of stock for controlling feature of stake of equity is in the limits from 30 up to 40 %, the discount to the value of for non–controlling feature of stake of equity is about of 23 %.

3. According to «Mergerstat Review» (/www.mergerstat.com/) the premium amendment to the value of stock for controlling feature of stake of equity is in the range from 20 up to 40 %.


6. According to the recommendations of Ivanov A.M., Ivanova N.S. and Perevozhikov A.G. [4] the ranges of discounts and premiums to the value of stocks in package are: the premium allowance for the controlling feature is in the range from 0 up to 75 %, the discount for non–controlling feature is from 0 up to 50 %.

It would be noted that the given ranges of amendments to the value of share (equity stake) in some comparisons are confirmed in pairs, which, apparently, can be considered as factor of their reliability. On the other hand, these ranges are quite wide and are not given in the dependence on share size and on the availability of the other shares in the composition of capital, which is the obstacle of the listed recommendations.
The dependences of sum of premiums and discounts with the account of size of stake of equity unquoted stocks of public joint stock companies according to the Scientific and Practical Center of Professional Evaluation [11] are on the graph (Fig. 2) to analyze the form of these dependencies. The graph shows that the highest amendments to the value correspond to control packages at their size close to 50 % плюс 1 share. One can also see some downward trend of the discounts for the value of the «blocking» equity stocks (the discounts are in the range from 20 down to 40 %, minus). With the use of the considered graph (Fig. 2 a) it authors found the intervals of values and the average amendments dependent on share size, that are indicated in the following graph in the digitized form (Fig. 2 b and following graphs). It should be noted that on the digitized charts on the x-axis are duplicated the values of share 50 % (+) and 50 % (−), which correspond to the application of positive and negative amendments to the value of the corresponding share.

![Graph](image)

**Fig. 2.** The dependence of the amendment to the value of share \(a\) (of unquoted stocks of public joint stock companies) according to the Scientific and Practical Center of Professional Evaluation in it original schematic form (a) and in the digitized form (b).

The legend: - Average - Upper limit - Lower limit

The presented graph (Fig. 2 a and b) characterizes the dependence of the amendment to the value of share (equity stake) on it size, what the listed before authors don't have. At the same time the following comments may occur:

1. It is required to balance the values of amendments so that the sum of the value of shares (the equity stocks) would be equal the value of equity, as far as the distribution of equity is concerned in to shares, wherein the value and the composition of real assets in the property complex impacting on the value of the company are not dependent on the composition of shares of equity. There is the mismatch on the assumption in the graphs.

In particular, if equity would include two approximately equal shares (49 % and 51 %), then the amount of values of shares with the account of the amendments (−0.095 and 0.5) to their values would be different from the equity value (100 %) and is: \(0.49\times0.905+0.51\times1.50=1.2085\), that is about 121 %, which is more than the value of previously evaluated equity (100 %) and is
methodological contradiction of this choice of amendments to the value of shares. Moreover, in order to match the value of equity for the both approximately equal shares (49 % and 51 %) the amendments to the values having the opposite sign would be approximately equal in size.

2. The increase of the discount with the decrease of the minoritary share size is difficult to explain logically. From the point of view of the parties when selling of small minoritary non–blocking shares their sizes conceptually would not impact on the interests regard one unit of share (or one stock). That is the minoritary non–blocking shares could be attributed to the single set, which when evaluated is characterized by the same relative discount amendment to the value. When attributing of minoritary non–blocking shares to this set the discount to their value will be dependent on the availability of controlling and blocking shares. If in the rest of the equity the controlling and/or blocking shares are available, then the amendment to the values of the minoritary shares will be commensurate with this values set, which level below the minoritary shareholders lose the interest in the investing (in the purchase–selling of the minoritary shares).

The amendment to the value of minoritary non–blocking share depends not so much on size of this share, as on the availability of controlling and blocking shares in the composition of equity. In particular, if the equity consists of the minoritary shares the only and controlling and blocking shares are absent, then the discount to the value of each minoritary share is zero.

Apparently, the form of dependences of the discount on reducing of the minoritary share (Fig. 2 a and b) shows not so much an increase of size of the negative amendment (that is the discount), as growing with the decrease of the minoritary share of the error of determining of discount to the value of share, since the errors of finding of amendments to the value of the controlling and blocking shares in the same time corresponds to the error of determination of discount to the value of minoritary non–controlling share being with the indicated errors in the balance. As the minoritary non–controlling share is lower than the controlling share and blocking ones, then the error of finding of this discount to the value of minoritary non–blocking share increases by the relative magnitude with decreasing of size of this share. This serves as the basis for the assumption that with the decrease of minority share the shape of the curves (Fig. 2 a and b) does not show the growing size of the discount, and the increasing error of it finding.

The balance assumption of equity value and the sum of the equity shares values

To eliminate the considered contradictions one would adhere to the balance assumption, which for the total value of equity after the introduction of all required amendments used in it valuation related to operating, financial and investing activities, the amendments for the availability of privileged shares, the amendments by the factor of participants’ shares unsatisfactory composition which could cause difficulties affecting the value of equity and thus proportionally affecting the value of each it share and other required amendments corresponds to the value of all shares in the composition of equity (of 100 % shares)

\[(1+x)\times a + (1+y+z)\times b + (1+z)\times (1-a-b) = 1,\]  

(1)

where \(a\) – controlling share (the size of controlling equity stake), which is usually more than 50 % of all stocks or \(1/2\) part, in the formulas and on the graphs in the absence of controlling share the variable \(a\) denotes the variable value of the equity share; \(b\) – the set of minoritary blocking shares, the size of blocking share is usually more than 25 % of stocks, or more than \(1/4\) part of equity; \((1-a-b)\) – the set of minoritary non–blocking shares, that is, the total size of all shares besides of controlling and blocking; \(x\) – the relative amendment to the value of controlling share (the premium amendment for control); \(y\) – the amendment to the value of blocking shares; \(z\) – the
amendment to the values of minoritary shares, which has negative sign and serves as the
discount to the value of share. When accounting the amendments to the value of in the equation (1)
the following rule is observed: the amendment to control is taken into account when finding the
value of controlling share, for blocking minoritary shares counted the amendment for the block-
ing feature and the discount to the value taken into account for non–controlling feature of the
non–blocking minoritary shares.

For the determining of amendments to the value of share, that is for finding of amendments
for controlling feature \( x \), for blocking one \( y \) and the discount \( z \) for non–controlling feature the
balance equation (1) is transformed

\[
x = \frac{-z \times (1-a) - y \times b}{a}, \quad y = \frac{-z \times (1-a) - x \times a}{b} \quad \text{and} \quad z = \frac{-x \times a - y \times b}{1-a}.
\]  

(2)

Assuming the simpler composition of equity, if there are no blocking shares \((y = 0)\), and
there are only the controlling share and the minoritary shares the balance equation (1, 2) has the
form

\[(1 + x) \times a + (1 + z) \times (1-a) = 1 \quad \text{that is} \quad z = a \times x / (a-1) \quad \text{or} \quad x = z \times (a-1) / a.
\]  

(3)

On the graphs (Fig. 3 a and b) the balance equation (3) is used to assess the conformity of
amendments to the value of controlling share according to the data of the Scientific and Practical
Center of Professional Evaluation [11] and according to the discount to the value of minoritary
non–blocking shares, which values are shown on the graph above (Fig. 2). The dependence of
the amendments according to the specified data is alternately combined with the balance equa-
tion (3) on the premiums site (Fig. 3 a), which for on this graph the agreed values of the discount
found are solutions of the equation of balance, which for the sum of the values of controlling and
minoritary shares is the equity value, and on site of discounts (Fig. 3 b), which gives the allow-
ances for the control, consistent with the amendments of the discount for non–controlling fea-
ture.

Since the dependence of discounts for non–controlling feature of minoritary shares (Fig. 2 a
and b) apparently shows an increase of the error of finding discounts at the decreasing of share
size, rather than growing discount, which in the set with non–blocking small minoritary shares
perhaps does not depend on size of each share, because it depends on the composition of the rest
of equity, that is on the availability and sizes of controlling and blocking shares, then it is logical
to use for determining of the discount to the value of it interdependence on the allowance to the
value of share of the other category, in particular the allowance to the value of controlling share,
as shown on the graph (Fig. 3 a).

With the account of the balance assumption (1–3) which for the sum of the value of all
shares is equal to the value of equity and using the data of Scientific and Practical Center of Pro-
fessional Evaluation of amendments to the value of controlling share one can find consistent val-
ues of the discount to the value of minoritary share, which is also shown on the graph. The exist-
ing discrepancy of the graphs may indicate error in the determination of amendments to the value
in the dependence on share size (Fig. 2 a).

To evaluate the assumption that the dependence of the discount to the value of minoritary
shares growing with decreasing of share size characterizes more the increase of the error of the
discount finding, than the magnitude of the discount, the graphic dependences of discounts for
non–controlling feature of minoritary share found using the balance equation (3) are done consis-
tently with variations of the amendments set in the form of deviations (retreats) to the value of
controlling share from it specified values (Fig. 4).
Fig. 3. Combining the data of Scientific and Practical Center of Professional Evaluation on the sites: (a) of amendments to the value of share for controlling feature and (b) discounts for non–controlling feature of share. The legend:
- The data of the Scientific and Practical Center Professional Evaluation
- The agreed values of allowances and discounts on equity stake \ share

Fig. 4. The dependences of discounts found consistently for non–controlling feature of minoritary share with variations of controlling allowances. The legend:
- The agreed values of allowances and discounts
- The discount minus 40 %
- The discount minus 50 %
- The discount minus 20 %
- The discount minus 30 %
- The discount minus 60 %

Fig. 5. The dependences of the amendment to the value of share on it size with variations of the discount to the value of minoritary shares. The legend:
- The agreed values of allowances and discounts
- The discount 3/4 for the non-controlling nature
- The discount 1/2 for the non-controlling nature
- The discount 1/3 for the non-controlling nature
- The discount 1/4 for the non-controlling nature
- The discount 2/3 for the non-controlling nature

The done graphic dependences show that variation of retreats of the allowance to the value of controlling share corresponds to significant variations of agreed with it discounts for non–controlling feature of minoritary share. This can serve as proof that the growing dependence of
the discount to the values with the decrease of minoritary non–blocking share size characterizes more the increase of the error of the discount finding rather than increase of the discount magnitude with the reduce of the minoritary share.

Since there is no reason to believe that the discount to the value of minoritary non–blocking shares (the equity stocks) depends on their size, as this discount depends on the availability of controlling and blocking shares, then it makes sense to check whether constant value of the discount independent on the size of minoritary non–blocking share matches satisfactory the agreed allowance found using the balance equation (3). To answer this question the set of graphic dependencies of the amendment to the value of share on it size is done (Fig. 5), and for each of the graphs the size of discount $z$ within the limits of minoritary shares (from 0 up to 50 %) is constant and does not depend on the size of minoritary share, and the premium to value of controlling share is agreed with the discount, using the balance assumption (1–3).

The graphs show that the discount to the value of minoritary non–blocking shares in the interval from 50 % up to 70 % (that is approximately from $\frac{1}{2}$ up to $\frac{3}{5}$) in the assumption of independence of the discount on size of minoritary share and in the consistence with the use of balance assumption (1–3) the amendments to the value of controlling share correspond to the observed data [11] with acceptable accuracy. Although for this correspondence on the same data it was not necessary to follow the dependences of discounts to the value for non–controlling feature on size of minoritary shares according to the indicated observations.

The compliance of the allowance to the controlling share value confirms the ability to adhere the discount to the value of minoritary non–blocking share in the given interval from 50 % up to 70 % independent on size of this share, although this range of the discount exceeds those for some recommendations, besides of the data of Ivanov A.M., Ivanova N.S. and Perevozhikov A.G. [4], as well as the data of the Scientific and Practical Center of Professional Evaluation [11], which corresponds to the specified interval (from $\frac{1}{2}$ up to $\frac{3}{5}$) of the magnitude of the discount to the value of minoritary share.

The dependence of the amendment to the value of share on it size

Assuming the absence of blocking share ($y=0$) the amendment $x$ to the value of controlling share (the premium for control) in the dependence on the discount $z$ to the values of minoritary shares and on controlling share $a$ is possible to find with use of the balance assumption (1–3), as $x = -z/a + z$, when the amount of controlling share $1/2 \leq a < 1$. (4)

The dependence of the amendment $x$ to the value of controlling share and of the discount $z$ to the value of minoritary shares on the discount $z$ and on share size $a$ is in the form of functional surface (Fig. 6) assuming the absence of blocking share.

On the resulting graph you can see the characteristic feature of dependence of the amendment on share size in the vicinity of the line of the discontinuity of functional surface. The amendment $x$ to the value of controlling share near the critical size of this share ($a \approx 1/2$) in accordance with balance assumption (1–3) is equal in the magnitude and opposite in sign to the discount $z$ for non–controlling feature of minoritary shares. The balance assumption corresponds to the symmetry feature of dependencies to the value on share size in the vicinity of their mathematic discontinuities, as well as to the symmetry of dependencies of amendments in the remaining intervals of share size $a$ for the scale associated with this size: $1+1/(a-1)$ or $1–1/a$. 

10
Assuming the absence of the controlling and blocking shares \((x=y=0)\) the discount to the value of non–blocking shares corresponds to zero \((z=0)\). The limiting size of discount to the value of non–blocking shares, related to the availability of controlling and blocking shares and the corresponded premiums to their values can be set at the level which, if exceeded, would lead to the increase of discount to the value of minoritary shares and to the loss of interest by minoritary owners for the equity. That is the proportion of amendments to the values of the controlling and blocking shares, on one hand, and the discount to the value of minoritary non–blocking shares, on the other one, depends on the compromise of the parties’ interests and on the appropriate limit of the discount to value of the minoritary non–blocking shares. With the account of the balance assumption \((1–3)\), which for the total value of all shares is equal to the value of equity and no «financial bubbles» are supposed for the considered example (Fig. 6), the obstacle to an increase of the premium to the value of controlling share is the limiting size of the discount to value of minoritary non–blocking shares.

Assuming the absent of blocking share \((y=0)\) the premium to the value of controlling share one could find using of the balance assumption \((1–3)\) with the account of the discount \(z\) to the value of non–blocking share independent on share size and matching the interval from 50 % up to 70 % of the discount, as
\[
x \approx -(50 \div 70\%) \times (a-1) / a \quad \text{at} \quad z \approx -(50 \div 70\%) , \quad 1/2 \leq a < 1.
\]
(5)

Assuming the absent of controlling share \((x=0)\) the amendment \(y\) to the value of blocking share (Fig. 7) in the dependence on the discount \(z\) to the value of minoritary shares and on size of blocking share \(b\) one could find with use of the balance assumption \((1–3)\), as
\[
y = -z / b + z \quad \text{for the constraints of blocking share} \quad 1/4 < b \leq 1/2.
\]
(6)
The dependence of amendment (allowance) $y$ to the value of blocking share and of the discount $z$ to the values of minoritary shares on this discount $z$ and on size of blocking share $b$ assuming the absence of the controlling share ($x=0$)

**The correlation of amendments to the values of controlling and blocking shares and the discount to value of minoritary share**

The amendments to the value of controlling $x$ and blocking $y$ shares in the dependence on terms of the discount ($z = z_x + z_y$): the term $z_x$, corresponded with the amendment to the value of controlling share, and the term $z_y$, corresponded with the amendment to the value blocking shares, are determined with use of balance assumption (1–3), as

$$x = -z_x / a + z_x \text{ at } 1/2 \leq a < 1; \quad y = -z_y / b + z_y + z_x \text{ at } 1/4 < b \leq 1/2. \quad (7)$$

As the illustrative example, without going into it analysis from the economic side, for finding of terms ($z_x$, $z_y$) of the discount $z$ to the value of minoritary non–blocking shares one could use the assumption that the total amendment ($y + z$) to the values of minoritary shares is the discount, that is

$$y + z < 0 \text{ if the size of the share (equity stake) is less than 50 \%.} \quad (8)$$

This assumption (8) is performed if

$$y + z = 0 \text{ at } b = 1/4, \quad z_x = z \times 3/4, \quad z_y = z / 4 \text{ accordingly } z = z_x + z_y. \quad (9)$$

The dependence of amendments: to the value of controlling share $x$, to the value of blocking share $y$ and the discount $z$ to the values of minoritary shares on this discount $z$ and on share size $(a, b)$ is presented in the form of functional surface (Fig. 8).

The analysis of worked out graph (Fig. 8 a and b) and the assumptions used for this graph (the formulas 7, 8 and 9) allows come to the conclusions:

– about the general compliance of the form of dependence of amendment to the value of share (Fig. 8 a and b) on share size $(a, b)$ obtained using the balance assumption (1–3) with the observations of the Scientific and Practical Center of Professional Evaluation [11] (Fig. 2 a and b);

– about the advisability of finding the terms of the discount to the values of minoritary non–blocking shares, which would serve the data for finding of amendments to the values of the controlling and blocking shares in accordance with the balance assumption, which for the total value of shares of all participants is equal to the value of the company equity.
The dependence of value of share on the discount $z$ and on the size of the share $(a, b)$ is in the form of functional surface (Fig. 9). According to this graph one can see the disproportionate nature of variation of share value (equity stake) in the composition of equity in the dependence on nominal size of the share.

Since the forms of dependencies of amendments to the values of the controlling and blocking shares $(x$ and $y$) largely caused by the discount $z$ to the value of minority non–blocking share, both by the size of discount and by the proportion of terms in it, specifically: the first term of the discount $z_x$, which is corresponded with the amendment to the value of controlling share, and the second term $z_y$, corresponded with the amendment to the value of blocking shares, then for the analysis the graphic dependences (Fig. 10 a and b, Fig. 12 a and b) of amendments $x$ and $y$ to the value of the controlling or the blocking shares $(a, b)$ are given below on share size and
on term $z_x$ of the discount $z$ to the value of minoritary non–blocking share, namely on that term of the discount, which is correlated with the amendment $x$ to the value of controlling share in accordance with balance assumption (1–3, 7).

The dependence of the amendments to the value of controlling share $x$, of blocking share $y$ and the discount $z$ to the value of minoritary non–blocking share on the term $z_x$ of this discount, corresponded with the amendment $x$ to the value of controlling share, and on share size $(a, b)$ for total value of the discount ($z = z_x + z_y$) equal 70 % is in the form of functional surface (Fig. 10).

![Fig. 10. The dependence of amendments in the projections (a and b): to the value of controlling share $x$, of blocking share $y$ and of the discount $z$ to the value of minoritary shares on term of the discount $z_x$, corresponded with the amendment $x$ to the value of minoritary shares on term of the discount $z_x$, and on share size $(a, b)$ for total value of the discount $z$ assumed 70 %](image)

The total discount $z$ to value of minoritary non–blocking shares specified in the above observed interval from 50 % (Fig. 12 a and b) up to 70 % (Fig. 10 a and b) for each of the worked out graphs is constant and independent on size of this share. The varying of the term $z_x$ of the discount $z$ to the value of minoritary non–blocking share, corresponded with the amendment $x$ to the value of controlling share in accordance with the balance assumption (1–3, 7), for each magnitude of term $z_x$ means the change of the ratio of it and the other term of the discount $z_y$, corresponded with the amendment $y$ to the value of blocking share.

Despite the fact that the considered example of the dependence of amendment to the value on share size (Fig. 10) is released from the assumption that the total amendment $(y + z)$ to the values of minoritary shares is the discount, which was previously used in the illustrative example (Fig. 8), however, an analysis of these dependencies also serves the confirmation of general consistency with available observational data [11] (Fig. 2 a and b) of the form of dependence of amendment to the value of share (equity stake) on the size of this share. Although the dependence of the amendment to share value on the size of minoritary share with the account of the availability of blocking shares and with the balance assumption (1–3, 7) has more complex form than in the above observations, which apparently reflect not so much the dependence of the dis-
count to the value on size of minoritary share, as the error in the determination of the discount to the value, which increases with the decrease of this share.

**The discount and the amendments to the value in the dependence on proportion of sizes of the shares**

For logical finding of amendments to the value of controlling and blocking shares the opportunity is of interest of correlation between the terms of the discount $z$: the term $z_x$, corresponded with the amendment to the value of controlling share, and the term $z_y$, corresponded with the amendment to the value of blocking shares, in the proportion of sizes of blocking and controlling shares $(b/a)$

$$z_y/z_x = b/a.$$  \(10\)

Both of the terms $(z_x, z_y)$ of discount $z$ to the value of the minoritary non–blocking share in given proportion $(b/a)$ are

$$z_y = \frac{z}{1+a/b} \quad \text{and} \quad z_y = \frac{z}{1+b/a}. \quad (11)$$

When using the resulting formulas $(11)$ together with the balance assumption $(1–3, 7)$ one needs to observe the intervals of variation of controlling and blocking shares $(a, b)$ and their proportion $(b/a)$

$$1/3 < b/a < 1 \quad \text{at} \quad \frac{1}{2} < a \leq 1,0 \quad \text{and} \quad \frac{1}{4} < b \leq \frac{1}{2}, \quad (12)$$

as well as to adhere the restrictions of the domain with the account of joint variation of controlling $a$ and the blocking $b$ shares

$$a+b \leq 1 \quad \text{which for} \quad a \leq 1-\frac{z_y}{z} \quad \text{and} \quad a > 1/2 \quad \text{that is} \quad b/(b/a) > 1/2. \quad (13)$$

The dependence of amendment to the value of controlling share $x$, of amendment to the value of blocking share $y$ and of the discount $z$ to the values of minoritary shares on the proportion of sizes of blocking and controlling shares $(b/a)$ and on sizes of the shares $(a, b)$ is done within the variation of intervals of the share size $(12)$ and the restrictions of the domain $(13)$ is in the form of functional surface in the above observed interval of total discount $(z = z_x + z_y)$ from $50 \%$ (Fig. 11 a–c) up to $70 \%$ (Fig. 11 d).

The use of graphic functional surface for the evaluation of amendments to the value of shares is possible as follows. For the specified sizes of shares $(a, b)$ to find their proportion $(b/a)$, to select the level of the discount $z$ to the value of minoritary non–blocking shares, to select the appropriate graph in the observed interval of the discount from $50 \%$ (Fig. 11 a–c) up to $70 \%$ (Fig. 11 d) and to determine the amendments: to the value of controlling share $x$ and to the value of blocking share $y$, which are appropriate to the balance assumption $(1–3, 7)$ for given values of these shares and for their proportion found.

In addition to the graphs, one can use the formula $(7 \text{ and } 11)$ with the account of the constraints $(12 \text{ and } 13)$ in spreadsheets by the categories: $(a, b)$ and $(b/a)$, similar to that performed when making functional surfaces (Fig. 11), wherein as the initial data the discount $z$ to the value of minoritary non–blocking shares is specified, used for graphs in the constraints of range of values of the discount $z$ from $50 \%$ up to $70 \%$.

The analysis of graphic functional surfaces (Fig. 11 a–d) makes it possible to see that the amendments to the value of minoritary share, as for the blocking one in the restrictions of the
domain, including the discount for non–controlling feature and the allowance for blocking feature (1), so as for the non–blocking share, serve the discount, that is have negative or zero value of the total amendment.

![Diagram](image)

Fig. 11. The dependence in the projections (a, b, c and d) of the amendment to the value of share on proportion of blocking and controlling shares \((b/a)\) and on share sizes \((a, b)\) with the account of the restrictions domain of these variables: \(a + b \leq 1\) which for \(a \leq 1 - z, z\) and \(a > 1/2\) that is \(b/(b/a) > 1/2\) in the observed interval of the total discount \(z\) in the range from 50 % \((a–c)\) up to 70 % for the value of minoritary non-controlling share \((d)\)

Under certain proportions of blocking and controlling shares \((b/a)\) with the account of their sizes \((a, b)\) the amendments to the value are zero \((x=y=z=0)\): on the constraint of blocking domain of the share size \(b\) (that is \(b/(b/a) = 1/2\)), at which may appear the controlling share \((a>1/2)\) besides of the available blocking one, and in case of two existing shares equality \((b/a = 1, b = a)\), which for minoritary shares in the composition of equity are absent consequently.

The amendments to the value of shares are zero also if it is:
– the minoritary non–blocking shares the only are available in the composition of the equity, if controlling and blocking shares are absent;
– the availability of blocking shares in the composition of the equity, with the absence of controlling share and minoritary non–blocking ones;
– the availability the only of one share (100 %-th).

The dependence of amendment to the value of share shown on graphs (Fig. 11 a–d) on shares sizes \((a, b)\) and on proportion of blocking and controlling shares \((b/a)\) is based on the following assumptions:

1) the total value of all shares is the equity of stock company, and the limiting size of discount to the value of minoritary non–blocking shares, corresponded by the balance assumption \((1–3, 7)\) to the amendments to the value of the controlling and blocking shares;

2) the supposition \((10\) and \(11)\) of the possibility of correlation among the terms \((z_x, z_y)\) of the discount \(z\) to the value of minoritary non–blocking shares in the proportion of sizes of blocking and controlling shares \((b/a)\);

3) the limiting discount \(z\) to the value of minoritary non–blocking shares assuming at the level that, if exceeded, would lead to the loss of interest of minority owners to the participation in the business, this level corresponds with the availability the controlling and blocking shares;

4) the sizes of blocking and controlling shares \((a, b)\) with the account of their proportion \((b/a)\) are within the ranges of variation and in the domain of definition \((12\) and \(13)\).

Outside the domain of definition \((13)\), that is assuming the absence in the composition of equity of controlling share \((x=0)\) or of blocking shares \((y=0)\) and upon the availability of just one of the specified shares one can find an amendment to the value with use of balance assumption, converted to the form, which corresponds to the availability of only controlling share \((4)\) or blocking shares the only \((6)\).

For logical finding of amendments to the value of controlling and blocking shares it is of interest to determine the set of discount to the values of minoritary shares, in particular, to do the research of terms of this discount: the term corresponded with the amendment to value of controlling share and the term corresponded with the amendment to value of blocking share in the proportion of sizes of the controlling and blocking shares or any.

**The amendments to the value in the dependence on increments of shares in relation to critical magnitudes of shares**

When discussing the impact of the amount and the size of shares on the amendments to the values of the shares an illustrative example could be given sometimes of the composition of capital in two equal shares of 49 % each, which belong to independent owners, and one share of 2 %, which is subject to the assumptions of the example, valued highly because buying one of the parties of this small share would serve the emergence of controlling share. For this abstract example one need to clarify, that each of the shares of 49 % apparently is blocking, and therefore the purchase of the share of 2 % by one of these parties is excluded, except by agreement between the owners of two 49 %-th shares with the corresponding settlements between them. Although in the composition of capital the presence of two blocking shares, owned by independent owners, who are in competitive relationship, means the absence of controlling share and could be considered as the obstacle of share composition, affecting the value of equity, that is proportionally reducing the value of all shares, what is mentioned above.

Perhaps, it is a more characteristic the example of the share composition, when the equity consists of shares of the different sizes, with the availability of controlling share or close to the size, and the blocking shares or close to them, as well as the minoritary non–blocking shares, which could be combined with the shares close to blocking ones or to the controlling, thereby
increasing their categories up to the controlling or blocking. For this example circumstances, it would be useful to define controlling and blocking shares not by their absolute value, but as the size of the increment to their critical values: for controlling share as the increment to \( \frac{1}{2} \), for blocking shares – to \( \frac{1}{4} \). The balance assumption of the equality of shares values total to the equity, used above for outputting of the applied formulas and for working out of graphs, can be defined in the function of these shares sizes in relation to their critical values which are indicated. Such change of variables in the balance equation (1) looks like

\[
(1 + x) \times (1/2 + a_{1/2}) + (1 + y + z) \times (1/4 + b_{1/4}) + (1 + z) \times (1/4 - a_{1/2} - b_{1/4}) = 1,
\]

where \( a_{1/2} \) – the excess of size of controlling share (stake of equity) beyond its critical value \( \frac{1}{2} \):

\( 1/2 + a_{1/2} = a \); \( b_{1/4} \) – the excess of size of blocking stake of equity beyond critical value \( \frac{1}{4} \):

\( 1/4 + b_{1/4} = b \).

In the domain of definition: \( 0 < b_{1/4} \leq 1/4 \) at \( 1/4 < b \leq 1/2 \) and \( 0 \leq a_{1/2} < 1/2 \) at \( 1/2 \leq a < 1 \) the amendments \( x, y \) and \( z \) to value are

\[
x = \frac{-z \times (1/2 - a_{1/2}) - y \times (1/4 + b_{1/4})}{1/2 + a_{1/2}}, \quad y = \frac{-z \times (1/2 - a_{1/2}) - x \times (1/2 + a_{1/2})}{1/4 + b_{1/4}}, \quad z = \frac{-x \times (1/2 + a_{1/2}) - y \times (1/4 + b_{1/4})}{1/2 - a_{1/2}}.
\]

Assuming he absence of blocking share \( (y = 0) \) the balance equation (1) with the change of variables is

\[
(1 + x) \times (1/2 + a_{1/2}) + (1 + z) \times (1/2 - a_{1/2}) = 1.
\]

The amendment to the value of controlling share and the discount to the value of minoritary one, assuming the absence of blocking share, with change of variables \( (1/2 + a_{1/2} = a \) and \( 1/4 + b_{1/4} = b \) are determined as

\[
x = \frac{1 - (1 + z) \times (1/2 - a_{1/2})}{1/2 + a_{1/2}} - 1, \quad z = \frac{1 - (1 + x) \times (1/2 + a_{1/2})}{1/2 - a_{1/2}} - 1,
\]

at \( a_{1/2} = 0 \), \( x = -z \); at \( a_{1/2} = \frac{1}{2} \), \( x = 0 \).

With change of variables, assuming the absence of controlling share (at \( x = 0 \), the balance equation (1) looks like

\[
(1 + y + z) \times (1/4 + b_{1/4}) + (1 + z) \times (3/4 - b_{1/4}) = 1,
\]

and the amendment to the value the blocking minoritary share is

\[
y = \frac{1 - (1 + z) \times (3/4 - b_{1/4})}{3/4 + b_{1/4}} - z - 1; \quad y = -z + \sqrt{3} \text{ at } b_{1/4} = 0; \quad y = \sqrt{3} + z/2 \text{ at } b_{1/4} = \frac{1}{4}.
\]

The formulas (14–19) above are variations of the balance assumption (1–3, 7) and the methodical recommendation when using in the calculations of the increments \( (a_{1/2} \) and \( b_{1/4} \) of sizes of controlling and the blocking shares instead absolute ones \( (a, b) \).

The conclusions and the recommendations

1. The amendments to the value of share (stake of equity), recommended by several authors (Rutgaiser V.M., Gryaznova A.G., Fedotova M.A., Ivanov A.M., Ivanova N.S., Perevozhikov A.G., «Mergerstat Review», Drezdner Kleinwort Benson, etc.), are confirmed in pairs in certain intervals, which could be considered as the factor of their reliability, at the same time the inter-
vals of the amendments to the value are rather broad and are presented usually regardless of size of the share and on the availability of the other shares in the composition of equity.

2. For determining of the amendment to value of controlling share, to value of blocking share and the discount to the value of minoritary shares one has to follow the balance assumption, wherein the value of the equity found is equal to the total of all it shares values (100 %), that serves as logical postulate and confirmed by the observations of available data.

3. The analysis of data of the allowance to the value of controlling share (stake of equity) agreed with the amendment to the value of minoritary non–blocking shares confirms the possibility to adhere the values of discount in the interval 50 % up to 70 % independent on size of this share. Wherein the amendments to the value of controlling share and blocking shares depend on the discount to the value of minoritary shares having in the account the balance of the amendments.

4. The discount to the value of minoritary non–blocking shares, which is sustainable subject to the compromise of motives of owners of controlling and blocking shares, on the one hand, and the loss of interest by owners of minoritary non–blocking shares for participation in the business, on the another one, serves as the data when finding of amendments to the value of controlling and blocking shares having in account the balance assumption, wherein total value of all shares is equal to equity.

5. The discount to the value of minoritary non–blocking shares for their non–controlling non–blocking feature is associated with the availability of controlling and minoritary blocking shares in the equity composition, in the absence of which the discount for the non–controlling feature to the value of minoritary non–blocking shares corresponds to zero.

6. The balance assumption, at which the total value of all shares (stocks of equity) is the equity value, need to be considered not as the method of functional dependencies deriving of the amendments to the value on their sizes using spreadsheets, but as the approach, fit to use in manual counting mode or otherwise (without limitation of the technique of computation) for finding of the amendment to the value of share in the dependence on share size agreed with the amendments to values of other shares and with the sizes of these shares.

7. If the sizes of the other shares and the amendments to their values are unknown, then the use of balance assumption for determining of the amendment to value in the dependence on size of the share means the match of found amendment with the assumption, that the amendments to value of the other shares also correspond to the assumption of balance.

References

6. Markus Ya.I., Sivets S.A. How much are the stocks today? The estimation of equity stocks (http://binfo.zp.ua/articles/06.shtml)


Michael Voyloshnikov, e-mail: mikhael.vladlen@gmail.com
Valeria Krivets, e-mail: krivet@mail.ru