Taxing Emissions When They are Already Regulated

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Abstract: The efficiency properties of emission taxes are well-known when they are used in isolation. Also well-known are the required modifications to the optimal tax rule when other taxes are present. But it is common for emission taxes to be applied in the presence of other emission regulations and the implications of this combination have largely been ignored. I develop a second-best general equilibrium model with two polluting firms and look at how optimal emission pricing is affected by two types of pre-existing regulations: an emissions cap on only one firm, and an economy-wide emission intensity rule (or, equivalently, an energy efficiency standard). In the first case the changes to the optimal tax depend on the relative energy intensities of the regulated versus unregulated sectors and the effect of the regulation on the size of the tax base. Under an intensity rule, the tighter the standard, the higher the marginal welfare cost of the first unit of the emission tax and the less likely it can be welfare-improving even when marginal damages are positive.

Key words: Emission taxes; tax interactions; damage thresholds; climate policy

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1 INTRODUCTION

The efficiency case for emission taxes presupposes that command-and-control regulations are absent. In practice, however, the two types of instruments are usually applied in combination. For example, in 2017 the Government of Canada announced a minimum national carbon dioxide price of ten dollars per tonne to begin in 2018, rising to $50 per tonne by 2022, which provincial governments must implement either as a tax or a tradable permit system.[1] But Canadian federal and provincial governments have already implemented numerous sectoral carbon dioxide regulations, such as coal phase-outs in Ontario and Alberta, a hard cap on carbon dioxide emissions from the Alberta oil sands, energy efficiency standards, national ethanol blending requirements in gasoline, etc., none of which are to be repealed as a result of the introduction of the pricing requirement. Similarly, carbon permit trading systems in California, the European Union and elsewhere operate in conjunction with, not instead of, myriad climate-related regulations.

The interaction of instruments raises second-best issues comparable with, but not identical to, those associated with tax interactions. It is well-known that, in tax-distorted economies, revenue from a pollution charge can fund tax reductions elsewhere, but the tax itself increases the marginal distortions associated with the overall tax system. A standard result originally due to Sandmo (1975) is that the optimal pollution tax should, as a result, be equated to the marginal social damages of emissions (*MD*) deflated by the marginal cost of public funds (*MCPF*, Sandmo 1975,

[1] See Government of Canada website http://news.gc+.ca/web/article-en.do?nid=1132149 accessed May 4, 2017.

Bovenberg and Goulder 1996, Parry et al. 1999, Schöb 2003),[2] a result I refer to herein as the Sandmo rule. The intuition is that there is a wedge between the marginal private and the social costs of abatement. It is the latter, not the former, that needs to be equated to *MD*, but since polluters respond to the emission tax according to their marginal private costs, the optimal tax needs to be adjusted to account for the difference.

This paper extends the second-best emission tax literature by examining the effects of pre-existing emission *regulations* on the welfare properties of such taxes. The motivation of the analysis is practical, not prescriptive. We are considering a situation that clearly departs from textbook ideal, but its ubiquity in practice makes it worthwhile understanding the implications. Turvey (1963) voiced an early caution against introducing Pigovian taxes without first considering prior regulations or Coasian bargaining. Numerical simulations have shown that under non-revenue raising instruments, or revenue-raising instruments refunded in a lump-sum fashion, the optimal emissions tax is no longer merely a multiple of marginal damages but can include an additive term as well, which can give rise to an outcome in which the optimal emission tax remains zero unless marginal damages exceeds a positive threshold value (Bovenberg and Goulder 1996, Goulder et al. 1997, Bento and Jacobsen 2007). These cases resemble regulatory instruments, in that they are non-revenue raising. Bovenberg and Goulder (1996) and Goulder et al. (1997) estimated that, in a model calibrated to the US economy, *MD* would have to be relatively high in the case of carbon dioxide (over $50 US per ton in 1996 dollars) and sulfur dioxide (over $100 per ton in 1997 dollars) for any non-revenue raising policy to be welfare-improving. Fullerton and Metcalf (2001) argued

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[2] The marginal cost of public funds is the dollar-valued welfare cost of raising an additional dollar for government spending. Unless the tax system is first-best (non-distorting) it exceeds unity.

that the key mechanism is not revenue-raising *per se* but whether the policy creates scarcity rents that are left in private hands. If another fiscal instrument captures the rents and uses them to offset tax interactions the threshold effect disappears.

I model herein a second-best (tax-distorted) economy with two polluting firms and a government sector funded by a labour tax. I briefly recapitulate some conventional results, showing that the optimal emissions tax rate is an affine transformation of MD. I then examine how several types of pre-existing regulation affect the welfare properties of emission taxes, specifically an economy-wide emission cap, a cap only on one of the firms, and an economy-wide emission-intensity standard. In each case the regulation affects both the scaling and translation coefficients in complex ways. The marginal welfare gain from the emission tax is typically reduced, with the overall effect dependent on the nature and coverage of the regulation, the effect on the tax base and, in the case of partial regulation, the relative emission intensity of the regulated sector.

The next section sets up the model and reproduces some basic second-best results. Section 3 then looks at cases with pre-existing regulations. Section 4 offers discussion and conclusions.

2 SECOND-BEST EMISSION TAX MODEL

2.1 BASIC SET-UP

There are N identical households, two goods each produced by a separate firm, an energy sector, a labour market and a government. Household-specific goods consumption is denoted x_i, $i = (1,2)$, the corresponding prices are p_i, and aggregate demand is denoted $X_i = Nx_i$. Households also consume e_{hh} units of energy at price p_E and aggregate household energy demand is $E_{hh} = Ne_{hh}$. Households each have a time endowment t which can be allocated to labour l or leisure

h, so the aggregate labour supply is $L = Nl$, aggregate leisure is $H = Nh$ and the aggregate time endowment is $T = Nt$. The before-tax nominal wage rate is w. We will set w as the numeraire so it is constant and equal to unity but for notational clarity I retain it in the derivations.

Energy is produced by a sector that uses only labour L_E so its production function is $E = F^E(L_E)$ and its profits are $\pi^E = p_E F^E(L_E) - wL_E$. Use of energy causes emissions $C = cE$ where c denotes the emissions intensity of energy and is assumed constant. Emissions are taxed at τ_C per unit so the tax-inclusive price of energy is

$$p'_E = p_E + c\tau_C. \quad (1)$$

Note that $'$ throughout denotes a tax-inclusive term.

Each goods-producing firm has a single unit of fixed capital K_i equal to unity and a production function $F^i(L_i, E_i)K_i$ where the first argument denotes labour usage and the second denotes energy. As in Bento and Jacobsen (2007) assume that F^i is strictly concave and has decreasing returns to scale in L_i and E_i. The profit function for firm *i* is

$$\pi_i = p_i F^i(L_i, E_i) - wL_i - p'_E E_i$$

where $F_L^i > 0$ and $F_E^i > 0$. The first-order conditions imply $F_L^i = w/p_i$ and $F_E^i = p'_E/p_i$. Decreasing returns to scale imply that profits are positive and represent the return to capital for each firm. We assume shares in firms are distributed equally among all households.

We will assume that prices are initially normalized so that $p_i = p_E = w = 1$. The tax rate on household income is τ_Y and net income is

$$y' = \left(\frac{\pi_1+\pi_2+\pi_E}{N} + wt\right)(1-\tau_Y). \tag{2}$$

The household budget constraint is $p_1 x_1 + p_2 x_2 + p_E' e_{hh} + w'h = y'$ where $w' = w(1-\tau_Y)$. The corresponding national budget constraint (NBC) is

$$p_1 X_1 + p_2 X_2 + p_E' E_{hh} + w'H = Y' \tag{3}$$

where $Y' = (\pi_1 + \pi_2 + \pi_E + wT)(1-\tau_Y)$.

The government does not use energy but purchases some of the available production of good 2 and gives it to households in equal shares. It finances this through the tax τ_C on emissions C and the income tax τ_Y. Hence the Government Budget Constraint (GBC) is

$$p_2 G = \tau_Y B + \tau_C C \tag{4}$$

where the income tax base B equals $\pi + wL$ and $\pi = \pi_1 + \pi_2 + \pi_E$.

Goods Market Equilibrium (GME) occurs where $X_1 = F_1$ and $X_2 + G = F_2$. Energy Market Equilibrium (EME) occurs where $E_{hh} + E_1 + E_2 = E$. Labour Market Equilibrium (LME) occurs where $L_1 + L_2 + L_E = T - H$. It is straightforward to show that imposing GME, LME and EME on the NBC implies the GBC holds; likewise any four implies the fifth.

We assume that tax rates are adjusted to hold G constant. Differentiating Equation (4) and rearranging yields the G-neutrality tax condition

$$\frac{d\tau_Y}{d\tau_C} = \frac{1}{B}\left(G\frac{dp_2}{d\tau_C} - \tau_Y\frac{dB}{d\tau_C} - \tau_C\frac{dC}{d\tau_C} - C\right). \tag{5}$$

Since the emissions tax raises the cost of providing the public good and causes the income tax base and emissions to decline, the first three terms in the brackets must sum to a positive number. We will assume that we are operating in a region of the economy for which the new tax revenue (represented by the fourth term) is sufficiently large as to make whole derivative negative, meaning that an increase in emission taxes permits a reduction in the income tax.

Household utility is $u(x_1, x_2, e_h, h) + \frac{\alpha G}{N} - \delta C$ where α is the positive welfare weight on the public good G and δ is the marginal welfare cost of each unit of emissions. We will use the indirect utility function $v(p_1, p_2, p_E, w', y')$ to define the national social welfare function

$$W = Nv(p_1, p_2, p_E', w', y') + \alpha G - \delta NC. \tag{6}$$

The planner's problem is to choose τ_C to maximize (6). Since G is assumed fixed and given, τ_Y is then determined by equation (4).

2.2 Derivation of Optimal Taxes

The marginal welfare effect of emission taxation is (see Appendix)

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = \frac{dC}{d\tau_C}\left(\tau_C - \frac{\delta N}{v_y}\right) - Q + \tau_Y R \tag{7}$$

where the first term on the right side is the primary welfare effect of the emissions tax, represented by the change in emissions times the difference between the tax τ_C and $\delta N/v_Y$, which is the marginal external cost of emissions, or *MD*, the tax interaction effect Q is

$$Q = \left(F^1 \frac{dp_1}{d\tau_C} + F^2 \frac{dp_2}{d\tau_C}\right) - \tau_Y w \left(\frac{\partial L}{\partial p_E'} c + \frac{\partial L}{\partial p_1}\frac{\partial p_1}{\partial \tau_C} + \frac{\partial L}{\partial p_2}\frac{\partial p_2}{\partial \tau_C}\right) \tag{8}$$

and the revenue-recycling effect R is

$$\tau_Y R = -w^2 \tau_Y \frac{\partial L}{\partial w'}\frac{\partial \tau_Y}{\partial \tau_C}. \tag{9}$$

Equation (7) thus decomposes into standard components (compare to Bento and Jacobsen 2007, Parry et al. 1999 and others). The tax interaction effect captures the welfare costs resulting from price changes induced by the emissions tax. The first term measures costs to households and the government of the increased output prices resulting from the emissions tax. The second term represents the lost tax revenue due to the labour market effects of the emissions tax. On the supply side, an increase in prices reduces real wages and thus reduces labour supply. On the demand side, under strict concavity each firm's demand for labour must decline when the price of energy rises (Henderson and Quandt 1980 p. 81). Consequently, the change in L must be negative. Combining these yields $Q > 0$. $\tau_Y R$ represents the benefit of the reduction in the income tax rate financed by the emission tax, which induces an increase in the labour supply. Examination of the derivatives shows $R > 0$.

We can evaluate equation (7) at the unregulated emissions level by setting $\tau_C = 0$, which yields

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -\frac{dC}{d\tau_C}MD - Q + \tau_Y R \qquad (10)$$

The first unit of abatement will yield a positive increment of welfare only if the value of avoided damages exceeds the tax interaction effect net of the revenue-recycling effect. Hence we have the standard result (see, e.g., Fullerton and Metcalf 2001):

Result 1. If the emissions policy fails to capture scarcity rents, abatement policy will only raise welfare if MD exceeds the threshold $-Q\left(\frac{dC}{d\tau_C}\right)^{-1}$ which is strictly positive.

Proof: With no revenue-recycling effect we have $Q > 0$ and $R = 0$. Rearrange equation (10) to get the result. ■

Note that, *ceteris paribus*, the less price-elastic emissions are the higher will be the threshold which MD must exceed for emissions policy to be welfare-enhancing. Furthermore, note that in the Sandmo (1975) framework, if the government revenue requirement is low enough to be fully satisfied by an externality tax, the optimal policy would entail a tax on the dirty good equal to marginal social damages and no other tax. But here if we set $\tau_Y = 0$ and set equation (7) to zero we obtain

$$\tau_C = MD + (F^1 \frac{dp_1}{d\tau_C} + F^2 \frac{dp_2}{d\tau_C}) \left(\frac{dC}{d\tau_C}\right)^{-1}$$

which is strictly less than MD. The reason for the difference is that this model allows producer prices to change whereas in the Sandmo framework they are determined by fixed input-output coefficients. If prices were similarly fixed herein the second term would disappear and the classical solution would emerge.

It is common to derive the optimal emissions tax from equation (7) by setting it equal to zero and rearranging to yield $\tau_C = MD + Q/\frac{dC}{d\tau_C} - \tau_Y R/\frac{dC}{d\tau_C}$ (see, e.g. Bento and Jacobsen 2007 equation 2.12). This is an incomplete solution here since the income tax rate is determined by τ_C, so τ_Y must be substituted out. Carrying out the substitution yields the following:

Result 2. In an economy as described above and given equations (1—6) the optimal tax on emissions is

$$\tau_C^* = aMD - b \tag{11}$$

where

$$a = \frac{\frac{dC}{d\tau_C}}{\frac{dC}{d\tau_C} - \frac{RC}{B}} \tag{12}$$

and

$$b = a\left(\frac{p_2 G}{B} R - Q\right)\left(\frac{dC}{d\tau_C}\right)^{-1}. \tag{13}$$

Proof: See Appendix.

Since $R > 0$ and $dC/d\tau_C < 0$, the weight a on MD must be positive and less than unity. In the Appendix I discuss how a relates to the marginal cost of public funds in this model. Also since a and b are determined by optimization with respect to the emissions tax the envelope theorem implies they are invariant to τ_C in equation (11).

The additive term b cannot be signed unambiguously. Recall that B is the income tax base and $p_2 G$ is total government spending, so at $\tau_C = 0$ the ratio of the two equals the income tax rate τ_Y, making the expression in the first pair of brackets $\tau_Y R - Q$, or the revenue recycling effect minus the tax interaction effect. If they equal each other then $b = 0$ and the optimal tax follows the Sandmo rule $\tau_C = aMD$. Otherwise the sign of b depends on the relative magnitudes of $\tau_Y R$ and Q. If the tax interaction effect is larger than the revenue recycling effect, $b > 0$.

The damage threshold Z can be derived by setting $\tau_C^* = 0$ and solving for MD. This yields $Z = b/a$, and represents the amount marginal damages must exceed before the first unit of the emissions tax is welfare-improving. As discussed in the introduction, in the case of carbon dioxide emissions Z was found in Bovenberg and Goulder (1996) to depend on the form of the policy, and can potentially be a large number relative to common estimates of marginal damages.

Equation (11) can be used to compare the marginal private and social abatement costs by noting that firms respond to an emissions tax by equating it to MAC_p and the social optimum occurs where $MAC_s = MD$, so rearranging these yields

$$MAC_s = \frac{1}{a} MAC_p + \frac{b}{a}. \tag{14}$$

In Figure 1 the classical Pigovian outcome is shown as the case where $MD = MAC_P$ at emissions E_P. In the presence of tax distortions but with $b = 0$ the marginal welfare costs of emission reductions follow MAC_{1S}, but firms still respond to a tax according to their private marginal abatement costs, implying an optimal emissions tax rate $a\tau_P$. Also shown is the case where $b > 0$ which implies marginal welfare costs of abatement along MAC_{2S} and a threshold value Z showing the value of MD below which the optimal emissions tax would remain zero.

Non-optimized tax system and the damage threshold

The solution at equation (11) assumes both taxes are chosen optimally. Bovenberg and Goulder (1996) noted that the optimal pollution tax only corresponded to the Sandmo result when the rest of the tax system was optimized. We can examine a non-optimal outcome by returning to equation (7) but instead of setting it equal to zero, set it equal to some other value ρ:

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = \frac{dC}{d\tau_C}\left(\tau_C - \frac{\delta N}{v_y}\right) - Q + \tau_Y R = \rho \tag{15}$$

The benefits threshold becomes

$$Z^{\rho} = \frac{b}{a} - \frac{\rho}{dC/d\tau_C} \tag{16}$$

This implies:

Result 3. *Ceteris paribus*, when the tax system is not optimized, such that labor is over-taxed, the benefits threshold Z is unambiguously raised .

Proof: The GBC implies that if the income tax is raised relative to the optimum, the emissions tax must be reduced. Since the welfare function is convex, $\tau_C^{\rho} < \tau_C^{*}$ implies a positive value of ρ, hence $Z^{\rho} > Z$. ■

3 EMISSION TAXES UNDER PRE-EXISTING REGULATIONS

3.1 ECONOMY-WIDE EMISSION CAP

We now turn to cases in which emissions are regulated prior to emission taxes being introduced. If emissions from both sectors are restricted to a total of $\hat{C}$ which has an associated shadow price $\hat{\tau}_C$, and both firms operate with equal marginal abatement costs MAC_p, the policy is identical to tradable quotas which, as noted above, do not capture scarcity rents. The planner's problem can be re-stated as an optimization of W by choice of $\hat{C}$. The marginal welfare effect with respect to $\hat{C}$ is (see Appendix)

$$\frac{dW}{d\hat{C}}\frac{1}{v_Y} = MAC_P - MD - \left(F^1\frac{dp_1}{d\hat{C}} + F^2\frac{dp_2}{d\hat{C}}\right) + \tau_Y w\frac{dL}{d\hat{C}} \tag{17}$$

An increase in $\hat{C}$ reduces costs for firms, so $F^1 \frac{dp_1}{d\hat{C}} + F^2 \frac{dp_2}{d\hat{C}} < 0$. Since an increase in allowed emissions (as opposed to a change in the relative cost of labour and emissions) raises employment we have $\frac{dL}{d\hat{C}} > 0$. This immediately implies:

Result 4. The optimal level of $\hat{C}$ occurs where $MAC_p < MD$.

(Proof omitted).

Obtaining the conventional textbook optimum at $MAC_P = MD$ requires either $\tau_Y = 0$ or a fixed labor supply, and fixed prices, which are very restrictive conditions.

The marginal welfare cost of the first unit of emission reductions is given by equation (17) evaluated where $MAC_P = 0$, which means MD must exceed $-\left(F^1 \frac{dp_1}{d\hat{C}} + F^2 \frac{dp_2}{d\hat{C}}\right) + \tau_Y w \frac{dL}{d\hat{C}}$, which is strictly positive, for the first unit of emission reductions to be welfare-improving. This threshold will be larger, the higher is the labor tax and the effect on prices of changes in $\hat{C}$.

However, once emissions are capped, as long as the emission tax is below the shadow price associated with $\hat{C}$, prices and outputs no longer respond to changes in τ_C so the derivative of the welfare function with respect to the emissions charge reduces to $\tau_Y R$ which is positive. Thus:

Result 5. Under a binding economy-wide emissions cap $\hat{C}$, the first unit of the emissions tax is always welfare-improving.

(Proof omitted).

This result reflects the fact that the tax is now a rent capture mechanism that funds reductions in the income tax rate.

3.2 Partial Emission Cap

We now turn to the case in which pre-existing command-and-control regulations cover emissions from only part of the economy before an emissions tax is introduced. The regulation freezes activity in one sector, which as we will see reduces the magnitude of the tax interaction effect based the coverage of the regulation (the fraction of the economy regulated), while reducing the revenue recycling effect according to the stringency of the regulation (the percentage cut in total emissions). The outcome under this case will be denoted with ~. Suppose that firm 2 is required to reduce emissions to a fixed target level $c\tilde{E}_2$ which is below the level associated with any proposed emissions tax rate.[3] Both firms pay the tax but only firm 1 freely chooses its emissions level. The first order conditions remain the same for firm 1, but for firm 2 only that related to labour remains the same. The profit function for firm 2 is now

$$\pi_2 = p_2 F^2\left(L_2, \tilde{E}_2\right) - wL_2 - (p_E + \tau_C c)\tilde{E}_2.$$

The emissions charge is a lump-sum rent capture for firm 2, which will help finance reductions in τ_Y, but the regulation itself restricts the production function and results in an upward shift of its

[3] If $\tilde{E}_2$ exceeded this level then the constraint would not bind and we would simply be back to the case of a uniform emissions tax across both sectors.

supply curve, by an amount exceeding what would have been experienced under the emission tax. The price charged under the regulation is denoted $\tilde{p}_2$ and is strictly greater than p_2. Firm 2 profits $\tilde{\pi}_2$ are also lower than in the previous case. The regulation does not change the relative price of the inputs, instead it forces down output and profitability and has a negative effect on labour demand, leading to $\tilde{L}_2 < L_2$. Since profits and labour earnings are reduced we expect $\tilde{y}' < y'$. These changes make it unambiguous that $v(p_1, \tilde{p}_2, p_E, \widetilde{w}', \tilde{y}') < v(p_1, p_2, p_E, w', y')$. Also the tax base shrinks, i.e. $\tilde{B} < B$.

The coverage of the regulation is C_2/C. Total emissions are $\tilde{C} = C_1 + \tilde{C}_2$ which is strictly less than C. The stringency of the regulation is measured as $\tilde{C}/C$. Because both firms pay the emission tax we have $\frac{d\tilde{\pi}}{d\tau_C} = -\tilde{C}$. The NBC is $p_1 X_1 + \tilde{p}_2 X_2 + (p_E + \tau_C c)E_{hh} + \widetilde{w}' H = (\pi_1 + \tilde{\pi}_2 + wT)(1 - \tau_Y)$. The GBC is $\tau_C \tilde{C} + \tau_Y \tilde{B} = \tilde{p}_2 G$. Since the emissions level is fixed for firm 2, $\frac{d\tilde{p}_2}{d\tau_C} = 0$ and the G-neutrality condition becomes

$$-\tilde{B}\frac{d\tau_Y}{d\tau_C} = \tau_C \frac{dC_1}{d\tau_C} + \tilde{C} + \tau_Y \frac{d\tilde{B}}{d\tau_C}.$$

The derivative of W with respect to τ_C now becomes:

$$\frac{d\widetilde{W}}{d\tau_C}\frac{1}{v_y} = \left(\tau_C - \frac{\delta N}{v_y}\right)\frac{dC_1}{d\tau_C} - F_1 \frac{d\tilde{p}_1}{d\tau_C} + \tau_Y w \frac{d\tilde{L}}{d\tau_C} \qquad (18)$$

where $\tilde{L} = L_1 + \tilde{L}_2$. Starting at $\tau_C = 0$, an emissions tax in the presence of partial regulations will raise welfare only if

$$-\frac{\delta N}{v_y}\frac{dC_1}{d\tau_C} - F_1\frac{d\tilde{p}_1}{d\tau_C} + \tau_Y w\frac{d\tilde{L}}{d\tau_C} > 0.$$

Comparing this to equation (10) all three terms get smaller, but not by the same amount. The first term, which is positive, shrinks according to the regulatory coverage since marginal damages are only reduced from abatement activity by firm 1. The second term, which is negative, shrinks according to the output share of firm 1. The third term changes according to the shrinkage of the labor supply in response to the emission regulation. This will be the smallest of the three adjustment factors. Suppose, for instance, the regulation requires a 30 percent emission reduction from a sector responsible for half of total emissions and a third of total output. After implementing the regulation, total employment in the economy will be reduced but not likely by half or a third.

We can now note the following:

Result 6. For a given level of labor, when emissions are partially regulated the marginal welfare benefit of the first unit of the emissions tax will fall by more, the greater the emissions intensity of the regulated sector relative to the unregulated sector.

Proof. Denote the unregulated fraction of emissions as $c_u = 1 - C_2/C$, the unregulated fraction of goods output as $d_u = 1 - F_2/F$ and $d_u = c_u + \epsilon$. Note that $\epsilon > 0$ implies the unregulated sector is relatively less emissions intensive than the regulated sector. Comparing equation (18) to equation (7), holding labor constant and setting $\tau_C = 0$, to a first approximation we have $\frac{d\widetilde{W}}{d\tau_C}\frac{1}{v_y} =$

$-c_u \frac{dC}{d\tau_C}\frac{\delta N}{v_y} - d_u Q = c_u \frac{dW}{d\tau_C}\frac{1}{v_y} - \epsilon Q$. Since $0 < c_u < 1$ the marginal welfare effect is now smaller and the reduction in dW is larger if $\epsilon > 0$. ■

Related to this we have:

Result 7. For a given level of total emissions, when emissions are partially regulated, the marginal welfare benefit of the first unit of the emissions tax will fall by more, the lower the labor intensity of the regulated sector relative to the unregulated sector.

Proof. As before denote the unregulated fraction of goods output as $d_u = 1 - F_2/F$; also denote the unregulated fraction of labor as $l_u = 1 - L_2/L$ and denote $l_u = d_u + \epsilon$. Note that $\epsilon < 0$ implies the unregulated sector is relatively less labor intensive than the regulated sector. Comparing equation (18) to equation (7), holding emissions constant and setting $\tau_C = 0$, to a first approximation we have $\frac{d\widetilde{W}}{d\tau_C}\frac{1}{v_y} = d_u \frac{dW}{d\tau_C}\frac{1}{v_y} + \epsilon\tau_Y R$. Since $0 < d_u < 1$ the marginal welfare effect is now smaller and the reduction in dW is larger if $\epsilon > 0$. ■

Hence, if the preliminary regulations target the more emissions-intensive and/or the less labor intensive sector, the marginal welfare benefit of any subsequent emission tax will be smaller than would otherwise be the case. If, for instance, the difference in emission intensities is large enough, the outcome could arise in which the marginal welfare effect of the emission tax goes negative.

In the Appendix I show that the optimal tax rate is now

$$\tilde{\tau}_C = \tilde{a}\frac{\delta N}{v_Y} - \tilde{b} \tag{19}$$

where

$$\tilde{a} = \frac{\frac{dC_1}{d\tau_C}}{\frac{dC_1}{d\tau_C} - \frac{\tilde{R}\tilde{C}}{\tilde{B}}}$$

$$\tilde{b} = \tilde{a}\left(\frac{\tilde{p}_2 G}{\tilde{B}}\tilde{R} - \tilde{Q}\right)\left(\frac{dC_1}{d\tau_C}\right)^{-1}$$

$$\tilde{Q} = F_1\frac{dp_1}{d\tau_C} - \tilde{\tau}_Y w\left(\frac{\partial L_1}{\partial p_E'}c + \frac{\partial L_1}{\partial p_1}\frac{\partial p_1}{\partial \tau_C}\right)$$

and

$$\tilde{R} = -\frac{\partial \tilde{L}}{\partial w'}\frac{\partial \tilde{\tau}_Y}{\partial \tau_C}.$$

Since the tax base has shrunk and the cost of government spending has risen, either or both of the income and emission taxes must be higher compared to the no-regulation case.

Equation (19) yields a number of useful comparisons, stated as the following results.

Result 8. The weight $\tilde{a}$ on marginal damages in the optimal tax rule will decline compared to the unregulated case if the proportional change in R/B is smaller than the proportional change in the percent response to the emission tax.

Proof. Divide the top and bottom of $\tilde{a}$ by $-\tilde{C}$ to get $\tilde{a} = \mu/(\mu + \tilde{R}/\tilde{B})$ where μ is the absolute value of the percentage reduction in $\tilde{C}$ due to a marginal change in the tax rate. Then $\tilde{a}^{-1} = 1 + \frac{1}{\mu} \times \frac{\tilde{R}}{\tilde{B}}$. The latter term gets larger as long as the proportional decrease in μ exceeds the proportional decrease in $\tilde{R}/\tilde{B}$, which implies a gets smaller. ∎

The importance of this result is as follows. μ will tend to shrink based on the coverage of the regulation: the larger the fraction of emissions that are frozen by the regulation, the smaller the proportional economy-wide response to the emission tax. The terms that make up R (see equation 9) include the numeraire, the slope of the labor supply function and the change in τ_Y as τ_C changes. The first two aren't affected by the regulation. The last will become a bit smaller because the shrinkage of the tax base B reduces the funds available to pay for a cut in the income tax: but this factor is also in the denominator of $\tilde{R}/\tilde{B}$, hence there will be a cancelling out. So the largest effect of partial emission regulations will likely be that on μ, which means imposition of regulations covering part of the economy will cause a to go down.

It is not possible to determine whether b or Z become larger or smaller under partial regulations. Regarding the latter we have, prior to the introduction of the emission tax, $\tilde{Z} = (\tilde{\tau}_Y \tilde{R} - \tilde{Q})/(dC_1/d\tau_C)$. The denominator is unambiguously smaller than before. In the numerator, the income tax rate is larger than before because the tax base is smaller and p_2 is higher, while $\tilde{R}$ is smaller, so the change in the magnitude of the revenue-recycling term is ambiguous. $\tilde{Q}$ is smaller than before because only the unregulated sector contributes to the tax interaction effect. Combining these, the magnitude and even the sign of the numerator may change depending on the specific elasticities of the unregulated sector versus the economy as a whole. 999999

3.3 EMISSION INTENSITY STANDARD

Suppose the pre-existing regulation takes the form of an emissions intensity rule stating that emissions per unit of output in sectors 1 and 2 must be below a level z, which we write as $cE_i \leq zF^i(L_i, E_i)$. Note that in the present context this is equivalent to an energy efficiency standard. We assume that the constraint binds with equality over the relevant range of any emission tax. It can be shown that the firm uses less energy and less labor, and each firm moves to a point on the isoquant where the ratio of labor to energy is greater relative to the unconstrained case.

Denote variables at the new starting point with #. If we introduce an emissions tax the firm's profit function becomes

$$\pi_i^{\#} = p_i^{\#} F^{\# i} - (p_E^{\#} \frac{z}{c} + z\tau_C) F^{\# i} - wL_i^{\#}.$$

We have seen in earlier analyses that the profit function affects the choice of the optimal tax rule by its effect on the derivative of profits with respect to the emissions tax rate. By the envelope theorem, $d\pi_i^{\#}/d\tau_C = -zF_i^{\#}$. This is the same expression as we would get if, instead of imposing an emissions tax, we imposed an output tax $\tau_F = z\tau_C$ on the firm, making the profit function

$$\pi_i^{\#} = (p_i^{\#} - \tau_F) F^{\# i} - p_E^{\#} E_i^{\#} - wL_i^{\#}.$$

Consequently, the effect of the binding intensity rule is to convert the emission tax into an output tax on goods, from the perspective of the social planner. The resulting optimal value of τ_F has a form very similar to equation (11), but now z appears in it as does the slope of the aggregate goods production function $dF/d\tau_F$ where $F = F^1 + F^2$. Specifically, we have

Result 9. In the presence of a binding emissions intensity constraint the optimal emission charge is now equivalent to an output tax of the form

$$\tau_F^{\#} = a^{\#} z \frac{\delta N}{v_Y} - b^{\#}$$

where (suppressing the # superscripts on the right hand side)

$$a^{\#} = \left(\frac{\frac{dF}{d\tau_F}}{\frac{dF}{d\tau_F} - \frac{RF}{B}} \right)$$

and

$$b^{\#} = a^{\#} \left(R \frac{p_2 G}{B} - Q \right) \left(\frac{dF}{d\tau_F} \right)^{-1}.$$

Proof: See Appendix.

Two interesting results immediately follow.

Result 10. If the economy is subject to a binding emission intensity constraint $C = zF$, the magnitude of the threshold value $Z^{\#}$ which marginal damages must exceed in order for an emissions tax to be welfare-improving is increasing in the stringency of the standard.

Proof: Solve $\tau_F^{\#} = 0$ to get $Z^{\#} = \frac{b^{\#}}{za^{\#}} = \left(\frac{1}{z}\right)\left(R\frac{p_2 G}{B} - Q\right)\left(\frac{dF}{d\tau_F}\right)^{-1}$ which represents the level of marginal damages below which the optimal output tax is non-positive. The envelope theorem implies that as z gets smaller, $Z^{\#}$ gets larger. ■

Result 12. If the economy is subject to a binding emission intensity constraint $C = zF$, *ceteris paribus* the magnitude of the threshold value $Z^{\#}$ is larger, the steeper is the aggregate goods production function F.

Proof: Using the expressions for $a^{\#}$ and $b^{\#}$ we have $Z^{\#} = \left(\frac{1}{z}\right)\left(R\frac{p_2 G}{B} - Q\right)\left(\frac{dF}{d\tau_F}\right)^{-1}$ from which the result follows immediately. ■

4 Discussion and Conclusions

The optimal emission fee in a second-best (tax-distorted) economy is of the form $\tau = aMD - b$ where $a < 1$ except under restrictive conditions and the sign of b depends on the relative magnitudes of the tax interaction and revenue-recycling effects. The marginal social welfare cost of the first unit of emission reduction is $Z = b/a$, and if this is positive it creates a threshold which MD must exceed for emission reductions to improve welfare. The model developed herein has two polluting sectors, decreasing returns to scale and variable prices, and yields a second-best optimum in which $a < 1$ and b is typically non-zero.

I examine a second-best economy with both a tax and a regulatory distortion prior to implementing an emissions tax. If emissions are controlled by an economy-wide cap the damage threshold Z is strictly positive, which implies the optimal emissions cap is greater than the emissions level associated with an optimal emissions tax. However, once a binding emissions cap is in place, an emissions tax becomes a rent-capture device and the first unit is strictly welfare-improving.

If regulations only partially cover emissions the effects are more complex and depend on the technologies of the regulated versus unregulated sectors. The marginal welfare gain from emission taxes is smaller compared to the unregulated case, and the reduction is larger if the regulations target the relatively more emissions-intensive, or relatively less labor-intensive, firm. If the regulation consists of an intensity limit (a cap on emissions per unit of output), which in this model is equivalent to an energy efficiency standard, an emissions tax becomes formally equivalent to an output tax on the goods producing sectors. The more stringent the intensity rule, the higher the threshold Z becomes, which makes it less and less likely that the first unit of an emissions tax will be welfare-improving.

The vast literature on emission taxes has assumed that they are used instead of, not in addition to, regulations. But in practice the two types of instruments are almost always used in combination, especially in the case of climate policy and carbon taxes. An influential literature has previously shown that pre-existing factor taxes strongly affect the marginal welfare properties of emission taxes, and this has been taken into account in many quantitative analyses including computable general equilibrium simulations. But there has been little discussion of the effects of pre-existing regulations. The analysis herein establishes that they also affect the marginal welfare properties of emission taxes and need to be considered in quantitative policy analysis.

5 REFERENCES

Bento, Antonio and Mark Jacobsen (2007) "Ricardian Rents, Environmental Policy and the 'Double-Dividend' Hypothesis." *Journal of Environmental Economics and Management* 53 pp. 17—31.

Bovenberg, A. Lans and Ruud A. de Mooij (1994) "Environmental Levies and Distortionary Taxation" *The American Economic Review* Vol. 84, No. 4 (Sep., 1994), pp. 1085-1089

Bovenberg, A. Lans and Lawrence H. Goulder (1996). "Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses." *American Economic Review* 86(4) 985—1000.

Fullerton, Don and Gilbert E. Metcalf (2001) "Environmental Controls, Scarcity Rents and Pre-Existing Distortions." *Journal of Public Economics* 80 pp. 249—267.

Goulder, Lawrence H. (1998) "Environmental Policy Making in a Second-Best Setting." *Journal of Applied Economics* I(2) November 1998, 279—328.

Goulder, Lawrence H. (2013) "Climate Change Policy's Interactions with the Tax System." *Energy Economics* http://dx.doi.org/10.1016/j.eneco.2013.09.017

Helfand, Gloria (1991) "Standards versus Standards: The Effects of Different Pollution Restrictions." *American Economic Review* 81(3) 622—634.

Metcalf, Gilbert (2003) "Environmental Levies and Distortionary Taxation: Pigou, Taxation and Pollution." *Journal of Public Economics* 87 (2003) 313-322.

Parry, Ian W.H. (1995) "Pollution taxes and revenue-recycling" *Journal of Environmental Economics and Management* 29 S64-S77.

Parry, Ian W.H. (1997). "Environmental taxes and quotas in the presence of distorting taxes in factor markets." *Resource and Energy Economics* (19), 203–220.

Parry, Ian, Roberton C. Williams III and Lawrence H. Goulder (1999). "When Can Carbon Abatement Policies Increase Welfare? The Fundamental Role of Distorted Factor Markets." *Journal of Environmental Economics and Management* 37: 52—84.

Sandmo, Agnar (1975) "Optimal Taxation in the Presence of Externalities." *Swedish Journal of Economics* 77 (1), 86–98.

Schöb, Ronnie (2003) "The Double Dividend Hypothesis of Environmental Taxes: A Survey." Fondazione Eni Enrico Mattei Working Paper No. 60.2003, 11 June 2003 http://papers.ssrn.com/abstract=413866

Turvey, Ralph (1963) "On Divergences Between Social Cost and Private Cost" *Economica* August 1963, reprinted in Dorfman R. and N. Dorfman *Economics of the Environment: Selected Readings* 2nd Ed., New York: Norton, 1977.

6 Figure

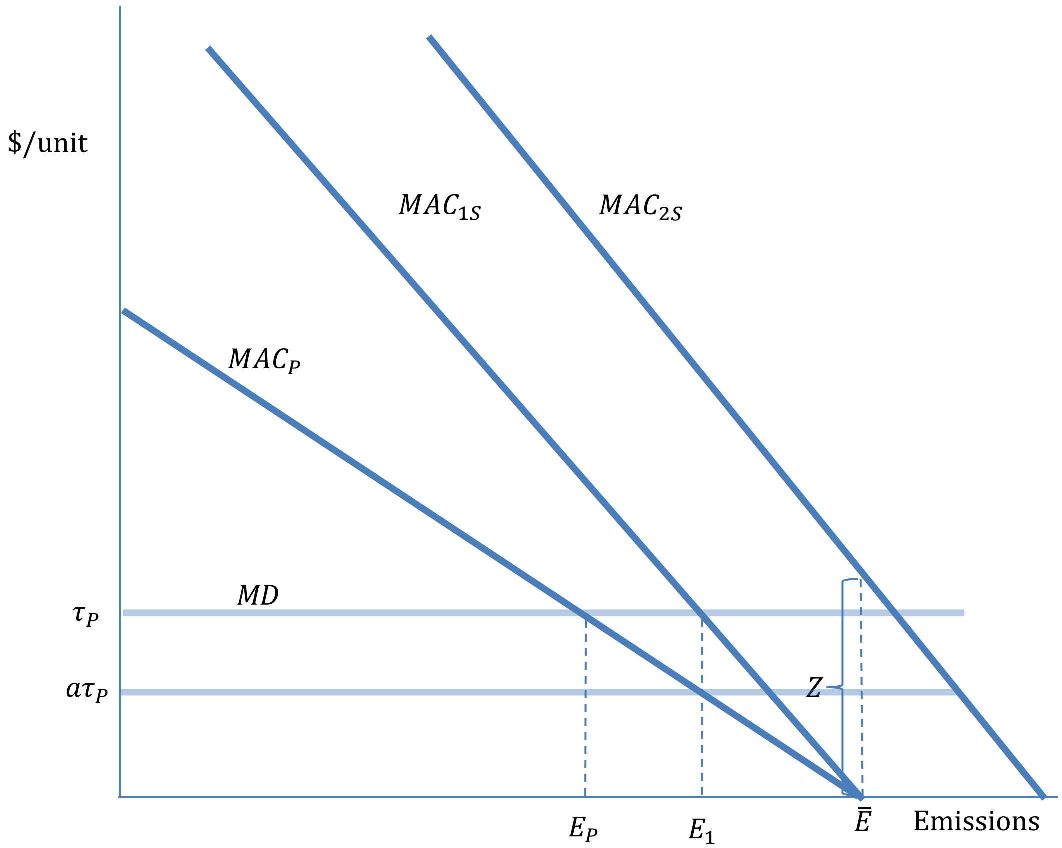

FIGURE 1: The classical Pigovian tax τ_P equals MD (here assumed constant), yielding equivalence between MD and private MAC (MAC_P) at emissions E_P. But when social MAC is rotated out to MAC_{1s} the optimal emissions tax is at $a\tau_P$ and optimal emissions is at E_1. If a positive threshold exists (Z) the marginal welfare costs of emission reductions follow MAC_{2S} and the optimal tax is below $a\tau_P$. As drawn the optimal emission tax is zero even though MD is positive.

7 APPENDIX

Derivation of Equation (7)

The first derivative of the social welfare function (equation 6) with respect to τ_C is

$$\frac{dW}{d\tau_C} = N\left(v_1\frac{dp_1}{d\tau_C} + v_2\frac{dp_2}{d\tau_C} + v_E\frac{dp_E'}{d\tau_C} + \ v_w\frac{dw'}{d\tau_C} + v_y\frac{dy'}{d\tau_C}\right) - \delta N\frac{dC}{d\tau_C}$$

where derivatives of v are subscripted in order of the arguments. Divide this equation by v_y and apply Roy's theorem to obtain

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - E_{hh}\frac{dp_E'}{d\tau_C} - H\frac{dw'}{d\tau_C} + \frac{dY'}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}.$$

Use $\frac{dY'}{d\tau_C} = (1-\tau_Y)\frac{d\pi}{d\tau_C} - \pi\frac{d\tau_Y}{d\tau_C} + H\frac{dw'}{d\tau_C} + L\frac{dw'}{d\tau_C}$ and note that because w is the numeraire, $\frac{dw'}{d\tau_C} = -w\frac{d\tau_Y}{d\tau_C}$ to obtain

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - cE_{hh} - H\frac{dw'}{d\tau_C} + H\frac{dw'}{d\tau_C} + L\frac{dw'}{d\tau_C} + (1-\tau_Y)\frac{d\pi}{d\tau_C} - \pi\frac{d\tau_Y}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - cE_{hh} - wL\frac{d\tau_Y}{d\tau_C} + (1-\tau_Y)\frac{d\pi}{d\tau_C} - \pi\frac{d\tau_Y}{d\tau_C} - \frac{\delta N}{v_y}\frac{dE}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - cE_{hh} - \frac{d\tau_Y}{d\tau_C}(\pi + wL) + (1-\tau_Y)\frac{d\pi}{d\tau_C} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}.$$

When firms optimize inputs the envelope theorem implies $\frac{d\pi^i}{d\tau_C} = -cE_i$. Note also that $\pi + wL = B$, so $\frac{dB}{d\tau_C} = -c(E_1 + E_2) + w\frac{dL}{d\tau_C}$. Use these and equation (5)

$$\frac{d\tau_Y}{d\tau_C} = \frac{1}{B}\left(G\frac{dp_2}{d\tau_C} - \tau_Y\frac{dB}{d\tau_C} - \tau_C\frac{dC}{d\tau_C} - C\right)$$

to obtain

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - cE_{hh} - B \times \frac{1}{B}\left(G\frac{dp_2}{d\tau_C} - \tau_Y\left(w\frac{dL}{d\tau_C} - c(E_1+E_2)\right) - \tau_C\frac{dC}{d\tau_C} - C\right) +$$

$$(1-\tau_Y)\frac{d\pi}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - cE_{hh} - G\frac{dp_2}{d\tau_C} + \tau_Y\left(w\frac{dL}{d\tau_C} - c(E_1+E_2)\right) + \tau_C\frac{dC}{d\tau_C} + C + \frac{d\pi}{d\tau_C} - \tau_Y\frac{d\pi}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} - (X_2+G)\frac{dp_2}{d\tau_C} - cE_{hh} - c(E_1+E_2) + C + \tau_Y w\frac{dL}{d\tau_C} + \tau_C\frac{dC}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}.$$

Expand the derivative of labour as $\frac{dL}{d\tau_C} = \frac{\partial L}{\partial p_E'}c + \frac{\partial L}{\partial p_1}\frac{\partial p_1}{\partial \tau_C} + \frac{\partial L}{\partial p_2}\frac{\partial p_2}{\partial \tau_C} + \frac{\partial L}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C}$. Use this, GME and the emissions function to reduce the above to

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -F^1\frac{dp_1}{d\tau_C} - F^2\frac{dp_2}{d\tau_C} + \tau_Y w\left(\frac{\partial L}{\partial p_E'}c + \frac{\partial L}{\partial p_1}\frac{\partial p_1}{\partial \tau_C} + \frac{\partial L}{\partial p_2}\frac{\partial p_2}{\partial \tau_C}\right) + \tau_Y w\left(\frac{\partial L}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C}\right) + \tau_C\frac{dC}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}.$$

The rest follows immediately. ■

Proof of Result 2

Rearrange $\frac{dW}{d\tau_C}\frac{1}{v_y} = -F^1\frac{dp_1}{d\tau_C} - F^2\frac{dp_2}{d\tau_C} + \tau_Y w\frac{dL}{d\tau_C} + \tau_C\frac{dC}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C} = 0$ to get

$$\tau_C = \frac{\delta N}{v_Y} + \frac{1}{dC/d\tau_C}Q - \frac{\tau_Y R}{dC/d\tau_C}.$$

Using equation (4) we have $\tau_Y R = \left(\frac{p_2 G}{B} - \frac{\tau_C C}{B}\right)R$. Making the substitution yields

$$\tau_C = \frac{\delta N}{v_Y} + \frac{1}{\frac{dC}{d\tau_C}}\,Q - \frac{Rp_2G}{B\frac{dC}{d\tau_C}} + \frac{RC}{B\frac{dC}{d\tau_C}}\tau_C$$

$$\tau_C\left(1 - \frac{RC/B}{\frac{dC}{d\tau_C}}\right) = \frac{\delta N}{v_Y} + \frac{1}{\frac{dC}{d\tau_C}}\,Q - \frac{Rp_2G}{B\frac{dC}{d\tau_C}}$$

$$\tau_C = \frac{\delta N}{v_Y}\left(\frac{\frac{dC}{d\tau_C}}{\frac{dC}{d\tau_C}-\frac{RC}{B}}\right) + \left(\frac{Q}{\frac{dC}{d\tau_C}} - \frac{Rp_2G}{B\frac{dC}{d\tau_C}}\right)\left(\frac{\frac{dC}{d\tau_C}}{\frac{dC}{d\tau_C}-\frac{RC}{B}}\right).$$

The rest follows immediately.

Interpretation of Equation (12)

Multiply the top and bottom by τ_C then note that $dC/d\tau_C = cdE/d\tau_C$ and $\partial L/\partial\tau_C = -\partial H/\partial\tau_C$, which yield

$$a = \frac{c\tau_C \frac{dE}{d\tau_C}}{c\tau_C \frac{dE}{d\tau_C} + \tau_C \frac{C}{B} w \frac{\partial H}{\partial \tau_C}}.$$

Recall from equation (1) that $c\tau_C$ is the wedge between the supply price of energy and the marginal willingness to pay, hence the numerator is the welfare loss associated with a reduction in energy consumption due to an incremental increase in the emissions tax for the purpose of funding additional government spending. The same term appears in the denominator. The second term is the decline in leisure due to the emission tax weighted by $\tau_C Cw/B$. To understand this term, note that solving the GBC for τ_Y would break it down to two components: p_2G/B and $-\tau_C C/B$. The first is the portion required to cover government spending and the second is the offsetting reduction permitted by emission tax revenues. If government spending were zero but marginal damages necessitated $\tau_C > 0$ we could use the emission tax revenue to subsidize labour at the rate $-\tau_C C/B$. This represents the opportunity cost of needing to fund G. Hence the decline of leisure is weighted by the nominal wage rate times the portion of the income tax rate that represents the opportunity cost of needing to fund the government. Consequently, the denominator of a is the marginal (with respect to τ_C) opportunity cost of financing government spending through τ_C, and the inverse of a is this amount relative to the direct economic cost of the emission tax increase, giving a an interpretation similar to the inverse-MCPF weights found in previous models.

Derivation of Equation (17)

The derivative of W with respect to the emissions constraint yields

$$\frac{dW}{d\hat{C}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{C}} - X_2\frac{dp_2}{d\hat{C}} - H\frac{dw'}{d\hat{C}} + \frac{dY'}{d\hat{C}} - \frac{\delta N}{v_y}.$$

Since the policy does not raise revenue the GBC is $p_2 G = \tau_Y B$ which implies $-B\frac{d\tau_Y}{d\hat{C}} = (-G\frac{dp_2}{d\hat{C}} + \tau_Y\frac{d\pi}{d\hat{C}} + \tau_Y w\frac{dL}{d\hat{C}})$. Also note that $\frac{dY'}{d\hat{C}} = \frac{d\pi}{d\hat{C}}(1-\tau_Y) - \pi\frac{d\tau_Y}{dC} + T\frac{dw'}{d\hat{C}}$. Combining these yields:

$$\frac{dW}{d\hat{C}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{C}} - X_2\frac{dp_2}{d\hat{C}} - H\frac{dw'}{d\hat{C}} + H\frac{dw'}{d\hat{C}} + \frac{d\pi}{d\hat{C}}(1-\tau_Y) - \pi\frac{d\tau_Y}{d\hat{C}} + L\frac{dw'}{d\hat{C}} - \frac{\delta N}{v_y}$$

$$\Rightarrow \frac{dW}{d\hat{C}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{C}} - X_2\frac{dp_2}{d\hat{C}} - G\frac{dp_2}{d\hat{C}} + \tau_Y w\frac{dL}{d\hat{C}} + B\frac{d\tau_Y}{d\hat{C}} - \pi\frac{d\tau_Y}{d\hat{C}} - Lw\frac{d\tau_Y}{d\hat{C}} + \left(\frac{d\pi}{d\hat{C}} - \frac{\delta N}{v_y}\right)$$

$$= -F^1\frac{dp_1}{d\hat{C}} - F^2\frac{dp_2}{d\hat{C}} + \frac{d\tau_Y}{d\hat{C}}(B - \pi - wL) + \tau_Y w\frac{dL}{d\hat{C}} + \left(\frac{d\pi}{d\hat{C}} - \frac{\delta N}{v_y}\right)$$

$$= -F^1\frac{dp_1}{d\hat{C}} - F^2\frac{dp_2}{d\hat{C}} + \tau_Y w\frac{dL}{d\hat{C}} + \left(\frac{d\pi}{d\hat{C}} - \frac{\delta N}{v_y}\right).$$

Using $\frac{d\pi}{d\hat{C}} = MAC_p$ and $\frac{\delta N}{v_y} \equiv MD$ yields the result. ■

Derivation of Equation (19)

Variables modified to take account of the change in their initial values due to the partial regulation are denoted with ~. The derivative of W with respect to the emissions constraint yields

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - cE_{hh} - H\frac{dw'}{d\tau_C} + \frac{d\tilde{Y}'}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}.$$

Substitute in $\frac{d\tilde{Y}'}{d\tau_C} = (1-\tau_Y)\frac{d\tilde{\pi}}{d\tau_C} - \tilde{\pi}\frac{d\tau_Y}{d\tau_C} + H\frac{dw'}{d\tau_C} + \tilde{L}\frac{dw'}{d\tau_C}$ to obtain

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - cE_{hh} - H\frac{dw'}{d\tau_C} + H\frac{dw'}{d\tau_C} + \tilde{L}\frac{dw'}{d\tau_C} + (1-\tau_Y)\frac{d\tilde{\pi}}{d\tau_C} - \tilde{\pi}\frac{d\tau_Y}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} - cE_{hh} - w\tilde{L}\frac{d\tau_Y}{d\tau_C} + (1-\tau_Y)\frac{d\tilde{\pi}}{d\tau_C} - \tilde{\pi}\frac{d\tau_Y}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} - cE_{hh} - \frac{d\tau_Y}{d\tau_C}(\tilde{\pi} + w\tilde{L}) + (1-\tau_Y)\frac{d\tilde{\pi}}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} - cE_{hh} - cE_1 - c\tilde{E}_2 - \frac{d\tau_Y}{d\tau_C}\tilde{B} - \tau_Y\frac{d\tilde{\pi}}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}.$$

Following similar steps as before while noting that $\tilde{p}_2$ does not change in response to the emission tax but labor supply responds to changes in τ_Y, we have $-\tilde{B}\frac{d\tau_Y}{d\tau_C} = \tau_Y\frac{d\tilde{B}}{d\tau_C} + \tau_C\frac{dC_1}{d\tau_C} + \tilde{C}$ and $\frac{d\tilde{B}}{d\tau_C} = \frac{d\tilde{\pi}}{d\tau_C} + w\frac{d\tilde{L}}{d\tau_C}$. Substitute these in to obtain

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - \tilde{C} + \tilde{C} + \tau_Y\frac{d\tilde{B}}{d\tau_C} + \tau_C\frac{dC_1}{d\tau_C} - \tau_Y\frac{d\tilde{\pi}}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} + \tau_Y\frac{d\tilde{\pi}}{d\tau_C} + \tau_Y w\frac{dL}{d\tau_C} - \tau_Y\frac{d\tilde{\pi}}{d\tau_C} + \frac{dC_1}{d\tau_C}\left(\tau_C - \frac{\delta N}{v_y}\right)$$

$$= -X_1\frac{dp_1}{d\tau_C} + \tau_Y w\left(\frac{\partial L_1}{\partial p_E'}c + \frac{\partial L_1}{\partial p_1}\frac{\partial p_1}{\partial \tau_C}\right) + \tau_Y w\frac{\partial \tilde{L}}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C} + \frac{dC_1}{d\tau_C}\left(\tau_C - \frac{\delta N}{v_y}\right)$$

Set this to zero to obtain $\tilde{\tau}_C = \frac{\delta N}{v_y} + \frac{\tilde{Q}}{dC_1/d\tau_C} - \frac{\tau_Y R}{dC_1/d\tau_C}$ where $\tilde{Q} = F_1\frac{dp_1}{d\tau_C} - \tau_Y w\left(\frac{\partial L_1}{\partial p_E'}c + \frac{\partial L_1}{\partial p_1}\frac{\partial p_1}{\partial \tau_C}\right)$ and $\tau_Y\tilde{R} = \tau_Y w\frac{\partial \tilde{L}}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C}$. The modified GBC yields $\tau_Y\tilde{R} = \left(\frac{p_2 G}{\tilde{B}} - \frac{\tau_C\tilde{C}}{\tilde{B}}\right)\tilde{R}$. Substituting as before yields

$$\tilde{\tau}_C = \frac{\delta N}{v_Y} + \frac{1}{\frac{dC_1}{d\tau_C}}\ \tilde{Q} - \frac{\frac{p_2 G}{\tilde{B}}}{\frac{dC_1}{d\tau_C}}\tilde{R} + \tilde{\tau}_C \frac{\frac{\tilde{C}}{\tilde{B}}\tilde{R}}{\frac{dC_1}{d\tau_C}}$$

$$\tilde{\tau}_C\left(1 - \frac{\frac{\tilde{C}}{\tilde{B}}\tilde{R}}{\frac{dC_1}{d\tau_C}}\right) = \frac{\delta N}{v_Y} + \frac{1}{\frac{dC}{d\tau_C}}\ \tilde{Q} - \frac{\frac{p_2 G}{\tilde{B}}}{\frac{dC_1}{d\tau_C}}\tilde{R}$$

which solves to equation (19). ■

Proof of Result 9

The profit function is now $\pi_i = (p_i - \tau_F)F^i - p_E E_i - wL_i$. The Social Welfare Function is $W = Nv(p_1, p_2, p_E, w', y') + \alpha G - \delta NC$ and the GBC becomes $p_2 G = \tau_Y B + \tau_F F$. Using $\frac{dp_E}{d\tau_F} = 0$ the first derivative of the social welfare function with respect to τ_F is

$$\frac{dW}{d\tau_F} = N\left(v_1 \frac{dp_1}{d\tau_F} + v_2 \frac{dp_2}{d\tau_F} + \ v_w \frac{dw'}{d\tau_F} + v_y \frac{dy'}{d\tau_F}\right) - \delta N \frac{dC}{d\tau_F}.$$

Follow the same steps as before to obtain

$$\frac{dW}{d\tau_F}\frac{1}{v_y} = -F^1 \frac{dp_1}{d\tau_F} - F^2 \frac{dp_2}{d\tau_F} + \tau_Y w \frac{dL}{d\tau_F} + \tau_F \frac{dF}{d\tau_F} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}.$$

Expand the derivative of labour as before and rearrange to get

$$\frac{dW}{d\tau_F}\frac{1}{v_Y} = \tau_F \frac{dF}{d\tau_F} - \frac{\delta N}{v_Y}\frac{dC}{d\tau_F} - Q + \tau_Y R$$

where $Q = F^1 \frac{dp_1}{d\tau_F} + F^2 \frac{dp_2}{d\tau_F} - \tau_Y w \left(\frac{\partial L}{\partial p_1}\frac{\partial p_1}{\partial \tau_F} + \frac{\partial L}{\partial p_2}\frac{\partial p_2}{\partial \tau_F}\right)$ and $R = -\tau_Y w^2 \frac{\partial L}{\partial w'}\frac{\partial \tau_Y}{\partial \tau_F}$. Set this equal to zero and use the GBC to substitute out τ_Y to obtain

$$\tau_F^{\#} = a^{\#} z \frac{\delta N}{v_Y} - b^{\#}$$

where (suppressing the # superscripts on the right hand side)

$$a^{\#} = \left(\frac{\frac{dF}{d\tau_F}}{\frac{dF}{d\tau_F} - \frac{RF}{B}} \right)$$

and

$$b^{\#} = a^{\#} \left(R \frac{p_2 G}{B} - Q \right) \left(\frac{dF}{d\tau_F} \right)^{-1}.$$

■