# Optimal Monetary Policy, Currency Unions, and the Eurozone Divergence

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**Abstract**

What is the optimal inflation rate in an open economy, and when are currency unions a good idea? I investigate these questions using a monetary open economy model where firms have market power because of search frictions. Consumers respond to inflation by increasing their search effort, and as a result, inflation has real and non-monotonic effects. The optimal inflation rate depends on the fundamentals of the economy, such as the disutility of search or the cost of firm entry, and on the inflation rates of trading partners. When countries coordinate their monetary policies, inflation will be lower and welfare will be higher than in non-cooperative equilibrium. However, *coordinating* policy is not the same as conducting the *same* policy, and the welfare effects from joining a currency union are asymmetric. Preliminary analysis suggests that the model can account for some features of the macroeconomic divergence within the Eurozone in the 1990s and 2000s.

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# 1 Introduction

What is the optimal monetary policy in an open economy? Extensive research has addressed the question of how monetary policy should respond to temporary shocks, domestic or foreign, but has ignored the effects of structural factors on the optimal long-run money growth rate, and therefore the optimal long-run inflation rate. Models of such effects exist, but have previously been limited to closed economies. However, the question of optimal money growth may be especially relevant in an international context. First, there may be benefits from coordinating long-run inflation rates between countries. Second, there may be costs to giving up an independent choice of the money growth rate, such as when pegging the exchange rate or forming a currency union.

In order to address these questions, I use a model of a monetary economy with search frictions and endogenous search effort. Specifically, I build on the optimal inflation model of Head and Kumar (2005), which studies the endogenous choice of search effort by consumers in a monetary-search model with price posting. I extend this model in three directions: ex-ante firm heterogeneity, free entry of firms, and international trade among an arbitrary number of countries. International trade provides a channel through which monetary policy in one country affects another. Firm heterogeneity and free entry allow me to link the results of the model with the recent experience of the Eurozone.

In a closed-economy version of the model, the results are similar to those of Head and Kumar (2005). The optimal long-run inflation rate is robustly above the Friedman rule (deflation at the rate of time preference), for the following reason: inflation acts as a tax on real balances, but for low levels of inflation, other channels can outweigh this negative effect. The principal channel through which inflation has positive effects is the choice of search effort by consumers. Search effort limits market power in the aggregate, but individual consumers do not take this externality into account. By increasing price dispersion, inflation stimulates search effort, which reduces market power and enhances welfare. This effect dominates for low levels of inflation, but not for high ones. As a result, the inflation rate which maximizes welfare is positive. Extending the original results, I characterize how this optimal rate of inflation depends on structural parameters. Most notably, it is an increasing function of the disutility of search effort, and of the cost of firm entry.

Next, I solve a two-country version of the model. Firms can sell at home or abroad, but they are subject to iceberg trade costs and to local-currency pricing. I find that inflation is a beggar-thy-neighbor policy for two reasons: first, some of the inflation tax is passed on to foreign consumers; second, inflation increases search effort at home, which limits the market power of foreign firms. Consequently, when trading partners cooperate to choose jointly optimal inflation rates, inflation will be lower and welfare will be higher than in non-cooperative equilibrium. Choosing the *same* inflation rate, however, cannot be optimal when countries have structural differences, such as in the disutility of search effort or the cost of firm entry. This implies that currency unions exert a welfare cost, but the effects are necessarily asymmetric. As a benchmark case, assume that prospective members of a currency union are coordinating their inflation rates optimally. If they agree to a common inflation rate that is optimal for the union as a whole, then countries who initially had lower inflation will gain, while countries who initially had higher inflation will lose.

These results add to our understanding of optimum currency areas. Starting with Mundell (1961), the theory has focused largely on the adjustment of the economy to temporary shocks, and on how currency unions can affect this adjustment. Similarly, papers studying optimal monetary policy in an open economy typically model an environment where the only role of monetary policy is to respond to fluctuations. For example, Corsetti and Pesenti (2009) find that under local currency pricing, any type of increase in the money growth rate is a beggar-thy-neighbor policy, but with transitory effects. In the present paper, on the other hand, unexpected increases have no effects at all, but permanent increases have permanent effects.

Such permanent effects of changes in long-run money growth rates are common to monetary-search models with endogenous search effort. The debt to Head and Kumar (2005) has already been acknowledged. In an early contribution, Li (1995) introduced the "hot potato effect", where inflation increases the frequency of trading. Liu, Wang and Wright (2011) study the effect further, but in contrast to the present paper, they find that search effort decreases with inflation and only participation increases. The externality due to endogenous search effort (or entry) is studied by Shi (2006), Berentsen, Rocheteau and Shi (2007), and Craig and Rocheteau (2008). In these papers, the externality of endogenous search could be positive or negative; this arises from the bargaining mechanism that determines the terms of trade in their models. In my paper, on the other hand, the terms of trade are determined by price posting, and the externality of search effort is always positive: more search effort puts pressure on firms to reduce markups, but individual buyers do not take this into account.

So far, the monetary-search literature has mostly focused on environments with homogeneous firms or sellers. However, firm heterogeneity is important for the following reason. Higher search effort always shifts production to the cheaper firms, and when firms are heterogeneous, there is a direct welfare gain because the cheaper firms are more efficient. Two consequences are apparent. First, in models of a single homogeneous good, price dispersion is associated with a welfare loss, and is one of the reasons for the welfare cost of inflation.[1] This is not true in the present paper, because prices should reflect costs, and low price dispersion indicates that the efficient firms charge high markups. Second, total factor productivity is affected by search effort, and hence by monetary policy.

The premise of my paper is that inflation affects the real economy through price dispersion, to which consumers respond by choosing their search effort (Head and Kumar, 2005). Recently, this premise has received some empirical support. Caglayan, Filiztekin and Rauh (2008) find that price dispersion is V-shaped as a function of inflation and that the amount of dispersion is related to search costs. Using European Union price data, Becker and Nautz (2010) and Becker (2011) find this V-shaped relationship disappears in highly integrated markets, where search costs are presumably low.

To my knowledge, the conclusion that very low rates of inflation are associated with reductions in output, as proposed by Head and Kumar (2005) and supported in my model, has not received direct empirical attention in the existing literature. However, it is certainly consistent with the literature on persistent large output gaps (surveyed by Meier, 2010). Consider for example Meier's observation that "disinflation has tended to taper off at very low positive inflation rates, arguably reflecting downward nominal rigidities and well-anchored inflation expectations", which is also consistent with a model in which expectations of slower money growth initially bring on disinflation, but then cause output to fall and prices to rise when search effort responds. In this sense, monetary-search models

[1] Examples include Head and Kumar (2005) and Wang (2014).

may provide an alternative explanation as to why output losses and/or sluggish growth would coexist with low inflation and interest rates, without appealing to sticky prices, and without implying liquidity traps or paradoxes of thrift.

A final remark is in order. The present paper argues that the policy instrument of the long-run money growth rate is important and that giving up this instrument may have serious side-effects. However, I do not claim to make a full cost-benefit analysis. Realistically, there are many channels through which common currencies can have positive effects: risk-sharing (Mundell, 1973) is an old example, and the fact that recognizable currencies promote efficient trade (Devereux and Shi, 2008; Zhang, 2012) is a more recent one. Any cost-benefit analysis of currency unions would have to take these channels into account.

The remainder of the paper is organized as follows. Section 2 describes and solves the model. Section 3 analyzes the welfare effects of money growth. Section 4 concludes and briefly discusses how the model can help explain the macroeconomic imbalances in the run-up to the Euro crisis. Proofs and graphs are provided in the appendix.

# 2 Model

The model is a specialization of the workhorse model developed in Herrenbrueck (2015), built on the optimal inflation framework introduced by Head and Kumar (2005). That framework was based on the monetary search economy of Shi (1997) with price posting by firms (Burdett and Judd, 1983) and endogenous search effort by consumers. Aside from some technical innovations, the main contribution of Herrenbrueck (2015) is to solve the model for general cost distributions of firms (ex-ante heterogeneity), for free entry of firms, and for an arbitrary number of countries with local-currency pricing but perfectly competitive foreign exchange markets. In what follows, however, the analysis is simplified by considering steady states alone, and time subscripts will be suppressed whenever possible.

## 2.1 Environment

Time is discrete and infinite, and there is a measure 1 of households. Members can be either buyers or workers; there is a positive measure of each type, and they sum to measure 1. There exists a monetary authority that supplies a stock $M_t$ of perfectly durable, infinitely divisible asset called money at time $t$, and augments this stock at the beginning of each period with lump-sum transfers $T_t$ to the households. Household members do not have independent utility but share equally in the utility of the household (Shi, 1997). Households value the streams of consumption $c$, search effort $s$, and work effort $n$ according to the separable utility function:

$$U(c, s, n) = (1 - \beta) \sum_{t=0}^{\infty} \beta^t \left[u(c) - \mu s_t - n_t\right] \tag{1}$$

Assume $\beta \in (0, 1)$ and $\mu > 0$, with the disutility of labor normalized to 1. $u(\,\cdot\,)$ is strictly increasing, strictly concave, and satisfies the Inada conditions. Furthermore, $u'(c)c$ is non-increasing in $c$.

So far, the set-up is similar to that of Head and Kumar (2005), but for concreteness I will additionally assume that there exists a measure $N$ of firms, owned by households, that hire workers in a perfectly competitive labor market in order to produce goods.[2] In order to trade these goods, firms and buyers enter an anonymous and memoryless goods market characterized by search frictions. The infinitely many buyers of a household must search separately and, once in the market, cannot coordinate with their siblings.

The trading period proceeds as follows: Firms learn their costs, as well as all macroeconomic variables, and post prices. As they can perfectly forecast demand (by the law of large numbers applied to the mass of buyers), they then hire workers and produce. Households learn the price distribution and decide how much money $m_t$ each member carries into the market, and how much effort $s$ should be spent on obtaining price offers. Once in the market, each buyer receives $k$ random quotes (Burdett and Judd, 1983), i.e. is able to observe the prices of $k$ firms, where $k$ is drawn from a distribution $Q(k|s)$ with support $\{1, 2, \ldots\}$. The distribution of quotes with higher search effort strictly first order stochastically dominates the one with lower search effort: informally, more effort supplies more quotes.

A buyer can then purchase goods from any of the firms whose prices he has observed, but he cannot spend more money than he carries. To keep the analysis tractable, buyers cannot coordinate with one another, cannot spend any extra ef-

[2] This is not a critical assumption. A frictional labor market does not change any of the qualitative results.

fort after receiving quotes, and cannot recall any quotes from a previous period. Similarly, buyers remain anonymous to firms, so firms will only accept payment in cash. At the end of the period, firms pay their workers and remit the profits to their owners. Workers take home their pay, buyers take home their goods, and the members of the household share equally in consumption and earnings.

To simplify notation, all monetary variables will be expressed in stationary "constant-money" terms, which means the nominal value divided by the money stock $M_t$. I avoid the term "real" in order to prevent confusion, because "real" is commonly understood to mean the nominal value divided by the price level $P_t$. In my paper, money growth has nonmonotonic effects even in steady-state, so dividing by the money stock is different from dividing by the price level, and more precise.

### Matching

Let $q_k(\eta),\ k = 1, \dots, K$ be the probability that a buyer who searches with *success rate* $\eta$ observes exactly $k$ prices. Assume that $\eta = s \cdot N$, where $s$ is search intensity and $N$ is the mass of active firms; it seems reasonable that matching success should depend both on buyer effort and on firm presence. Define the functions $J :\ [0,1] \times [0,\infty) \to [0,1]$ and $a :\ [0,1] \times [0,\infty) \to [0,\infty)$:

$$J(F, sN) = \sum_{k=1}^{K} q_k(sN)\,(1 - (1 - F)^k)$$
$$a(F, sN) = \sum_{k=1}^{K} q_k(sN)\,k\,(1 - F)^{k-1}$$

Let the cumulative distribution function of prices posted by sellers be $F(p)$ on support $\mathcal{F}$. The good is homogeneous, so buyers who observe more than one price will only buy from the cheapest firm. Then the c.d.f. of the transactions prices is the c.d.f. of the lowest price observed by a buyer:

$$J(F(p), sN) \quad \forall p \in \mathcal{F}.$$

As a consequence, how many transactions can a firm with price $p$ expect when all buyers search with success rate $sN$? The answer is:

$$J_1(F(p), sN) = a(F(p), sN)) \quad \forall p \in \mathcal{F},$$

where the subscript $J_1$ denotes the derivative with respect to the first argument. For this reason, I call $a(F, sN)$ the *arrival function* in analogy to the *arrival rates* common in search theory.

Throughout this paper, quantitative results are derived using the "geometric" matching process $q_k(\eta) = \frac{\eta^{k-1}}{(1+\eta)^k}$ with upper limit $K = \infty$ and expectation $\bar{q} = 1 + \eta$ (a geometric distribution of the number of prices observed). Intuitively, the process can be characterized by the buyer flipping a coin (loaded by $\eta$) until it comes up tails, and then getting observing one price for each flip. Appendix B discusses other reasonable matching processes, including those used in the literature, and contrasts them with the geometric process.

### Households

Households take the distribution of prices $F(p)$ and the wage $w$ (all expressed in constant-money terms) as given, and choose the search effort and expenditure strategies for its buyers, as well as the work effort of its workers. As the sub-utility of consumption $u(c)$ is strictly concave, the household will treat all of its buyers the same. Therefore, all buyers pursue the same expenditure strategy $x(p)$. With household money stock $m$ and aggregate money stock $M$, the household optimizes

$$v(m, M) = \max_{m', x(\cdot), c, s, n} \{u(c) - \mu s - n + \beta \mathbb{E}\{v(m', M')|M\}\} \tag{2}$$

subject to

$$\frac{m'}{m} = 1 - \int_{\mathcal{F}} \frac{x(p)}{m} a(F(p), sN)\, dp + \Pi(M) + wn + \frac{T}{m} \tag{3}$$

$$c = \int_{\mathcal{F}} \frac{x(p)}{p} a(F(p), sN)\, dp \tag{4}$$

where $\Pi(\,\cdot\,)$ denotes aggregate profits.

Denote the Lagrange multiplier on the budget constraint (3) by $\Omega$. (The budget constraint is expressed in constant-money terms to make $\Omega$ stationary.) As discussed in the section on firms, a buyer carrying $m$ units of currency cannot buy more than $\frac{m}{Mp}$ of goods at nominal price $Mp$ ($p$ in constant-money terms). Because all buyers share equally in their household's utility and are too small to influence consumption $c$, they will pursue a reservation price strategy, such that:

$$x(p) = \begin{cases} m & \text{if } p \leq \bar{p} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The key constraint is that once in the market, they cannot transfer the money to another buyer. Going home means leaving the money idle until next period; and the value of a marginal unit of idle money at the end of the period is $M\Omega$, hence the nominal reservation price satisfies:

$$M\,\bar{p} = M\,\frac{u'(c)}{\Omega} \tag{6}$$

The labor supply is perfectly elastic: households will meet any demand for labor as long as the wage satisfies $w \geq 1/\Omega$.

Let $c(s)$ denote the consumption resulting from searching with effort $s$. By the law of large numbers, this is a deterministic function. If it is also concave, then the optimal choice of search effort implies that $u'(c(s))c'(s) = \mu$.

Finally, the key intertemporal variable, $\Omega$, is determined by the first-order condition for $m'$ together with the envelope condition for $m$, iterated forward by one period:

$$\Omega = \mathbb{E}\left\{\frac{\beta m}{m'} u'(c')c'\right\}$$

**Firms**

Let there be a mass $\bar{N}$ of firms. In this section, suppress time subscripts, and write $a(F, sN)$ as simply $a(F)$.

Assume that a firm requires $\phi$ units of labor to produce one unit of output. In a perfectly competitive labor market, firms can hire any quantity of labor at the nominal wage $Mw$. With nominal price $Mp$, they will attract $a(F(p))$ buyers (the price distribution $F(p)$ is expressed in constant money terms so as to be stationary). If $p \leq \bar{p}$, each buyer will spend all the money they carry, and as shown above, all buyers carry the same amount of money, $M$. Therefore, the nominal profits of a firm with marginal cost $\phi$ are determined by:

$$M\pi(\phi) = \max_p \left\{ M\left(1 - \frac{\phi w}{p}\right)\frac{a(F(p))}{N} \mid p \leq \bar{p} \right\}. \tag{7}$$

As shown in Head and Kumar (2005), when all firms have the same marginal cost $\phi$, they must all make the same profits. When firms have heterogeneous costs, however, clearly they do not all make the same profits. The firm's problem must be solved in the following way: assume that each firm draws a value of $\phi$, the marginal labor cost of production, from a distribution $G(\phi)$ with support $\mathcal{G} \subset \mathbb{R}$ and upper bound $\bar{\phi}$. (The nominal marginal cost of production is then $\phi M w$.) Abusing notation, $\phi$ will sometimes identify a representative seller with disutility $\phi$.[3]

**Lemma 2.1.** *Assume $p_1$ and $p_2$ are solutions to profit maximization with $\phi_1$ and $\phi_2$, respectively (we do not require them to be unique solutions). If $\phi_1 < \phi_2$, then $p_1 < p_2$.*

*Proof.* See appendix A. □

Consequently, firms' prices are completely ranked by their costs. If the support of $G$ includes $\bar{p}/w$, i.e. $\bar{\phi}w > \bar{p}$, some firms (with mass $(1-G(\bar{p}/w))$) will not be able to sell their products for any profit at all. In particular, I assume that $\bar{\phi} = \infty$ in all quantitative applications. To simplify matters, I assume that all such noncompetitive firms are simply inactive, and are ready to produce as soon as the reservation price rises. Define the cost distribution conditional on producing:

$$\tilde{G}(\phi) = \frac{G(\phi)}{G(\bar{p}/w)}$$

Assuming that $G$ is differentiable, guessing that $F(p)$ is differentiable, and guessing that the solution to profit maximization is a differentiable function which we denote by $p(\phi)$, this implies the *ranking conditions*:

$$\begin{aligned} F(p(\phi)) &= \tilde{G}(\phi) \quad \text{for } \phi w \leq \bar{p} \\ F'(p(\phi))p'(\phi) &= \tilde{G}'(\phi) \end{aligned}$$

[3] In monopolistic competition models, firms are usually identified by their productivity, often using the letter $\varphi$, the inverse of marginal cost.

Profit maximization implies the following first-order condition:

$$\frac{\partial \pi}{\partial p}(p;\phi) \;=\; \left(1-\frac{\phi w}{p}\right)\frac{a_1(F(p))}{N}F'(p)+\frac{\phi w}{p^2}\frac{a(F(p))}{N} \;=\; 0$$

Together, the ranking and first-order conditions determine a differential equation

$$p'(\phi)=\left(\frac{p(\phi)^2}{\phi w}-p(\phi)\right)\left(\frac{-a_1(\tilde{G}(\phi))}{a(\tilde{G}(\phi))}\right)\tilde{G}'(\phi) \tag{8}$$

with boundary condition

$$p(\bar{p}/w)=\bar{p}. \tag{9}$$

Equation (8) is a special case of the *Riccati equation*, which has the following solution.

**Lemma 2.2.** *Let* $p(\phi)=\infty$[4] *if* $\phi w>\bar{p}$ *and*

$$p(\phi)=\frac{a(\tilde{G}(\phi))\bar{p}}{a(1)-\frac{\bar{p}}{w}\int_{\phi}^{\bar{p}/w}a'(\tilde{G}(t))\frac{1}{t}\tilde{G}'(t)\;dt} \tag{10}$$

*otherwise. Then* $p(\phi)$ *solves the system* $\{$(8), (9)$\}$ *and therefore maximizes profits* (7).

*Proof.* See appendix A. □

## 2.2 Equilibrium in autarky

Even without free entry, the mass of active firms can potentially vary with the reservation price. Recall that the number of potential firms is $\bar{N}$, and the mass of active firms is $N\equiv\bar{N}G(\bar{p}/w)$. Define the gross money growth rate $\gamma=1+\frac{T}{M}$, and assume that $\gamma$ is given and constant.

**Definition 1.** A *steady-state monetary search equilibrium* (SSMSE) is a collection of a value function $v(m,M)$, policy functions $m'(m,M)$, $x(\,\cdot\,;m,M)$, $s(m,M)$, $n(m,M)$, common expenditure rule, $X(\,\cdot\,;M)$, common search effort $S(M)$, a wage $w(M)$, and a distribution of posted prices, $F(\,\cdot\,;M)$, such that

1. The value function $v(m,M)$ solves (2) with the associated policy functions $m'(m,M)$, $x(\,\cdot\,;m,M)$, $s(m,M)$, and $n(m,M)$, and Lagrange multiplier $\Omega(M)$.

[4]Or any price above $\bar{p}$.

2. The price distribution $F(\,\cdot\,; M)$ satisfies the ranking conditions together with Equation (10).
3. Aggregate (constant-money) profits in (3) are

$$\Pi(M) = \int_{\mathcal{F}} \left(1 - \frac{\phi w(M))}{p}\right) a(F(p; M), S(M)N)\; dp.$$

4. The money market clears: $w(M)n(M, M) + \Pi(M) = 1$. By Walras' Law, this is equivalent to labor market clearing.
5. Individual choices equal aggregate quantities: $x(p; M, M) = X(p; M)$ for all $p$, $s(M, M) = S(M)$, and $m'(M, M) = \gamma M$.
6. Money has value: $F(p) < 1$ for some $p < \infty$.

As money grows at constant rate $\gamma$, $\Omega$ satisfies in steady state:

$$\Omega = \frac{\beta}{\gamma} u'(c)c. \tag{11}$$

As the labor market is perfectly competitive, the wage must just compensate workers for their work effort:

$$w = \frac{1}{\Omega}. \tag{12}$$

**Fixed search**

Equation (4) can be combined with the optimal pricing formula to express aggregate consumption as a function of aggregate search effort $s$ (more details in Herrenbrueck, 2015). Denote the infimum of the support of $G(\cdot)$ by $\underline{\phi}$. The upper bound is given by the reservation cost $\bar{p}/w$. Further substituting (6) and (12), and rearranging, we can derive the *market equilibrium* equation:

$$u'(c)c = \Omega \left[ a(1, sN(c)) + \int_{\underline{\phi}}^{u'(c)} \frac{u'(c)}{t} \left(-a_1(\tilde{G}(t), sN(c))\right) \tilde{G}(t)\tilde{G}'(t)\; dt \right] \tag{13}$$

The number of active firms is written as $N(c) \equiv \bar{N}\, G(u'(c))$; the reservation cost equals $\bar{p}/w = u'(c)$ because the disutility of labor was normalized to 1.

In order to obtain a relationship in $(s, c)$-space, we have to replace $\Omega$ using (11).

The steady-state market equilibrium equation becomes:

$$\frac{\gamma}{\beta} = a(1, sN(c)) + \int_{\underline{\phi}}^{u'(c)} \frac{u'(c)}{t} \left(-a_1(\tilde{G}(t), sN(c))\right) \tilde{G}(t)\tilde{G}'(t)\, dt \tag{14}$$

It can be shown that this curve is upward-sloping in $(s, c)$-space: for a given money growth rate, more search effort implies more consumption. One complication arises through the endogenous reservation price, which implies that not all firms will be able to compete. However, this complication is not severe if the cost distribution $G(\phi)$ is well-behaved and does not put too much weight on the tails. Furthermore, the market equilibrium curve shifts down and right as $\gamma$ increases, implying that the Friedman rule $\gamma = \beta$ achieves the maximum level of consumption when search effort is fixed (see figure 1).[5]

### Optimal search

The Friedman rule is still the optimal monetary policy when search effort is *socially* chosen, by maximizing $u(c) - \mu s$ subject to the constraint (14). However, the result breaks down when households *privately* choose their search effort. The reason is that search effort accomplishes two things: it allows buyers to pick up low-price offers, and through that, it constrains the market power of the firms. The key is that because each household is small, only the former effect is internalized. As a result, loose monetary policy creates price dispersion and forces buyers to search harder. The inflation tax is Pigovian for small levels of inflation.

Solving the households' problem (equation (2)), privately optimal search implies that $u'(c(s))c'(s) = \mu$, taking the distribution of prices $F(p)$ as given. Details of the derivation are in Herrenbrueck (2015); we need to define the auxiliary function

$$h(F, \eta) = \int_0^F \frac{a_2(z, \eta)}{a(z, \eta)}\, dz$$

(the index $a_2$ refers to the derivative with respect to the second argument), and can

[5] Formally, the Friedman rule is defined by a nominal interest rate of zero, which can be implemented in a variety of ways (Lagos, 2010), but which is commonly taken to be equivalent to shrinking the money supply at the rate of time preference.

then derive the *optimal search* equation

$$\mu = \Omega\, N(c) \left[ a(1, sN(c))h(1, sN(c)) \; + \right.$$
$$\left. \int_{\underline{\phi}}^{u'(c)} \frac{u'(c)}{t} \Big( -a_1(\tilde{G}(t), sN(c)) \Big)\, h(\tilde{G}(t), sN(c))\, \tilde{G}'(t)\; dt \right] \tag{15}$$

We can use the market equilibrium equation (13) to divide out $\Omega$ and obtain a relationship independent of money growth $\gamma$. If $u'(c)c$ is non-increasing in $c$,[6] the behavior of this relationship in $(s, c)$-space is driven by the term $-a(1, sN)h(1, sN)$. The term is positive for $s > 0$, zero for $s = 0$, and converges to zero for $s \to \infty$. It must therefore strictly increase for low levels of search effort, and decline back to zero for high levels of search effort. As a result, the relationship describes an inverted u-shaped curve in $(s, c)$-space (see figure 1). This curve is unaffected by monetary policy and reflects both market equilibrium and the optimal choice of search effort.

**Theorem 2.1.** *If the following conditions hold then an SSMSE* $\{c, s\}$ *exists, is unique, and its components* $\{\Omega, c, s\}$ *are fully described by* (11), (13) *and* (15) *for each time period* $t$.

*E1. The matching process satisfies assumptions M1–6 in appendix B.*

*E2. The sub-utility of consumption is such that* $u'(c)c$ *is strictly decreasing in* $c$.

*E3. The money growth rate* $\gamma$ *is constant and not less than* $\beta$.

*E4. The cost distribution* $G(\phi)$ *is well-behaved: it is unimodal and it does not put too much weight on the tails.*

*Proof.* See Herrenbrueck (2015). Conditions E1 and E2 could be relaxed, if desired, for fortuitous combinations of parameters. □

For suitable matching processes (see appendix B), this optimal choice of search effort is a stable interior equilibrium, unlike in the original model of Head and Kumar (2005).[7]

[6] Head and Kumar (2005) make this assumption throughout.

[7] The reason is that consumption as a function of search effort, taking prices as given, is strictly concave for certain matching processes. At this point this is a conjecture, but it is easy to verify graphically.

**Free entry**

Define the function $\Gamma(\phi) = \int_{\underline{\phi}}^{\phi} t\,\tilde{G}'(t)\,dt$ (the conditional mean below $\phi$). Then aggregate profits are, in constant-money terms:

$$\Pi = 1 - a(1, sN(c))\frac{\Gamma(u'(c))}{u'(c)} - \int_{\underline{\phi}}^{u'(c)} \frac{\Gamma(t)}{t}\left(-a_1(\tilde{G}(t), sN(c))\right)\tilde{G}'(t)\;dt \tag{16}$$

Firms can enter into the market if they pay a fee fee $k_E < 1$, assessed in constant-money terms[8] and each period.[9] Assume that this entry fee is distributed among households, i.e. added to equation (3). Firms will enter if their expected profits exceed the entry fee, and exit otherwise. Crucially, firms will only learn their cost after paying the entry fee. Therefore, the free-entry condition is:

$$\begin{aligned}\Pi &= N(c)k_E \\ &= \bar{N}G(u'(c))k_E\end{aligned} \tag{17}$$

The SMSE is now summarized by $\{\Omega, c, s, \bar{N})\}$, which solve equations (11), (13), (15), and (17). Dividing $\Omega$ and $\bar{N}$ out of equation (15) using equations (13) and (17) yields a curve in $(sN, c)$-space that is unaffected by money growth. The market equilibrium equation (13) depends only on $sN$, not on $s$ or $N$ directly. The only difference is that the new optimal search curve is a bit flatter than previously, but it has the same shape, and the SMSE exists and is unique under the same conditions as before, with the addition of $k_E < 1$ to ensure that some firms do enter.

**Effects of inflation**

In a flexible-price model like this one, money is neutral and only *expectations* of future money growth affect the value of money, $\Omega$, and therefore the real economy. This is easily obscured when focusing on steady states alone, but worth keeping in mind for the following analysis. More details and proofs are provided in Herrenbrueck (2015).

As shown in appendix A.1, there exists a consumption-maximizing rate of

[8] The qualitative results are the same if the cost is assessed in labor terms.

[9] Like in Melitz (2003), this simplifies the analysis, but ignores the effect of interest rates on funding costs.

money growth $\hat{\gamma} > \beta$. Consumption is increasing in $\gamma$ for $\beta < \gamma < \hat{\gamma}$ and decreasing in $\gamma$ otherwise. Search effort is increasing in $\gamma$ throughout. With free entry, the number of firms is decreasing in $\gamma$ (see figure 2).

If we define the aggregate nominal price level by $P = \frac{M}{c}$, we can also discuss the effects of money growth on inflation. Clearly, in a steady state, money growth must equal inflation. Considering a transition from one steady state into another, however, they may be quite separate, because of the effect on consumption growth. Consider a world where expectations of money growth are so low that consumption would increase with higher money growth ($\gamma < \hat{\gamma}$). In a new steady state with lower expectations of money growth, consumption would be lower, too. By the definition of the price level $P$, price inflation then falls by less than money growth, and may even rise in rare cases, between the two steady states.

Lastly, it is worth paying attention to the interaction between the search cost $\mu$ and the optimal rate of money growth in steady state. A higher $\mu$ shifts the optimal search curve down in $(s, c)$-space, implying a lower level of consumption for a given search effort. Assume that $\gamma$ was previously such that the consumption maximum was achieved. As the market equilibrium curve slopes up for any level of $\gamma$, the new consumption maximum is to the right and below the formerly optimal market equilibrium curve. Consequently, the optimal rate of inflation is increasing in the search cost $\mu$; and the same thing is true for the cost of firm entry, $k_E$, when there is free entry of firms.

## 2.3 The open economy

Let there be two countries, Home and Foreign, and denote Home variables with the subscript $H$ and Foreign variables with the subscript $F$. Define the nominal exchange rate $E$ in terms of Home currency divided by Foreign currency, and define the real exchange rate using the Home and Foreign price levels: $\varepsilon = \frac{E\,P_F}{P_H}$ (in terms of Home consumption divided by Foreign consumption). By the definition of the price level $P$, this implies:

$$\varepsilon = E\,\frac{M_F\,c_H}{M_H\,c_F}$$

The model is much easiest to solve in terms of the wage-based exchange rate (in terms of Home labor divided by Foreign labor, analogous to using a PPI to deflate the nominal exchange rate instead of the CPI), because this is the exchange rate relevant for international competition between firms. Define

$$\begin{aligned} v &= E\,\frac{M_F\,w_F}{M_H\,w_H} \\ &= \varepsilon\,\frac{c_F\;\Omega_H}{c_H\;\Omega_F}. \end{aligned} \tag{18}$$

Trade proceeds as follows. Firms can advertise prices in either country, but denominated in local currency.[10] From the point of view of the consumer, domestic and imported goods are indistinguishable. There is also no bias on how likely a consumer is to observe a domestic or importing firm. Whenever a consumer likes the price and wants to purchase the good, the firm produces the good and ships it to they buyer. Of course, goods cannot be shipped costlessly. Trade costs arise in the familiar iceberg form: in order to sell $x$ units of a good in a foreign market, a firm needs to ship $\tau x$ units from home, with $\tau > 1$. So while there is no bias against imported goods, domestic firms are able to charge lower prices on average than the importing competition.

In order to simplify the analysis, I assume that the cost distribution of domestic firms is the same in each country, and that any firm can export if is able to compete abroad. Assume that the mass of potential firms is $N_H$ at Home and $N_F$ in the Foreign country. The distribution of active firms in either country is therefore:

$$\tilde{G}_H(\phi) = \frac{N_H}{N_H + N_F}\;\frac{G(\phi)}{G(u'(c_H))} \;+\; \frac{N_F}{N_H + N_F}\;\frac{G(\frac{\phi}{v\tau})}{G(\frac{u'(c_H)}{v\tau})} \tag{19}$$

$$\tilde{G}_F(\phi) = \frac{N_H}{N_H + N_F}\;\frac{G(\frac{\phi v}{\tau})}{G(\frac{u'(c_F)v}{\tau})} \;+\; \frac{N_F}{N_H + N_F}\;\frac{G(\phi)}{G(u'(c_F))} \tag{20}$$

As before, the distribution of quotes a buyer receives depends not only on his search effort, but also on the mass of active firms:

$$N_H(c_H, v) = N_H\,G(u'(c_H)) \;+\; N_F\,G(\frac{u'(c_H)}{v\tau}) \tag{21}$$

$$N_F(c_F, v) = N_H\,G(\frac{u'(c_F)v}{\tau}) \;+\; N_F\,G(u'(c_F)) \tag{22}$$

[10] This assumption is not essential for the the results of the model, but convenient. It can be motivated by assuming that households only recognize local currency as counterfeiting-proof. Alternative motivations, such as an infinitesimal cost of conducting transactions in any foreign currency (e.g. in Geromichalos and Simonovska, 2014), would deliver the same results.

To avoid having to track the bounds of the support of the cost distribution, I assume that $G(\phi) \in (0,1)$ for all $\phi > 0$. Existence of an equilibrium does require that the density $G'(\phi)$ declines to zero quickly enough as $\phi \to 0$. In practice, a log-normal distribution of costs seems to work well, and so would a log-logistic or Weibull distribution.

**Balance of Payments**

After trading goods, firms can visit an instantaneous, perfectly competitive foreign exchange market in order to obtain the domestic currency valued by its workers and owners. In steady state (or without capital markets), currency flows must exactly offset each other. The value of Foreign currency revenue gained by Home exports must equal the value of Home currency revenue paid for Home imports. After substituting equation (18) for the nominal exchange rate, we obtain the balance of payments relationship:

$$v = \frac{\Omega_H}{\Omega_F} \frac{\int_0^{\frac{u'(c_H)}{v\tau}} a(\tilde{G}_H(v\tau t), s_H N_H(c_H, v))\, G'(t)\, dt}{\int_0^{\frac{v}{\tau}u'(c_F)} a(\tilde{G}_F(\frac{\tau}{v}t), s_F N_F(c_F, v))\, G'(t)\, dt} \frac{N_F\, G(\frac{v}{\tau}u'(c_F))}{N_H\, G(\frac{u'(c_H)}{v\tau})} \tag{23}$$

**Equilibrium, solving for equilibrium, and comparative statics**

**Definition 2.** An open-economy steady-state equilibrium consists of the seven endogenous sequences $\{\Omega_{Ht}, \Omega_{Ft}, c_{Ht}, c_{Ft}, s_{Ht}, s_{Ft}, v_t\}_{t=0}^{\infty}$ which solve the following equations:

I. Equation (11) for Home. It links $\Omega_H$ and $c_H$.

II. Equation (11) for Foreign. It links $\Omega_F$ and $c_F$.

III. Equation (13) for Home, replacing $\tilde{G}(\,\cdot\,)$ by (19) and $N(c)$ by (21). It links $\Omega_H$, $c_H$, $s_H$, and $v$.

IV. Equation (13) for Foreign, replacing $\tilde{G}(\,\cdot\,)$ by (20) and $N(c)$ by (22). It links $\Omega_F$, $c_F$, $s_F$, and $v$.

V. Equation (15) for Home, replacing $\tilde{G}(\,\cdot\,)$ and $N(c)$ as for (III.). It links $\Omega_H$, $c_H$, $s_H$, and $v$.

VI. Equation (15) for Foreign, replacing $\tilde{G}(\cdot)$ and $N(c)$ as for (IV.). It links $\Omega_F$, $c_F$, $s_F$, and $v$.

VII. Equation (23), which links all seven endogenous variables.

A numerical solution to this system involves six numerical integrals, which may have to evaluated many times as a solution is approached. This procedure can be sped up dramatically by evaluating each integral on a grid of endogenous variables and interpolating the equation on this grid. Because of the curse of dimensionality, the number of endogenous variables involved in each equation should be as small as possible.

For example, steady state solutions can be found as follows. Firstly, replace all instances of $\Omega_H$ and $\Omega_F$ using equations (I.) and (II.). Secondly, while equation (VII.) still involves five endogenous variables, it can be split into two parts, the first involving the home endogenous variables plus $v$, the second one involving the foreign endogenous variables plus $v$. There are six integrals left, each needing to be evaluated on a three-dimensional grid. If the grid contains $p$ points in one dimension, $6 \times p^3$ integrals have to be evaluated in order to create a manageable system of five interpolated equations in five variables. After this procedure, solutions can be found easily. The productivity parameters have to be hard-coded into the interpolation, but the money growth and search cost parameters can be introduced into the last stage, which makes their comparative statics very easy to examine.

Unfortunately, free entry would require two additional integrals (for profits), and the dimension of each integral would go up to five when $N_H$ and $N_F$ are made endogenous. So a grid that contains $p$ points in each dimension would require $8 \times p^5$ integral evaluations. For this reason, free entry would complicate the computation considerably.

Turning to the effects of money growth: An increase in the Home money growth rate $\gamma_H$ increases both Home and Foreign search effort (although naturally, Home search effort responds more strongly if $\tau > 1$). It increases Home consumption when $\gamma_H$ is low and decreases it when $\gamma_H$ is high, just as in autarky. It increases the real exchange rate (a Home depreciation), because it increases Home search effort which reduces the market power of Foreign firms selling at Home, reducing their revenue, and balance of payments then forces a depreciation (see figure 3).

As the increase in Home money growth always increases Home search effort, it shifts production to the more efficient firms. As a result, higher Home money growth reduces Home labor demand and increases Home productivity (defined as production divided by labor input; see appendix A.2) throughout. The effect on Foreign productivity is ambiguous and small.

Unlike in the closed economy, higher Home money growth only has a small, but still negative, effect on Home profits. It has a much larger negative effect on Foreign profits, however. (With free entry, "profits" could be substituted by "the number of firms", and the rest of the results would not change qualitatively.)

When search effort is fixed, which is probably the correct model in the short run, the effects are very different. Higher Home money growth $\gamma_H$ decreases Home consumption throughout, barely affects Foreign consumption, and decreases both the constant-money nominal and real exchange rates (Home imports fall, but their value relative to Home exports has to be constant to maintain balance of payments). Because they rely on the strong assumption of balance of payments, these short-run exchange-rate effects probably would not survive the introduction of capital markets into the model.

With the effect of Home money growth on home consumption as described, consumption is maximized with a money growth rate above the Friedman rule. This consumption-maximizing Home money growth rate increases in $\mu_H$ (as was the case in autarky), is barely affected by $\mu_F$, and decreases in Foreign money growth $\gamma_F$. Consequently, when countries play a non-cooperative game, each choosing the money growth rate with the goal of maximizing domestic consumption, the game has an interior Nash equilibrium.

# 3 Welfare implications

In this section, I analyze the steady-state effects of constant money growth on the household utility function (1). Steady state implies that we can focus on a single representative period.

## 3.1 Costs and benefits of money growth

Higher steady-state money growth has the following effects in a closed economy:

- The inflation tax increases market power, which implies lower consumption and/or more search effort.
- The inflation tax increases the real wage, because payment compensates workers for the fixed disutility of production, and payment loses value. This higher real wage implies lower production.
- When households choose their search effort, they only consider the internal effect of better offers, but not the external effect of lower market power. They will tend to search too little. Through stimulating search effort, money growth increases consumption.
- Higher search effort directly decreases utility ("shoe-leather cost").
- Higher search effort shifts matches to the more efficient firms, which implies higher consumption and/or lower labor effort.

Two second-order effects need to be mentioned, too:

- With heterogeneous firms, higher consumption lowers the reservation wage, which allows fewer firms to compete and reduces matching. This will tend to dampen the effects of money growth on consumption.
- With free entry, lower search effort implies more profits and induces entry, and vice versa. This will tend to dampen the effects of money growth on consumption and search effort.

In conclusion, very low money growth implies very low consumption and therefore very low utility. Moderate money growth yields a maximum of consumption, and even higher money growth reduces consumption again. As money growth increases search effort throughout, the resulting increase in "shoe-leather costs" suggests that the optimal rate of optimal money growth is below the level that maximizes consumption. On the other hand, when firms are heterogeneous, higher money growth increases productivity and reduces the amount of labor required in production, which suggests that the optimal rate of money growth is above the level that maximizes consumption. Which effect dominates can only be decided numerically and empirically.

In the open economy, the following effects of an increase in the Home money growth rate need to be considered additionally:

- Higher Home search effort reduces the market power of Foreign exporters, and does not affect Home exports abroad. The result is an improvement in the terms of trade.
- Higher real wages at Home exploit market power abroad.

Both effects increase Home consumption and reduce Foreign consumption. As a result, the optimal money growth rate in the open economy will be higher that in the closed economy, but at the expense of trading partners (see figure 4).[11]

## 3.2 Coordination

As discussed among the comparative statics of the international economy, the consumption-maximizing Home money growth rate is decreasing in the Foreign money growth rate. One can verify numerically that this also holds for the welfare-maximizing Home money growth rate, although only globally. This implies that the joint welfare function obtained by adding both countries' welfare functions is generally submodular in $(\gamma_H, \gamma_F)$, and while it may have multiple Nash equilibria in principle, they must lie close together and may be indistinguishable in practice.

Higher Home money growth unambiguously decreases Foreign consumption, increases Foreign search effort, and increases Foreign labor use. The outcome is unambiguously negative for the Foreign country. The Home welfare function is smooth, at least in the neighborhood of any equilibrium, if the functions $u(c)$, $G(\phi)$, and $a(z, s)$ are smooth. Therefore, the Home country would increase joint welfare by reducing money growth a little bit from any Nash equilibrium level. At very low levels of home money growth, however, Home welfare rises faster than Foreign welfare falls with a small increase in Home money growth. Consequently, there exists at least one combination of money growth rates that maximizes joint welfare, and at least one country must set money growth below the level that would, given the other country's money growth rate, maximize national welfare.

Clearly, monetary coordination is then the best outcome for the two countries, and the difference may be significant. As an example, consider the parameters $\{G \sim \mathrm{logN}(1, \frac{1}{6})$, $\tau = 1.3$, $\sigma = 2$, $\beta = 0.98$, $\mu_H = \mu_F = 0.008\}$. The unique Nash

[11] In the words of Corsetti (2006), inflation is a beggar-thy-neighbor policy.

equilibrium is $\gamma_H = \gamma_F = 1.1$ (nominal interest rates of 12%), while the joint optimum is $\gamma_H = \gamma_F = 1.04$ (nominal interest rates of 6%, half as high).

### 3.3 Currency union

Consider another example, with two countries that are of equal size and alike in every way, except that $\mu_H > \mu_F$.[12] Then any Nash equilibrium and any joint optimum must satisfy $\gamma_H > \gamma_F$. With the same parameters as above, except that $\mu_H = 0.016$ is twice as high as $\mu_F = 0.008$, the optimal money growth rates are as follows:

- Nash equilibrium: $\gamma_H = 1.16$, $\gamma_F = 1.05$.
- Joint optimum: $\gamma_H = 1.12$, $\gamma_F = 1.02$.
- Symmetric joint optimum: $\gamma_H = \gamma_F = 1.05$.

A currency union is defined by the criterion $\gamma_H = \gamma_F$.[13] The numerical example above demonstrates why a currency union can be disadvantageous in the wrong circumstances. At the joint symmetric optimum, the welfare difference between home and foreign is an order of magnitude larger than in the joint optimum (see figure 4). The joint welfare loss is small, but the foreign country gains from the currency union and the home country loses, even when joint welfare is the criterion used to determine the money growth rate.

If we believe that search effort does not respond quickly, because consumers need to keep learning the price distribution and firms need to keep learning the sales function (which depends on search effort), the transition dynamics into currency union become interesting. With the given parameters, consider a move by the Home country from $\gamma_H = 1.12$ to $\gamma_H = 1.05$, while the Foreign country moves from $\gamma_F = 1.02$ to $\gamma_F = 1.05$. As long as search effort remains fixed, Home consumption rises by a dramatic 7 or 8%, while Foreign consumption declines very slightly (probably too slightly to be distinguished from noise).

[12] While $\mu$ is the disutility of search, it has similar static effects as the cost of firm entry $k_E$. As the two-country model is harder to solve with free entry, I understand $\mu$ as standing in for $k_E$, too, in this exercise.

[13] The exact conversion rate $E$ does not affect the steady state, as in the long run, households holds the relative amount of currency that will achieve balance of payments. Certainly, the transition path may matter for welfare.

As soon as search effort adjusts, however, Foreign search effort rises slightly while Home search effort collapses by more than two thirds. As a result, Foreign consumption increases by 2 or 3%, while Home consumption falls by 8 or 9%, below the initial level. In addition, Foreign productivity increases and Home productivity falls. Together, these effects imply the result discussed above: Foreign wins, Home loses, even when joint welfare is used to determine the monetary growth rate in the currency union (see figures 3 and 4).

To make matters worse, the outcome is easily mis-diagnosed as due to different rates of productivity growth at home and abroad. What really happens is that the decline in Home search effort leads to less efficient matches, causing measured Home productivity to fall, while the increase in Foreign search effort causes Foreign productivity to rise.

# 4 Conclusion and discussion

In this paper, I use the monetary search model of Herrenbrueck (2015) to study monetary policy in an international setting. The optimal steady-state money growth rate depends on the fundamentals of the economy, and it can be calculated numerically. Higher inflation necessarily has adverse effects on trading partners (a "beggar-thy-neighbor" policy), so inflation will be lower and welfare will be higher when policy is coordinated.

However, when countries with different fundamentals join a currency union which chooses its inflation rate to maximize joint welfare, at least one country must raise its inflation rate relative to the coordination outcome, and at least one country must lower it. A country that raises its inflation rate gains twice: first, a higher inflation rate relative to optimal coordination must raise its domestic welfare, while second, lower inflation in its trading partner countries improves its terms of trade. On the other hand, by the same mechanisms, a country that lowers its inflation rate to join a currency union loses twice. Certainly, a currency union may or may not improve welfare compared with a non-cooperative Nash equilibrium, or when all members reduce their inflation rates, so it may still be a good idea when coordination is not feasible for some reason.

In future work, I plan to calibrate the model to historical episodes of pegged exchange rates (such as the Bretton Woods system) or currency unions (such as the Eurozone), and compute the welfare outcomes precisely. Preliminary evidence presented in Figure 5 suggests that the model can contribute to explaining the see-saw macroeconomic divergence of two parts of the Eurozone since the early 1990s (here named North and South, defined next to the graph), when plans to form a currency union congealed into fact. Panel (a) shows that until 1994, the economies of the North and the South moved together compared to a rest-of-the-developed-world aggregate, but that after 1995 (when the Euro was formally announced), South first catches up to North but then falls far behind. This is exactly consistent with my model; if search effort and the number of firms take a few years to adjust to the new steady state, a reduction in inflation in the South relative to the North will initially result in relative growth in the South as the inflation tax is reduced, but will ultimately result in relative decline in the South as search effort falls.

Panel (b) shows that indeed, monetary policy tightened in the South relative to the North from the early 1990s until convergence was achieved in 1998. Furthermore, the financial crisis of 2008-09 or the worries about a potential sovereign debt crisis, which began in 2009 and accelerated in 2010, appear to be consequences rather than causes of the macroeconomic imbalances. Relative GDP of the South peaked in 2002, and relative consumption (not graphed) peaked in 2006. Instead, I propose that the turnaround of relative output may be a late consequence of the relative monetary tightening in the 1990s.[14] Currently, however, I have no microeconomic evidence that search effort was the channel of this see-saw divergence. In future work, I plan to study data on price dispersion, sales dispersion, and firm entry, which together should provide a clearer picture of search effort and firm entry in the crucial period of the 2000s.

Whereas the disutility of search effort seems hard to reliably compare across countries, data on the cost of firm entry and operation is readily available. According to the Ease of Doing Business Index (World Bank, 2011), the countries making up the North as defined in Figure 5 average a worldwide rank of 24.5, and the countries of the South average a rank of 63.6 (weighted by 2008 nominal GDP). The conclusions of the model are clear: the optimal inflation rate is higher in the South than in the North, and while the overall downward trend of nominal interest rates in the 1990s may have been good for both regions, the convergence of interest

[14] The German welfare reforms of 2002 are also an unlikely cause of the turnaround; the growth pattern looks identical if Germany is excluded.

rates was likely harmful for the South.

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# A Appendix: Proofs

*Proof of lemma 2.1:* (Based on equation (34) in Burdett and Mortensen (1998).) We can rank

$$\pi(p_1;\phi_1) \ \geq \ \pi(p_2;\phi_1) \ > \pi(p_2;\phi_2) \ \geq \ \pi(p_1;\phi_2) \tag{24}$$

The first and last inequality follow from profit maximization. The middle inequality follows from $\phi_1 < \phi_2$. Subtracting the fourth term from the first, and the third term from the second, and rearranging, we get

$$\frac{p_2}{a(p_2)} \ \geq \ \frac{p_1}{a(p_1)} \tag{25}$$

As $F(p)$ is a continuous c.d.f. with connected support, it is strictly increasing on its support. From its definition, it is easy to see that $a(F)$ is a strictly decreasing function of $F$. Therefore, $a(F(p))$ is a strictly decreasing function of $p$ on the support of $F$. □

*Proof of lemma 2.2:* $p(\phi)$ solves the FOC. Regarding the boundary condition: either $\bar{p}$ is binding for some sellers, in which case the marginal seller will charge $\bar{p}$ and make zero profits. Or the least efficient seller is still able to compete, but must charge the highest price by the ranking condition; then, however, no buyer with another choice will buy from this seller. Therefore, a buyer who is willing to buy from this seller will accept any price up to $\bar{p}$, and the least efficient seller will charge $\bar{p}$. □

## A.1 Effects of inflation

**Lemma A.1.** *In a steady state of a SSMSE, the Friedman rule $\gamma = \beta$ implies $c = 0$, $\gamma > \beta$ implies $c > 0$ and $\gamma \to \infty$ implies $c \to 0$. Therefore, the consumption-maximizing rate of money growth satisfies $\gamma > \beta$.*

*Proof.* For simplicity, keep the number of firms fixed. But the lemma also holds with free entry.

Divide optimal search equation (15) by the market equilibrium equation (13) to obtain (writing $N$ instead of $N(c)$ for simplicity):

$$\mu = Nu'(c)c \, \frac{a(1,sN)h(1,sN) + \int_{\underline{\phi}}^{u'(c)} \frac{u'(c)}{t} \left(-a_1(\tilde{G}(t),sN)\right) h(\tilde{G}(t),sN)\, \tilde{G}'(t)\, dt}{a(1,sN) + \int_{\underline{\phi}}^{u'(c)} \frac{u'(c)}{t} \left(-a_1(\tilde{G}(t),sN)\right) \tilde{G}(t)\tilde{G}'(t)\, dt} \tag{26}$$

Consider this "equilibrium optimal search equation" together with the market equilibrium equation (14) in $(c, s)$-space. The market equilibrium curve slopes upwards, crosses the equilibrium optimal search curve exactly once, and shifts right as money growth $\gamma$ increases: faster money growth reduces the value of money, which creates market power. Constant consumption would therefore require ever-increasing search effort. As $\gamma \to \beta$, it becomes $\Gamma$-shaped and converges to the $c$-axis on the left and the $c = c^*$-line on top (in case of common costs, with $u'(c^*) = \phi$); as $\gamma \to \infty$, it converges to the $s$-axis. The graph of the equilibrium optimal search equation, on the other hand, has an inverted u-shape in $(s, c)$-space, and does not depend on $\gamma$ (or any other expectations), so it contains the set of possible equilibria. It approaches $c \to 0$ both for $s \to 0$ and $s \to \infty$, because of assumptions E2, M2, and M3, and achieves a maximum $\hat{c}$ for $s > 0$. As a result, increasing money growth *traces out* the optimal search curve, and achieves the maximum $\hat{c}$ for $\gamma > \beta$. Higher money growth also reduces profits, which implies exit of firms, and vice versa. This tends to dampen, but not reverse, the effects of money growth on consumption and search. □

**Lemma A.2.** *Let $\mu_2 > \mu_1$.*

*(i) Say that the graph of the equilibrium optimal search equation* (26) *achieves the maximum $\hat{c}$ for $\hat{s}$. Then $\hat{c}_2 < \hat{c}_1$ and $\hat{s}_2 > \hat{s}_1$, that is, the maximum point shifts down and to the right in $(s, c)$-space.*

*(ii) The consumption-maximizing rate of money growth is higher for $\mu_2$ than for $\mu_1$.*

*Proof.* For (i), consider equation (26) above. Conjecture that higher $\mu$ implies lower $c$ due to the $u'(c)c$-term. Thus, $\frac{u'(c)}{\phi}$ is lower, too, so $s$ must rise a bit to make up for it. As a result, the entire curve shifts down and just a bit right. For (ii), note that the market equilibrium curve (14) is upward sloping in $(s, c)$-space. Therefore, if money growth was optimal for $\mu_1$, it must cross the new curve (26) to the left of both the new and the old $\hat{s}$, and higher money growth is necessary to achieve $(\hat{s}_2, \hat{c}_2)$. □

## A.2 Derivation: profits, labor demand, and productivity

Individual firm profits are given by (7). Using the policy function $p(\phi)$ of (10), and defining $\Gamma(\phi) = \int_{\underline{\phi}}^{\phi} t\,\tilde{G}'(t)\,dt$ (the conditional mean below $\phi$), the aggregate profit level given by equation (16) can be derived as follows:

$$\begin{aligned}
\Pi &= \int_{\underline{\phi}}^{u'(c)} \pi(\phi)\,\tilde{G}'(\phi)\,d\phi \\
&= \int_{\underline{\phi}}^{u'(c)} \left(1 - \frac{\phi}{\Omega p(\phi)}\right) a(\tilde{G}(\phi), sN(c))\,\tilde{G}'(\phi)\,d\phi \\
&= 1 - \int_{\underline{\phi}}^{u'(c)} \left[\frac{\phi}{u'(c)} a(1, sN(c)) - \int_{\phi}^{u'(c)} \frac{\phi}{t} a_1(\tilde{G}(\phi), sN(c))\tilde{G}'(t)\right] \tilde{G}'(\phi)\,d\phi \\
&= 1 - a(1, sN(c))\frac{\Gamma(u'(c))}{u'(c)} - \int_{\underline{\phi}}^{u'(c)} \frac{\Gamma(t)}{t} \Big(-a_1(\tilde{G}(t), sN(c))\Big)\,\tilde{G}'(t)\,dt.
\end{aligned}$$

A firm with cost $\phi$ and constant-money price $p$ will sell $1/p$ units of output per consumer, and will therefore have a labor demand of $L(\phi) = \frac{\phi}{p} a(F(p), sN(c))$. Integrating over all firms, it is easy to see the similarity with the previous calculation for profits. Indeed, each firms' profits can be expressed as revenue—wage*labor, and since total revenue is 1 (in constant-money terms), total labor $L$ satisfies: $\Pi + wL = 1$, or:

$$L = \Omega\,(1 - \Pi)\,, \tag{27}$$

with $\Pi$ as above.

Define productivity to be consumption divided by labor input, $\varphi \equiv c/L$. We obtain the full expression for aggregate productivity (reintroducing $N(c)$):

$$\varphi(s, c) = \frac{a(1, sN(c))\frac{\Gamma(u'(c))}{u'(c)} + \int_{\underline{\phi}}^{u'(c)} \frac{\Gamma(t)}{t} \Big(-a_1(\tilde{G}(t), sN(c))\Big)\,\tilde{G}'(t)\,dt}{a(1, sN(c)) + \int_{\underline{\phi}}^{u'(c)} \frac{u'(c)}{t} \Big(-a_1(\tilde{G}(t), sN(c))\Big)\,\tilde{G}(t)\tilde{G}'(t)\,dt}, \tag{28}$$

which is not directly dependent on expectations, hence easy to calculate given an equilibrium solution $(s, c)$.

# B Appendix: Matching

The assumptions on the matching process $Q(k|\eta)$ sufficient for the results in this paper are:

M1. $Q(k|\eta)$ is a cumulative distribution function with support $\{1 \dots K\}$, where $K \in \{2 \dots \infty\}$. $q_k(\eta)$ is used to denote the probability mass function.

M2. $q_1(\eta) \in (0, 1)$ for all $\eta > 0$, and $q_1(0) = 1$.

M3. When $\eta' > \eta$, $Q(k|\eta') < Q(k|\eta)$: the distribution of quotes with higher search effort strictly first order stochastically dominates the one with lower search effort.

M4. $Q(k|\eta)$ is differentiable in $\eta$.

M5. $q_1'(\eta)$ is bounded as $s \to 0$.

M6. There exists an $\varepsilon > 0$ such that the series $\sum_{k=1}^{K} k^{2+\varepsilon} \left(q_k'(\eta)\right)^2$ converges for any $\eta > 0$, and that the series limit approaches 0 as $\eta \to \infty$.

A particularly tractable family of matching processes can be derived in the following way. Start by assuming that the number of quotes $k$ follows a negative binomial distribution on $k \in \{0, 1, \dots\}$, parametrized in the usual way with "number of failures" $r > 0$ and "success probability" $p \in (0, 1)$. (In this interpretation, $r$ is an integer but as we see below, the extension to non-integer values of $r$ is smooth.) In that case, the probability mass function is:

$$q_k = \frac{\Gamma(k+r)}{\Gamma(k+1)\Gamma(r)}(1-p)^r p^k$$

The mean number of quotes is $\bar{k} = rp/(1-p)$. Here, a more convenient parametrization is using the unconditional mean $\eta \equiv \bar{k}$ and a dispersion parameter $\rho \equiv 1/r$ to replace $(r, p)$.[15] We obtain the following probability mass function:

$$q_k = \frac{\Gamma(k+1/\rho)}{\Gamma(k+1)\Gamma(1/\rho)}(1+\rho\eta)^{-1/\rho}\left(\frac{\rho\eta}{1+\rho\eta}\right)^k$$

[15] This parametrization is often used in econometrics when estimating count data.

Using the definitions of the transactions function $J$ and the arrival function $a$, we can compute them in closed form:

$$
\begin{aligned}
J(F) &= \sum_{k=1}^{\infty} \frac{q_k}{1-q_0} \left(1-(1-F)^k\right) \\
&= \frac{1-(1+\rho\eta F)^{-1/\rho}}{1-(1+\rho\eta)^{-1/\rho}},
\end{aligned}
$$

which clearly satisfies $J(0) = 0$ and $J(1) = 1$, and:

$$
\begin{aligned}
a(F) &= \sum_{k=1}^{\infty} \frac{q_k}{1-q_0} k\,(1-F)^{k-1} \\
&= \frac{\eta}{1-(1+\rho\eta)^{-1/\rho}} (1+\rho\eta F)^{-1-1/\rho},
\end{aligned}
$$

which is clearly positive and decreasing, as required. (Because these functions are defined as proportions of those matched, they are automatically conditional on $k > 0$, so for all of the cases discussed below, whether we allow $k = 0$ only matters for a household's decisions and not for those of the firms.) A very nice feature is that this solution to the matching process extends more generally than $\rho \in (0, \infty)$. We can distinguish four special cases:

**Binomial:** Let $\rho < 0$ and $\eta \in (0, -1/\rho)$. If $-1/\rho$ is a positive integer, then the number of quotes follows a binomial distribution with maximum $K = -1/\rho$, success probability $-\rho\eta$, and mean $\eta$. A subcase is when $K = 2$ (shoppers have either zero, one, or two quotes): this is the most commonly used case in the literature building on Burdett and Judd (1983). It has an appealing foundation (Head and Kumar, 2005): if shoppers could choose the probability weights $q_k$ subject to linear costs of receiving quotes, they would choose to mix between $q_1$ and $q_2$ only. But for the purposes in this paper, negative $\rho$ is inconvenient: household consumption becomes a *convex* function of search effort, requiring a mixed strategy equilibrium for the choice of search effort. The other cases (which put more weight on receiving large numbers of quotes) do not have this problem.

**Poisson:** Taking the limit as $\rho \to 0$, for fixed $\eta > 0$, the number of quotes follows a Poisson distribution with intensity $\eta$ (Mortensen, 2005; Head and Lapham, 2006), and the arrival function becomes exponential: $a(F) \propto \exp(-\eta F)$.

**Negative Binomial:** As discussed in the derivation, when $\rho > 0$ and $\eta > 0$, the number of quotes follows a negative binomial distribution with number of failures $1/\rho$ and success probability $\rho\eta/(1+\rho\eta)$.

**Logarithmic:** Taking the limit as $\rho \to \infty$, the probability $q_0$ of receiving no quotes at all converges to 1 for any fixed mean. So the distribution only converges to a well-defined limit if we condition on $k > 1$ and if $\eta$ is rescaled appropriately. In that case, the limit is the discrete logarithmic distribution $q_k = \left(\frac{\eta}{1+\eta}\right)^k \frac{1}{\log(1+\eta)}/k$, with mean $\eta$ and $k \in \{1, \ldots, \infty\}$.[16] The associated transactions functions are:

$$J(F) = \frac{\log(1+\eta F)}{\log(1+\eta)}$$
$$a(F) = \frac{\eta}{\log(1+\eta)}\left(1+\eta F\right)^{-1}$$

Because of its scope, I refer to this family of matching processes as the **generalized binomial** family. [17] It has one property that could be very useful for empirical applications: if $\rho > 0$, the distribution of customers per firm is a truncated Pareto distribution with shape parameter $\rho/(1+\rho)$, and the ratio of the upper to the lower bound of the distribution is $a(0)/a(1) = (1+\rho\eta)^{1+1/\rho}$.

Specifically, as $a(F)$ is the proportion of customers that buy from a firm with position $F$ in the price distribution, the inverse of $a(F)$ describes the fraction of firms charging a lower price than a firm with $a(F)$ customers, which is also the fraction of firms with more customers than $a(F)$. If we allow $q_0 > 0$ (some searchers go unmatched), the CDF of the *number* of customers per firm is:

$$F(x) = \frac{1+\rho\eta}{\eta} - \frac{1}{\rho\eta}\left(\frac{x}{\eta}\right)^{-\rho/(1+\rho)}.$$

[16] To distinguish this distribution from the continuous logarithmic distribution, some authors call it the "log-series distribution" because it can be derived from the power series of the logarithm.

[17] The fact that the region $\rho \in (-\infty, -1/\eta)$ is excluded has an intuitive interpretation, too. It corresponds to the restriction in Burdett and Judd (1983) that shoppers must receive more than one quote with positive probability; otherwise, all firms will charge the reservation price, and due to the lack of price dispersion, no equilibrium with positive search intensity exists.

# C Appendix: Illustrations

For all calculations, the representative utility function is:

$$U\left(\{s_t, c_t\}_{t=0}^{\infty}\right) = (1-\beta)\sum_{t=0}^{\infty}\beta^t\left[\frac{c_t^{1-\sigma}-1}{1-\sigma} - \mu s_t - L(s_t, c_t)\right].$$

The matching process is $q_k(\eta) = \frac{\eta^{k-1}}{(1+\eta)^k}$, which yields $\bar{k} = 1+\eta$ and $a(F,\eta) = \frac{1+\eta}{(1+\eta F)^2}$.

Aggregate work effort $L(s_t, c_t)$ is defined in (27). The disutility of work effort is normalized to 1. Steady state is equivalent to a single representative period. Home and Foreign have the same size and the same number of firms $N_H = N_F = 1$ (except where "free entry" is indicated).

Figure 1: **General equilibrium in autarky.** The parameters are $\beta = 0.98$, $\sigma = 2$, $\mu = 0.008$, and $G(\phi) \sim \mathrm{logN}(1, \frac{1}{6})$. The continuous lines represent steady-state market equilibrium (equation 14), and the contours are such that steady-state nominal interest rates would be {2%, 4%, 8%, 16%, 36%} (left to right, light to dark). The dashed line represents equilibrium optimal search (equation 26).

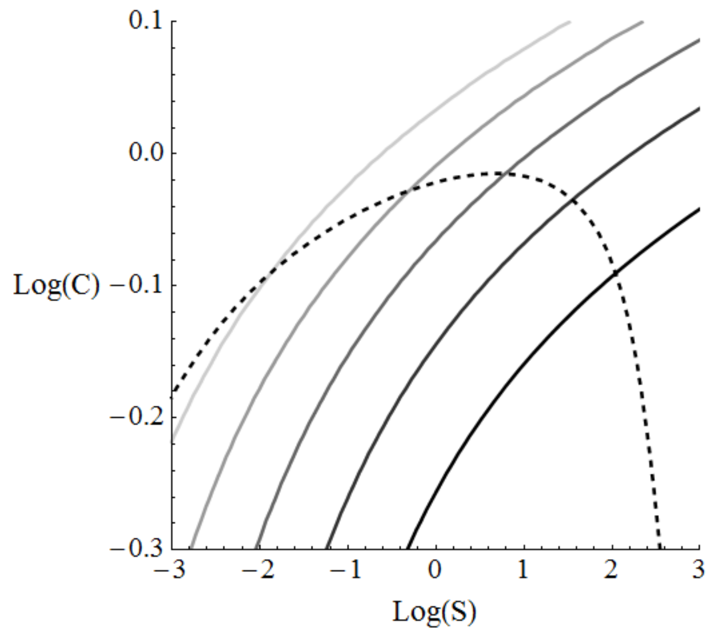


Figure 2: **Effects of inflation in autarky.** The parameters are $\beta = 0.98$, $\sigma = 2$, $\mu = 0.008$, and $G(\phi) \sim \mathrm{logN}(1, \frac{1}{6})$ for both panels, and $k_E = 0.2$ for panel (b).

(a) Effects of inflation (no entry)

(b) Effects of inflation (free entry)

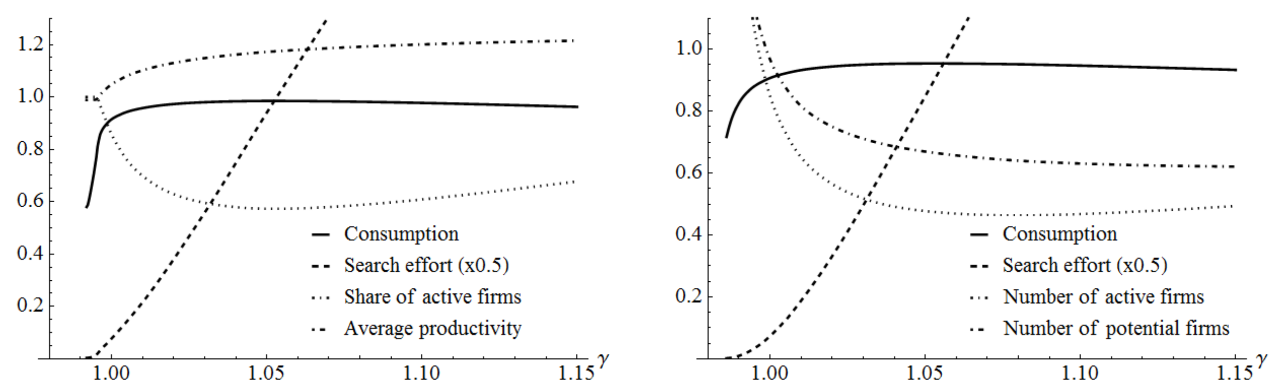

Figure 3: **Effects of inflation in the open economy, without free entry.** The parameters are $\beta = 0.98$, $\sigma = 2$, $\mu_F = 0.008$, and $G(\phi) \sim \mathrm{logN}(1, \frac{1}{6})$ for all panels; $\mu_H = 0.008$ for panels (a) and (b); $\mu_H = 0.016$ for panels (c) and (d). The vertical line represents Foreign money growth $\gamma_F$.

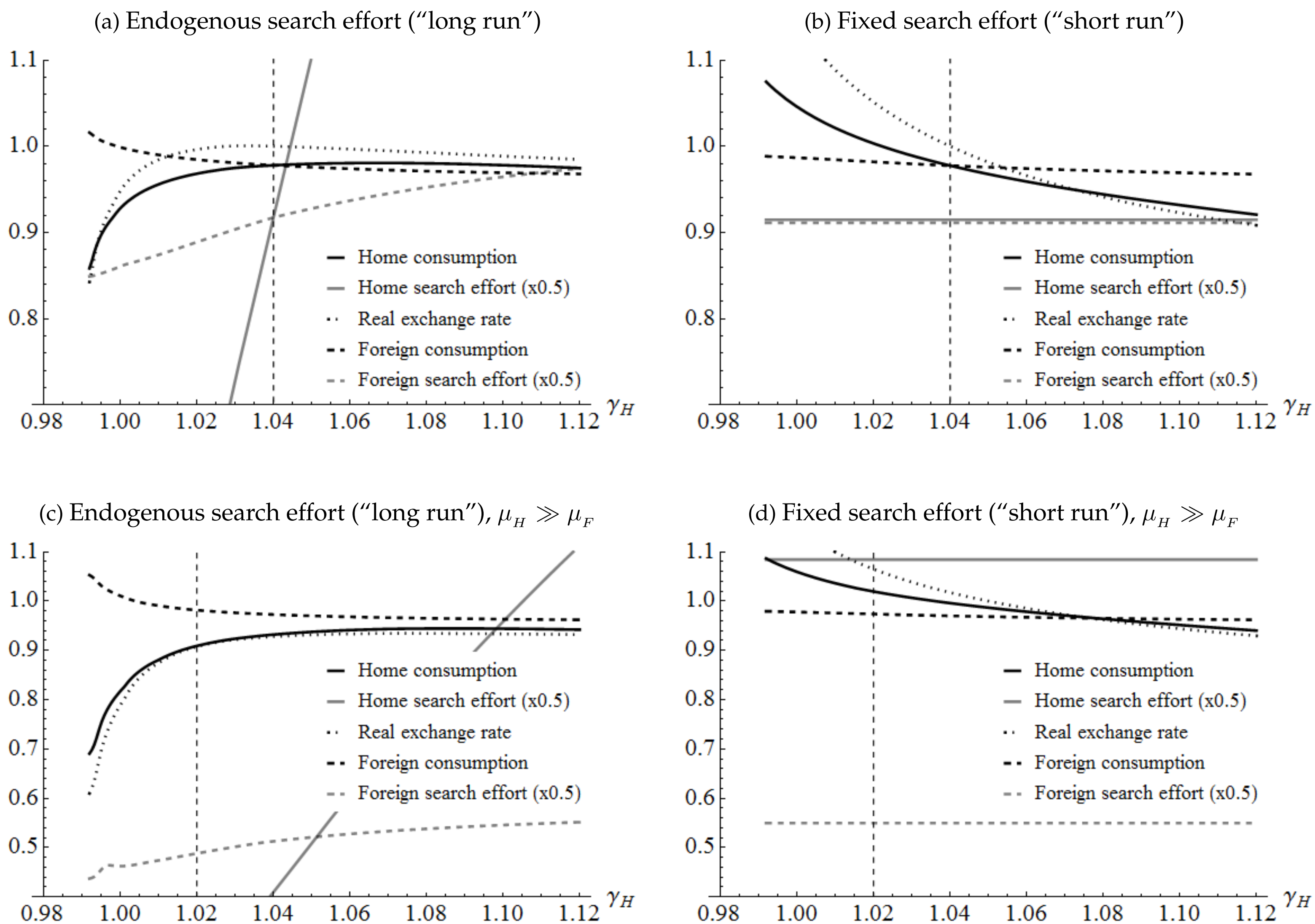

Figure 4: **Welfare effects of inflation in the open economy, without free entry.** Welfare is measured as the natural logarithm of equivalent consumption. The parameters are $\beta = 0.98$, $\sigma = 2$, $\mu_F = 0.008$, and $G(\phi) \sim \log\mathrm{N}(1, \frac{1}{6})$ for all panels; $\mu_H = 0.008$ for panel (a); $\mu_H = 0.016$ for panels (b) and (c). The vertical line represents Foreign money growth $\gamma_F$.

(a) Symmetric case: $\mu_H = \mu_F$

(b) Asymmetric case: $\mu_H \gg \mu_F$

(c) Asymmetric case with a currency union: $\mu_H \gg \mu_F$ and $\gamma_H = \gamma_F$

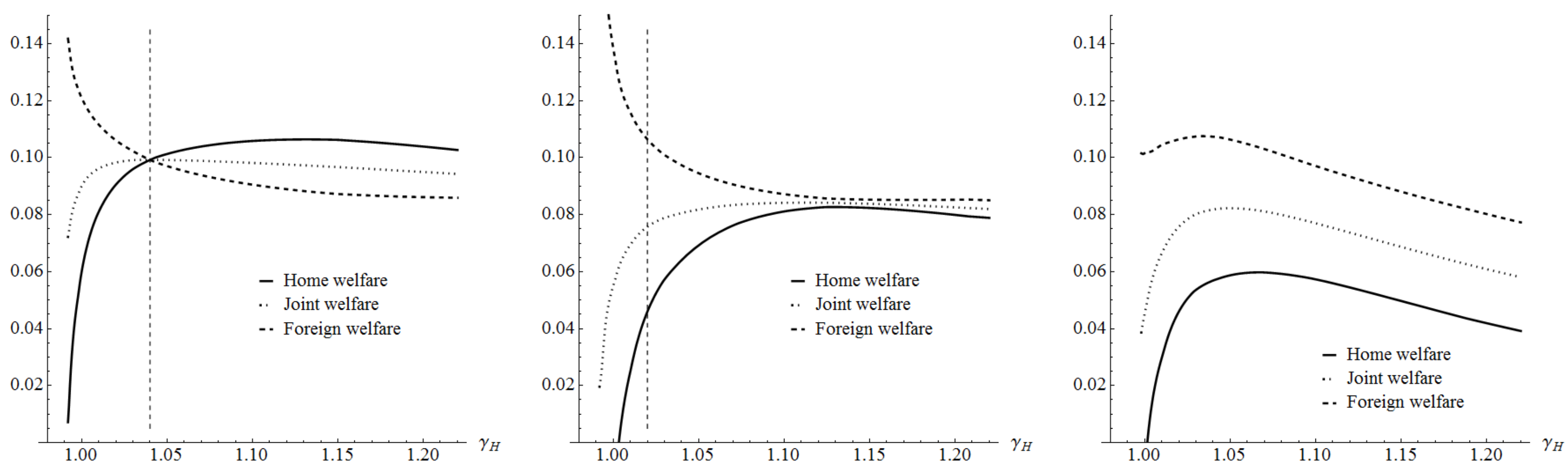

Figure 5: **The Eurozone divergence.** Real GDP (source: IMF) and interest rates on 10-year government bonds (source: Eurostat), weighted by 2008 nominal GDP, for the following three regions:
South: Ireland, Italy, Portugal, Spain.
North: Austria, Belgium, Finland, France, Germany, Netherlands.
Rest-of-world: Canada, Denmark, Japan, Norway, Sweden, Switzerland, UK, USA.
Excluded due to insufficient data: Luxemburg, Greece, Eastern Europe.

(a) Real GDP comparisons (detrended by subtracting a geometric trend) (b) Real GDP (detrended) and nominal interest rates