Entrepreneurial Finance: Role of
Financier Risk Aversion Heterogeneity & Search by Entrepreneurs

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This Version: December 2015

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This research is supported by a grant from the Canadian Social Science & Humanities Research Council (SSHRC). The authors would like to thank various conference and seminar attendants for useful comments and discussion, and in particular, Jim Brander, Romesh Chandra, Douglas Cumming, Joanne Oxley, Michael Shih and Joseph Vu.

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Abstract

This paper studies the role of heterogeneity in financier risk preferences and competitive searching in venture finance deals. We analyze a double-hazard agency model in which entrepreneurs search for the best deal from financiers who maximize the expected utility of their investment return to determine equity shares that induce optimal efforts. Model results generate several novel insights on characteristics influencing expected investment returns and equity shares and on the coexistence of angels/VCs and the venture funding cycle. Without search, lower risk-aversion financiers have an advantage as their expected investment return is higher, the equity share given lower, and they can profitably fund higher risk projects. With search, an equilibrium emerges that reduces the advantage (disadvantage) of lower (higher) risk-aversion financiers. Since VCs can reasonably be associated with lower risk-aversion financiers relative to angels, the model predicts that angels/VCs with diverse risk preferences can coexist profitably in an industry that would otherwise favor VCs. The model further predicts that larger financiers (VCs) will prefer to finance at a later stage because their expected return rises with the venture's initial value and financier productivity. Hence, the model theoretically justifies the pattern of funding activity by angels/VCs in the venture finance industry.

Keywords: financier risk preferences, angels/VCs funding, competitive search, double-sided moral hazard, expected investment returns, equity shares.

JEL Codes: G24, G32, D53, D83, L26

I. INTRODUCTION

Entrepreneurial finance remains an active and topical area of research. New ventures formed by entrepreneurs typically obtain financing from private investors such as angels and venture capitalists (VCs) instead of traditional sources such as banks or public stock markets. An extensive theoretical literature seeks to explain the nature of venture capital contracts, and explores the implications for entrepreneurs, venture capitalists (VCs) and policy-makers. As well as theories about why start-ups may find equity preferable to debt contracts (Ueda (2004), Chemmanur and Chen (2006), De Bettignies and Brander (2007)) and the nature of projects financed by equity (Winton and Yerramilli (2008)), researchers have attempted to offer theoretical rationales for a range of specific equity contracts. These include convertible debt-equity contracts (Berglöf (1994); Hellmann (2006); Hellmann (1998); Casamatta (2003), Kaplan and Stromberg (2003)); staged financing whereby venture capitalists release funds in discrete tranches (Bergemann & Hege, (1998), Li (2008)); and vesting and control rights (Gompers and Lerner (1999)). Other theoretical work explores strategic interactions between entrepreneurs and financiers, including the possibility of opportunistic behavior (Cornelli & Yosha (2003), Schmidt (2003), Schure (2003), Elitzur & Gavious (2003), Fairchild (2011)). This work provides a rich theoretical landscape that helps us to comprehend a wide variety of equity contract forms and their role in financing entrepreneurial activity.

Plain vanilla equity contracts, which stipulate a simple division of future venture value between entrepreneur and financier, remain in widespread use around the world (Cumming, 2005). They are especially common in the context of angel finance, where these investors typically invest smaller amounts at early stages of venture development. Studies report that angel financing is significant relative to venture capital financing both in terms of the number of new firms receiving funding and the value of financial investment. For example, Wong, Bhatia and Freeman (2009) note that the National Venture Capital Association assesses the size of the angel market in the United States to be $100 billion and the institutional venture capital market to be $48.3 billion. Further, recent empirical research suggests that angel investors and VCs can act as ‘dynamic substitutes’ in that companies that obtain VC funding are less likely to obtain subsequent angel funding and vice versa (Hellmann, Schure and Vo (2013)). Since both types of financiers can in principle provide funding at the early stage, a salient question is what factors lead to concentration of angel funding at early stages of venture finance? Furthermore, prior to settling on a particular deal, entrepreneurs search for the best funding deal from several potential financiers (angels, VCs). Yet relatively little is known about how heterogeneity in financier risk preferences and competitive searching interact with venture characteristics to determine critical funding outcomes such as the expected investment return and equity division. This is somewhat surprising in view of the central importance of these important funding parameters to financiers and entrepreneurs. Therefore, we would expect angels/VCs, entrepreneurs and practitioners to be keenly interested in gaining a deeper understanding of factors that endogenously determining expected investment returns and equity shares.

The purpose of this paper is to fill these gaps. We do so by analyzing a double-hazard agency model in which entrepreneurs search for the best financing deal from financiers who maximize the expected utility of their investment return and determine the equity contract to induce optimal efforts from both the entrepreneur and the financier. Results from the model generate several insights on how the financier’s expected investment return and equity shares depend on various venture characteristics (e.g. project risk, initial project value, funding size, financier/entrepreneur productivities) and on the coexistence of angels/VCs and the venture funding cycle.

The model predicts that, without search, lower risk-aversion financiers have an advantage in the venture finance market: their expected investment return is higher, the equity share given to entrepreneurs is lower, and they can more profitably finance higher risk projects. In the presence of competitive search for the best funding deal by entrepreneurs, an equilibrium emerges that allows a broader and more diverse set of risk-averse financiers to operate profitably by reducing the advantage in expected investment return of lower risk-aversion financiers and the disadvantage of their higher risk-aversion counterparts. Since VCs can reasonably be associated with lower risk-aversion financiers relative to angels (see discussion below in Section III.B regarding differences in diversified portfolio of venture investments and funding capacity), these results suggest that searching enables both angels and VCs with heterogeneous risk preferences to coexist in an industry that would otherwise favor VCs.

The model further predicts that financiers with access to larger investment funds (e.g. VC firms) will benefit by funding ventures at the later stage of development because their expected investment return rises with the venture's initial value and financier productivity. At the later stage, there is an increase in both the ventures' initial value and venture capitalists' ability to add value by providing advice and management services (e.g. Wong, Bhatia and Freeman (2009), Shane (2009)). Hence, the model theoretically supports the pattern of funding activity by angels/VCs observed in the venture finance industry.

Many of our results depend on some new and unique features of the model. This includes allowing the generality of a risk-averse financier, competitive search by entrepreneurs, using the investment return as the key analysis variable (it is independent of investment scale), and modeling the venture's future value as a continuous random variable that depends functionally on a rich set of inputs (e.g. initial venture value, funding size, entrepreneur/financier productivities and efforts). Venture finance agency models typically assume risk-neutral financiers and outcomes take binary values (with effort affecting the probability of the higher outcome). This is also among the first papers in entrepreneurial finance that incorporates equilibrium over financiers with heterogeneous risk preferences by entrepreneurs.

As briefly surveyed earlier, a broad theoretical literature has developed to explain the nature of venture capital contracts and their implications. We discuss below some relevant aspects of this literature and relate it to the model and analysis of this paper.

One of these streams explores the boundaries between VCs and bank financing. Ueda (2004) assumes that VCs have greater capabilities to screen entrepreneurial opportunities than banks and, therefore, need to rely less on collateral to finance entrepreneurs. Their model predicts that VCs are more likely than banks to finance companies with high return and risk profiles; and entrepreneurs who possess relatively little collateral will be likelier to seek VC financing. De Bettignies and Brander (2007) point to financier effort as a major distinction between VCs and banks and that equity contracts best balance the incentives for both contracting parties when the marginal value of VC effort is sufficiently high. In a similar spirit, this paper extends our understanding of the coexistence of financiers with heterogeneous risk preferences and differing funding capacity (e.g. angels, VCs) in a context of searching by entrepreneurs for the best funding deal.

The literature has also examined strategic issues at the exit stage of the venture. Berglöf (1994) analyzes the potential conflict of interests between entrepreneurs and VCs that arises from a future sale of the company to a third party. The entrepreneur values private benefits of control that increase with firm value and is afraid of a sale that compensates him insufficiently for their loss while the VC is afraid of a premature 'cheap exit' that lead to poor investment returns. Hellmann (1998) studies why entrepreneurs give up control rights to VCs who expend costly effort to search for a superior management team. This exposes entrepreneurs to the risk of being fired before their shares are fully vested. In subsequent work, Hellmann (2006) shows why venture capitalists use contracts with convertible preferred equity to mitigate concerns regarding exit decisions as they allocate different cash flow rights for acquisitions and IPOs. With a similar focus on exit, Elitzur & Gavious (2003) analyze a sequential investment game with moral hazard comprising entrepreneurs, angels and VCs where angels invest before VCs and are concerned of value transfer to VCs; all agents are risk-neutral.[1] Inderst & Müller (2004) adopt a bargaining perspective and argue that bargaining power between entrepreneurs and VCs, as reflected in equity shares inter alia, varies with technological and economic conditions. Fairchild (2011) allow for potential 'stealing' by either the entrepreneur or the financier at the IPO stage and show how entrepreneur/angel empathy relative to the VC's value-adding abilities affects financing choices. Our analysis incorporates double-sided hazard but abstracts over issues related to the mode of exit since our primary focus is on understanding how heterogeneity in financier risk preferences interacts with important venture characteristics (e.g. project risk, initial value, financier/entrepreneur productivities) to shape funding outcomes under competitive search by entrepreneurs.

[1]The empirical study of Kerr, Lerner and Schoar (2014) analyzes two prominent angel groups and find that the interest levels of angels at the stages of the initial presentation and due diligence are predictive of investment success, suggesting that angels engage in an efficient selection and screening process.

A number of studies rationalize the use of convertible securities in VC financing contracts (Casamatta (2003), Schmidt (2003), Baiman et al. (2008)). Schmidt (2003) analyzes a double moral hazard model where the entrepreneur provides effort first and the VC second. With conversion, the VC has stronger equity-like incentives and the entrepreneur works hard so that the VC sees enough upside potential to convert and become a common equity holder. Casamatta (2003) also considers the possibility that an outsider can provide value-adding advice in addition to effort by the entrepreneur and shows that the provision of advice creates information rents and that CPE is the optimal contracting security for sufficiently large investments. Baldenius and Meng (2010) consider how entrepreneurs can signal their firm's value to investors who have the expertise to contribute actively to the start-up companies operations and firm decision-making. As discussed earlier, angel financing typically involves equity deals and this provides a common benchmark for comparing the expected investment return and equity division in contracts designed by financiers with differing risk preferences and funding capabilities (e.g. angels, VCs).

In a different vein, Kanniainen & Keuschnigg (2003, 2004) study how a risk-neutral VC decides on the size of his portfolio of start-ups with costly advisory effort. The VC faces a tradeoff in that advice could be spread too thinly, thereby reducing the survival chances of all firms in the portfolio. As projects become riskier, the VC must cede a higher profit share to entrepreneurs to secure their effort that is critical for survival. With the corresponding erosion in the VC's own profit share, the VC eventually finds it unattractive to expand the portfolio further. Keuschnigg and Nielsen (2004) show that by failing to internalize effort costs of the other party, equity share divisions are generally inefficient, and require a corrective tax to restore efficiency. Sorensen (2007) considers a matching model in which more experienced VCs add more value and bring companies public at a higher rate.

The remainder of the paper is organized as follows. Section II presents the model and its solution. Discussion of model results and implications follow in Section III. Section IV concludes the paper.

II. THE MODEL

The model reflects the strategic interaction between an entrepreneur *E* (agent) and potential financiers *F* (principal) willing to provide venture financing in return for an equity position. Financiers are heterogeneous in their risk-aversion $\alpha_F \in [0, \alpha_{Max}]$ and their distribution over this support is given by the probability measure $\mathrm{P}(\alpha) = P(\alpha_F \leq \alpha)$. The entrepreneur does not know the type of the financier $F(\alpha_F)$, $\alpha_F \in [0, \alpha_{Max}]$, approached for funding. The timing of actions in the model are as follows. First, the entrepreneur *E* searches over financiers of type $F(\alpha_F)$, $\alpha_F \in [0, \alpha_{Max}]$, to obtain the best equity financing deal in return for an investment of size f. Second, each financier $F(\alpha_F)$ offers an equity contract that maximizes his expected utility of his investment return. Third, both *E* and $F(\alpha_F)$ simultaneously choose their levels of effort given the equity share. Fourth, the future value of the venture V is realized and payoffs based on the contract are realized.

A. Project Value and Financing

An entrepreneur *E* owns a project with initial value V_0 and seeks additional financing f from potential financiers at the start of the period $[0, t]$. Next, with efforts a and b provided by the entrepreneur and financier, respectively, the value of the venture at the end of the period at time t is

$$V = (V_0 + f)(1 + \beta a + \gamma b + \varepsilon), \qquad \varepsilon \sim N(0, \sigma^2) \tag{1}$$

where β represents the productivity of the entrepreneur's effort; γ is the productivity of the financier effort; and ε reflects the return uncertainty in the future value of the venture (assumed to be normally distributed). While various market scenarios can be imagined regarding the realization of future value V (e.g. IPO, acquisition, private equity buyout), we view V as simply the terminal value of the venture at time t.

Next, *E* searches for the best financing deal from financiers of types $F(\alpha_F)$, $\alpha_F \in [0, \alpha_{Max}]$. We describe the entrepreneur's search problem below after first solving for the optimal contract designed by a financier of type $F(\alpha_F)$. In exchange for funding f, the financier $F(\alpha_F)$ offers the entrepreneur an equity share $x(\alpha_F) \in [0,1]$ in the new venture with the remaining value $(1 - x(\alpha_F))V$ retained by the financier $F(\alpha_F)$. Given the equity share $x(\alpha_F)$, the entrepreneur determines his effort level to maximize his expected utility. Principal-agency model commonly use the exponential utility specification with constant absolute risk aversion (CARA) since it yields tractable expressions for incentives and effort that are independent of the agent's initial wealth (e.g. Grossman and Hart (1983), Jensen and Murphy (1990), Garen (1994), Gao (2010) and others). We adopt this specification as well and, accordingly, the entrepreneur determines her best effort response from maximizing the utility

$$U_E(x, a, b) = -\exp\left(-\alpha_E\left(w - \tfrac{1}{2} g a^2\right)\right) \tag{2}$$

where α_E is the entrepreneur's coefficient of risk aversion and the parameter g reflects the entrepreneur's cost of exerting effort.

B. The Financier's Investment Problem

The financier $F(\alpha_F)$'s investment return is given by

$$r_F = \frac{(1-x)V-(1+r)f}{f} = \frac{(1-x)(V_0+f)(1+\beta a+\gamma b)-(1+r)f}{f} \tag{3}$$

where r is the financier's borrowing rate (cost of capital). The utility of the financier from the venture's investment return is given by

$$U_F(x,a,b) = -\exp\left(-\alpha_F\left(r_F - \tfrac{1}{2}hb^2\right)\right) \tag{4}$$

where α_F is the financier's coefficient of risk aversion and the parameter h represents the financier's personal cost of exerting effort. For notational convenience, we suppress the type of the financier below when the distinction is not important.

The financier F maximizes the expected utility of his investment return $EU_F(x,a,b)$ by offering the equity contract $w = xV$ while taking into account the best-effort response of the entrepreneur and his own effort while ensuring that entrepreneur's participation. The entrepreneur has an outside alternative opportunity with monetary value $\overline{w}$ (in addition to the sale price of the current venture with initial value V_0).

The optimal financing problem of $F(\alpha_F)$ is as follows[2]:

$$\max_{\{x\}} EU_{F(\alpha_F)}(x,a,b) = \max_{\{x\}} \frac{(1-x)(V_0+f)(1+\beta a+\gamma b)-(1+r)f}{f} - \tfrac{1}{2}hb^2 - \alpha_F \frac{(1-x)^2(V_0+f)^2\sigma^2}{f^2} \tag{5}$$

subject to

[2]On the objective function (5a) and (5b), note that by the monotonicity of the exponential function, maximizing $EU_E(x,a,b)$ based on (4) is the same as maximizing $E(r_F) - \frac{1}{2}hb^2 - \frac{1}{2}\alpha_F V(r_F)$.

$$\max_{\{a\}} EU_E(x,a,b) = \max_{\{a\}} \left(x(V_0 + f)(1+\beta a + \gamma b) - V_0 - \tfrac{1}{2}\alpha_E x^2 (V_0 + f)^2 \sigma^2 - \tfrac{1}{2} g a^2 \right) \quad (6)$$

$$\max_{\{b\}} EU_{F(\alpha_F)}(x,a,b) \quad (7)$$

$$EU_E(x,a,b) \geq U(\overline{w} + V_0) \quad (8)$$

where $U(\overline{w} + V_0) = -\exp\left(-\alpha_E\left(\overline{w} + V_0\right)\right)$. For the discussion below, let $x^*(\alpha_F)$ generically denote the optimal equity share offered by the financier of type $F(\alpha_F)$ by solving (5) subject to (6)-(7). Further, let $a^*(\alpha_F)$ and $b^*(\alpha_F)$ represent the resulting optimal efforts, and from (1), $V^*(a^*(\alpha_F), b^*(\alpha_F)) = (V_0 + f)\left(1 + \beta a^*(\alpha_F) + \gamma b^*(\alpha_F) + \varepsilon\right)$ is the terminal venture value.

The weak inequality in (8) represents two possible situations. The first is that the constraint holds with equality, which implies that entrepreneurs receive the lowest equity valuation that makes them indifferent between pursuing the entrepreneurial venture and their best alternative monetary opportunity. It also implies that the equity share x will be determined purely by the entrepreneur's reservation wage, making the financier's problem (5) irrelevant.[3] Furthermore, forcing equality in (8) also assumes that the financier knows the best alternative opportunity $\overline{w}$ of the entrepreneur (who is not his employee) with a fair degree of precision. Given these concerns, the more realistic and palatable assumption is to assume slackness in the inequality (8). Accordingly, the financier determine the equity division $x^*(\alpha_F)$ that maximizes the expected utility of his investment return (5) while taking into account the best effort responses of both the entrepreneur and the financier from (6)-(7), respectively. The entrepreneur then accepts the equity

[3]In another version of the model, we considered the standard compensation contract $w = s + xV$ with both a fixed salary amount s and an equity stake x in the problem (5)-(8) with an *equality* participation constraint in (8): $E\left\{-\exp\left(-\alpha\left(w(a) - \tfrac{1}{2} g a^2\right)\right)\right\} = U(\overline{w} + V_0)$. This leads to more complex expressions, but essentially the same comparative statics as reported in Table 1 for the equity stake and efforts (results available from the authors on request). However, equity finance deals typically do not offer a fixed salary component, making this a less realistic contract type.

contracts only if the expected utility gain exceeds the one derived from his (privately known) outside opportunity as described in Definition 1.

Definition 1. ***The Optimal Feasible Contract***

A optimal contract $x^*(\alpha_F)$ is feasible if it satisfies the following conditions:

i) The equity stake $x^*(\alpha_F)$ maximizes financier $F(\alpha_F)$'s investment return utility (5) subject to best effort responses (6)-(7).

ii) The entrepreneur's participation inequality (8) holds, meaning that the contract $x^*(\alpha_F)$ is selected if it offer a larger utility gain over the best alternative opportunity $\overline{w}$.

$$E\,U_E(x^*, a^*, b^*) - U(\overline{w} + V_0) > 0.$$ [4]

Our interest is naturally on feasible deals that the entrepreneur will accept (i.e. satisfy Definition 1). The expressions for $x^*(\alpha_F)$, $a^*(\alpha_F)$ and $b^*(\alpha_F)$ in Proposition 1 and 2 below provide a direct way for the entrepreneur to evaluate condition ii) in Definition 1 or (8).

C. Optimal Equity Contract & Efforts

The problem (5)-(7) for financier $F(\alpha_F)$ is solved backwards and requires first obtaining the best effort responses. The first-order conditions for (6)-(7) lead to the following best effort responses for the entrepreneur and financier given the contract $w = xV$ (see Appendix A):

$$a = \frac{x(V_0 + f)\beta}{g}. \tag{9}$$

$$b = \frac{(1-x)(V_0 + f)\gamma}{fh}. \tag{10}$$

[4] If a deal does not satisfies (8), it means that either the entrepreneur's outside opportunity is too valuable or the future productivity of the project is too low (e.g. low β and γ). In either of these cases, the entrepreneur will decide to pursue his alternative opportunity.

Both efforts increase with productivity, incentive share, initial project value and funding size while they decrease with the agent's cost of effort. Incorporating the best effort response (9)-(10) into (5), the first-order-condition for the equity share leads to the equity division in Proposition 1 (see Appendix A for details).

Proposition 1. ***Optimal Equity Division – Entrepreneur & Financier***

a) *The optimal equity share for the entrepreneur that maximizes* $F(\alpha_F)$ *'s expected utility (4) from the investment return* r_F *is*

$$x^*(\alpha_F) = \frac{(V_0 + f)\left(\beta^2 fh + \alpha_F \sigma^2 gh - \gamma^2 g\right) - fgh}{(V_0 + f)\left(2\beta^2 fh + \alpha_F \sigma^2 gh - \gamma^2 g\right)} \tag{11}$$

where $\gamma^2 < 2f\beta^2(h/g) + \alpha_F \sigma^2 h$.

b) *Financier* $F(\alpha_F)$ *'s optimal equity share is*

$$1 - x^*(\alpha_F) = \frac{f\left(\beta^2(V_0 + f) + g\right)h}{(V_0 + f)\left(2\beta^2 fh + \alpha_F \sigma^2 gh - \gamma^2 g\right)}. \tag{12}$$

The optimal efforts for the entrepreneur and financier in Proposition 2 follow from using the equity shares (11)-(12) in (9)-(10).

Proposition 2. ***Return-Maximizing Efforts***

The optimal effort levels of the entrepreneur and financier $F(\alpha_F)$ *are*

$$a^*(\alpha_F) = \frac{\left(\beta^2 f(V_0 + f)h + \alpha_F(V_0 + f)\sigma^2 gh - \gamma^2(V_0 + f)g - fgh\right)\beta}{(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g)g} \tag{13}$$

$$b^*(\alpha_F) = \frac{\left(\beta^2 f(V_0 + f)h + g\right)\gamma}{\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g}. \tag{14}$$

Lastly, Proposition 3 reports the optimal expected investment return for the financier. The expression follows directly from substituting the financier's equity share (12) and efforts (13)-(14) into the financier's investment return (3) and simplifying.

Proposition 3. ***Financier's Optimal Investment Return***

Financier $F(\alpha_F)$ *'s optimal expected investment return is*

$$Er_F(\alpha_F) = \frac{\left(\beta^2(V_0+f)+g\right)h}{\left(2\beta^2 fh+\alpha_F\sigma^2 gh-\gamma^2 g\right)^2}\left[\begin{matrix}1+\beta^2\begin{pmatrix}\beta^2 f(V_0+f)h+\alpha_F(V_0+f)\sigma^2 gh\\ -\gamma^2(V_0+f)g-fgh\end{pmatrix}\\ +\gamma^2\left(\beta^2 f(V_0+f)h+g\right)\end{matrix}\right]-(1+r). \quad (15)$$

D. The Entrepreneur's Equilibrium Search Contract

The entrepreneur's equilibrium contract arises from optimally searching over the best financing deal available from all possible financiers of type $F(\alpha_F)$, $\alpha_F \in [0, \alpha_{Max}]$. Let $W(x(\alpha))$ be the maximized expected value to the entrepreneur of either accepting the equity share $x(\alpha_F)$ currently offered by a financier $F(\alpha_F)$ or performing another search for a better deal across financiers of different types. $W(x(\alpha))$ solves the following problem:

$$W(x(\alpha_F)) = \max\left(x(\alpha_F)EV(a(\alpha_F), b(\alpha_F)), \beta\int_0^{\alpha_{Max}} W(x(\alpha_F))d\mathrm{P}(\alpha)\right) \quad (16)$$

where $x(\alpha_F)EV(a(\alpha_F), b(\alpha_F))$ is the expected value of the current equity contract $x(\alpha_F)$ to the entrepreneur, $\int_0^{\alpha_{Max}} W(x(\alpha_F))d\mathrm{P}(\alpha)$ represents the expected value from continuing the searching for a better deal, and $\beta \le 1$ is the discount factor (time-value of money to entrepreneur in waiting for the next search). Using standard arguments, the contract $x(\overline{\alpha})$ with $\overline{\alpha} \in [0, \alpha_{Max}]$ that makes the entrepreneur indifferent between accepting the current deal $x(\overline{\alpha})$ and performing another search is given by the condition

$$x(\overline{\alpha})EV(a(\overline{\alpha}),b(\overline{\alpha}))=\beta\int_0^{\alpha_{Max}}x(\alpha)EV(a(\alpha),b(\alpha))d\mathrm{P}(\alpha)\,. \qquad (17)$$

Equation (17) implicitly defines the critical level of financier risk-aversion $\overline{\alpha}\in[0,\alpha_{Max}]$ and corresponding financier contract $x(\overline{\alpha})$ that solves the search problem of finding the best deal available from various types of financiers $F(\alpha)$ of measure $\mathrm{P}(\alpha)$.

In solving (17), Assumption 1 below ensures that the optimal solution for the equity share in Proposition (1) hold during the search by entrepreneurs across financiers of different types.

Assumption 1. ***Excess Funding Demand***

The total demand for venture funds by entrepreneurs exceeds their supply available from each type of financiers $F(\alpha_F)$.

Assumption 1 reflects the realism that funds available from financiers are a much scarcer resource than entrepreneurial projects seeking funding. It excludes perfect competition among potential financiers of the same type $F(\alpha_F)$ when the entrepreneur is searching for the best deal. This ensures that financiers with the same risk-aversion $\alpha_F\in[0,\alpha_{Max}]$ will not undercut each other and deviate from the optimal feasible contract for that type of financier $F(\alpha_F)$ in Proposition 1 and Definition 1. Under Assumption, the financier would simply wait for one of many entrepreneurs to approach them and provide funding that yields the expected return of Proposition 3. If there was excess supply of venture funds, perfect competition will lead financiers to increase the entrepreneur's equity stake beyond the optimal level in Proposition 1 until their expected utility in (5) goes to zero.[5] It is also useful to note that Assumption 1 is implicitly made in other models where the analysis is based on a contract designed by a risk-neutral financier ($\alpha_F = 0$, single type).

Proposition 4 characterizes the equilibrium search contract under a particular assumption regarding the distribution of financier types $\mathrm{P}(\alpha)$ (see Appendix B).

Proposition 4. ***Equilibrium Search Contract***

If financiers $F(\alpha)$ *are uniformly distributed over* $\alpha \in [0, \alpha_{Max}]$*, then, the equilibrium search contract solving (18) is given by the feasible contract* $x^*(\overline{\alpha})$ *in (11) with*

$$\overline{\alpha} \approx \frac{\beta}{2}\alpha_{Max} - (1-\beta)\frac{x(0)EV(a(0),b(0))}{\frac{\partial\{x(\alpha)EV(a(\alpha),b(\alpha))\}}{\partial\alpha}\Big|_{\alpha=0}} \qquad (18)$$

where $x(0)EV(a(0),b(0)) = x(0)(V_0 + f)\big(1+\beta a(0)+\gamma b(0)\big)$ *and*

$$\frac{\partial\{x(\alpha)EV(a(\alpha),b(\alpha))\}}{\partial\alpha}\Big|_{\alpha=0} = \big\{x'(0)(V_0+f)\big(1+\beta a(0)+\gamma b(0)\big) + x(0)(V_0+f)\big(\beta a'(0)+\gamma b'(0)\big)\big\}\alpha .$$

If searches are conducted simultaneously by the entrepreneur, there is no time-value loss ($\beta = 1$ *) and (18) reduces to* $\overline{\alpha} \approx \frac{1}{2}\alpha_{Max}$. (19)

If the equilibrium search contract $x^*(\overline{\alpha})$ is offered to the entrepreneur, she will accept the funding deal because the expected value from another search would give the same expected value. Although more complex distributions of financiers $\mathrm{P}(\alpha)$ will lead to somewhat differing results for position of $\overline{\alpha}$ (and the contract $x^*(\overline{\alpha})$), these solution will generally satisfy $\overline{\alpha} \in (0, \alpha_{Max})$. Therefore, our discussion and insights in Section III below on how the equilibrium search contract

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[5]From equation (5) in the model, the most competitive contract in this scenario will be that of the risk-neutral financier ($\alpha_F = 0$) and the equilibrium equity share will be set to make the expected utility of the financier zero.

affects the dynamics of expected investment returns among financiers of different types is largely robust to specific assumptions regarding the probability measure $P(\alpha)$.

E. Comparative Statics

The model's comparative statics are summarized in Table 1. The signs of the partial derivatives for the entrepreneur's equity share, expected investment return, and entrepreneur/financier effort (columns) are reported with respect to various model parameters (rows). To aid in the analysis, the signs are reported in separate columns for both a risk-neutral financier ($\alpha_F = 0$) and a risk-averse financier ($\alpha_F > 0$). These results lead to the various insights and implications in Section III below.

[Table 1]

III. DISCUSSION & IMPLICATIONS

We discuss below insights and predictions from the model on how heterogeneity in financier risk preferences and competitive searching by entrepreneurs determine funding outcomes such as expected investment returns and equity shares. Model results also have implications regarding the coexistence of the angel and VC markets and the venture funding cycle.

A. Financier Expected Returns & Equilibrium Search

We first consider show the equilibrium search contract reduces the comparative advantage that lower risk-aversion financiers over higher risk-aversion financiers. This enables a broader spectrum of financiers with differing risk preferences to operate in the venture finance market.

Model results show that, in the absence of search, venture financiers with lower levels of risk-aversion benefit as they have higher expected investment returns. This follows directly from the negative relationship between the financier's expected investment return and risk-aversion: $\frac{\partial Er_F}{\partial \alpha_F} < 0$ (Table 1 and (D.1) in Appendix C). Meanwhile, higher risk-averse financiers experience lower expected investment returns and also provide a larger equity stake to entrepreneurs ($\frac{\partial x}{\partial \alpha_F} > 0$, Table 1 and (C.1) in Appendix C). This encourages greater effort from the entrepreneur ($\frac{\partial a}{\partial \alpha_F} > 0$) that positively impacts the size of the total venture 'pie' (V), but also reduces the financier's share and the financier's exposure to venture risk (second term of (5)). The risk reduction effect is larger for more risk-averse financiers and increases their expected utility. Therefore, it is utility enhancing for higher risk-aversion financiers to offer a higher equity share to entrepreneurs but this comes at a cost of lower expected investment returns. Similarly, the expected investment return also declines with venture risk ($\frac{\partial Er_F}{\partial \sigma} < 0$). This has the implication that less risk-averse financiers are more capable of financing riskier projects relative to their higher risk-averse counterparts.

Next, we show that the equilibrium search contract $x^*(\overline{\alpha})$ with $\overline{\alpha} = \frac{1}{2}\alpha_{Max}$ (Proposition 4) reduces the higher expected return of lower risk-aversion financiers while improving the profitability of higher risk-aversion financiers. More specifically, for financiers with higher levels of risk-aversion $\overline{\alpha} < \alpha_F$, the equilibrium search contract $x^*(\overline{\alpha})$ generates a windfall gain because $\frac{\partial Er_F}{\partial \alpha_F} < 0$ implies that their expected return rises: $Er_F(\alpha_F) < Er_F(\overline{\alpha})$. They also need to offer a lower equity share to entrepreneurs since $\frac{\partial x}{\partial \alpha_F} > 0$ implies that $x(\overline{\alpha}) < x(\alpha_F)$ for $\overline{\alpha} < \alpha_F$. On the other hand, financiers with lower levels of risk-aversion $\alpha_F < \overline{\alpha}$ will face a declining expected investment return as $Er_F(\overline{\alpha}) < Er_F(\alpha_F)$ they need to increase the equity share to entrepreneurs (since $\frac{\partial x}{\partial \alpha_F} > 0$, $x(\alpha_F) < x(\overline{\alpha})$ for $\alpha_F < \overline{\alpha}$). Hence, the search for the best venture deal by entrepreneurs leads to an equilibrium that enables financiers with a broader range of risk preferences to coexist and operate profitably in the venture finance market.

Proposition 5. ***Financier Expected Returns - Equilibrium Search***

The equilibrium search contract $x^*(\overline{\alpha})$ *with* $\overline{\alpha} = \frac{1}{2}\alpha_{Max}$ *(Proposition 4) enables a broader set of financiers with higher levels of heterogeneous risk aversions to operate profitably in the venture market by making their expected investment returns and equity shares similar. The equilibrium contract* $x^*(\overline{\alpha})$*:*

a) *Reduces the higher expected investment return of lower risk-aversion financiers* $\alpha_F \in [0, \overline{\alpha}]$ *and increases the equity share of entrepreneurs:* $Er_F(\overline{\alpha}) < Er_F(\alpha_F)$, $x(\overline{\alpha}) > x(\alpha_F)$.

b) *Raises the lower expected investment return of higher risk-aversion financiers* $\alpha_F \in [\overline{\alpha}, \alpha_{Max}]$ *and decreases the equity share to entrepreneurs:* $Er_F(\alpha_F) < Er_F(\overline{\alpha})$, $x(\overline{\alpha}) < x(\alpha_F)$.

Further, due to the negative relationship between financier expected return and venture risk ($\frac{\partial Er_F}{\partial \sigma} < 0$), financiers in the lower risk-aversion group $\alpha_F \in [0, \overline{\alpha}]$ are more capable of financing higher risk ventures relative to the higher risk-aversion group $\alpha_F \in [\overline{\alpha}, \alpha_{Max}]$. As project risk increases, entrepreneur effort is discouraged as venture outcomes become more random and the certainty value of the entrepreneur's equity stake declines. Therefore, to elicit higher effort, risk-averse financiers need to offer a higher equity share to entrepreneurs ($\frac{\partial a}{\partial \sigma} > 0$). Consequently, the financier's expected investment return declines with project risk ($\frac{\partial Er_F}{\partial \sigma} < 0$ if $\alpha_F > 0$). Therefore, there is a risk *cutoff* beyond which financing risky projects becomes unprofitable for higher risk-aversion financiers $\alpha_F \in [\overline{\alpha}, \alpha_{Max}]$. Hence, the model predicts that ventures with very high uncertainty can only be feasibly financed by lower risk-aversion financiers.

Proposition 6. ***Financing of High Risk Ventures***

Higher risk ventures can only be profitably financed by less risk-averse investors since their expected investment return declines with project risk.

B. Implication for Angel & VC Markets

The model results above, along with the effect of initial project value and financier productivity on expected investment returns, have some interesting implications for the coexistence of the angel and VCs in the venture finance industry. Since both types of financiers can potentially provide entrepreneurial finance at earlier stages (Hellmann, Schure and Vo (2013)), why is angels funding concentrated at early stages while VCs participate at later stages?

In mapping our theoretical model to the profile of participants in the venture industry, we note that the risk preferences of angel investors as a group are more likely to be similar to those of higher risk-averse financiers. VCs frequently consist of institutional firms that have larger funding capacity and hold a diversified portfolio of venture investments in industry sectors (Kanniainen & Keuschnigg (2003, 2004)). In contrast, angels are frequently wealthy individuals who deploy their own personal wealth to finance a single or few start-ups. Hence, in terms of our model, VCs correspond more closely to the financiers whose risk-aversion largely below that of the equilibrium search financier $F(\overline{\alpha})$: $F(\alpha_F), \alpha_F \in [0, \overline{\alpha}]$. Meanwhile, angels are morel likely to be associated with financiers whose risk-aversion is clustered above the search equilibrium financier: $F(\alpha_F), \alpha_F \in [\overline{\alpha}, \alpha_{Max}]$.

Given this topology, our analysis above shows that VCs would tend to dominate in the absence of searching by entrepreneurs: they have higher expected investment returns relative to angels (since $\frac{\partial Er_F}{\partial \alpha_F} < 0$) and they will also offer deals with lower equity stakes to entrepreneurs (since $\frac{\partial x}{\partial \alpha_F} > 0$). This also means that VCs are in a better position to finance higher-risk ventures. Under competitive search by entrepreneurs (where the equilibrium contract $x^*(\overline{\alpha})$ is accepted over continuing the search), Proposition 5 shows that the competitive advantage of VCs shrinks as their expected return declines ($Er_F(\overline{\alpha}) < Er_F(\alpha_F)$) and the equity share to entrepreneurs increases ($x(\overline{\alpha}) > x(\alpha_F)$ for $\overline{\alpha} > \alpha_F$). Similarly, angels with the disadvantage of lower expected returns without search, benefit as their expected return rises ($Er_F(\overline{\alpha}) > Er_F(\alpha_F)$) and the equity share to entrepreneurs declines ($x(\overline{\alpha}) < x(\alpha_F)$ for $\alpha_F > \overline{\alpha}$). This equilibrium dynamics under search means that a larger set of financiers clustered around the equilibrium level of risk aversion $0 \le \overline{\alpha} \le \alpha_{Max}$ can continue to operate profitably in the venture finance industry. Proposition 7 is a corollary to Proposition 5 in the context of the venture industry.

Proposition 7. Angel and VC Financiers - Equilibrium Search

The equilibrium search contract $x^*(\overline{\alpha})$ with $\overline{\alpha} = \frac{1}{2}\alpha_{Max}$ (Proposition 4) enables a broader mix of angel and VC investors with higher levels of heterogeneous risk aversions to operate in the venture finance market by making their expected investment returns and equity shares similar. The equilibrium contract $x^*(\overline{\alpha})$:

a) Reduces the higher expected investment return of lower risk-aversion VC investors $\alpha_F \in [0, \overline{\alpha}]$ and increases the equity share of entrepreneurs: $Er_F(\overline{\alpha}) < Er_F(\alpha_F)$, $x(\overline{\alpha}) > x(\alpha_F)$.

b) *Raises the lower expected investment return of higher risk-aversion angel investors* $\alpha_F \in [\overline{\alpha}, \alpha_{Max}]$ *and decreases the equity share to entrepreneurs:* $Er_F(\alpha_F) < Er_F(\overline{\alpha})$, $x(\overline{\alpha}) < x(\alpha_F)$.

Further, the negative relationship between financier expected return and venture risk ($\frac{\partial Er_F}{\partial \sigma} < 0$) predicted by the model means that VCs are in a stronger position to finance ventures with higher uncertainty since angel expected returns will be lower. Further, there is a risk threshold beyond which angel financing becomes infeasible while the same projects are profitable to VCs.

Proposition 8. ***VC Financing of High Risk Ventures***

Beyond a certain level of venture risk, only VCs can profitably finance ventures due to the negative relationship between expected investment return and venture risk ($\frac{\partial Er_F}{\partial \sigma} < 0$*).*

The results above are largely consistent with the funding arrangement observed in the venture finance market where angels tend to finance less risky projects and institutional VCs take on more high risk projects (Gompers & Lerner (1999)).

C. The Venture Funding Cycle

The venture's initial value V_0 and financier productivity γ also play an important role in differentiating the profitability of financiers. This has direct implications for the participation of angels and VCs at different stages of venture development.

First note that the financier expected investment return rises with initial project value and financier productivity: $\frac{\partial Er_F}{\partial V_0} > 0$ and $\frac{\partial Er_F}{\partial \gamma} > 0$ (Table 1). This means that it is in the interest of venture investors to finance projects with high initial V_0 and where the financier can add greater value to the venture following funding (high financier productivity γ). The model's contract also provides greater incentives to entrepreneurs of projects with higher initial value ($\frac{\partial x}{\partial V_0} > 0$) and this increase is greater for financier who are less risk-averse ($\frac{\partial^2 x}{\partial V_0 \partial \alpha_F} < 0$; (C.6) in Appendix C). When V_0 is high, the likelihood of a higher future project values (and higher return r_F) increases due to the 'scale effect' of effort (see (1)). The financier will accordingly seek greater effort from the entrepreneur when initial value V_0 is higher by providing stronger equity incentives ($\frac{\partial a}{\partial V_0} > 0$). On the other hand, higher financier productivity reduces the equity offered to entrepreneurs ($\frac{\partial x}{\partial \gamma} < 0$). The model also predicts that entrepreneurs with high productivities will receive a higher equity stake ($\frac{\partial x}{\partial \beta} > 0$) but only from lower risk-averse financiers (e.g. VCs, see Table 1).

Model results on project size and financier productivity support the emergence of a venture capital cycle (Gompers and Lerner, 1999). In this cycle, new start-ups often raise relatively modest sums initially, and then seek larger funding in later rounds when the VC is in a position to add more value. In applying the model in this direction, note that the model (5)-(8) with equilibrium search (17) reflects the strategic interaction in any independent funding round between the entrepreneur and potential financiers (angel or VC). At the start of each such round, the initial value of the venture V_0 will change. Since the initial value of the start-up V_0 increases as it moves to later stages of development, the model predicts that the financiers' expected investment return will increase because $\frac{\partial Er_F}{\partial V_0} > 0$. Therefore, financiers with larger pools of available funds (e.g. institutional VC firms) will prefer to participate in later stages because their expected investment returns will be larger due to the higher V_0, as predicted by the model. Further, VCs are also in a better position at later stages of venture development to provide value-adding management advice which also positively impacts the profitability of financing later stage ventures since $\frac{\partial Er_F}{\partial \gamma} > 0$.

Proposition 9. ***VC Timing & Financing Rounds***

Since $\frac{\partial Er_F}{\partial V_0} > 0$ and $\frac{\partial Er_F}{\partial \gamma} > 0$, financiers with access to larger investment funds (e.g. VCs) can increase their expected investment return by investing in larger ventures (higher V_0) at later stages when they can also provide more value-adding advice and management services (higher γ).

As shown above, the model provides theoretical support for the venture capital cycle that associates business angels with small deal sizes initially and VCs with larger deals at later stages when they can make greater contributions (Gompers and Lerner (1999), Shane (2009)). The higher initial value V_0 of the venture at the later stage positively affects the expected investment return of VCs. The model is also consistent with the characteristic of VC firms to concentrate and focus their investment portfolio on specific sectors (e.g. biotech, social-media, IT). This enables them to develop more focused capabilities and networks that can increase the value of their advisory and management services at the late stage (increase γ).

IV. CONCLUSION

This paper expands our understanding of the role of heterogeneity in financier risk preferences and search by entrepreneurs in entrepreneurial finance. Our analysis is based on an double-hazard agency model in which entrepreneurs search for the best deal from financiers who maximize the expected utility of their investment return to determine equity shares that induce optimal efforts. Model results provide several novel insights on characteristics influencing expected investment returns and equity shares and on the coexistence of angels/VCs and the venture funding cycle.

Without search, lower risk-aversion financiers have an advantage as their expected investment return is higher, the equity share given lower, and they can profitably fund higher risk projects. With search, an equilibrium emerges that reduces the advantage (disadvantage) of lower (higher) risk-aversion financiers. Financiers with risk-aversions above the equilibrium level experience an increase in their lower expected investment return and they reduce the equity share given to entrepreneurs. Similarly, there is a decline in the higher expected return of lower risk-aversion financiers and the equity to entrepreneurs increases. This search dynamics allows a more heterogeneous set of financiers with higher levels of risk-aversion to coexist and operate profitably in the venture finance market.

As discussed earlier, in the industry context, it is reasonable to associate VC firms with lower risk-aversion financiers (relative to angels) as they tend to hold a more diversified portfolio of venture investments and have larger funding capacity. Meanwhile, angel investors are generally wealthy individuals who focus on a single or few start-up companies. Given this topology, our model results suggest that competitive searching by entrepreneurs enables both angels and VCs of differing risk preferences to operate profitably in an industry which would otherwise favor VCs (because their expected investment returns would be higher).

The model further predicts that financiers with access to larger investment funds (e.g. VC firms) will benefit by waiting to fund larger venture at later stages of their development. This occurs because the expected investment return is positively related to the initial venture value and financier productivity. Hence, the model can theoretically justify the pattern of funding activity observed in the venture finance industry with angels funding start-ups in their earlier stage and VCs engaging at the later stage when the venture's initial value is higher and when they have greater capability to provide value-adding advice and management services (e.g. Wong, Bhatia and Freeman (2009), Shane (2009)). Here, the tendency of VC firms to concentrate their investment portfolio on specific sectors (e.g. biotech, IT, social media) allows then to further developed more focused capabilities and networks.

The paper also contributes to the venture finance modeling literature by introducing some unique features and many of our results above rely on these innovations. The model allows the generality of a risk-averse financier and our analysis is based on the financier's investment return, which is independent of scale, and is a primary measure of performance in the venture finance industry. Further, the venture's future value is modeled as a continuous random function that incorporates a rich set of inputs (initial venture value, funding size, entrepreneur/financier productivities and efforts). Lastly, our analysis incorporates equilibrium search by entrepreneurs over financiers with heterogeneous risk preferences.

There is considerable potential to extend the model and analysis in a number of promising directions. First, the model provides a rich set of hypotheses and predictions for future empirical studies (see Table 1). For example, studies could investigate if equity shares and realized investment returns from financing deals by angels and VC differ as predicted by the model. We await the outcomes of this empirical analysis with interest. Second, model results provide useful guidance on the design of venture finance deals by relate the optimal equity division and expected investment return to a number of venture characteristics. Lastly, while the pure-equity contract is commonly observed in angel deals, it is well known that VC deals include convertible features. Incorporating such option-like features to the paper's modeling framework with search (see (5)-(8) and (17)) can provide additional insights on mitigating asymmetric risks.

Lastly, although we do not explicitly explore implications for public policy in this paper, the model and its extensions can shed useful light in this direction. For example, the model shows that the positive impact of initial project value on financier expected returns and the negative impact of project risk is a source of disadvantage to investors with lower access to funding and higher risk aversion (e.g. angels). Targeted subsidies, preferential tax treatment and provision of partial insurance may further enhance financing opportunities in the angel market for smaller entrepreneurs with riskier projects.

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Table 1. Comparative Statics

The table displays the signs of the partial derivatives for the entrepreneur's equity stake, expected investment return, entrepreneur effort and financier effort (columns) with respect to various model parameters (rows).

	Entrepreneur's Equity Stake		**F's Expected Investment Return**		**Entrepreneur's Effort**		**Financier's Effort**	
	x		Er_F		a		b	
	VC $\alpha_F = 0$	Angel $\alpha_F > \alpha_F^*$	VC $\alpha_F = 0$	Angel $\alpha_F > \alpha_F^*$	VC $\alpha_F = 0$	Angel $\alpha_F > \alpha_F^*$	VC $\alpha_F = 0$	Angel $\alpha_F > \alpha_F^*$
α_F	+	+	−	−	+	+	−	−
σ	0	+[1]	0	−[1]	0	+[1]	0	−[1]
β	+	−[2]	−	?	+	?[1]	−	+[2]
γ	−	−	+	+	−	−	+	+
V	+	+	+	+	+	+	+	+
f	+	−[3]	−	?	+	?[1]	−	+[4]

[1]Holds for $\alpha_F^* = 0$; [2]Holds for $\alpha_F^* = \frac{\gamma^2 (V_0 + f) + 2fh}{(V_0 + f)\sigma^2 h}$; [3]Holds for

$\alpha_F^* = \frac{\gamma^2 (\beta^2 (V_0 + f)^2 + V_0 g) + 2\beta^2 f^2 h}{(\beta^2 (V_0 + f)^2 + V_0 g)\sigma^2 h}$; [4]Holds for $\alpha_F^* = \frac{\beta^2 g + 2(\beta^2 V_0 + g)h}{\sigma^2 gh}$.

Appendix C: Sensitivity of Entrepreneur's Equity Stake

It is easy to show using (A.6) that the partial derivatives of the entrepreneur's equity stake with respect to various model parameters as given below.

i) Financier's Risk Aversion

$$\frac{\partial x}{\partial \alpha_F} = \frac{f\left(\beta^2 (V_0 + f) + g\right)\sigma^2 gh^2}{(V_0 + f)\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} > 0 \tag{C.1}$$

since $\gamma^2 < 2f\beta^2 (h/g) + \alpha_F \sigma^2 h$.

ii) Project Risk

$$\frac{\partial x}{\partial \sigma} = \frac{2\alpha_F \gamma f\left(\beta^2 (V_0 + f) + g\right)\sigma^2 gh}{(V_0 + f)\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} = \begin{cases} = 0 & \text{if } \alpha_F = 0 \\ > 0 & \text{if } \alpha_F > 0 \end{cases} \tag{C.2}$$

iii) Entrepreneur's Productivity

$$\frac{\partial x}{\partial \beta} = \frac{2\beta f\left(-\beta^2 (V_0 + f)h\sigma^2 \alpha_F + \gamma^2 (V_0 + f) + 2fh\right)gh}{(V_0 + f)\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} = \begin{cases} \geq 0 & \text{if } \alpha_F \leq \alpha^* \\ < 0 & \text{if } \alpha_F > \alpha^* \end{cases} \tag{C.3}$$

$$\text{where } \alpha^* = \frac{\gamma^2 (V_0 + f) + 2fh}{\beta^2 (V_0 + f)h\sigma^2 \alpha_F}. \tag{C.4}$$

iv) Financier's Productivity

$$\frac{\partial x}{\partial \gamma} = -\frac{2\beta f\left(\beta^2 (V_0 + f) + g\right)}{(V_0 + f)\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} < 0 \tag{C.5}$$

v) Initial Value

$$\frac{\partial x}{\partial V_0} = \frac{fgh}{(V_0 + f)\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} > 0 \tag{C.6}$$

where $\frac{\partial^2 x}{\partial V_0 \partial \alpha_F} < 0$ so that E's equity stake increases at a lower rate when the financier is more risk-averse.

vi) Funding Size

$$\frac{\partial x}{\partial f} = \begin{cases} > 0 & \text{if } \alpha_F < \alpha^* \\ \leq 0 & \text{if } \alpha_F \geq \alpha^* \end{cases} \tag{C.7}$$

where $$\alpha^* = \frac{\beta_2^2\left(\beta_1^2 (V_0 + f)^2 + gV_0\right) + 2\beta_1^2 f^2 h}{\sigma^2\left(\beta_1^2 (V_0 + f)^2 + gV_0\right)h}. \tag{C.8}$$

Appendix D: Sensitivities of Financier's Expected Investment Return

The partial derivatives of the financier's investment return (15) with respect to various project-financier-entrepreneur characteristics are reported below (for complicated expressions, only the sign is reported below; the complete expression is available from the authors).

i) Entrepreneur's Risk Aversion

$$\frac{\partial E(r_F)}{\partial \alpha_F} = -\frac{2\beta f\left(\beta^2 (V_0 + f) + g\right)^2\left(\sigma^2 h\alpha_F + \gamma^2\right)\sigma^2 gh^2}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^3} < 0 \tag{D.1}$$

ii) Project Risk

$$\frac{\partial E(r_F)}{\partial \sigma} = -\frac{2\alpha_F\left(\beta^2 (V_0 + f) + g\right)^2\left(\sigma^2 h\alpha_F + \gamma^2\right)\sigma g}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^3} = \begin{cases} = 0 & \text{if } \alpha_F = 0 \\ < 0 & \text{if } \alpha_F > 0 \end{cases} \tag{D.2}$$

i) Entrepreneur's Productivity

$$\frac{\partial E(r_F)}{\partial \beta} = \begin{cases} < 0 & \text{if } \alpha_F = 0 \\ ? & \text{if } \alpha_F > 0 \end{cases} \tag{D.3}$$

i) Financier's Productivity

$$\frac{\partial E(r_F)}{\partial \gamma} = \frac{4\gamma\left(\beta^2 (V_0 + f) + g\right)^2 \left(\sigma^2 gh\alpha_F + \gamma^2\right)h^2}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^3} > 0 . \tag{D.4}$$

ii) Initial Value

$$\frac{\partial E(r_F)}{\partial V_0} = \frac{2\beta^2 \left(\beta^2 (V_0 + f) + g\right)\left(\sigma^2 g\alpha_F + \beta^2\right)h^2}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^3 g} > 0 \tag{D.5}$$

$$\frac{\partial^2 E(r_F)}{\partial V_0 \partial \alpha_F} = -\frac{2\beta^2 \sigma^4 \alpha_F \alpha_E g\left(\beta^2 (V_0 + f) + \alpha_E g\right)}{(2\beta^2 f + \alpha_E \alpha_F \sigma^2 g)^3} < 0 \tag{D.6}$$

iii) Funding Size

$$\frac{\partial E(r_F)}{\partial f} = \begin{cases} \leq 0 & \text{if } \alpha_F = 0 \\ ? & \text{if } \alpha_F > 0 \end{cases} . \tag{D.7}$$

Appendix E: Optimal Effort Sensitivities

Further, from the optimal efforts (A.7)-(A.8), the sensitivities of the entrepreneur's optimal effort level to various model parameters are provide below (for complex expressions with a large number of terms, only the sign is reported below; complete expressions are available from the authors).

i) Financier's Risk Aversion

$$\frac{\partial a}{\partial \alpha_F} = \frac{\beta^2 f\left(\beta^2 (V_0 + f) + g\right)\sigma^2 h^2}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} > 0 \tag{E.1}$$

$$\frac{\partial b}{\partial \alpha_F} = -\frac{\gamma\left(\beta^2 (V_0 + f) + g\right)\sigma^2 gh}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} < 0 \tag{E.2}$$

ii) Project Risk

$$\frac{\partial a}{\partial \sigma} = \frac{2\alpha_F \beta \; f\left(\beta^2 (V_0 + f) + g\right)\sigma \; h^2}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} \begin{cases} = 0 & \text{if } \alpha_F = 0 \\ > 0 & \text{if } \alpha_F > 0 \end{cases} \tag{E.7}$$

$$\frac{\partial b}{\partial \sigma} = -\frac{2\alpha_F \gamma \left(\beta^2 (V_0 + f) + g\right)\sigma \; gh}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} \begin{cases} = 0 & \text{if } \alpha_F = 0 \\ < 0 & \text{if } \alpha_F > 0 \end{cases} \tag{E.8}$$

iii) Entrepreneur's Productivity

$$\frac{\partial a}{\partial \beta} = \begin{cases} > 0 & \text{if } \alpha_F = 0 \\ ? & \text{if } \alpha_F > 0 \end{cases} \tag{E.3}$$

$$\frac{\partial b}{\partial \beta} = \frac{2\beta\gamma\left((V_0 + f)\sigma^2 h \alpha_F - \gamma^2 (V_0 + f) - 2fh\right)}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} = \begin{cases} \leq 0 & \text{if } \alpha_F \leq \alpha^* \\ > 0 & \text{if } \alpha_F > \alpha^* \end{cases} \tag{E.4}$$

where $\alpha^* = \dfrac{\gamma^2 (V_0 + f) + 2fh}{(V_0 + f) h \sigma^2 \alpha_F}$.

iv) Financier's Productivity

$$\frac{\partial a}{\partial \gamma} = -\frac{\beta \gamma f \left(\beta^2 (V_0 + f) + g\right)h}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} < 0 \tag{E.5}$$

$$\frac{\partial b}{\partial \gamma} = \frac{\left(\beta^2 (V_0 + f) + g\right)\left(\alpha_F \sigma^2 gh + 2\beta^2 fh + \gamma^2 g\right)}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)^2} > 0 \tag{E.6}$$

v) Initial Value

$$\frac{\partial a}{\partial V_0} = \frac{\beta\left(\alpha_F \sigma^2 gh + \beta^2 fh - \gamma^2 g\right)}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)g} > 0 \tag{E.9}$$

$$\frac{\partial b}{\partial V_0} = \frac{\beta^2 \gamma}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g\right)} > 0 \tag{E.10}$$

vi) Funding Size

$$\frac{\partial a}{\partial f} = \begin{cases} > 0 & \text{if } \alpha_F = 0 \\ ? & \text{if } \alpha_F > 0 \end{cases} \tag{E.11}$$

$$\frac{\partial b}{\partial f} = -\frac{\beta^2 \gamma \left(-\alpha_F \sigma^2 gh + 2(\beta^2 V + g)h + \gamma^2 g \right)}{\left(\alpha_F \sigma^2 gh + 2\beta^2 fh - \gamma^2 g \right)^2} \begin{cases} \leq 0 & \text{if } \alpha_F \leq \alpha^* \\ > 0 & \text{if } \alpha_F > \alpha^* \end{cases} \tag{E.12}$$

where $\alpha^* = \dfrac{2(\beta^2 V + g)h + \gamma^2 g}{\sigma^2 gh}$.