Uncertainty and Export Status: Theory and Evidence

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Abstract

I develop an export-decision model in which risk-neutral firms choose their export status and total output prior to the realization of destination-specific idiosyncratic shocks. I show that if firms can shuffle output across destinations in response to shock realizations (ex-post allocation), exporting increases their expected profit by more than if they must commit to the allocation of output prior to the realization of shocks (ex-ante allocation). This implies that the ability to allocate output in response to shock realizations constitutes a motivation for exporting even for risk-neutral firms. Furthermore, while the likelihood of exporting decreases with foreign uncertainty under both ex-post and ex-ante allocation, the likelihood of exporting increases with home uncertainty and decreases with the correlation between home and foreign shocks under ex-post allocation but is independent of both under ex-ante allocation. Using French firm-level data, I show that the data is consistent with ex-post allocation of output, but inconsistent with ex-ante allocation of output, thus confirming that uncertainty both at home and abroad play a role even in risk-neutral firms' export decisions.

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1 Introduction

On its most basic level, exporting allows a firm to access an entirely new customer base for its goods. This not only gives the firm access to *more* customers thereby increasing the overall level of demand for its goods, but it also gives the firm access to *different* customers thereby giving the firm varied sources of demand for its goods. Access to more customers is the motivation for exporting that has received most attention in the international trade literature. However, although it has received little attention in the literature, access to customers different than the firm's domestic customers may motivate the firm to export if it can benefit from a more varied customer base.

That exporting can have a stabilizing effect on a firm's revenue by varying its sources of demand has already been demonstrated by Hirsch and Lev (1971). More recently, Vannoorenberghe (2012) has shown that, particularly for small exporters, exporting reduces revenue volatility. However, when firms are risk neutral, as they are in my analysis, there is no benefit to this reduction in volatility.[1] But by what mechanism would acquiring more varied sources of demand rather than simply more sources of demand via exporting benefit a risk-neutral firm? That is, how can a more variegated customer base increase a firm's expected profit?

A hint can be found in the mounting evidence of a negative correlation between home and foreign sales.[2] This is unlikely to be attributable to a negative correlation between home and foreign demand shocks since firms tend to export to countries similar to their own.[3] Rather, the finding of a negative correlation between home and foreign sales may indicate that firms cannot fully adjust quantities in one market independently of the other.[4] After all, if firms were able to do so, home and foreign sales would be positively correlated reflecting the positive correlation between home and foreign demand shocks.

Instead, the negative correlation suggests that firms can adjust quantities at home (abroad) at the expense of quantities abroad (at home). This interpretation is supported by the findings of Almunia, Antràs, Morales and Lopez-Rodriguez (2018) who find that when hit with a negative domestic shock, Spanish firms increased their export flows. The authors conclude that goods intended for sale at home were instead shipped abroad due to unfavorable domestic demand, implying both that firms produce their output prior to the realization of demand shocks and that firms have the flexibility to adjust the final destination of their output in response to shock realizations. As will become clear, this ability to

[1] Esposito (2017) explores the potential gains from exporting for risk-averse entrepreneurs.

[2] See Nguyen and Schaur (2010), Vannoorenberghe (2012), Blum, Claro and Horstmann (2013), Haddad, Lim, Pancaro and Saborowski (2013), Soderbery (2014) and Ahn and McQuoid (2017). All of these papers take the firm's export status and export destinations as exogenously given.

[3] See, for example, Eaton, Kortum and Kramarz (2011).

[4] The papers cited in footnote 2 rely on convex production costs to explain the dependence between the home and foreign quantities. This could either be due to capacity constraints (as in Ahn and McQuoid (2017)) or because firms must make investments in capital prior to the realization of shocks (as in Vannoorenberghe (2012)), which leads to diminishing returns in labor after the realization of shocks. As will become clear, my approach is similar since firms are capacity constrained after the realization of shocks, but are free to choose that capacity prior to the realization of shocks.

shuffle output across destinations in response to demand shocks is the source of the gains for an exporter of having varied sources of demand for its good.

Guided by these findings, I develop and empirically test a two-country Melitz-type model with the goal of answering the following two questions: (1) Do risk-neutral exporters benefit from having varied sources of demand? and (2) if so, does this benefit translate into a motive for exporting that can be detected in the data?

In the model, risk-neutral firms face idiosyncratic market-specific demand shocks.[5] Crucially, prior to learning the realization of these shocks firms must choose both export status, which requires payment of a fixed cost, and total output, which is produced at constant marginal cost. In one version of the model, the ex-ante allocation version, the firm commits not only to total output but also to the allocation of this output across destinations prior to the resolution of uncertainty. In an alternative version of the model, the ex-post allocation version, the firm only commits to total output prior to the realization of shocks, but is free to decide the allocation of output across markets in response to the shock realizations.

The timing of the allocation of output across markets is important. While both the ex-ante and ex-post allocation versions of the model are consistent with a reduction in revenue volatility attributable to exporting (as long as foreign uncertainty is not too great), only in the ex-post allocation version of the model does an exporter benefit from having varied sources of demand. Specifically, when ex-post allocation is possible, an exporter benefits in the sense that the expected profit of an exporter exceeds the sum of the expected profit of an identical firm that serves only the home market and an identical firm that serves only the foreign market. In contrast, when the firm must commit to allocation of output ex ante, its expected profit is equal to this sum so that there is no benefit to having more varied sources of demand. The intuition is that ex-post allocation of output allows the firm to more efficiently allocate its output by diverting more of its output to the destination with relatively favorable shock realizations and away from the destination with relatively unfavorable shock realizations, an option unavailable to the firm in the ex-ante allocation version of the model.

Since taking advantage of the ability to allocate output ex post is only possible when the firm serves more than just its domestic market, i.e., when the firm exports, the ability to allocate output in response to shock realizations constitutes a motive for exporting. As a result, in the ex-post allocation version of the model firms are more likely to export than in the ex-ante allocation version of the model. While this prediction alone does not readily lend itself to testing which of the two versions of the model is supported by the data, the model provides other predictions that are more amenable to empirical testing. In particular, the model predicts that, on the one hand, when ex-post allocation is possible, the probability of exporting increases with the variance of home shocks and decreases with both the variance of foreign shocks and the covariance between home and foreign

[5] The importance of idiosyncratic shocks is supported by Eaton, Kortum and Kramarz (2011) and Munch and Nguyen (2014) who find that firm-destination-specific shocks (rather than productivity or other firm-specific shocks) explain most of the sales variation within particular export markets for a given firm.

shocks. On the other hand, when ex-post allocation is not possible, the probability of exporting also decreases with the variance of foreign shocks, but is independent of both the variance of home shocks and the covariance between home and foreign shocks. The logic is that while foreign uncertainty makes the foreign market less attractive regardless of the timing of the final allocation of output, the benefit from more varied sources of demand only exists when firms can adjust the destination of their output in response to shock realizations. The greater is the uncertainty at home, the greater is the benefit from an additional market to absorb variations in home demand, and the less correlated are the home and foreign shocks, the greater is the foreign country's ability to absorb these variations.

In order to use these predictions to test which of the versions of the model is more compatible with the data, I use French firm-level data from the ORBIS database. I begin by showing how to use the model together with the data to derive model-consistent measures of uncertainty, i.e., the second moments of the home and foreign shock distributions implied by the data if the model were the true data-generating process. The ultimate goal of deriving these model-consistent measures of uncertainty is to test the model, but they are interesting in their own right. For instance, they show both that shocks at home and abroad are positively correlated (correlation coefficient of 0.37) and, as some authors have conjectured, that foreign shocks are more uncertain than home shocks (coefficient of variation of 0.51 abroad vs. 0.25 at home).

Using these measures of uncertainty, I test the model's predictions regarding the relationship between the probability that a firm exports and the second moments of the home and foreign shocks distributions. I find that the data strongly supports the ex-post allocation version of the model and rejects the ex-ante allocation version of the model. Thus, the answer to the questions stated above are both affirmative - firms do indeed benefit from having varied sources of demand, and this benefit constitutes a motive for exporting that shows up in the data. Because this benefit and the resulting motive for exporting stems from the firm's ability to allocate output ex post in response to shock realizations, I refer to it as the ex-post allocation motive for exporting. Thus, while the findings of Almunia, Antràs, Morales and Lopez-Rodriguez (2018) mentioned in the opening paragraphs of this paper (that firms have the flexibility to adjust the final destination of their output in response to shock realizations) show that the data matches the *assumptions* of the ex-post allocation version of the model, the current paper shows that the data also matches the *predictions* of the ex-post allocation version of the model.

The results also indicate that the effect of uncertainty on a firm's decision to export is far from trivial. Indeed, for a manufacturing firm with an initial 10% probability of exporting, moving up one standard deviation in the home uncertainty distribution increases the chance of exporting by 8.0 percentage points. For a similar firm, a movement of one standard deviation up the foreign uncertainty distribution or the distribution of covariances between home and foreign shocks leads to a decrease in the probability of exporting by 3.9 and 3.4 percentage points, respectively.

This paper is part of growing literature that emphasizes uncertainty and volatility as important determinants of observed trade patterns.[6] One strand of this literature seeks to explain the constant churning of firms into and out of export markets, which has been shown to be an important driver of observed trade flows.[7] A common explanation is that firms serve a market in periods when demand is relatively high, but not in periods when demand is relatively low.[8] Thus, if demand is volatile, so too is a firm's exporting decision, leading firms to repeatedly enter and exit export markets. In this class of models uncertainty is always resolved before the decision to export is made. As a result, it is the realizations of the shocks that determine whether or not a firm exports rather than the very existence of uncertainty.[9] This contrasts with my model in which the decision to export precedes the resolution of uncertainty, reflecting the fact that it can be time consuming for a firm to set up the necessary distribution infrastructure in a foreign country or to register and license its products for exporting. As a result, in my model it is the *distribution* of shocks that matters for the firm's decision to export, and individual shock *realizations* have no effect on the firm's decision to export. In other words, while shock realizations affect the intensive margin of exporting, only the shock distributions affect the extensive margin of exporting.

Only few papers have examined how uncertainty per se, that is, how shock distributions rather than a particular shock realization, affects the decision to export. One approach has focused on the role of uncertain future foreign profitability. For example, Handley (2014) and Handley and Limão (2015) show that policy uncertainty regarding tariffs can dissuade firms from incurring a sunk exporting cost. In a similar vein, Impullitti, Irarrazabal and Opromolla (2013) show that, in the presence of sunk exporting costs, productivity uncertainty (which makes the profitability of exporting uncertain as well) can delay both entry into and exit out of export markets. These papers apply the insights on optimal investments in the presence of uncertainty and sunk costs to the case of exporting.

By assuming that output is fully adjustable *after* the realization of shocks, these papers break the dependence between the home and foreign countries. As a result, in those papers home uncertainty plays no role in a firm's decision to export. Interestingly, this conclusion

[6] A related strand of the literature, which mainly focuses on macroeconomic patterns, examines how trade openness affects volatility. See, for example, Giovanni and Levchenko (2010). The focus in the current paper is the causality in the opposite direction, that is, I examine the impact of volatility on a firm's decision to export and, by extension, trade openness.

[7] Eaton, Kortum and Kramarz (2004), Bernard, Jensen, Redding and Schott (2007) and Eaton, Eslava, Kugler and Tybout (2008) have shown that individual firms' entry into and exit out of export markets are important drivers of observed trade flows.

[8] Another common interpretation is that firms experiment by exporting to different markets as a way of learning about foreign demand for their product, as in Akhmetova and Mitaritonna (2012). However, in these papers production always takes place *after* uncertainty is resolved and, therefore, the ex-post allocation motive is absent.

[9] See Vannoorenberghe, Wang and Yu (2014). Blum, Claro and Horstmann (2013) show that in the presence of increasing costs a negative shock realization at home can incentivize a firm to export. Since these papers highlight the role of shock realizations, they naturally focus on why firms switch into and out of exporting. In contrast, since I focus on the effect of shock distributions, which are a more persistent characteristic of the firm than transitory shock realizations, my focus is on firms' long term export status.

remains unchanged with the opposite assumption, that output is adjustable only *before* the realization of shocks, as in the ex-ante allocation version of my model. However, as I show, the data supports a middle ground between these two extreme assumptions - total output is fully adjustable only *before* the realization of shocks, but the destination of that output is adjustable *after* the realization of shocks. This timing restores the link between the home and foreign countries and with it the role that home uncertainty plays in the firm's decision to export.[10]

Some papers have maintained the link between home uncertainty and the decision to export by assuming that firms (or the entrepreneurs that own them) are risk averse. For example, Esposito (2017) studies the exporting decisions of risk-averse entrepreneurs who must choose export destinations prior to the realization of shocks but choose output after the realization of shocks. In this approach access to more varied markets can lower the volatility of realized profits, but cannot increase those profits relative to access to larger but identical markets. As a result, the impact of uncertainty on the incentive to export operates only through risk aversion. Indeed, one of the key insights in my paper is that positing that production occurs prior to the realization of shocks obviates the need to appeal to risk aversion to explain why firms may benefit from having varied sources of demand, while also explaining other features of the data such as the negative correlation of sales across a firm's home and foreign markets.[11]

Importantly, the assumption in my paper that production takes place prior to the realization of shocks is well supported by the data. Besides the indirect evidence provided by this paper and the aforementioned paper of Almunia, Antràs, Morales and Lopez-Rodriguez (2018), both Evans and Harrigan (2005) and Hummels and Schaur (2010) provide more direct evidence of this timing of events. Indeed, these authors find evidence that some firms delay production until after the resolution of demand uncertainty, either by producing in a more costly location that is geographically closer to the point of sale or by paying extra to ship goods faster. The upshot is that were it not for these costly measures to delay production, demand shocks would occur after production takes place but before goods are shipped. Thus, the results in these papers lend further empirical support to my findings.[12]

[10] Another way to restore the link between the home and foreign countries while allowing production to occur after the resolution of uncertainty is to assume that production costs are convex. However, such an assumption would not alter my results because firms would derive a benefit due to the ability to allocate productive capacity across markets. This would be similar to the mechanism in my model in which firms derive a benefit due to the ability to allocate previously produced output across markets.

[11] The related problem of how uncertainty influences the tradeoff between exporting and foreign direct investment is analyzed in Ramondo, Rappoport and Ruhl (2013) for the case of shocks to aggregate productivity; in Lewis (2014) for the case of nominal shocks; and in Fillat, Garetto and Oldenski (2015) and Fillat and Garetto (2015) for the case of shocks to aggregate demand. Using a real option-value model with risk-averse agents, the latter two papers show that multinational activity offers diversification benefits (in the sense of volatility reduction rather than an increase in expected profit) which are reflected in the risk premium for firms and hence in the expected returns to investors. However, if agents were risk neutral as the firms are in my model, then these effects would disappear.

[12] The assumption that production occurs after the realization of shocks may be appropriate when ana-

The remainder of this paper is organized as follows: Section 2 introduces the ex-ante and ex-post allocation versions of the theoretical model. Section 3 shows that there is an ex-post allocation motive for exporting, while Section 4 develops the model's predictions for the relationship between uncertainty and a firm's export status. Section 5 derives model-consistent measures of uncertainty and uses them to test the model's predictions. In Section 6, a series of robustness checks confirms that the results in the previous section are not driven by any one particular assumption. Finally, Section 7 concludes.

2 Model

In this section, I propose two versions of an export-decision model. The firm-level data I will use to test the alternative versions of the model contains information on a firm's total exports, but not on destination-specific exports. Therefore, to maintain consistency with the data, I model the world as consisting of two countries indexed by $c \in \{h, f\}$, a home country and one foreign country that can be thought of as the rest of the world.[13] In what follows, I develop the model from the perspective of a single home-country firm. Demand for the firm's good in the two countries is (q_c is demand in country c)

$$q_h = A\gamma_h p_h^{-\sigma} \text{ and } q_f = AB_f\gamma_f p_f^{-\sigma}, \tag{1}$$

where A is a global demand factor; B_f represents demand abroad relative to demand at home; γ_c is a firm-specific demand shock in country c with a unit mean; p_c is the country c consumer price of the good produced by the firm; and $\sigma > 1$ is the price elasticity of demand.[14]

Access to the foreign market requires payment of a fixed cost, f_x. Once the firm pays the fixed cost, it draws its B_f from a cumulative distribution function $\mathscr{B}(\cdot)$.[15] In addition to the fixed cost, an exporter incurs an iceberg cost so that it must ship $\tau > 1$ units of its good for each unit sold abroad. The marginal cost of production for the firm

lyzing shocks whose impact on demand may be lagged, such as trade agreements whose implementation can take several years. However, with shocks whose impact on demand is more immediate, such as with changing fads or imposition of tariffs in a trade war, it may be more appropriate to model production as occurring before the realization of shocks. This is particularly so for firms in countries with rigid labor markets, where the hiring and firing process can be protracted so that even if the impact of shocks can be anticipated, output cannot easily be adjusted.

[13]For an analysis of uncertainty and the ex-post allocation motive for exporting in a multicountry setting see Danziger and Danziger (2018).

[14]The assumption that the shocks, γ_h and γ_f, have a unit mean is made for simplicity and is without loss of generality because any mean different than one in country c can be absorbed by A and B_f.

[15]That the firm does not learn the realization of B_f prior to payment of a fixed cost captures the fact that learning about demand in a new market is not costless as it requires the firm to expend resources for such things as market research, legal fees, etc. Notwithstanding the logic underlying this timing assumption, it does not affect any of the theoretical results. Rather, this timing means that the *realization* of B_f has no bearing on the firm's decision to export, which will prove useful in the empirical analysis. This is because it is impossible to directly control for a nonexporters' (hypothetical) realized openness.

is $1/\varphi$. Therefore, the total cost of a firm that produces a total quantity of $Q = q_h + \tau q_f$ is $Q/\varphi + \mathbf{1}_{X=1} f_x$, where $X \in \{0,1\}$ indicates whether (1) or not (0) the firm exports.

After choosing its export status and the quantity it wishes to produce, the firm draws a pair of demand shocks, (γ_h, γ_f), for the home and foreign countries from the joint cumulative distribution function, $G(\cdot,\cdot)$. Since the firm must make irreversible decisions prior to the realization of shocks, the uncertainty regarding these shocks is detrimental even for a risk-neutral firm. In contrast, if, as is assumed in much of the literature, these decisions were made after the resolution of uncertainty, then the fact that the shocks are uncertain has no negative impact on the firm's expected profit and are therefore only detrimental if firms are risk averse. At this point, I consider two alternative versions of the model, indexed by $v \in \{I, II\}$:[16]

Version I - Ex-Ante Allocation In this version, the firm not only chooses total output, Q, but must also decide how to allocate this output between the home and foreign countries prior to learning the realization of shocks, that is, q_h and q_f are allocated ex ante.

Version II - Ex-Post Allocation In this version, the firm only needs to commit to total output, Q, prior to learning the realization of shocks. After learning the realization of shocks the firm may choose to allocate its output as it sees fit subject to the constraint $Q \geq q_h + \tau q_f$, that is, q_h and q_f are allocated ex post.[17]

2.1 Ex-Ante Allocation

I begin by analyzing the ex-ante allocation version of the model. After having chosen export status and quantities for each market it serves, the firm is entirely at the mercy of the shock realizations, γ_h and γ_f. Therefore, given export status (X) and the relative size of the foreign market (B_f), the firm's realized revenue for any choice of q_h and q_f and any pair of realized shocks is determined entirely by the demand function (Equation (1)), which yields

$$R\left(q_h, q_f, X, B_f; \gamma_h, \gamma_f\right) = A^{1/\sigma}\left[\gamma_h^{1/\sigma} q_h^{1-1/\sigma} + \mathbf{1}_{X=1} {B_f}^{1/\sigma} {\gamma_f}^{1/\sigma} q_f^{1-1/\sigma}\right]. \tag{3I}$$

[16]Except where clarity necessitates it, I will refrain from using indexes to differentiate the results from each version of the model.

[17]An alternative to the ex-post allocation version of the model would be to assume that firms needed to allocate output between the home and foreign countries before the realization of shocks but could reallocate that output at a cost after the shocks are realized. The theoretical results would remain unchanged, although the mathematics would be greatly complicated.

Taking this result into account, the firm chooses q_h and q_f to maximize expected variable profit given X and B_f

$$\pi\left(X,B_f\right)=\max_{q_h,q_f}E\left[R\left(q_h,q_f,X,B_f;\gamma_h,\gamma_f\right)-\frac{\left(q_h+\mathbf{1}_{X=1}\tau q_f\right)}{\varphi}\right].$$

Standard profit maximization yields

$$q_h=A\rho^{\sigma}\varphi^{\sigma}\left[E\left(\gamma_h^{1/\sigma}\right)\right]^{\sigma}\qquad\text{and}\qquad q_f=AB_f\tau^{-\sigma}\rho^{\sigma}\varphi^{\sigma}\left[E\left(\gamma_f^{1/\sigma}\right)\right]^{\sigma},\tag{4I}$$

where $\rho=(\sigma-1)/\sigma$. The firm's expected variable profit is therefore

$$\pi\left(X,B_f\right)=\frac{A\rho^{\sigma-1}\varphi^{\sigma-1}}{\sigma}\left[\left[E\left(\gamma_h^{1/\sigma}\right)\right]^{\sigma}+\mathbf{1}_{X=1}\mathscr{O}_f\left[E\left(\gamma_f^{1/\sigma}\right)\right]^{\sigma}\right],\tag{5I}$$

where $\mathscr{O}_f=\tau^{1-\sigma}B_f$ is a measure of the firm's openness taking into account both the iceberg cost (lower iceberg cost implies greater openness) and the size of the foreign market relative to the home market (larger foreign market implies greater openness). A key feature of this version of the model is that there is no interaction between the two markets, and therefore the firm's total profit is equal to the sum of the profits earned in each market. Therefore, the firm chooses to export if and only if the expected profit attributable to the foreign country exceeds the fixed cost of exporting:

$$\frac{A\rho^{\sigma-1}\varphi^{\sigma-1}}{\sigma}E_{\mathscr{O}_f}\left(\mathscr{O}_f\right)\left[E_{\gamma_f}\left(\gamma_f^{1/\sigma}\right)\right]^{\sigma}>f_x,\tag{6I}$$

where the expectation operator on $\mathscr{O}_f$ reflects the fact that the firm draws B_f only after deciding whether or not to export (see footnote 15). This condition defines a cutoff productivity φ^* above which firms export and below which they serve only the home market. So far the analysis is almost identical to the standard analysis in the existing literature with the exception of the introduction of uncertainty. The ex-post allocation version of the model represents a more substantial departure from the existing literature.

2.2 Ex-Post Allocation

In the ex-post allocation version of the model the firm only needs to commit to the total quantity it produces, Q, prior to learning the realization of shocks. However, the ultimate destination of that output can be chosen after shocks are realized. The timing of this version of the model can be divided into three stages. In the first stage (export-decision stage) the firm chooses its export status and, if it chose to export, learns B_f. In the second stage (production stage) the firm chooses the total quantity it produces, Q. Finally, in the third stage (allocation stage) the firm learns the realization of the demand shocks, γ_h and γ_f, and, if the firm is an exporter, decides how to divide its output, Q, chosen in the second stage, between the home and foreign countries subject to the constraint that $Q\geq q_h+\tau q_f$. The firm's problem can be solved backwards in three steps corresponding to the three stages of the model.

Allocation Stage The firm finds the maximal profit for any potential choices of Q and export status, as well as any possible draw of B_f and realizations of the demand shocks, γ_h and γ_f. Since at this point all costs are sunk (apart from the iceberg cost), the firm maximizes its revenue by choosing q_h and q_f as follows:

$$\begin{aligned} R\left(Q, X, B_f; \gamma_h, \gamma_f\right) &= \max_{q_h, q_f}\left[q_h p_h + \mathbf{1}_{X=1}\left(q_f p_f\right)\right] \\ &\quad \text{subject to } Q \geq q_h + \mathbf{1}_{X=1}\left(\tau q_f\right) \text{ and demand given by Equation (1).} \end{aligned}$$

This is a trivial decision for nonexporters, and their solution is $q_h = Q$. Exporters, however, achieve the optimal division of output by equalizing marginal revenue across the two markets, which yields:

$$q_h\left(Q, 1, B_f; \gamma_h, \gamma_F\right) = \frac{\gamma_h}{\Gamma} Q \qquad \text{and} \qquad q_f\left(Q, 1, B_f; \gamma_h, \gamma_f\right) = \frac{\mathscr{O}_f \gamma_f}{\Gamma} Q, \tag{2II}$$

where $\Gamma = \gamma_h + \mathscr{O}_f \gamma_f$ and, as in the ex-ante allocation version of the model, $\mathscr{O}_f = \tau^{1-\sigma} B_f$ is a measure of openness of the firm taking into account both the foreign relative to home market size as well as the additional cost associated with supplying the foreign relative to home country. The solution for the optimal division of output across markets is to sell output in each market in proportion to its realized market size, that is, when the shock is favorable in the home country the firm should sell proportionally more of its output domestically, and when it is favorable in the foreign country it should sell proportionally more abroad. The solution to the firm's problem implies that total realized revenues are

$$R\left(Q, X, B_f; \gamma_h, \gamma_f\right) = A^{1/\sigma}\left(\gamma_h + \mathbf{1}_{X=1} \mathscr{O}_f \gamma_f\right)^{1/\sigma} Q^{1-1/\sigma}. \tag{3II}$$

Production Stage Taking into account the results of the allocation stage, the firm chooses the total quantity to produce, Q, that maximizes its expected profit given export status for any given draw of B_f. Thus, the firm solves the maximization problem

$$\pi\left(X, B_f\right) = \max_Q E_{\left(\gamma_h, \gamma_f\right)}\left[R\left(Q, X, B_f; \gamma_h, \gamma_f\right) - Q/\varphi\right],$$

where the subscript on the expectation operator indicates the random variable over which the expectation is calculated. The solution is

$$Q\left(X, B_f\right) = A \rho^\sigma \varphi^\sigma \left[E_{\left(\gamma_h, \gamma_f\right)}\left(\gamma_h + \mathbf{1}_{X=1} \mathscr{O}_f \gamma_f\right)^{1/\sigma}\right]^\sigma. \tag{4II}$$

The firm's expected variable profit is then given by

$$\pi\left(X, B_f\right) = \frac{A \rho^{\sigma-1} \varphi^{\sigma-1}}{\sigma}\left[E_{\left(\gamma_h, \gamma_f\right)}\left(\gamma_h + \mathbf{1}_{X=1} \mathscr{O}_f \gamma_f\right)^{1/\sigma}\right]^\sigma. \tag{5II}$$

Export-Decision Stage Using the results of the allocation and production stages, the firm chooses whether or not to incur the fixed cost of exporting to the foreign country. The firm will choose to export if and only if the expected export profit, i.e., the expected profit from serving both markets less the expected profit from serving only the domestic market, exceeds the fixed cost of exporting. That is, the firm will export if and only if

$$\frac{A\rho^{\sigma-1}\varphi^{\sigma-1}}{\sigma}E_{\mathscr{O}_f}\left(\left[E_{(\gamma_h,\gamma_f)}\left(\gamma_h+\mathscr{O}_f\gamma_f\right)^{1/\sigma}\right]^{\sigma}-\left[E_{\gamma_h}\left(\gamma_h\right)^{1/\sigma}\right]^{\sigma}\right)>f_x. \tag{6II}$$

As was the case in the ex-ante allocation version of the model, this condition defines a cutoff, φ^*, such that a firm exports if and only if its productivity exceeds this cutoff.

3 The Ex-Post Allocation Motive for Exporting

A comparison of Inequalities (6I) and (6II) shows one of the key differences between the two versions of the model. In the ex-ante allocation version of the model, the export profit is simply the profit attributable to the foreign country, i.e., foreign revenue less the cost of producing the output shipped abroad. However, in the ex-post allocation version of the model, attributing profit solely to the foreign country is no longer possible. This is because in addition to the profit associated with sales in the foreign country, an exporting firm also profits from the flexibility of adjusting the location of its sales to account for the shock realizations. Because this latter component of the profit stems from the ability to allocate output ex post, I define the ex-post allocation profit as the difference between the expected profit of a firm in the ex-post allocation version of the model and an identical firm in the ex-ante allocation version of the model.

For a nonexporter there is no difference between the two versions of the model since it must sell all its output domestically, and therefore for such a firm the ex-post allocation profit is zero. For an exporter, the ex-post allocation profit is (for any realization of B_f)

$$\pi^{ex-post}=\frac{A\rho^{\sigma-1}\varphi^{\sigma-1}}{\sigma}\left\{\left[E\left(\gamma_h+\mathscr{O}_f\gamma_f\right)^{1/\sigma}\right]^{\sigma}-\left[E\left(\gamma_h\right)^{1/\sigma}\right]^{\sigma}-\mathscr{O}_f\left[E\left(\gamma_f\right)^{1/\sigma}\right]^{\sigma}\right\}. \tag{7}$$

The ex-post allocation profit must be nonnegative since a firm cannot be worse off when it can allocate its output ex post. After all, nothing prevents a firm from committing to an ex-ante allocation of output and forgoing the option of ex-post allocation if it were desirable to do so. Indeed, Equation (2II) shows that only when shocks are proportional, i.e., there exists an $\alpha>0$ such that $\gamma_f=\alpha\gamma_h$ for all possible $\left(\gamma_h,\gamma_f\right)$ pairs, do the particular shock realizations have no bearing on a firm's choice of q_h and q_f. However, so long as home and foreign shocks are not proportional to one another, the firm's ex-post allocation depends on the particular shock realizations. It therefore follows that as long as shocks are not proportional, the ex-post allocation profit is strictly positive.

Since nonexporters earn the same expected profits in the two versions of the model, the ex-post allocation profit is also equal to the amount by which the expected export profit in the ex-post allocation version of the model exceeds the expected export profit in the ex-ante allocation version of the model. Since the ex-post allocation profit is positive (assuming nonproportionality of shocks), it follows that the export profit is greater in the ex-post as compared to the ex-ante allocation version of the model, and hence $\varphi_I^* > \varphi_{II}^*$ (see Inequalities (6I) and (6II)). Thus, the model predicts that in addition to exporters that would export independently of the ability to allocate output ex post ($\varphi > \varphi_I^*$), there are also exporters that would export only when given the ability to allocate output ex post ($\varphi \in [\varphi_{II}^*, \varphi_I^*]$). This implies that if firms do, in fact, have the ability to allocate output ex post, then this ability constitutes a motive for exporting. I refer to this as the ex-post allocation motive for exporting.

The prediction that $\varphi_I^* > \varphi_{II}^*$, i.e., if ex-post allocation is possible then firms are more likely to export than if allocation is only possible ex ante, is a major theoretical finding of this paper. However, this finding only predicts a quantitative difference between the two versions of the model (more firms export in one version of the model) rather than a qualitative difference (the impact of φ on exporting has the same sign in both versions). As a result, this prediction is not useful for testing which of the two versions of the model the data supports. Fortunately, the two versions of the model offer other predictions that are better suited to empirically testing which of the two versions of the model provides a better fit for the data. I now turn to these predictions.

4 The Impact of Uncertainty on Exporting

In this section, I address the following question: what characteristics of the distribution of the home- and foreign-country shocks affect a firm's incentive to export? Since the answer to this question depends on whether or not ex-post allocation of output is possible, the answer will provide useful testable hypotheses, which I will exploit in the empirical portion of the paper. A simple numerical example will serve to illustrate.

4.1 A Simple Numerical Example

Suppose the two countries are symmetric and that there is no iceberg trade cost, i.e., $A = B_f = \tau = 1$, and that $\sigma = 2$ and $\varphi = 4$. These numbers are chosen for simplicity and result in an expected variable profit in the ex-ante allocation version of the model given by (see Equation (5I))

$$\pi_I(X) = \left[E\left(\gamma_h\right)^{1/2}\right]^2 + \mathbf{1}_{X=1}\left[E\left(\gamma_f\right)^{1/2}\right]^2, \tag{8I}$$

and in the ex-post allocation version of the model given by (see Equation (5II))

$$\pi_{II}(X) = \left[E\left(\gamma_h + \mathbf{1}_{X=1}\gamma_f\right)^{1/2}\right]^2. \tag{8II}$$

The function $g\left(\gamma_h, \gamma_f\right)$ denotes the probability that a firm receives the shock-realization pair $\left(\gamma_h, \gamma_f\right)$. In all five cases considered below, the expected shock realization is one in both countries, that is, $E(\gamma_h) = E(\gamma_f) = 1$. This ensures that the results in the different cases depend on higher moments of the shock distribution rather than on the first moment. The relevant results for the cases discussed below are summarized in Table 1 and can be easily calculated from Equations (8I) and (8II) and the information in the table (see table notes).

Case 1 is a baseline case with no uncertainty where both γ_h and γ_f equal one. In this scenario a firm always sells half of its output in each destination and there is no advantage to having the ability to allocate output ex post. Indeed, as Table 1 shows there is no ex-post allocation profit, and there is no difference in expected profits in the two versions of the model.

Case 2 introduces uncertainty in the home country (γ_h equals either zero or two with equal probability), while maintaining certainty in the foreign country (γ_f always equals one). In the ex-ante allocation version of the model, where the firm has to commit to q_h and q_f before learning the realization of shocks, the realized profit is always one in the foreign country, just as in the no uncertainty case. However, in the ex-post allocation version of the model, the firm can do better by adjusting the allocation of its output in response to shock realizations. In particular, when $\gamma_h = 0$, the firm will sell all of its output abroad, and when $\gamma_h = 2$, it will only sell one third of its output abroad. As Table 1 shows, this ex-post allocation leads to an additional expected profit of $\sqrt{3}/2 - 1/4$, bringing the total expected export profit in the ex-post allocation version of the model to $\sqrt{3}/2 + 3/4$ which is greater than the expected export profit in Case 1. The introduction of uncertainty in the home country makes exporting more likely even while reducing the overall expected profit of the firm.

When ex-post allocation is possible, not having access to a foreign market hurts a nonexporter both when the home-country shock is favorable and when it is unfavorable. On the one hand, a nonexporting firm is forced to sell all of its output at home even when $\gamma_h = 0$. On the other hand, when $\gamma_h = 2$, a nonexporting firm cannot sell more than it initially intended at home to take advantage of the high demand. However, with access to the additional market the firm benefits in both scenarios. When $\gamma_h = 0$, the firm can sell the output abroad instead of being forced to sell it for nothing at home. Moreover, when $\gamma_h = 2$, the firm can divert more of its output away from the foreign market and towards the more favorable home market.

Case 3 reverses the roles of the home and foreign countries. In this case, exporting remains more likely in the ex-post than in the ex-ante allocation version of the model because the firm recoups some of the loss from uncertainty via the ex-post allocation profit, which, as a consequence of the symmetry between the two countries, is the same as in Case 2. However, even when ex-post allocation is possible, a firm is less likely to export compared to the full-certainty case. This is because the benefit from mitigating foreign risk, i.e. the ex-post allocation profit, cannot possibly make up for the loss from introducing the risk in the first place, i.e. the drop in the ex-ante allocation version expected export profit from one to one quarter.

Cases 4 and 5 illustrate the impact of the correlation between home and foreign shocks on a firm's incentive to export. In both cases both γ_h and γ_f are either zero or two with

Table 1: The Impact of Uncertainty on Exporting

	Probabilities $g\left(\gamma_h, \gamma_f\right)$	Profit I & II: Nonexporter	Profit I: Exporter	Profit II: Exporter	Export Profit I	Export Profit II	Ex-Post Allocation Profit
Case 1: No Uncertainty	$g(1,1)=1$	1	2	2	1	1	0
Case 2: Home Uncertainty	$g(0,1)=\frac{1}{2}$ $g(2,1)=\frac{1}{2}$	$\frac{1}{4}$	$1+\frac{1}{4}$	$1+\frac{\sqrt{3}}{2}$	1	$\frac{3}{4}+\frac{\sqrt{3}}{2}>1$	$\frac{\sqrt{3}}{2}-\frac{1}{4}$
Case 3: Foreign Uncertainty	$g(1,0)=\frac{1}{2}$ $g(1,2)=\frac{1}{2}$	1	$1+\frac{1}{4}$	$1+\frac{\sqrt{3}}{2}$	$\frac{1}{4}$	$\frac{\sqrt{3}}{2}<1$	$\frac{\sqrt{3}}{2}-\frac{1}{4}$
Case 4: Positive Correlation	$g(0,0)=\frac{1}{2}$ $g(2,2)=\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	0
Case 5: Negative Correlation	$g(0,2)=\frac{1}{2}$ $g(2,0)=\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{7}{4}$	$\frac{1}{4}$	$\frac{3}{2}$	$\frac{5}{4}$

The expected profit for nonexporters is identical in the two models and are reported in the second column. Expected profit for exporters differ in the ex-ante and ex-post allocation versions of the model and are reported in columns 3 and 4, respectively. The expected export profit is the expected profit of an exporter less that of a nonexporter, i.e. column 3 (4) minus column 2 in the ex-ante (ex-post) allocation version of the model. This is reported in column 5 (6) for the ex-ante (ex-post) allocation version of the model. Finally, the ex-post allocation profit is the difference in the expected export profit between the two versions of the model (column 6 minus column 5), and it is reported in the last column.

equal probability. In Case 4 the shocks are perfectly positively correlated, so that always $\gamma_h = \gamma_f$, and the ex-post allocation profit is zero. Indeed, the firm cannot do better than always selling half of its output in the home country and half of its output abroad regardless of the shock realization, even when ex-post allocation is possible. Therefore, just as in Case 1, there is no ex-post allocation profit, and expected profits are the same in the two versions of the model, albeit lower than the expected profits in the full-certainty scenario.

Finally, in Case 5 the home and foreign shocks are perfectly negatively correlated so that the firm knows that the shock will be zero in one market and two in the other. It just does not know which until after the shocks are realized. With no possibility of ex-post allocation, an exporter will sell half of its output in each destination leading to results identical to the case of positive correlation. However, when ex-post allocation is possible, an exporter always sells all of its output in the market that receives the favorable shock thereby eliminating all demand uncertainty. Thus, the ex-post allocation profit makes up for the uncertainty-induced losses at home and abroad suffered by an equivalent firm in the ex-ante allocation version of the model. As a consequence, while the correlation between the home and foreign shocks has no impact on the incentives of firms in the ex-ante allocation version of the model, in the ex-post allocation version of the model, firms are more likely to export when shocks are less (positively) correlated as this increases the potential benefits of ex-post allocation of output.

4.2 The General Case

The numerical example highlights the way in which the probability that a firm exports depends both on uncertainty and on the ability to mitigate the negative impact of this uncertainty through ex-post allocation of output. In so doing, the numerical example demonstrates the key theoretical finding that distinguishes the two versions of the model. The following proposition generalizes the results illustrated in the numerical example and shows that the intuition borne out in the numerical example is not contingent on its specifics (proof in Appendix A):

Proposition 1. *The export cutoff:*

1. *decreases with the variance of home-country shocks if and only if the firm has the ability to allocate output ex post.*
2. *increases with the variance of foreign-country shocks regardless of the firm's ability to allocate output ex post.*
3. *increases with the covariance between the home- and foreign-country shocks if and only if the firm has the ability to allocate output ex post.*

While uncertainty abroad disincentivizes exporting in both versions of the model, the two versions differ in how both home uncertainty and the correlation between home and foreign shocks affect the incentive to export. In the ex-ante allocation version of the model, neither has any impact on exporting, but in the ex-post allocation version of the model, the former increases the attractiveness of exporting while the latter decreases it. In other words, uncertainty at home creates an incentive for the firm to export as a means of varying its sources of demand, and the smaller the correlation between the home and foreign shocks the greater is the increase in variety achieved by exporting. Importantly, because its qualitative predictions diverge for the two versions of the model, Proposition 1 provides testable hypotheses that I will exploit in the empirical portion of the paper to show that the data supports the ex-post allocation version of the model.

5 Empirics

The remainder of the paper is devoted to testing the two alternative versions of the model proposed in the theoretical portion of the paper. In Section 5.1, I introduce the French data I use. In Section 5.2, I show how to use this data to infer model-consistent measures of the second moments of the shock distribution, i.e., the second moments implied by the data if the model were the true data-generating process. In Section 5.3, I provide some descriptive statistics of the model-consistent shock distributions faced by firms. In Section 5.4, I use these shock distributions to show that the data rejects the ex-ante allocation version of the model in favor of the ex-post allocation version of the model, which is the main empirical result in this paper. Finally, in Section 6, I show that this result remains unchanged even when I allow for a variety of alternative specifications.

5.1 Data

I carry out the empirical analysis using French firm-level data from the ORBIS database. This database contains worldwide firm-level data, but the availability of particular variables varies widely across countries. The French subset of this data is particularly useful because, unlike the data for other countries, it contains firm-level export data. In particular, the database contains annual data on French firms from 2006 ($t = 1$ in what follows) through 2014 ($t = T = 9$ in what follows), including information on each firm's NAICS industry, revenue and proportion of that revenue that comes from exporting. Although the dataset does not contain a wide range of variables, the structure of the model is rich enough that the aforementioned variables suffice for my purposes.

I drop firms that are missing data for any of the nine years from 2006 to 2014. Because at its core the model is static, it is not intended to capture temporary entry into export markets. I therefore consider only firms that either export in all nine years or never export.[18] This also ensures that each firm's status as exporter or nonexporter is unambiguous. For reasons that I will discuss in footnote 29, and with the exception of

[18] As discussed in the Introduction, a growing literature seeks to explain why many firms switch between exporting and nonexporting. These explanations include experimental entry (see footnote 8) and volatility in foreign demand (see footnote 9). The former explains why firms export only temporarily before permanently exiting the foreign market, and the latter explains why some firms enter and exit export markets repeatedly.

Table 2: Data on Number of Firms and Exporters

	MFG	T&T	I&F	Total
Number of Firms	17,248	44,381	38,717	100,346
Percentage that Export	36.2%	20.8%	9.0%	18.9%
Export Share among Exporters	26.1%	21.6%	25.8%	23.8%

The export share among exporters is calculated as the mean share of revenue that comes from exporting across exporters without weighting by firm size. This is the relevant statistic in the scenario that all firms within a sector draw their openness from the same distribution.

some robustness checks, I use only firms in the manufacturing (MFG); trade and transport (T&T); and information and financial activities (I&F) supersectors, corresponding to the 1-digit NAICS supersectors 3, 4 and 5, respectively. Table 2 provides information on the number of firms and exporters in these supersectors in the data.

5.2 From Theory to Data

My strategy for testing the two alternative versions of the model laid out in Section 2 is to exploit variations in firms' export status in conjunction with variations in the joint distribution of home and foreign shocks faced by firms. This presents an immediate challenge because, while firms' export status is readily available in the data, the distribution of shocks faced by firms is only indirectly observed via firm-level revenue volatility. In what follows, I first describe how to interpret the time series data to be consistent with my static model. I then discuss the role that the various model parameters play in inferring shock realizations and ultimately shock distributions from the data. Finally, I show how to use the theoretical model together with the revenue data to extract information on the model parameters including measures of uncertainty. Importantly, because the two versions of the model differ in their predictions of firm behavior, the same revenue data implies different parameter values for each version of the model.

A Static Model with Time-Series Data

While the theoretical model was formulated as a one-period model, the data covers nine years. I assume that each firm in my dataset makes a one-time decision at $t = 0$, i.e., prior to 2006, whether or not to export. At the time of the export decision, the firm knows A_{it}, φ_i, $G_i(\cdot,\cdot)$, and f_{xi}, where i indexes the firm. A firm that chooses to export draws a B_{fi} from the distribution $\mathscr{B}_i$, and this remains the firm's B_{fi} throughout. Since a firm can equivalently be considered to draw $\mathscr{O}_{fi}$, I will consider $\mathscr{O}_{fi}$ rather than B_{fi} for the remainder of the paper.[19] In each period, in addition to drawing the shocks γ_{hit} and, if exporting, γ_{fit} from $G_i(\cdot,\cdot)$, each firm makes production, allocation and pricing decisions given its particular set of parameters as described in Section 2.

The Role of the Model's Parameters

Although not all the parameters play an important role in the theoretical results, they play an integral role in the empirical analysis. The parameters φ_i and $\mathscr{O}_{fi}$ are important because in their absence all revenue variation in the data across firms and destinations would be attributable to the firm-specific demand shocks, γ_{hit} and γ_{fit}. However, this would almost certainly be erroneous since much (if not most) of a firm's revenue is driven by persistent firm characteristics rather than transitory shocks. The parameters φ_i and $\mathscr{O}_{fi}$ capture these persistent firm characteristics, with φ_i capturing persistent differences in firms' abilities to generate revenue and with $\mathscr{O}_{fi}$ capturing persistent differences across firms in the share of revenue generated abroad.

In a similar vein, not all revenue variation across time for a single firm is attributable to unpredictable shocks, i.e., γ_{hit} and γ_{fit}. Indeed, some revenue variation is predictable to the firm when it makes its production decision, e.g., variations due to expected changes in demand. The role of A_{it} is to account for this predictable variation in revenue across time.[20] Thus, by accounting for both persistent firm features (φ_i and $\mathscr{O}_{fi}$) and predictable changes in a firm's ability to generate revenue (A_{it}), I can attribute the remaining revenue variation across firms and across time to the shock realizations.

Derivation of Model Parameters from the Data

Substituting Equations (4I) and (4II) into Equations (3I) and (3II), respectively, yields, for $v \in \{I, II\}$, realized revenue in country c for firm i in period t:

$$r^v_{cit} = K^v_{ci} A_{it} \psi^v_c\left(\gamma_{hit}, \gamma_{fit}\right) \tag{9}$$

where K^v_{ci} and ψ^v_c are defined by

$$\begin{aligned} K^I_{ci} &= \mathscr{O}_{ci}\left[\rho\varphi_i E\left(\gamma_{cit}\right)^{1/\sigma}\right]^{\sigma-1} &\qquad \text{and} \qquad \psi^I_c\left(\gamma_{hit}, \gamma_{fit}\right) &= \gamma_{cit}^{1/\sigma} \\ K^{II}_{ci} &= \mathscr{O}_{ci}\left[\rho\varphi_i E\left(\Gamma_{it}\right)^{1/\sigma}\right]^{\sigma-1} &\qquad \text{and} \qquad \psi^{II}_c\left(\gamma_{hit}, \gamma_{fit}\right) &= \Gamma_{it}^{1/\sigma-1}\gamma_{cit}, \end{aligned} \tag{10}$$

[19] In all the relevant results B_{fi} always appears together with τ in the form of $\mathscr{O}_{fi} = B_{fi}\tau^{1-\sigma}$ (see Equations (3I) , (3II), (4I) and (4II) as well as Conditions (6I) and (6II)). Indeed, none of the results would change if the model included only B_{fi} or only τ. The inclusion of τ as an iceberg cost common to all firms was solely for the purpose of maintaining consistency with the traditional notation in the literature.

[20] Like B_{fi} and τ, A_{it} and φ_i are interchangeable. However, for simplicity and without loss of generality, in the empirical analysis I assign the persistent element of revenue to φ_i and the time-varying but predictable element to A_{it}. Indeed, this is why below I normalize A_{i2006} to one, since A_{it} is only meant to capture variations across time for a given firm.

where $\Gamma_{it} = \gamma_{hit} + \mathbf{1}_{X_i=1}\mathcal{O}_{fi}\gamma_{fit}$, $\mathcal{O}_{hi} = 1$ and $\mathcal{O}_{fi} = \tau^{1-\sigma}B_{fi}$, just as in the theoretical portion of the paper. The term K^v_{ci} is a firm-destination-specific term that is time invariant and therefore does not contribute to variations in revenue over time.[21] Rather, variations in revenue over time are due to the two remaining terms, A_{it} and $\psi^v_i(\cdot,\cdot)$. The former leads to predictable variations in revenue, while the latter is a function of shocks leading to unpredictable variations in revenue.

Since the goal is to use the variations in the revenue data to infer the shocks realizations faced by the firm, it is necessary to purge the revenue data of variations in revenue unrelated to the shocks, i.e., variations caused by A_{it}. In order to do so, I assume that A_{it} can be approximated by a firm-level trend, that is, $A_{it} = (1+a_i)^{t-t_1}$, where a_i represents the trend of firm i. This assumption does not preclude the possibility of a sector- or economy-wide trend that is predictable from a firm's point of view since such a trend would be captured by the firm-level trend. In Section 6, I consider an alternative in which there is a sector-level component which is not constrained to follow a trend, but that is nevertheless predictable to the firm. As I show, this has almost no bearing on the results.

Given the firm-level trend, Equation (9) and the fact that total realized revenue is $R^v_{it} = r^v_{hit} + r^v_{fit}$, the log difference (with respect to time) of R^v_{it} is

$$\log R^v_{it+1} - \log R^v_{it} = \alpha^v_i + \varepsilon^v_{it}, \tag{11a}$$

where $\exp(\alpha_i) = (1+a_i)$.[22] The OLS estimate of a_i represents changes in demand (and, as a result, also revenue) that the firm foresees prior to making its export-status and output decisions. After controlling for the firm-level trend, the remaining revenue volatility is attributable to home and foreign shocks.

Having estimated A_{it} for all i and t, the next step is to back out the firm-destination-specific productivity factors (K^v_{ci}), which determine the level of firm i's revenue, and shock realizations (γ^v_{cit}), which determine the volatility around this level. For a given pair (K_{hi}, K_{fi}), Equations (9) and (10) together with data on firm revenue in each market yield shock realizations $(\gamma_{hit}, \gamma_{fit})$ for each period, t.[23] Since the best estimate for the sample mean of shocks is the population mean, the pair (K_{hi}, K_{fi}) is chosen so that the mean of shock realizations for each firm is one, i.e., $\sum_{t=1}^{T} \gamma_{hit}/T = \sum_{t=1}^{T} \gamma_{fit}/T = 1$ for all i.[24] While the assumption of a unit mean is without loss of generality (see footnote 14), it

[21] A natural interpretation of K^v_{ci}, based on Equation (10), is that it represents the firm's ability to generate revenue in country c taking into account both its productivity and the average demand for its good. In line with this, the firm's openness is the ratio of the firm's ability to generate revenue abroad and at home, that is, $K^v_{fi}/K^v_{hi} = \mathcal{O}_{fi}$.

[22] The error term is given by $\exp(\varepsilon^I_{it}) = \sum_{c\in\{h,f\}} \mathcal{O}_{ci}\gamma^{1/\sigma}_{cit+1} / \sum_{c\in\{h,f\}} \mathcal{O}_{ci}\gamma^{1/\sigma}_{cit}$ and $\exp(\varepsilon^{II}_{it}) = \Gamma^{1/\sigma}_{it+1}/\Gamma^{1/\sigma}_{it}$, for the ex-ante and ex-post allocation versions of the model, respectively. Since the shocks are independently and identically distributed (with respect to different periods), it follows that $E(\varepsilon^v_{it}) = 0$.

[23] In the main specification, I use $\sigma = 2.7$ for the demand elasticity, which is the median estimate in Broda and Weinstein (2006). In Section 6, I consider alternative values of the demand elasticity, and the results are very similar.

[24] In total for each exporting firm this procedure backs out twenty variables, K^v_{ci} for each country and eighteen γ_{cit} shocks (one for each country for each of the nine years in the data). To back out these vari-

has the advantage that the standard deviations and coefficients of variation of shocks are equal.

The procedure outlined above yields the shock realizations for every firm in every year from 2006 to 2014 in each market served by the firm as well as K_{hi}^{v} for every firm and K_{fi}^{v} for every exporter. However, to test the theory, I need data on the shock distributions faced by firms rather than on the particular shock realizations. To infer a shock distribution from the shock realizations, I assume that all firms within a 6-digit NAICS industry face the same shock distribution. I therefore compute the second moments of the shock distribution for each 6-digit industry by pooling all the realizations of all the firms in that 6-digit industry and computing statistics from this pooled sample.[25] In Section 6, I consider a specification in which every firm has its own unique shock distribution. However, since the hypothetical shocks that nonexporters would have received had they exported are unobserved, that specification only allows me to infer a home shock distribution for all firms. Finally, K_{hi}^{v} together with the shock realizations pooled at the 6-digit level are sufficient to compute the productivity, φ_i of every firm (by calculating the sample means of $\gamma_{cit}^{1/\sigma}$ and $\Gamma_{it}^{1/\sigma}$, see Equation (10)).

Importantly, the inferred shock realizations and by extension the inferred shock distributions (as well as destination-specific factors, K_{ci}^{v} and productivity measures φ_i) differ across the two versions of the model. This is because when firms can only allocate output ex ante, revenue in the home (foreign) country is determined solely by home (foreign) shock realizations. In contrast, when firms can allocate output ex post, home and foreign revenues are both determined by both shock realizations simultaneously. Formally, this can be seen from the definitions of $\psi_c^{v}(\cdot,\cdot)$. While $\psi_c^{I}(\cdot,\cdot)$ depends only on the shock realization in market c, $\psi_c^{II}(\cdot,\cdot)$ depends on the shock realizations in both markets. As an example, suppose observed revenue at home in a given year is above the firm's revenue trend. With ex-post allocation ruled out, the only possible explanation is a favorable home shock. However, when ex-post allocation is allowed, the high revenue at home could alternatively be the result of an unfavorable foreign shock which led the firm to allocate more output towards the home market.

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ables, I have twenty equations for each exporting firm, eighteen equations from substituting Equation (10) into Equation (9) (one for each country for each of the nine years in the data) and two equations from normalizing the mean of home and foreign shocks to one. Thus, formally backing out these variables involves solving twenty equations with twenty unknowns. Similarly, for nonexporters there are ten equations with ten unknowns.

[25] For concreteness, consider a typical manufacturing industry, NAICS industry 312130 that has a total of 237 firms of which 87 export. Since I have nine years of data, this gives me $237 \times 9 = 2133$ observed draws from the home shock distribution and $87 \times 9 = 783$ observed draws from the foreign shock distribution from which I can directly estimate the variance of home and foreign shocks. In addition, from the exporting firms I have 783 pairs of home and foreign shocks from which I can estimate the covariance between home and foreign shocks for this 6-digit industry.

5.3 Model-Consistent Second Moments

The procedure outlined in the previous section interprets the data through the lens of the model. Thus, the values derived in the previous section for the shock realizations, $\left(\gamma_{hit}, \gamma_{fit}\right)$, (as well as the destination-specific factors (K_{ci}^{v}) and productivities (φ_i)) are consistent with the model accurately describing the data-generating process. As a result, the estimates for the second moments of the shock distributions are also consistent with the model being the data-generating process, and hence the description of the second-moments as "model-consistent".[26] Indeed, all the parameters derived in the previous section are model consistent. This is important because testing the theory using these measures of uncertainty together with the other parameters is logically consistent in the sense that if the model's predictions are borne out in the data using these measures of uncertainty, then the model is consistent with the data.

Before testing the theory, I present some descriptive statistics of the second moments of the shock distributions implied by the model and the data. As mentioned above, I focus on NAICS supersectors beginning with 3, 4 and 5, which contain a total of 370 6-digit NAICS industries. I drop industries with no exporters since I have no information on foreign shocks in those industries. After dropping these industries, I am left with 351 6-digit manufacturing industries with a total of 100,346 firms (see Table 2 for more details).

Table 3 reports descriptive statistics of the second moments of the shock distributions derived in Section 5.2. These estimates, together with the information in Table 2, show that not only is there substantial variation in the proportion of exporting firms across industries, but also in the second moments of the distribution of home and foreign shocks. It is this variation that I will exploit to test which of the versions of the model accords better with the data.

The estimates in Table 3 provide some insight into the difference between the shock distributions implied by the two versions of the model. Specifically, the estimates make clear that the same revenue data implies higher variance of shocks in the ex-ante as compared to the ex-post allocation version of the model. To see why this is so, consider the shock realization implied by a given revenue increase in a particular market for an exporter. In the ex-ante allocation version of the model, this can only be explained by an increase in the shock realization by the same factor as the increase in revenue, i.e., if revenue doubled then so too did the shock realization. However, in the ex-post allocation version of the model, in addition to the direct effect of the shock realization on revenue, the revenue gets an additional boost as a result of ex-post allocation of output towards this market, i.e., a doubling of revenue implies less than a doubling of the shock realization. Analogously, a decrease in revenue in the ex-ante allocation version must be

[26]Using the shock realizations to estimate the second-moments of the (unobserved) shock distribution potentially introduces a finite-sample bias into the estimates of the second moments. However, most 6-digit industries have a sufficient number of firms to mitigate the risk of a substantial finite sample bias since I have nine years of shock realizations for each firm. The median 6-digit industry in terms of number of firms in my sample has 129 firms, and the median in terms of exporters has 20 exporters. This translates into a median of $129 \times 9 = 1161$ observations for estimating the home variance and $20 \times 9 = 180$ observations for estimating the foreign variance and the covariance between home and foreign shocks.

Table 3: Descriptive Statistics for the Second Moments of the Shock Distributions

		NAICS	Min.	Max.	Median	Mean	St. Dev.
Version I	H. Variance	31-33	0.05	4.05	0.27	0.33	0.33
		42-49	0.03	0.92	0.23	0.29	0.19
		51-56	0.08	1.69	0.35	0.39	0.25
	F. Variance	31-33	0.10	2.67	1.06	1.09	0.50
		42-49	0.05	6.04	1.38	1.40	0.71
		51-56	0.17	4.13	1.47	1.59	0.77
	Covariance	31-33	-0.84	0.47	-0.05	-0.05	0.12
		42-49	-0.31	0.30	-0.04	-0.05	0.08
		51-56	-0.66	0.53	-0.07	-0.08	0.16
Version II	H. Variance	31-33	0.05	0.73	0.19	0.21	0.10
		42-49	0.03	0.57	0.20	0.22	0.11
		51-56	0.08	1.69	0.31	0.35	0.23
	F. Variance	31-33	0.07	1.31	0.38	0.40	0.19
		42-49	0.02	1.45	0.49	0.51	0.21
		51-56	0.07	1.88	0.56	0.61	0.31
	Covariance	31-33	-0.17	0.64	0.10	0.12	0.09
		42-49	-0.03	0.40	0.12	0.12	0.08
		51-56	-0.04	0.90	0.15	0.18	0.16

The descriptive statistics are calculated at the 1-digit NAICS level. Shock distributions are common to all firms at the 6-digit NAICS level.

due to an equally large decrease in the shock realization, while in the ex-post allocation version part of the decrease in revenue is attributable to ex-post allocation of output away from the market. In other words, compared to the ex-ante allocation version of the model, the ex-post allocation version of the model requires a less favorable (unfavorable) shock to rationalize a given increase (decrease) in revenue. As a result, explaining the same variation in revenue data requires a greater variance in shock realizations in the ex-ante allocation version of the model compared to the ex-post allocation version of the model. A key lesson from Table 3 is that the shocks inferred from revenue data depend heavily on the specifics of the model, and, in particular, ignoring firms' ability to allocate output ex post leads to a substantial overstatement of the uncertainty firms face.

Besides being useful for testing the theory, the shock distributions are interesting in their own right. For example, some authors have conjectured that firms face greater uncertainty abroad than domestically because firms that export a large fraction of their output tend to also have more volatile revenue streams than either similar nonexporting firms or firms that export only a small fraction of their output.[27] My results confirm this view more directly. Indeed, according to the computations for the ex-post allocation version of the model, the variance of foreign shocks are on average 0.19, 0.29 and 0.26 higher than the variance of home shocks in the NAICS supersectors beginning with 3, 4 and 5, respectively. Moreover, the variance of foreign shocks is greater than the variance of home shocks in all but 3 out of the 351 6-digit industries.

While qualitatively the results for the variance of home and foreign shocks are similar in the two versions of the model, the same is not true for the correlation between home and foreign shocks. The results for the ex-ante allocation version of the model show that both the median and the mean covariances are negative in all three supersectors. This result stems from the negative correlation of sales across destinations, which has been found in several other datasets (see footnote 2). Since, as discussed in the Introduction, firms tend to serve export markets that are similar to their home market, we should expect a priori to observe a positive correlation between home and foreign shocks. Thus, the fact that the ex-ante allocation version of the model implies a negative correlation between home and foreign shocks is, at the very least, circumstantial evidence against that version of the model. In contrast, the ex-post allocation version of the model takes into account that output is allocated in response to shocks realizations, and, as a result, even though home and foreign sales are negatively correlated, the results show that shocks are positively correlated. Indeed, the correlation coefficient between home and foreign shocks overall is 0.37, and it is positive in 95.7% (336 out of 351) of 6-digit industries.

5.4 Evidence on the Ex-Post Allocation Motive for Exporting

Regardless of the timing of the allocation of output, a firm exports if and only if its expected export profit exceeds the firm-specific fixed cost of exporting (f_{xi}), i.e., if and only if the difference between the expected profit when exporting and when not exporting

[27] See, for example, Vannoorenberghe (2012).

exceeds the fixed cost of exporting. A log transformation of this export condition yields:

$$X_i = \begin{cases} 1 & y_i > 0 \\ 0 & \text{otherwise} \end{cases}, \tag{12}$$

where, as I show in Appendix B, the latent variable, y_i, can be linearly approximated by (using Condition (6I) or (6II) for the ex-ante and ex-post allocation versions of the model, respectively):

$$y_i = \beta_0 + \beta_1 \log \varphi_i + \sum_j \beta_j D_{ij} + D_{ik} \alpha_k Z + \varepsilon_i, \tag{13}$$

where D_{ij} (D_{ik}) is a dummy variable that equals one if firm i is in NAICS sector j (k) and

$$\alpha_k Z = \alpha_{1k} \mathrm{var}_i(\gamma_h) + \alpha_{2k} \mathrm{var}_i(\gamma_f) + \alpha_{3k} \mathrm{cov}_i(\gamma_h, \gamma_f), \tag{14}$$

where the second moments for firm i are determined by its 6-digit NAICS industry as explained in Section 5.2.[28] This specification, with interaction dummies for sector k, implies that the effect of the second moments of the shock distribution faced by a firm on the firm's export probability depends on the firms sector, and, in particular, the effect is not constrained to be common to firms in different sectors.[29] The interpretation of the β_j's is that they control for the distribution of $\mathcal{O}_f$ from which the firm draws its openness as well as the element of the exporting fixed cost that is common to firms at the 2-digit (3-digit) industry level. The error term, ε_i, accounts for the firm-specific element of the exporting fixed cost.[30]

[28] The j indexes 2-digit (or in an alternative specification 3-digit) NAICS industries. Dummies at more disaggregated levels are inappropriate because at the 4-digit NAICS level there are 171 industries as compared to 351 at the 6-digit NAICS level, with 71 of the 4-digit industries containing only a single 6-digit industry. This is problematic because in those cases the dummy variables will invariably pick up variations in the second moments of the shock distributions, which are common to all firms within a 6-digit industry.

[29] In the main specification, the k indexes 1-digit NAICS supersectors. For each of these supersectors, I estimate three coefficients (one for each second moment of the shock distribution). Since all firms within a 6-digit industry are assumed to have the same shock distribution, in order to reliably estimate α_k, sector k must contain a sufficient number of 6-digit NAICS industries. I set the minimum at forty 6-digit industries per supersector, which rules out supersectors other than 3, 4 and 5. This limitation also precludes estimating these coefficients at the 2-digit level except in the manufacturing sector. For robustness, in Section 6 I estimate the coefficients on the second moments of the shock distribution at the 2-digit level for the manufacturing sector, i.e., for sectors 31, 32 and 33. In addition, I also estimate one set of α coefficients for the entire economy, i.e., no dummies, which allows me to include data from the other supersectors. The results in both these cases do not change any of my conclusions.

[30] The fixed cost of exporting can be decomposed into a sector-j specific element (f_j^x) and a firm-specific element (f_i^x), so that $f_{xi} = f_j^x f_i^x$ with $E(\log f_i^x) = 0$. As I show in Appendix B, this implies that $\varepsilon_i = -\log f_i^x$ and that the sector-j fixed effect (β_j) is equal to $\log E_j(\mathcal{O}_{fi}) - \log f_j^x$ in the ex-ante allocation version of the model and $\log E_j[\mathcal{O}_{fi}/(1+\mathcal{O}_{fi})] + E_j\left[\mathcal{O}_{fi}^2/(1+\mathcal{O}_{fi})\right]/E_j[\mathcal{O}_{fi}/(1+\mathcal{O}_{fi})] - \log f_j^x$ in the ex-post allocation version of the model. The j subscript on the expectation operator denotes the fact that the distribution from which firms draw their openness is common to all firms in sector j. The decomposition of the exporting fixed cost is without loss of generality since the decomposition is always possible by setting $\log f_j^x = -E(\log f_{xi})$ for all j. The key to identification, therefore, is that the log of the firm-specific element of the exporting fixed cost ($\log f_i^x$) is uncorrelated with the second moments of the shock distribution.

I test the ex-ante allocation version against the ex-post allocation version of the model by running a logit regression defined by Equations (12), (13) and (14). As per Proposition 1, the ex-ante allocation version of the model predicts that $\alpha_2^I < 0$ and $\alpha_1^I = \alpha_3^I = 0$, while the ex-post allocation version of the model predicts that $\alpha_1^{II} > 0$ and $\alpha_2^{II}, \alpha_3^{II} < 0$. It is insufficient to run just one regression to test the two versions of the model, since, as explained in Section 5.2, the inferred shock distributions and therefore the data inputs into the regressions are model dependent. Rejecting the ex-ante allocation version of the model based on data implied by the ex-post allocation version would be inconsistent, as would accepting the ex-post allocation version of the model based on data implied by the ex-ante allocation version. Importantly, since the measures of productivity and uncertainty are model consistent (as discussed in Section 5.3), this implies that if for either version of the model the regression results agree with that version's predictions, then the data is consistent with that version of the model being the true data-generating process.[31] Therefore, in Table 4, I present two sets of results for the logit regression, one for each of the two versions of the model, where the data used to test each version is consistent with that version of the model.

The results for the ex-ante allocation version of the model (positive first row, negative second and third rows in Table 4) are at odds with that version's predictions (zero first and third rows, negative second row) for all three 1-digit supersectors regardless of whether the fixed effects are at the 2- or 3-digit level. Indeed, all but one of the estimates are significant at the 99% level.[32] The data, therefore, strongly rejects the the ex-ante allocation version of the model.

For the ex-post allocation version of the model, however, the results are exactly as the model predicts (positive first row, negative second and third rows) for all three supersectors when fixed effects are the 2-digit level. When fixed effects are at the 3-digit level the results are as the model predicts for two out of the three supersectors, and in the third supersector, two out of three coefficients have the correct sign. Again, all these results are significant at the 99% level. Moreover, the regression results also deliver a significant and positive effect of productivity on export status, which is also consistent with the model's predictions. On the basis of these results, therefore, I conclude that the data supports the hypothesis that firms have the ability to allocate output across markets in response to shock realizations and that the opportunity to do so constitutes a motive for exporting.

Finally, the results in Table 4 show that the impact of uncertainty and in particular the ability to allocate output across destinations in response to shock realizations has a substantial impact on a firm's export probability. For instance, in the manufacturing sector moving up the distribution of home-shock variances by one standard deviation increases the odds of exporting by 98%, which, for a firm with an initial 10% probability of exporting translates into a 8.0 percentage point increase in the probability of exporting.[33]

[31] Since the data inputs into the regressions differ for the two versions of the model, it is theoretically possible for the data to be consistent with both versions of the model.

[32] The one insignificant parameter estimate is the effect of foreign uncertainty on the probability of exporting, which is also inconsistent with the ex-ante allocation version of the model.

[33] This corresponds to an increase of 0.1 in the variance of home shocks faced by the firm (see Table 3).

Table 4: Logit Estimates for Ex-Ante & Ex-Post Allocation Versions of the Model

		Ex-Ante Allocation		Ex-Post Allocation	
		2-Digit FE	**3-Digit FE**	**2-Digit FE**	**3-Digit FE**
MFG, 31-33	$\text{var}(\gamma_h)$	3.89^{**}	2.58^{**}	6.84^{**}	5.32^{**}
		(0.189)	(0.250)	(0.323)	(0.418)
	$\text{var}(\gamma_f)$	-0.92^{**}	-1.10^{**}	-2.79^{**}	-2.82^{**}
		(0.050)	(0.063)	(0.153)	(0.195)
	$\text{cov}(\gamma_h, \gamma_f)$	-3.79^{**}	-4.56^{**}	-5.06^{**}	-6.80^{**}
		(0.330)	(0.378)	(0.445)	(0.558)
T&T, 42-49	$\text{var}(\gamma_h)$	4.10^{**}	3.84^{**}	7.26^{**}	6.35^{**}
		(0.114)	(0.153)	(0.233)	(0.280)
	$\text{var}(\gamma_f)$	-0.02	-1.01^{**}	-0.76^{**}	-4.13^{**}
		(0.035)	(0.072)	(0.118)	(0.209)
	$\text{cov}(\gamma_h, \gamma_f)$	-1.43^{**}	-1.37^{**}	-1.01^{**}	2.41^{**}
		(0.298)	(0.499)	(0.371)	(0.492)
I&F, 51-56	$\text{var}(\gamma_h)$	1.46^{**}	2.59^{**}	1.77^{**}	3.57^{**}
		(0.091)	(0.123)	(0.127)	(0.181)
	$\text{var}(\gamma_f)$	-0.52^{**}	-0.46^{**}	-1.02^{**}	-1.02^{**}
		(0.044)	(0.048)	(0.134)	(0.146)
	$\text{cov}(\gamma_h, \gamma_f)$	-2.28^{**}	-1.49^{**}	-1.48^{**}	-2.37^{**}
		(0.224)	(0.249)	(0.276)	(0.329)
	$\log \varphi_i$	0.55^{**}	0.57^{**}	0.53^{**}	0.54^{**}
		(0.007)	(0.007)	(0.006)	(0.007)

Statistical significance at the 99 percent level is denoted by ** and at the 95 percent level is denoted by *. Standard errors are in brackets.

Similarly, moving up by one standard deviation in the distribution of foreign-shock variances or covariances between home and foreign shocks leads to a decrease in the odds of exporting by 41% and 37%, respectively.[34] This translates into 3.9 and 3.4 percentage point decreases, respectively, in the probability of exporting for a firm with an initial 10% probability of exporting.[35]

As mentioned in footnote 30, given the structure of the model, the key to identification is that the log of the firm-specific element of the fixed cost of exporting is uncorrelated with the second moments of the firm's shock distribution. However, if the model omits sector characteristics that are correlated both with the second moments of the shock distribution and the propensity to export, such an omission could potentially call into question the conclusion that it is the ability to allocate output across destinations in response to shock realizations that provides the causal link between the propensity to export and the second moments of the shock distribution. One possibility is that innovative products are associated both with more demand uncertainty by dint of the inherent uncertainty surrounding new products and with more exporting if innovation is conducive to exporting.[36] Another logical possibility is that high levels of growth are associated with increased sales volatility but at the same time firms experiencing high levels of growth are also more likely to export.[37]

While either of these possibilities can explain the positive correlation between a firm's probability of exporting and the variance of its home shocks, they cannot explain the negative relationship between the propensity to export and the correlation between home and foreign shocks. Indeed, absent the ex-post allocation motive for exporting, i.e., if ex-post allocation were not possible, it would be reasonable to expect a positive relationship between the probability of exporting and the correlation between home and foreign shocks. Specifically, since firms are more likely to export to countries similar to their own (see footnote 3), we should expect similarity between countries, that is, a stronger positive

[34] This corresponds to an increase of 0.19 in the variance of foreign shocks and an increase of 0.09 in the covariance between home and foreign shocks faced by the firm (see Table 3).

[35] These results are based on the estimates with fixed effects at the 2-digit NAICS level. For a firm with an initial 10% probability of exporting in the trade and transport supersector (with corresponding results for the information and financial activities supersector in parentheses), moving up one standard deviation in the home-shock distribution increases the probability of exporting by 9.8 (4.3) percentage points, and a similar move in the distribution of foreign-shock variances or covariances between home and foreign shocks decreases the probability of exporting by 1.4 (2.6) and 0.8 (1.9) percentage points, respectively.

[36] The evidence on the link between innovation and exporting is far from conclusive. While some authors have found a positive correlation between innovation and a firm's export status, e.g., Caldera (2010), others have found a negative correlation, e.g., Wakelin (1998). Yet other authors have found no link between innovation and exporting, e.g., Damijan, Kostevc and Polanec (2008).

[37] In the main specification I purge the data from firm-level trends, and in the robustness checks I do so for sector-level volatility in addition to this trend. Nevertheless, it is possible that high-growth firms have more erratic growth, so that even when the trend is accounted for, their revenues are more volatile than low-growth firms. Whether or not high growth firms are more likely to export is unclear. On the one hand, smaller firms may grow faster than larger firms (e.g., Dunne, Roberts and Samuelson (1988)) but are less likely to export. On the other hand, exporting may drive growth, so that exporting firms, especially those expanding into new markets, may grow particularly fast (e.g., Bernard and Jensen (2004)).

correlation between home and foreign shocks, to be positively predictive of exporting.[38] That the opposite is the case points to there being a benefit, from the point of view of the firm, from differences between the home and foreign markets. A natural explanation is that the firm benefits from having varied sources of demand because this allows it to take advantage of the ability to allocate output in response to shock realizations.

The results in this section are the key results of this paper. By showing the effect of uncertainty both at home and abroad on the export-status decision of firms, the regression results demonstrate that the data is consistent with the ex-post allocation version of the model. This underscores the importance of the ability to allocate output in response to shock realizations as a factor in firms' export decisions. Thus, together with the theoretical findings in Section 3, the results in this section support the hypothesis that there is indeed an ex-post allocation motive for exporting.

6 Robustness

In this section, I provide some robustness checks on the results reported in Section 5. Here I report the major findings across various alternative specifications and assumptions, but I relegate the tables with the particular point estimates to Appendix C. The overarching theme in this section is that the findings are remarkably consistent across all specifications, with only minor deviations from the results reported in the previous sections. Thus, the results that follow provide additional empirical support for the ex-post allocation version of the model and the ex-post allocation motive for exporting.

Alternative Measure of Firm Productivity and Controlling for Firm-Level Openness

In the main specifications in Section 5, I control for firm productivity using the model-consistent measure of productivity, i.e., the firm's productivity if the model were the true data-generating process. To test the robustness of the results with respect to this measure of productivity, I instead use the log of the mean domestic revenue for each firm as that firm's measure of its productivity. The results are little changed by this alternative measure of productivity. Indeed, the measure of productivity implied by the model and this alternative measure of productivity are highly correlated (correlation coefficient equal to 0.97).

One of the inherent limitations of the data is the lack of information on the hypothetical openness of nonexporters, that is, it is impossible to know how open a nonexporter would be in the event that it had chosen to export. As a result, in the main specification, I assume that firms draw their openness only after deciding whether or not to export. Nevertheless, it is worthwhile considering whether or not the results are robust to controlling

[38] In addition, as discussed above, that firms export to countries similar to their home country should imply a positive correlation between home and foreign shocks, which is what the data implies for the ex-post allocation version of the model. However, in the ex-ante allocation version of the model the data implies a negative correlation, providing further evidence in favor of the ex-post allocation version of the model and against the ex-ante allocation version of the model.

for firm-level openness. I consider two ways of addressing the lack of data on openness for nonexporters. The first way is to compute the mean openness of all exporters within a 6-digit NAICS industry and assign all firms in the 6-digit industry, including nonexporters, that level of openness. An alternative approach is to assign to exporters the model-consistent value of openness ($\mathscr{O}_{fi}$) and use those values to impute openness for nonexporters by assuming a linear relationship between the log of productivity and openness.[39] Both approaches are problematic due to selection issues, i.e., nonexporters may have chosen not to export because they knew that they would have had a low level of openness had they chosen to export. However, this is unlikely to be a significant issue because openness is uncorrelated or very weakly correlated with the second moments of the shock distributions.[40] Even so, neither method of controlling for firm-level openness changes the results, and this is true regardless of whether the measure of openness used is the model-consistent measure ($\mathscr{O}_{fi}$) or the more commonly used foreign share of firm revenue (r_{fi}/R_i).

Alternative Assumption for Firm-Level Trend Next, I consider an alternative to the assumption that the predictable element of revenue variation follows a time trend. Here, in addition to a firm-level time trend, I allow for an industry-level component at either the 3-digit or 6-digit NAICS level which is predictable to the firm even though it does not necessarily follow a time trend. Formally, this means that $A_{it} = A_{st}(1+a_i)^{t-t_1}$, where a_i represents the trend of firm i and $A_{st} = \sum_{i \in NAICS} r_{it} / \sum_{i \in NAICS} r_{it_1}$ is the total revenue in year t relative to the total revenue in the first year in the data in either the 3-digit or 6-digit NAICS industry.

Thus, the sectoral factor, A_{st}, is normalized to unity in t_1 (2006), which means that this term captures variations in firm revenues attributable to sectoral changes relative to the initial year in the data. In this formulation, A_{st} captures all changes that affect every firm equally in sector s, such as sector-wide changes in demand or cost of inputs, while the firm-specific element captures each firm's trend relative to the rest of their sector.[41] The procedure for inferring shocks is identical to the one outlined in Section 5.2, except that Equation (11a) is replaced with Equation (11b):

$$(\log R_{it+1} - \log A_{st+1}) - (\log R_{it} - \log A_{st}) = \alpha_i + \varepsilon_{it}, \tag{11b}$$

where, as before, $\exp(\alpha_i) = (1+a_i)$ and $\exp(\varepsilon_{it}) = \Gamma_{it+1}^{1/\sigma} - \Gamma_{it}^{1/\sigma}$.

This alternative assumption regarding the structure of the predictable element of volatility has almost no effect on the specific values of the shock realizations that I infer for each

[39] I estimate this linear relationship at the 6-digit level by regressing log productivity on openness of exporters.

[40] Among exporters, the correlation coefficients between openness and the variance of home shocks, the variance of foreign shocks and the covariance between home and foreign shocks are 0.019, -0.002 and 0.009, respectively.

[41] The term A_{st} can be thought of as corresponding to the revenue factor in a general-equilibrium version of a Melitz model, $E(Pw)^{1-\sigma}$, where E is total consumer expenditure in the sector, P is the sectoral price index and w is the input price.

firm in each year, and hence on the inferred shock distributions faced by firms. Indeed, the correlation coefficient between the shock realizations inferred in the main specification and shock realizations inferred using the alternative specification considered here is 0.99 and 0.96 when the sectoral component, A_{st}, is at the 3-digit and 6-digit levels, respectively. As a result, none of my conclusions are altered by this alternative specification, and, in fact, the point estimates are almost identical to the point estimates in the main specification.

Alternative Values for the Demand Elasticity

The values for the shock realizations and subsequent results in Section 5, were obtained using $\sigma = 2.7$, which is the median value of the demand elasticity reported in Broda and Weinstein (2006). Since the problem of estimating the price elasticity of demand is a complex one, I do not take a stand on the correct estimates. Instead, I check the robustness of my results with respect to alternative values of the demand elasticity. Importantly, because σ enters the computations only in Equation (10), the choice of σ does not affect the theory in any qualitative way. Rather the choice of σ affects only the particular values of the shock realizations inferred from the data. The results remain largely unchanged when shocks are inferred using demand elasticities ranging from slightly greater than one to five. For larger values of the demand elasticity some of the results become statistically insignificant because, as explained below, as σ increases the variation in the data on second moments of the shock distribution decreases.

To see how the value of the demand elasticity affects the values of the shock realizations inferred from the revenue data, note that as $\sigma \to 1$, the revenue in each destination ceases to depend on the quantity sold in that destination, and therefore there is no advantage to ex-post allocation of output. In fact, the shocks inferred from the ex-post and ex-ante allocation versions of the model converge as σ approaches one (see Equation (10)). Conversely, as $\sigma \to \infty$, a firm with the ability to allocate output ex post will allocate as much output as possible to the destination with the more favorable shock, even if the shock is only marginally more favorable. The upshot is that, as σ increases, the variance of shocks needed to explain the variance in the revenue data decreases. As a result, for high values of σ there is very little variation in the second moments of the shock distributions across industries, which leads some of the results to become statistically insignificant.

Effect of Second Moments Interacted at Different NAICS Levels

In the specifications considered in Section 5, the coefficients on the second moments of the shock distribution interacted with dummies at the 1-digit level, i.e., the effect of the second moments of the shock distributions on the probability of exporting depended on the firm's 1-digit NAICS supersector. Here I consider the possibility that these dummies interact at the 2-digit level instead, i.e, the effect of the second moments depends on the firm's 2-digit NAICS sector. However, since I estimate three parameters for each 2-digit sector and all firms within a 6-digit industry face the same shock distribution, I consider only 2-digit sectors which contain at least forty 6-digit sectors (see footnote 29). This limits me to the three 2-digit manufacturing sectors (sectors 31, 32 and 33).[42] The results are consistent with the ex-post allocation version of the model and statistically significant at the 99% level for two out of the three sectors (31 and 33), while in the other sector the results are statistically insignificant.

As an alternative, I consider a scenario in which there are no interacting dummies on the coefficients on the second moments of the shock distribution, i.e., the effect of the second moments of the shock distribution on a firm's export probability is independent of the firm's sector. The advantage of this approach is that it allows me to incorporate more sectors into the analysis, since I am not forced to discard sectors with too few 6-digit industries. Therefore, in addition to considering results including only the firms in the three 1-digit supersectors that I have considered thus far (sectors beginning in 3, 4 and 5), I also consider results including all firms in the dataset (a total of 157,934 firms spread over 480 6-digit industries).[43] The results are exactly as the model predicts and statistically significant regardless of which sectors are included in the analysis.

Firm-Specific Shock Distributions

The final assumption that I relax is that all firms in a given 6-digit sector face the same shock distribution. Here I assume that each firm faces a firm-specific shock distribution. The second moments of each firm's shock distribution can then be estimated from the same shock realizations that were derived in Section 5.2.[44] The main problem with this approach is that it is impossible to estimate the variance of foreign shocks or the covariance between home and foreign shocks for nonexporters because I do not observe shock realizations for these firms. Indeed, one of the motivations for the assumption that the shock distribution was common to all firms at the 6-digit level was to circumvent this problem. Thus, I estimate Equation (13) with only the variance of home shocks while omitting the variance of foreign shocks and the covariance between home and foreign shocks. These omissions introduce omitted variable bias into the estimate of the coefficient on the variance of home shocks. Because the omitted variables are positively correlated with the variance of home shocks and have a negative effect on the probability of exporting, the estimate of the coefficient on the variance of home shocks will be biased downward.[45] However, since this militates against the model's prediction, finding a

[42] It is impossible to consider scenarios in which the effect of the second moments on the export probability depends on the firm's 3-digit sector because the 3-digit sector that nests the most 6-digit sectors (in the data) has only 21 6-digit sectors, with the majority having fewer than 10.

[43] This includes all firms which meet the criteria discussed in Section 5.1, i.e., firms that either export in all years in the data or never export.

[44] The difference is that in this case I do not pool all the shock realizations of firms in the 6-digit industry to estimate the second moments of the shock distributions.

[45] The correlation coefficient between the variance of home shocks and the variance of foreign shocks is 0.33, and the correlation coefficient between the variance of home shocks and the covariance between home and foreign shocks is 0.55.

positive coefficient despite the downward bias constitutes evidence in favor of the model.

One advantage of this approach is that the effect of the variance of home shocks can now differ across more disaggregated levels, rather than only at the 1- or 2-digit levels as discussed above. The reason is that since each firm now has its own shock distribution there is sufficient variation even within 2 and 3-digit sectors to estimate an industry-specific effect of the home variance, provided that there are enough firms in those sectors. Thus, when the effect of home variance is allowed to depend on the firm's 3-digit NAICS sector, the coefficient on the variance of home shocks is positive in twenty five of the thirty-three sectors in which the results are statistically significant at the 99 percent level. At the 2-digit level, eight out of the nine sectors with statistically significant coefficients are consistent with the model's prediction, and at the 1-digit level all three supersectors have a positive and statistically significant coefficient on the variance of home shocks at the 99% level.[46] Considering that the estimates are downward biased, these results strongly support the ex-post allocation version of the model.[47]

7 Conclusion

This paper explores the role that uncertainty plays in a firm's decision to export. In particular, I address the following questions: (1) Do risk-neutral firms benefit from varying their sources of demand via exporting? and (2) Does this benefit constitute a motive for exporting that shows up in the data?

In order to address these questions, I develop a Melitz-type model in which uncertainty is modeled as a market-specific demand shock whose realizations firms learn only after choosing their export status and level of output. In the ex-ante allocation version of the model, in addition to total output, firms must also choose the destination of that output prior to the realization of shocks. In contrast, in the ex-post allocation version of the model, firms need not commit to the final destination of their output until after the realization of shocks.

The timing of the allocation of output across destinations has important consequences. While in both versions of the model exporting benefits a firm by *increasing* its sources of demand, only in the ex-post allocation version of the model does exporting benefit a firm by *varying* its sources of demand. This benefit stems from the firm's ability to divert more output towards the market with relatively favorable shock realizations and away from the market with relatively unfavorable shock realizations, and it is therefore present only in

[46] Not surprisingly, at more disaggregated levels fewer of the sectors have statistically significant coefficients. At the 1-digit level all sectors are statistically significant at the 99% level, at the 2-digit level 64% of sectors are statistically significant, and for 3-digit sectors this drops to 54%. However, given the downward bias of the estimates, it should be expected that fewer estimates will be statistically significant than if no variables were omitted.

[47] When all sectors are included in the regressions rather than only those in supersectors 3, 4 and 5, the results remain largely unchanged. At the 3-digit (2-digit) level, of the thirty-eight (ten) sectors with statistically significant coefficients twenty seven (nine) have positive coefficients. At the 1-digit level, seven out of eight supersectors have positive coefficients with four of those being statistically significant.

the ex-post allocation version of the model but absent in the ex-ante allocation version of the model. Crucially, this ex-post allocation benefit stems from an increase in exporters' expected profit rather than from a decrease in the volatility of their revenue (or profit) so that even risk-neutral firms benefit. An important result is that, since this benefit from exporting is present only when ex-post allocation of output is possible, firms are more likely to export in the ex-post allocation as compared to the ex-ante allocation version of the model.

Besides predictions regarding the overall likelihood of exporting, the model also provides predictions on the effects of the second moments of the shock distribution on a firm's propensity to export. Specifically, the model provides three testable implications: (1) the propensity to export increases with home uncertainty only in the ex-post allocation version of the model; (2) the propensity to export decreases with foreign uncertainty in both versions of the model; and (3) the propensity to export decreases with the correlation between home and foreign shocks only in the ex-post allocation version of the model. That both versions of the model agree on the impact of foreign uncertainty is a reflection of the fact that uncertainty abroad is detrimental to a firm's profitability abroad when production decisions are made prior to the realization of shocks. However, that the two versions of the model disagree on the impact of home uncertainty and the correlation of shocks across destinations is a reflection of the fact that only in the ex-post allocation version of the model does the firm benefit from having varied sources of demand. Indeed, greater home uncertainty means greater benefits from ex-post allocation of output made possible by a foreign market, and this benefit is greater the more different the destinations, that is, the smaller the correlation of shocks between the two markets.

Armed with the above predictions, I use French firm-level data to test which of the two versions of the model better accord with the data. I begin by using the model to derive model-consistent measures of uncertainty, that is, the second moments of the shock distributions implied by the data if the model is the true data-generating process. Using these measures of uncertainty, I show that the data strongly supports the ex-post allocation version of the model and rejects the ex-ante allocation version of the model. A series of robustness checks confirms that this conclusion remains unchanged even when I allow for various alternative specifications. Moreover, the results show that the role that uncertainty plays in a firm's export decision is far from trivial. Indeed, my estimates imply that for a typical manufacturing firm with an initial 10% chance of exporting, moving up one standard deviation in the distribution of home uncertainty increases the probability of exporting by 8.0 percentage points. A similar movement in the distribution of foreign uncertainty or in the distribution of correlations between home and foreign shocks decreases the probability of exporting by 3.9 and 3.4 percentage points, respectively. Thus, the paper's key contribution is confirming that firms do indeed export as a means of varying their sources of demand and that this motive for exporting plays a nontrivial role in firms' export decisions.

In focusing on shocks that occur subsequent to the export and production decisions, I have abstracted from shocks that occur at other stages in the firm's decision-making process. As discussed in the Introduction, other authors have shown that shocks that occur either prior to production or prior to the export decision can also influence a firm's decision to export. While in some cases the impact on exporting is directly through the shock *realizations*, e.g., Blum, Claro and Horstmann (2013), in others it is through the shock *distributions*, e.g., Handley (2014), albeit by a mechanism that differs from the one analyzed in the current paper. Since in reality firms are likely subject to a constant stream of shocks, incorporating all of these different types of shocks in a single model will no doubt provide a fuller picture of the role that uncertainty plays in a firm's export decision.

The main conclusion of this paper, that firms export not only to access a larger market, but also to vary their sources of demand, opens the door to a whole slew of new issues. For instance, while this paper focused on a two-country setting, the ex-post allocation motive for exporting surely extends to a more general multicountry setting. As a result, the ex-post allocation motive will affect not only whether or not a firm exports, but also to which destinations it exports. Finally, and perhaps most importantly, the ability to allocate output in response to shock realization can have an important effect on the gains from trade. In addition to the gains due to a more efficient allocation of output, the resulting ex-post allocation motive for exporting may cause more firms to export, which may lead to additional gains for consumers from access to new varieties of goods. Incorporating these considerations into a general-equilibrium setting would help illuminate the extent to which the ex-post allocation of output and the concomitant ex-post allocation motive for exporting augment the gains from trade.

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A Proof of Proposition 1

Proof. We want to show that $\partial\left[E_{\mathscr{O}_f}\left[\pi\left(1,B_f\right)-\pi\left(0,B_f\right)\right]\right]/\partial x$ for $x\in\left\{\operatorname{var}\left(\gamma_h\right),\operatorname{var}\left(\gamma_f\right),\operatorname{cov}\left(\gamma_h,\gamma_f\right)\right\}$ all have the sign predicted by the proposition for each of the two versions of the model. It is straightforward to verify that the second-order Taylor approximation of a generalized mean with exponent $1/\sigma$ is given by

$$\left[E\left(z^{1/\sigma}\right)\right]^{\sigma}=\bar{z}-\frac{1}{2}\frac{\sigma-1}{\sigma}\bar{z}^{-1}\operatorname{var}(z)+P(z), \tag{15}$$

where z is a random variable and $P(z)$ is the remainder. We begin with Version I of the model. Substituting Equation (15) into the export condition for Version I (Condition (6I)) yields,

$$\bar{\gamma}_f-\frac{1}{2}\frac{\sigma-1}{\sigma}\bar{\gamma}_f^{-1}\operatorname{var}\left(\gamma_f\right)+P\left(\gamma_f\right)>\frac{\sigma f_x}{A_f\rho^{\sigma-1}\varphi^{\sigma-1}E\left(\mathscr{O}_f\right)}.$$

It is evident from inspection that the likelihood of exporting decreases with the variance of Foreign shocks. Moreover, since γ_h has no effect on the the export profit in the ex-ante allocation version of the model, neither the variance of home shocks nor the covariance between home and foreign shocks have any effect on the propensity to export. This completes the proof of the results relating to Version I of the model.

I now turn to Version II of the model. Substituting Equation (15) into the export condition for Version II (Condition (6II)) yields,

$$E_{\mathscr{O}}\left(\left[\bar{\Gamma}-\frac{1}{2}\frac{\sigma-1}{\sigma}\bar{\Gamma}^{-1}\operatorname{var}(\Gamma)+P(\Gamma)\right]\right. \tag{16}$$

$$\left.-\left[\bar{\gamma}_h-\frac{1}{2}\frac{\sigma-1}{\sigma}\bar{\gamma}_h^{-1}\operatorname{var}(\gamma_h)+P(\gamma_h)\right]\right)>\frac{\sigma f_x}{A_h\rho^{\sigma-1}\varphi^{\sigma-1}}, \tag{17}$$

where $\bar{\Gamma}=\bar{\gamma}_h+\mathscr{O}_f\bar{\gamma}_f$. Since $\operatorname{var}(\Gamma)=\operatorname{var}(\gamma_h)+\mathscr{O}_f^2\operatorname{var}\left(\gamma_f\right)+2\mathscr{O}_f\operatorname{cov}\left(\gamma_h,\gamma_f\right)$, this inequality can be rewritten as

$$E_{\mathscr{O}_f}\left[\bar{\Gamma}^{-1}\left[\bar{\Gamma}\left(\bar{\gamma}_h^{-1}-\bar{\Gamma}^{-1}\right)\operatorname{var}(\gamma_h)-\mathscr{O}_f^2\operatorname{var}\left(\gamma_f\right)-2\mathscr{O}_f\operatorname{cov}\left(\gamma_h,\gamma_f\right)\right]\right]>\frac{2}{\rho}\left[\frac{\sigma f_x}{A_h\rho^{\sigma-1}\varphi^{\sigma-1}}+C\right],$$

where $C=-\bar{\gamma}_f E\left(\mathscr{O}_f\right)+P(\Gamma)-P(\gamma_h)$. It is clear from inspection that the LHS decreases with the variance of Foreign shocks and covariance between Home and Foreign shocks and increases with the variance of Home shocks (because $\bar{\Gamma}>\bar{\gamma}_h$ for any realization of $\mathscr{O}_f$). This completes the proof. □

B Empirics

B.1 Estimating Equation

In this section, I derive the linearized form of the model which I use to test the model in Section 5.4.

Version I - Ex-Ante Allocation Condition (6I), Equation (15) and the fact that $\bar{\gamma}_f = 1$ imply that a firm will export if and only if

$$\frac{\rho^{\sigma-1}}{\sigma} A_i \varphi_i^{\sigma-1} E\left(\mathscr{O}_{fi}\right)\left[1-\frac{\rho}{2}\mathrm{var}_i\left(\gamma_f\right)\right] > f_{xi}, \tag{18I}$$

where $\mathscr{O}_i$ is defined in Section 5.2. Taking logs on both sides of the inequality yields

$$\rho \log \rho + \log A_i \varphi_i^{\sigma-1} + \log E\left(\mathscr{O}_{fi}\right) + \frac{\rho}{2}\mathrm{var}_i\left(\gamma_{fi}\right) > \log f_{xi}, \tag{19I}$$

where I have used the fact that $\log(1+x) \approx x$ for small x, which is valid if shocks are not too volatile. Decomposing the fixed cost into a firm-specific component (indexed by i) and a 2-digit (or 3-digit) NAICS industry-level component (indexed by j) so that $f_{xi} = f_j^x f_i^x$ with $E\left[\log f_i^x\right] = 0$, gives the latent variable in the logit regression

$$y_i = \beta_0 + \beta_1 \log A_i \varphi_i^{\sigma-1} + \sum_j \beta_j D_{ij} + \alpha_1 \mathrm{var}_i\left(\gamma_{fi}\right) + \varepsilon_i, \tag{20I}$$

where D_{ij} is a dummy for the firm's 2-digit (or 3-digit) NAICS industry, $\beta_j = \log E_j\left(\mathscr{O}_{fi}\right) - \log f_j^x$, $\varepsilon_i = -\log f_i^x$ and $E_j\left(\mathscr{O}_{fi}\right)$ is the expectation of $\mathscr{O}_{fi}$ for a firm in sector j (2-digit (or 3-digit) NAICS industry level). The variable $\log E_j\left(\mathscr{O}_{fi}\right)$ can be incorporated into the fixed effect even though the distribution $\mathscr{B}$ (from which the firm draws B_{fi}, which makes $\mathscr{O}_{fi}$ a random variable) is common for all firms in sector k. The reason is that in all specification the j sectors are more disaggregated than the k sectors, i.e., j-sectors are at the 2- or 3-digit level and k-sectors are at the 1-digit level.

Version II - Ex-Post Allocation Condition (6II), Equation (15) and the fact that $\bar{\gamma}_h = \bar{\gamma}_f = 1$ imply that a firm will export if and only if

$$\frac{\rho^{\sigma-1}}{\sigma} A_i \varphi_i^{\sigma-1} E_{\mathscr{O}_{fi}}\left[\left(\frac{\mathscr{O}_{fi}}{1+\mathscr{O}_{fi}}\right)\left[1+\mathscr{O}_{fi}+\frac{\rho}{2}\left[\mathrm{var}_i\left(\gamma_h\right)-\mathscr{O}_{fi}\mathrm{var}_i\left(\gamma_f\right)-2\mathrm{cov}_i\left(\gamma_h,\gamma_f\right)\right]\right]\right] > f_{xi}, \tag{18II}$$

where $\mathscr{O}_{fi}$ is defined in Section 5.2. This can be rewritten as

$$\frac{\rho^{\sigma-1}}{\sigma} A_i \varphi_i^{\sigma-1} \hat{\mathscr{O}}_{fi}\left[1+\frac{\tilde{\mathscr{O}}_{fi}}{\hat{\mathscr{O}}_{fi}}+\frac{\rho}{2}\left[\mathrm{var}_i\left(\gamma_h\right)-\frac{\tilde{\mathscr{O}}_{fi}}{\hat{\mathscr{O}}_{fi}}\mathrm{var}_i\left(\gamma_f\right)-2\mathrm{cov}_i\left(\gamma_h,\gamma_f\right)\right]\right] > f_{xi},$$

where $\hat{\mathscr{O}}_{fi} = E_{\mathscr{O}_{fi}}\left(\mathscr{O}_{fi}/1+\mathscr{O}_{fi}\right)$ and $\tilde{\mathscr{O}}_{fi} = E_{\mathscr{O}_{fi}}\left(\mathscr{O}_{fi}^{2}/1+\mathscr{O}_{fi}\right)$. Taking logs on both sides of this inequality and using the approximation $\log(1+x) \approx x$ (which is valid if shocks are not too volatile) yields

$$\rho\log\rho + \log A_i\varphi_i^{\sigma-1} + \log\hat{\mathscr{O}}_{fi} + \frac{\tilde{\mathscr{O}}_{fi}}{\hat{\mathscr{O}}_{fi}} + \frac{\rho}{2}\left[\mathrm{var}_i\left(\gamma_h\right) - \frac{\tilde{\mathscr{O}}_{fi}}{\hat{\mathscr{O}}_{fi}}\mathrm{var}_i\left(\gamma_f\right) - 2\mathrm{cov}_i\left(\gamma_h,\gamma_f\right)\right] > \log f_{xi}. \tag{19II}$$

Decomposing the fixed cost into a firm-specific component (indexed by i) and a 2-digit (or 3-digit) NAICS industry-level component (indexed by j) so that $f_{xi} = f_j^x f_i^x$ with $E\left[\log f_i^x\right] = 0$, gives the latent variable in the logit regression

$$y_i = \beta_0 + \beta_1\log A_i\varphi_i^{\sigma-1} + \sum_j \beta_j D_{ij} + \alpha_1\mathrm{var}_i\left(\gamma_h\right) + \alpha_2\mathrm{var}_i\left(\gamma_f\right) + \alpha_3\mathrm{cov}_i\left(\gamma_h,\gamma_f\right) + \varepsilon_i, \tag{20II}$$

where D_{ij} is a dummy for the firm's 2-digit (or 3-digit) NAICS industry, $\beta_j = \log\hat{\mathscr{O}}_{fi} + \tilde{\mathscr{O}}_{fi}/\hat{\mathscr{O}}_{fi} - \log f_j^x$ and $\varepsilon_i = -\log f_i^x$. The variable $\log E_j\left(\mathscr{O}_{fi}\right)$ can be incorporated into the fixed effect even though the distribution $\mathscr{B}$ (from which the firm draws B_{fi}, which makes $\mathscr{O}_{fi}$ a random variable) is common for all firms in sector k. The reason is that in all specification the j sectors are more disaggregated than the k sectors, i.e., j-sectors are at the 2- or 3-digit level and k-sectors are at the 1-digit level.

C Robustness

In this section I provide the numerical estimates for the robustness checks discussed in Section 6 of the main text. All the results below are for ex-post allocation version of the model.

Table 5: Logit Estimates with Alternate Measure of Firm Productivity and Controlling for Firm-Level Openness

		Alternate Productivity Measure		**Openness at 6-D Level**		**Imputed Openness**	
		2-D FE	**3-D FE**	**2-D FE**	**3-D FE**	**2-D FE**	**3-D FE**
MFG, 31-33	$\text{var}(\gamma_h)$	6.96^{**}	5.41^{**}	6.82^{**}	5.31^{**}	6.89^{**}	5.39^{**}
		(0.324)	(0.420)	(0.323)	(0.418)	(0.325)	(0.421)
	$\text{var}(\gamma_f)$	-2.75^{**}	-2.78^{**}	-2.78^{**}	-2.81^{**}	-2.81^{**}	-2.85^{**}
		(0.153)	(0.196)	(0.153)	(0.195)	(0.154)	(0.196)
	$\text{cov}(\gamma_h, \gamma_f)$	-5.06^{**}	-6.77^{**}	-5.07^{**}	-6.81^{**}	-5.05^{**}	-6.80^{**}
		(0.447)	(0.562)	(0.445)	(0.558)	(0.447)	(0.561)
T&T, 42-49	$\text{var}(\gamma_h)$	7.44^{**}	6.55^{**}	7.13^{**}	6.21^{**}	7.14^{**}	6.17^{**}
		(0.233)	(0.281)	(0.234)	(0.281)	(0.233)	(0.280)
	$\text{var}(\gamma_f)$	-0.66^{**}	-4.06^{**}	-0.76^{**}	-4.21^{**}	-0.60^{**}	-4.45^{**}
		(0.118)	(0.210)	(0.118)	(0.210)	(0.138)	(0.216)
	$\text{cov}(\gamma_h, \gamma_f)$	-1.10^{**}	2.33^{**}	-1.00^{**}	2.47^{**}	-1.48^{**}	2.61^{**}
		(0.372)	(0.494)	(0.372)	(0.493)	(0.377)	(0.489)
I&F, 51-56	$\text{var}(\gamma_h)$	1.94^{**}	3.68^{**}	1.75^{**}	3.51^{**}	1.80^{**}	3.18^{**}
		(0.126)	(0.180)	(0.127)	(0.181)	(0.129)	(0.183)
	$\text{var}(\gamma_f)$	-1.00^{**}	-1.01^{**}	-0.97^{**}	-0.96^{**}	-1.56^{**}	-1.62^{**}
		(0.134)	(0.145)	(0.134)	(0.145)	(0.142)	(0.159)
	$\text{cov}(\gamma_h, \gamma_f)$	-1.43^{**}	-2.22^{**}	-1.49^{**}	-2.35^{**}	-1.62^{**}	-2.33^{**}
		(0.275)	(0.327)	(0.275)	(0.328)	(0.287)	(0.340)
	$\log \varphi_i$			0.53^{**}	0.54^{**}	0.53^{**}	0.55^{**}
				(0.006)	(0.007)	(0.006)	(0.007)
	$\log rev_{hi}$	0.54^{**}	0.56^{**}				
		(0.006)	(0.007)				
	$\mathscr{O}_{fi}$			0.00^{**}	0.00^{**}	0.00^{**}	0.00^{**}
				(0.001)	(0.001)	(0.000)	(0.000)

Statistical significance at the 99 percent level is denoted by ** and at the 95 percent level is denoted by *. Standard errors are in brackets.

Table 6: Logit Estimates with Alternate Firm-Level Trends

		Sector Trend at 3-D Level		Sector Trend at 6-D Level	
		2-Digit FE	3-Digit FE	2-Digit FE	3-Digit FE
MFG, 31-33	$\mathrm{var}(\gamma_h)$	4.13^{**}	5.24^{**}	6.12^{**}	5.41^{**}
		(0.343)	(0.422)	(0.292)	(0.366)
	$\mathrm{var}(\gamma_f)$	-3.22^{**}	-2.78^{**}	-2.92^{**}	-3.23^{**}
		(0.154)	(0.194)	(0.148)	(0.191)
	$\mathrm{cov}(\gamma_h, \gamma_f)$	-4.19^{**}	-7.06^{**}	-3.41^{**}	-3.67^{**}
		(0.524)	(0.565)	(0.367)	(0.440)
T&T, 42-49	$\mathrm{var}(\gamma_h)$	5.84^{**}	6.35^{**}	4.99^{**}	5.50^{**}
		(0.222)	(0.280)	(0.219)	(0.279)
	$\mathrm{var}(\gamma_f)$	-1.00^{**}	-4.13^{**}	-1.08^{**}	-2.93^{**}
		(0.112)	(0.209)	(0.095)	(0.171)
	$\mathrm{cov}(\gamma_h, \gamma_f)$	-0.09	2.41^{**}	0.43	1.31^{**}
		(0.359)	(0.492)	(0.369)	(0.491)
I&F, 51-56	$\mathrm{var}(\gamma_h)$	1.85^{**}	3.57^{**}	2.11^{**}	4.23^{**}
		(0.123)	(0.181)	(0.121)	(0.180)
	$\mathrm{var}(\gamma_f)$	-1.01^{**}	-1.02^{**}	-0.46^{**}	-0.39^{**}
		(0.128)	(0.146)	(0.123)	(0.143)
	$\mathrm{cov}(\gamma_h, \gamma_f)$	-1.66^{**}	-2.37^{**}	-2.42^{**}	-2.92^{**}
		(0.267)	(0.329)	(0.278)	(0.325)
	$\log \varphi_i$	0.29^{**}	0.54^{**}	0.41^{**}	0.45^{**}
		(0.005)	(0.007)	(0.005)	(0.006)

Statistical significance at the 99 percent level is denoted by ** and at the 95 percent level is denoted by *. Standard errors are in brackets.

Table 7: Logit Estimates with Alternate Values for Demand Elasticity

		$\sigma = 2$		$\sigma = 4$		$\sigma = 5$	
		2-D FE	**3-D FE**	**2-D FE**	**3-D FE**	**2-D FE**	**3-D FE**
MFG, 31-33	$\text{var}(\gamma_h)$	9.06^{**}	6.83^{**}	4.02^{**}	3.12^{**}	2.77^{**}	2.15^{**}
		(0.447)	(0.563)	(0.202)	(0.267)	(0.153)	(0.206)
	$\text{var}(\gamma_f)$	-2.86^{**}	-3.07^{**}	-2.79^{**}	-2.62^{**}	-2.83^{**}	-2.56^{**}
		(0.155)	(0.201)	(0.153)	(0.194)	(0.153)	(0.193)
	$\text{cov}(\gamma_h, \gamma_f)$	-9.75^{**}	-11.48^{**}	-0.95^{**}	-2.45^{**}	0.52^{*}	-0.86^{**}
		(0.624)	(0.770)	(0.306)	(0.384)	(0.258)	(0.323)
T&T, 42-49	$\text{var}(\gamma_h)$	9.23^{**}	8.26^{**}	5.26^{**}	4.79^{**}	4.35^{**}	4.15^{**}
		(0.270)	(0.322)	(0.172)	(0.207)	(0.139)	(0.164)
	$\text{var}(\gamma_f)$	-0.76^{**}	-4.46^{**}	-0.81^{**}	-3.50^{**}	-0.95^{**}	-3.22^{**}
		(0.122)	(0.211)	(0.115)	(0.206)	(0.115)	(0.202)
	$\text{cov}(\gamma_h, \gamma_f)$	-1.23^{*}	1.98^{**}	-0.82^{**}	1.70^{**}	-0.48^{*}	1.46^{**}
		(0.531)	(0.704)	(0.237)	(0.338)	(0.191)	(0.281)
I&F, 51-56	$\text{var}(\gamma_h)$	3.08^{**}	5.33^{**}	0.95^{**}	2.10^{**}	0.69^{**}	1.59^{**}
		(0.186)	(0.243)	(0.080)	(0.115)	(0.065)	(0.091)
	$\text{var}(\gamma_f)$	-1.18^{**}	-1.15^{**}	-0.82^{**}	-0.90^{**}	-0.73^{**}	-0.82^{**}
		(0.139)	(0.146)	(0.125)	(0.144)	(0.126)	(0.148)
	$\text{cov}(\gamma_h, \gamma_f)$	-4.48^{**}	-5.33^{**}	-0.21	-0.63^{**}	0.04	-0.24
		(0.464)	(0.516)	(0.164)	(0.203)	(0.145)	(0.181)
	$\log \varphi_i$	0.53^{**}	0.55^{**}	0.53^{**}	0.54^{**}	0.53^{**}	0.54^{**}
		(0.006)	(0.007)	(0.006)	(0.007)	(0.006)	(0.007)

Statistical significance at the 99 percent level is denoted by ** and at the 95 percent level is denoted by *. Standard errors are in brackets.

Table 8: Logit Estimates with Interaction Dummies at 2-Digit Level

		Manufacturing - 2-Digit Level	
		2-Digit FE	**3-Digit FE**
NAICS=31	$\mathrm{var}(\gamma_h)$	15.38^{**}	12.68^{**}
		(1.165)	(1.496)
	$\mathrm{var}(\gamma_f)$	-1.39^{**}	-1.44
		(0.516)	(0.755)
	$\mathrm{cov}(\gamma_h, \gamma_f)$	-8.35^{**}	-6.66^{**}
		(1.025)	(1.472)
NAICS=32	$\mathrm{var}(\gamma_h)$	-1.16	-3.44^{*}
		(1.383)	(1.729)
	$\mathrm{var}(\gamma_f)$	-1.77^{**}	-0.50
		(0.623)	(0.795)
	$\mathrm{cov}(\gamma_h, \gamma_f)$	-6.32^{**}	-2.04
		(1.863)	(2.340)
NAICS=33	$\mathrm{var}(\gamma_h)$	7.16^{**}	5.72^{**}
		(0.650)	(0.955)
	$\mathrm{var}(\gamma_f)$	-3.69^{**}	-3.17^{**}
		(0.303)	(0.466)
	$\mathrm{cov}(\gamma_h, \gamma_f)$	-7.35^{**}	-5.03^{*}
		(1.282)	(2.019)
	$\log \varphi_i$	0.85^{**}	0.90^{**}
		(0.017)	(0.019)

Statistical significance at the 99 percent level is denoted by ** and at the 95 percent level is denoted by *. Standard errors are in brackets.

Table 9: Logit Estimates with No Interaction Dummies

	No Interaction Dummies			
	NAICS Supersectors 3-5		**NAICS Supersectors 1-7**	
	2-Digit FE	**3-Digit FE**	**2-Digit FE**	**3-Digit FE**
$\text{var}(\gamma_h)$	3.47^{**}	5.31^{**}	3.30^{**}	5.10^{**}
	(0.106)	(0.150)	(0.099)	(0.142)
$\text{var}(\gamma_f)$	-1.71^{**}	-2.65^{**}	-1.48^{**}	-2.10^{**}
	(0.079)	(0.104)	(0.072)	(0.093)
$\text{cov}(\gamma_h, \gamma_f)$	-1.39^{**}	-1.47^{**}	-1.07^{**}	-1.34^{**}
	(0.199)	(0.247)	(0.180)	(0.222)
$\log \varphi_i$	0.51^{**}	0.53^{**}	0.52^{**}	0.54^{**}
	(0.006)	(0.007)	(0.006)	(0.006)

Statistical significance at the 99 percent level is denoted by ** and at the 95 percent level is denoted by *. Standard errors are in brackets.