Suits' Watermelon Model:

The Missing Simultaneous Equations Empirical Application

Kenneth G. Stewart

Associate Professor
Department of Economics
University of Victoria
Victoria, British Columbia
Canada V8W 2Y2

email: kstewart@uvic.ca
phone: (250) 721-8534
fax: (250) 721-6214

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Abstract

Instructors of econometrics courses sometimes seek an empirical simultaneous equations application that, ideally, (i) goes beyond the two-equation case most often used in textbook examples, (ii) is based on available real-world data that can be used in hands-on exercises, (iii) replicates prominent published results, (iv) can be motivated as being of some historical importance, (v) uses accessible economic theory, and (vi) yields plausible empirical results understandable to students. The seminal but now-forgotten 1955 watermelon study of Daniel Suits is suggested as an empirical application that meets all these criteria.

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If one were to identify the single most important change in econometrics pedagogy of the past generation, it might well be the increased role of empirical application in illustrating econometric technique. The trend has been noted by Angrist and Pischke (2017, 135–6) who observe that "Traditional econometrics textbooks [of the 1970s] are thin on empirical examples." In contrast,

> Following a broader trend towards empiricism in economic research ... today's texts are more empirical than those they've replaced. In particular, modern econometrics texts are more likely ... to integrate empirical examples throughout, and often come with access to websites where students can find real economic data for problem sets and practice.
>
> But the news on the textbook front is not all good. Many of today's textbook examples are still contrived or poorly motivated.

One topic where empirical examples continue to be "contrived and poorly motivated" is surely the simultaneous equations model (SEM), where the illustrative applications have sometimes changed little in forty years.

Admittedly, SEMs no longer play as significant a role in econometrics instruction as they once did. Angrist and Pischke (Table 2) document that leading introductory textbooks of the 1970s devoted an average of 13.9 pages to simultaneous equations, whereas the average among contemporary texts is 3.6. Even so, SEMs remain the culmination of many econometrics courses, doubtless because the methodology falls into a natural topics ordering. SEM estimation combines instrumental variables (IV) estimation—in many textbooks introduced earlier in the single equation context prior to developing the SEM framework—with a system covariance structure that follows naturally from a discussion of generalized least squares (GLS). Both IV and GLS are mainstays of virtually all econometrics texts, and so the SEM is a natural follow-up.

A second reason why SEMs continue to be important is that they are among the very few contributions that econometrics as a discipline has made that go fundamentally beyond statistics proper. Most econometric methods are adaptations or extensions of techniques that have their origins in mainstream statistics. There are, arguably, just two exceptions. The first is SEMs, developed in the 1940s, '50s, and '60s. The second is cointegration, developed in the 1980s and '90s, prior to which there was no systematic framework for studying nonstationary time series.

Given that the SEM continues to have a place in many econometrics courses, this paper begins by documenting the dearth of SEM empirical applications in textbooks and the inadequacies of many of the examples that do appear. I make the case that there has long been a "missing empirical application" from textbooks that (i) goes beyond the two-equation case most often used in textbook examples, (ii) is based on available real-world data that can be used in hands-on student exercises, (iii) replicates prominent published results, (iv) can be motivated as being of some historical importance, (v) uses accessible economic theory, and (vi) yields plausible empirical results understandable to students. I suggest the seminal but now-forgotten 1955 watermelon study of Daniel Suits as an empirical application that meets all these criteria, yet does not have certain deficiencies from which some existing examples suffer.

Background and motivation

To document the inadequacy of existing textbook applications, consider the SEM treatments in the six contemporary texts that Angrist and Pischke identify as market leaders: Kennedy (2008), Gujarati and Porter (2009), Stock and Watson (2015), Wooldridge (2016), Dougherty (2016), and Studenmund (2017).

Kennedy (2008, Chap. 11) offers not a single empirical example, let alone an application with a data set.

Gujarati and Porter (2009, Chaps. 18, 19) provide two time series macroeconomic data sets in their end-of-chapter exercises: a GNP-consumption-investment data set for the estimation of an income expenditure model (Chap. 18), and a money-GDP-interest

rate-price level data set for the estimation of a two-equation money supply-demand system (Chap. 19). However, in addition to being relegated to the end-of-chapter exercises, neither is from any published study.

Stock and Watson (2015, Chap. 12) provide exercises based on several data sets from published studies. However they only discuss the single-equation IV case, without treating it explicitly as one equation from a multi-equation structural system with an associated reduced form, and their empirical applications are limited to this case.

Wooldridge (2016, Chap. 16) distinguishes between the structural and reduced forms of a multi-equation model, and provides a number of empirical exercises and data sets. However most of these exercises involve only the IV estimation of a single equation taken in isolation, and none goes beyond the two-equation case.

Dougherty (2016, Exercise 9.10) provides a single empirical exercise in which educational attainment and academic skills are determined simultaneously in a two-equation specification, and the identified equation is estimated by OLS and IV. In addition to only involving single-equation estimation, the analysis does not correspond to any published study.

Studenmund (2017) provides two empirical applications with data sets. The first (pp. 425–29) is a four-equation Keynesian model where the behavioral equations are a consumption function and an investment function. Annual data 1975–2007 are provided that are used to estimate each equation. The second (Exercise 6, p. 437) is a two-equation supply-demand model for oats estimated with ten observations of "hypothetical data." Again, neither application relates to any published study.

Casting the net beyond these leading texts to the market as a whole, one finds that textbook empirical applications invariably have at least one of the following limitations.

- They are not based on real-world data sets that students can use in hands-on exercises.

- Applications are not well motivated, in terms of replicating published work; or, if they do, that work is obscure and unrelated to what students are likely to encounter elsewhere in their studies.

- They are limited to the case of, at best, just two equations.

- When textbooks do venture beyond the two-equation case, they often do so by falling back on the same dated 1950s and '60s macroeconomic models—like Klein's Model I—that appeared in textbooks of the 1970s, models that today are of only historic interest. They bear little relation to what students are seeing in their macroeconomics courses, and are as irrelevant to policy as they are to contemporary theory.

In fact some textbook SEM empirical applications are not even methodologically sound. Wooldridge (2016, Secs. 16.1, 16.5) has discussions of the first two of the following points.

1. For a SEM system to make sense, the equations must be behavioral relations with ceteris paribus causal interpretations. The classic example is supply and demand, where the equations describe the decision processes of different economic agents, producers and consumers. Each equation therefore stands on its own as a behavioral construct, independent of the other. These independent behavioural relations are brought together in a SEM only because of the additional assumption that equilibration of the market results in observed price-quantity data that simultaneously satisfy both relations.

   This is in contrast to problems where "...the two endogenous variables are chosen by the same economic agent. Therefore, neither equation can stand on its own. ...just because two variables are determined simultaneously does *not* mean that a simultaneous equations model is suitable." (Wooldridge 2016, 503) Instead the choices should be modelled explicitly as a joint decision problem. Wooldridge gives two examples: household saving-housing choices, and students choices between hours spent working and hours spent studying.

2. 1960s-era macro models are not particularly good examples of SEM methodology because their exogeneity assumptions are often implausible. This is, of course, the famous Sims (1980, 1) critique, although it hardly originates with him: "...claims for identification in these models cannot be taken seriously."

3. Another problem with economic data observed over time, like the macroeconomic series used to estimate older Keynesian SEMs, is that some of the series may be nonstation-

ary. The first order of business should be to model or treat the nonstationarity; in comparison, endogeneity is a second-order issue. Without attention to the stationarity properties of the series, some posited behavioral equations may even be nonsensical "unbalanced regressions" that involve an unbalanced mixture of orders of integration.

Of course, students studying the SEM typically have not advanced to the point where they can appreciate these matters. Nevertheless, to have students proceed mechanically with SEM estimation without first considering the stationarity properties of the time series teaches bad habits and so is counterproductive. At a minimum, textbook SEM applications using time series data—as many do—should ideally be ones where the variables can reasonably be regarded as stationary.

Despite these issues, many instructors may find the applications that currently appear in textbooks to be perfectly adequate for their purposes. This is particularly so given the practical realities of finite weeks to the teaching term, competing topics, and end-of-term assignment exhaustion on the part of students. Other instructors, however, may want their students to have a better appreciation of how SEM methodology extends to more realistic settings. This brings us to Suits' model of the watermelon market.

Model specification

Sims (1980, 3) has observed that "The textbook paradigm for identification of a simultaneous equations system is supply and demand for an agricultural product." The first of many strengths of Suits' model for pedagogical purposes is that it falls within this paradigm. This section specifies the model using notation that is more suggestive than was available to Suits under the fairly crude typesetting capabilities of his day. The correspondence between the two sets of variable symbols is shown in Table 1.

As in any market for a particular commodity, Suits found that the market for watermelons has some idiosyncratic features that must be treated. With respect to supply, he found that it is important to distinguish between a season's *crop* of watermelons q_t available for harvest, and the amount h_t of that crop actually *harvested. Crop supply* depends mainly on planting

decisions made on information in the previous season:

$$\log q_t = \alpha_1 + \alpha_2 \log p_{t-1} + \alpha_3 \log p^c_{t-1} + \alpha_4 \log p^v_{t-1} + \alpha_5 d^c_t + \alpha_6 d^w_t + \varepsilon_{1t}. \tag{1}$$

The variables are:

q crop of watermelons available for harvest (millions)
p average farm price of watermelons (dollars per thousand)
p^c average annual net farm receipts per pound of cotton (dollars)
p^v average farm price of vegetables, called "commercial truck crops" by Suits (index)
d^c dummy variable for government cotton acreage allotment program, equals 1 in 1934–51
d^w dummy variable for World War II, equals 1 in 1943–46

The amount of the crop actually harvested, on the other hand, is dependent on current farm price p_t relative to prevailing farm wages w_t, wages being the major cost of harvesting. (*Farm price* is the price at the farm gate; that is, the price received by farmers, as opposed to the retail price paid by consumers. If farm price p_t is too low relative to wages w_t, farmers will lose money harvesting the crop. They are better off leaving the melons in the field to rot.) As well, the harvest h_t is of course limited by the available crop q_t. Suits therefore specified *harvested supply* as

$$\log h_t = \beta_1 + \beta_2 \log(p_t/w_t) + \beta_3 \log q_t + \varepsilon_{2t}. \tag{2}$$

The additional variables are:

h watermelons harvested (millions)
w farm wage rates in the South Atlantic States (index)

Turning to the demand side of the market, Suits specified farm price p_t as depending on per capita harvest, h_t/n_t, per capita income, y_t/n_t, and transportation costs:

$$\log p_t = \gamma_1 + \gamma_2 \log(h_t/n_t) + \gamma_3 \log(y_t/n_t) + \gamma_4 \log p^f_t + \varepsilon_{3t}. \tag{3}$$

The additional variables are:

y U.S. disposable income
n U.S. population
p^f railway freight costs for watermelons (index)

(a) Price p (dollars per thousand)
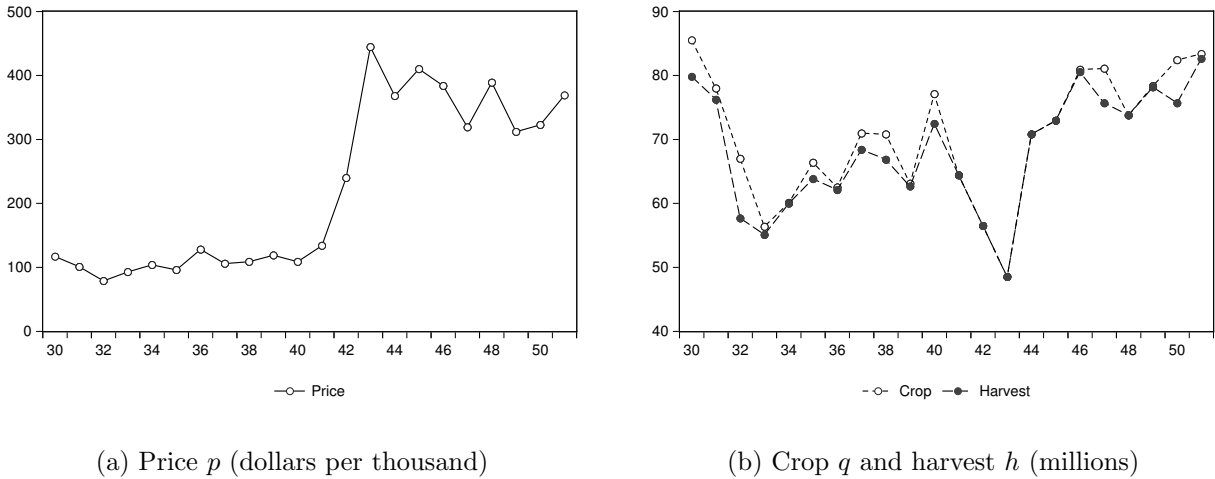
(b) Crop q and harvest h (millions)

Figure 1: Watermelon price and quantities, 1930–1951

To summarize, Suits' model of the watermelon market consists of three equations that jointly explain crop quantity q_t, harvest quantity h_t, and price p_t. Notice that, setting aside the dummy variables affecting crop supply, all equations are estimated as loglinear relations. The original article provides a detailed explanation of the many choices Suits made in specifying the model. For example, in common with all agricultural commodities there was a discrete jump in the price of watermelons beginning in 1942, shown in Figure 1a. The war dummy in crop supply (1) models this as arising from a leftward shift in supply, as opposed to a rightward shift in demand (3); and indeed the estimate of α_6 turns out to be negative and statistically significant. The intuition is presumably that supply was limited by a general shortage of factors of production during the war, labor being diverted to the war effort and land being bid away for other uses. Nevertheless, conditional on the discrete shift in average price being modeled in this way, price then varies around its pre- and post-1942 mean values in a way that—at least according to the eyeball metric—does not obviously violate stationarity. (Any more formal analysis of the time series properties of price p_t—at least, any meaningful one—would seem to be precluded by the small sample size.)

As one of the earliest estimated simultaneous equations systems, Suits' study was influential in subsequent work. Wold (1958) formulated a modification of the model to illustrate his famous "causal chain" or "recursive" system, under which the structural form can be estimated by equation-by-equation OLS. Suits' model was also used in the forecasting ex-

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ercise of L'Esperance (1964) and is among the historically important applications discussed by Wallis (1979). More recently, Murray (2006, 613) cites it as a "golden oldie greatest hit" piece of empirical work.

But if Suits' watermelon model was so seminal, why is it now largely forgotten by textbook writers? Probably because it has a feature that, if treated, introduces enough additional complication to make it unsuitable as a textbook application. Let us return to the crop supply and harvest supply equations. Clearly harvest quantity h_t is limited by crop quantity q_t, so $h_t \leq q_t$. In most years this constraint is not binding, but in some years (1941–45 and 1948; see the respective columns of Table 1 and their comparison in the plot of Figure 1b) the crop is fully harvested. Consequently the model has served as an application of econometric methods for the treatment of truncated variables, being used for this purpose by Goldfeld and Quandt (1975). It is in this context that the application is discussed by Maddala (1983, 312–315).

But although Suits' paid careful attention to this aspect of the data, it turns out that the substantive estimation results are little affected by ignoring it. For the purposes of student exercises, the model can be estimated using the standard SEM estimators without incorporating or treating the restriction $h_t \leq q_t$.

Data, estimation, and interpretation

Suits (1955) estimated the equations of his model with annual data using what he describes as "limited information" methods. From the references he cites, one understands this to mean limited information maximum likelihood (LIML) rather than two stage least squares (2SLS). He estimated crop supply for 1919–1951, demand and harvest supply for 1930–51. However Suits did not publish his data, and when Wold (1958, 22) came to formulate his recursive system he learned that "... Suits had cleared out his files for the original data ..." Wold therefore reconstructed the data for 1930–51, reporting them in his Table 1 (expressed as common logarithms, with values missing for farm wages and truck crop (vegetable) prices in the final year 1951). It is Wold's reconstructed dataset that is provided in my Table 1. Given the presence of lagged variables in crop supply, this means that the model as a whole can be estimated for the years 1931–50 using Wold's data. It is these 20 observations that

are used to obtain the modern estimation results of Table 2, where they are contrasted with the results originally reported by Suits. For a statement of the instrument set, see footnote b of Table 2. The estimation results were obtained with EViews and TSP, cross-checking between the two to ensure accuracy. (EViews is widely used in teaching, but the numerical accuracy of TSP's estimation algorithms has been more favourably assessed (McCullough 1999).)

Despite the reconstruction of the data, the change in sample period for the crop supply equation, the lack of treatment of the $h_t \leq q_t$ constraint, and the primitive computational technology available to Suits, my estimation results are remarkably similar to his. Equally surprising is that the modern results are, in terms of the substantive economic interpretations of coefficient signs, magnitudes, and test outcomes, fairly robust across alternative estimation methods.

Like great art, Suits' analysis can be appreciated at several levels of sophistication. This makes it useful for both introductory and more advanced econometrics courses.

Alternative estimation methods Table 2 reports estimation results for the standard estimators: OLS, 2SLS, LIML, single-equation GMM, iterative 3SLS, and FIML. The OLS, 2SLS, and LIML estimation results for crop supply are identical because it is a reduced form equation: the regressors are predetermined (the lagged prices) or exogenous (the dummy variables). In the case of the harvest supply and demand equations, the similarity of the OLS and IV-based estimates foreshadows the outcome of Hausman tests on the endogenous regressors, as we shall see.

The only standard estimator that the application does not admit is system GMM (based on either a White heteroskedasticity-robust or a Newey-West heteroskedasticity-and-auto-correlation-robust covariance estimator), because 20 observations is too few to estimate the system covariance matrix.

Coefficient interpretations and elementary hypothesis tests of fixed-value restrictions Suits' model yields a number of interesting and intuitively sensible elasticities and simple hypothesis tests that can be appreciated even by beginning students.

The crop supply equation (1) yields the following plausible results.

- Crop supply this season, q_t, responds positively to last season's price p_{t-1} with a statistically significant elasticity of around 0.58.

- Cotton and vegetables are the principal alternative crops that farmers could plant instead of watermelon. These cross-price supply elasticities are around -0.32 and -0.12 and are statistically significant. As substitute crops, it makes sense that crop supply this season responds negatively to their prices last season.

- Dummy variables for a federal cotton acreage allotment program and World War II are both statistically significant. (Not shown in Table 2.)

The harvest supply equation (2) yields the following plausible results.

- The own-price elasticity of harvest supply h_t is in the range of 0.09–0.16, so the harvest supply curve is steeply upward sloping. Based on the reported standard errors, this elasticity is less than unity to a degree that is statistically significant. Hence harvest supply is price inelastic, consistent with the intuition that it is largely predetermined by crop supply q_t.

- The elasticity of harvest supply h_t with respect to crop supply q_t is significantly different from zero but, for most of the estimates, not significantly different from unity: a two-standard-error confidence interval includes 1. It makes sense that there should be an essentially one-for-one percentage correspondence between crop and harvest.

The demand equation (3) is specified in per capita terms with price as the dependent variable. In order to interpret the estimation results most naturally, Suits restated the estimated equation with (log of per capita) quantity as the dependent variable:

$$\log(h_t/n_t) = -\frac{\hat{\gamma}_1}{\hat{\gamma}_2} + \frac{1}{\hat{\gamma}_2} \log p_t - \frac{\hat{\gamma}_3}{\hat{\gamma}_2} \log(y_t/n_t) - \frac{\hat{\gamma}_4}{\hat{\gamma}_2} \log p_t^{\mathrm{f}}.$$

Illustrating with the 3SLS estimates, the following intuitively plausible results are obtained.

- The own-price elasticity of demand is negative and (in absolute value) less than unity,

$$\frac{1}{\hat{\gamma}_2} = \frac{1}{-0.8891} = -1.125, \tag{4}$$

so watermelon demand is downward sloping and slightly price elastic.

- The income elasticity of demand is positive and greater than unity,

$$-\frac{\hat{\gamma}_3}{\hat{\gamma}_2} = -\frac{1.5563}{-0.8891} = 1.750 \tag{5}$$

so watermelon demand is substantially income elastic, consistent with the intuition that fresh fruit is a luxury to households.

- The cross-price elasticity of demand with respect to freight costs is negative:

$$-\frac{\hat{\gamma}_4}{\hat{\gamma}_2} = -\frac{-0.8604}{-0.8891} = -0.968. \tag{6}$$

Ceteris paribus, higher transportation costs shift the demand curve leftward. Reinterpreted spatially, the implication is the sensible one that, when consumers are located farther away from growing regions, their demand will be lower than if they were located closer.

Tests of within-equation linear restrictions   The demand and harvest supply equations specify the regressors $\log(p_t/w_t)$, $\log(h_t/n_t)$, and $\log(y_t/n_t)$ as log-ratios. This is testable. Specifically, the harvest supply equation (2) is a restricted form of

$$\log h_t = \beta_1 + \beta_2 \log p_t - \beta_2^* \log w_t + \beta_3 \log q_t + \varepsilon_{2t}$$

while watermelon demand (3) is a restricted form of

$$\log p_t = \gamma_1 + \gamma_2 \log h_t - \gamma_2^* \log n_t + \gamma_3 \log y_t - \gamma_3^* \log n_t + \gamma_4 \log p_t^{\text{f}} + \varepsilon_{3t}.$$

Under the single-equation estimators the restrictions $\beta_2 = \beta_2^*$, $\gamma_2 = \gamma_2^*$, and $\gamma_3 = \gamma_3^*$ are testable individually or, in the case of the latter two, jointly. Under systems estimation all three can be tested jointly. It is therefore possible to address systematically the question: Do the data support specifying these regressors as ratios? It also provides an opportunity to emphasize to students the distinction between individual and joint tests of restrictions, and that there is no simple relationship between the outcomes of individual versus joint tests.

Applications of Wald test routines

A standard device in the toolkit of applied econometricians is the use of Wald test routines to compute standard errors of nonlinear functions of parameters. Suits' watermelon model provides several examples, summarized in Table 3, that range from elementary to more sophisticated.

Elasticity expressions   Consider the nonlinear demand elasticity expressions (4), (5), and (6). Illustrating with the 3SLS estimates, a Wald test routine yields the standard errors 0.313, 0.496, and 0.360, shown in the first three rows of Table 3. Using these to construct two-standard-error confidence intervals around their respective demand elasticities reveals the following conclusions, which are fairly robust across the different estimation methods.

- The own-price elasticity of demand of -1.125 is significantly different from zero, so watermelon consumption *is* price elastic, but not significantly different from a unitary elasticity of -1.

- The income elasticity of demand of 1.750 is significantly different from zero, so watermelon consumption *is* income elastic, but not significantly different from unity. The point estimate is therefore less than compelling in establishing watermelon to be a luxury.

- The freight elasticity of demand of -0.968 is significantly different from zero, so watermelon consumption *is* reduced by higher transportation costs.

Dynamic stability   Given the presence of lagged price p_{t-1} in crop supply (1), Suits' model implies a dynamic path for the evolution of price p_t over time. Substituting equations (1) and (2) into (3), the difference equation governing the evolution of price is of the form (corresponding to equation (4a) on p. 245 of Suits)

$$\log p_t = \phi \log p_{t-1} + \text{ a function of exogenous variables and disturbances.}$$

Stability of the time path of price requires $|\phi| < 1$, which is a testable restriction. The expression for ϕ is

$$\phi = \frac{\gamma_2 \beta_3 \alpha_2}{1 - \gamma_2 \beta_2}. \tag{7}$$

Illustrating with the 3SLS results reported in the fourth row of Table 3, $\hat{\phi} = -0.494$ (Suits' value was -0.622) and a Wald test routine yields a standard error of 0.124, so a two-standard-error confidence interval around $\hat{\phi}$ does not include $|\phi| = 1$. Hence the estimated model is consistent with dynamic stability to an extent that is statistically significant, at least according to the 3SLS estimates. This conclusion is less strongly supported by the FIML results, where the estimate $|\hat{\phi}| = 0.560$ is within two standard errors of unity.

The half-life of a shock   Given the evidence that the estimated model is consistent with dynamic stability, it is of interest to ask: how long does convergence to equilibrium take? In the case of a first order difference equation, the standard expression for the half-life of a shock (which students may have seen in their courses on dynamics) is $\log(1/2)/\log(|\phi|)$. Illustrating with the 3SLS results reported in the fifth row of Table 3, this evaluates to

$$\frac{\log(1/2)}{\log(|\phi|)} = \frac{\log(1/2)}{\log(0.494)} = 0.982, \tag{8}$$

consistent with Suits' conclusion (p. 426) that "...the half life of this process is less than two years ...indicating a heavily damped oscillation with rapid approach to equilibrium." The standard error of 0.349 suggests that Suits' conclusion is supported to an extent that is statistically significant. The FIML evidence is weaker on this point: it yields an estimated half-life of 1.196 years, but with a larger standard error of 1.268.

Instrumental variables diagnostics

Teachers of IV estimation typically impress on students the scope for testing the implicit assumptions that underly these methods. Suits' 1955 study predated modern specification testing—not just by years, but by decades. How does his analysis stand up to modern diagnostic tools?

Three categories of IV-related diagnostics are typically discussed by instructors: the Hansen-Sargan test of the restrictions implied by an overidentifying instrument set; tests of

regressor endogeneity; and tests for weak instruments. Table 4 summarizes the results of the most common of these tests applied to the watermelon model.

Hansen-Sargan tests of overidentification When the instrument set is overidentifying, the "extra" instruments imply a set of parameter restrictions that are testable. The restrictions should hold as long as the instruments truly are exogenous. Hence a test of the restrictions can be interpreted as a test of the null hypothesis that the instruments are exogenous. The associated J statistic was originally proposed by Sargan (1958) in the context of 2SLS and later extended by Hansen (1982) to GMM.

The top section of Table 4 reports p-values for these tests, which do not reject at conventional significance levels. Hence by this criterion the model and instrument set are broadly supported.

Hausman tests of regressor endogeneity Is IV estimation really necessary? Only if the right hand side variables that are specified as endogenous truly are: these are price p_t and crop quantity q_t in harvest supply (2), and harvest quantity h_t in the demand equation (3). The endogeneity of these regressors can be tested with Hausman tests, the results of which are reported in the middle section of Table 4. The large p-values for both equations indicate that the null hypothesis of exogeneity is *not* rejected.

This is not terribly surprising for crop quantity q_t because, according to the crop supply equation (1), it is largely determined by information from the previous season. Crop quantity q_t is endogenous only via the correlation between its disturbance ε_{1t} and the other disturbances of the system.

Exogeneity is more surprising for price p_t and harvest quantity h_t, which are jointly determined by the demand and harvest supply equations. This result does explain, however, why the OLS estimates for these equations are so similar to those yielded by all the IV-based estimators. A Hausman test essential compares the OLS and IV estimators and asks: are they significantly different?

Weak instruments Inference with IV-based estimators hinges on their actual finite-sample distribution being well-approximated by their theoretical asymptotic distribution.

For this to be true the instruments should be strongly correlated with the endogenous regressors of an equation.

The simplest indicator of this is the Staiger-Stock F statistic—the F value from the "first stage" or "reduced form" regression of each 2SLS estimation, in which each of the endogenous variables is regressed on all the exogenous variables of the system. (I place "first stage" and "reduced form" in quotations because in a nonlinear model these would not be the literal interpretations.) The Staiger-Stock rule of thumb is that this F value should be at least 10 for the exogenous variables to be strong instruments for that endogenous variable. The bottom row of Table 4 (where each entry corresponds to the endogenous regressor used as the dependent variable in successive first-stage regressions) indicates that this criterion is easily satisfied in Suits' model.

Additional empirical issues

In addition to the results discussed so far, Suits' model provides the opportunity to expose students to several minor but nevertheless important practical issues.

Issues related to the use of logarithms   In introductory econometrics courses students learn that in a log-log regression the coefficients are interpretable as elasticities, while in a log-lin model—like a regression of log wages on education—the coefficient is a semielasticity: multiplied by 100, it is the percentage change in the dependent variable (like wages) associated with a unit change in the explanatory variable (like an additional year of schooling).

A more obscure point, but one that Wold's data provides instructors the opportunity to comment on, is the extent to which these interpretations are affected by the base of the logarithm. The elasticity interpretation is not: Wold's dataset is in common logarithms, but the results of a log-log regression are invariant to this, and the estimates of Table 2 are the same elasticities (and standard errors) that would be obtained were the data in natural logs. In contrast, the semielasticity interpretation assumes natural logs. For example, consider the war dummy in crop supply (1). The 3SLS estimate of its coefficient (not shown in Table 2) is $\hat{\alpha}_6 = -0.1567$ with a standard error of 0.0008, indicating that World War II significantly reduced crop supply. However the correct interpretation is *not* that this reduction was in

the amount of 15.7%, because Wold's data are in common rather than natural logs. The identity relating the two bases of logarithm is $\ln x = (\ln 10) \log_{10} x = 2.3026 \log_{10} x$, so using one logarithm rather than the other amounts to transforming the data by a scaling constant, in this case $1/2.3026$. The effect on the coefficient estimate is to change it by the reciprocal of that scaling constant. This means that, had Wold expressed his data in natural rather than common logs, the 3SLS estimate of the war dummy coefficient would have been $-0.1567 \times 2.3026 = -0.3608$.

Of course, instructors who find this a pointless digression can simply convert the data to natural logs, in which case the war dummy estimate of -0.3608 will be obtained directly.

However, to correctly interpret the economic meaning of this particular value -0.3608, a second issue arises that is unrelated to the conversion between common versus natural logarithms; that is, it would arise even had the model been estimated in natural logs. This is that the semielasticity -0.3608 is a continuously compounded growth rate. Simply multiplying by 100 to state it as a percentage change would be harmless enough were this a small change. But in the case of, as here, large changes—like the effect of race or gender dummies in wage equations—it is problematic. The role of the dummy variables in the crop supply equation provides instructors with a nice opportunity to make this point.

Specifically, it is incorrect to interpret the semielasticity of -0.3608 as implying that World War II reduced crop supply by 36%, because this is continuously compounded whereas normal parlance understands percentage changes in discretely compounded terms. The conversion from continuously to discretely compounded growth rates is

$$\exp(-0.3608) - 1 = -0.3029.$$

So the correct interpretation of the war dummy coefficient $\hat{\alpha}_6 = -0.1567$ is that World War II reduced watermelon crop supply by about 30%, not 36% (nor 15.7%).[1]

The role of lagged variables Yet another appealing feature of Suits' model for teaching is that it includes several categories of exogenous and predetermined variables: conventional exogenous variables (farm wages w_t in the harvest supply equation, population n_t and income y_t in the demand equation) and, in the crop supply equation, a lagged endogenous variable

(watermelon price p_{t-1}), lagged exogenous variables (the prices p^{c}_{t-1} and p^{v}_{t-1}), and dummy variables (d^{c}_{t}, d^{w}_{t}). Let us call these eight variables the *default instrument set.* It is useful for students to see—as few textbook examples allow them to—that all these types of variables can appear in a SEM and may serve as instruments.

The modern estimation results of Tables 2–4 follow Suits (as best I have been able to determine) in using this default instrument set. But the waters here run deep and, depending on the student audience, some instructors may wish to explore them.

Specifically, issues of both identification and estimation potentially arise in connection with some of these variable categories, questions that go back to the earliest work on SEMs and, in some cases, continue to this day. Sims (1980, 5) usefully summarized some of the early literature:

> J.D. Sargan several years ago considered the problem of simultaneous-equation identification in models containing both lagged dependent variables and serially correlated residuals. He came to the reassuring conclusion that, if a few narrow-looking special cases are ruled out, the usual rules for checking identification in models with serially uncorrelated residuals apply equally well to models with serially correlated residuals. In particular, it would ordinarily be reasonable to lump lagged dependent variables with strictly exogenous variables in checking the order condition for identification, despite the fact that a consistent estimation method must take account of the presence of correlation between the lagged dependent variables and the serially correlated residuals.
>
> ... work by Michio Hatanaka, however, makes it clear that this sanguine conclusion rests on the supposition that exact lag lengths and orders of serial correlation are known a priori. On the evidently more reasonable assumption that lag lengths and shapes of lag distributions are not known a priori, Hatanaka shows that the order condition takes on an altered form: we must in this case cease to count repeat occurrences of the same variable, with different lags, in a single equation. In effect, this rule prevents lagged dependent variables from playing the same kind of formal role as strictly exogenous variables in identification ...

The Sargan-Hatanaka controversy by no means exhausts the issues that arise in connection with the role of lagged variables in SEMs. For a recent contribution with a useful bibliography, see Reed (2015).

For pedagogical purposes the point is merely that, for the minority of instructors who wish to explore these issues, Suits' model offers scope for doing so. For the majority who prefer to circumvent them, as I have in Tables 2–4, this is easily and probably harmlessly done.

Pedagogical deficiencies

Are there any deficiencies of Suits' model as an example of SEM methodology? Two come to mind.

System GMM cannot be implemented because there is an inadequate number of observations to estimate the system covariance matrix (under either a White heteroskedasticity-robust or Newey-West heteroskedasticity-and-autocorrelation-robust specification).

Limited role for cross-equation restrictions Instructors often cite efficiency as the motivation for system estimation as an alternative to single-equation estimation, with the caveat that efficiency improvements are conditional on the model being correctly specified. Although this logic is correct, in practice the more important reason for system estimation is often the desire to impose and test cross-equation restrictions.

A limitation of Suits' model for pedagogical purposes is that the scope for this is modest, arising only tangentially in connection with the dynamic stability expression (7) and the half-life (8). Because these involve coefficients from both the demand and harvest supply equations, obtaining standard errors for them requires system estimation, as the construction of Table 3 emphasizes.

For instructors seeking an empirical application in which the role of cross-equation restrictions is more central, a better choice might be a system of demand equations—either consumer or factor demands—because the cross-equation restrictions of symmetry are predicted by microeconomic theory and can be imposed and tested. For example, Berndt (1991)

provides the data, including the instruments, needed to replicate the 3SLS translog factor demand system of Berndt and Wood (1975). Of course, this assumes a background in demand analysis that goes beyond what undergraduates usually see. The appeal of Suits' model is that it yields economically interpretable results that are interesting and intuitive, yet can be understood with only the most elementary background in economics.

Conclusions

I have advanced Suits' watermelon model as an almost ideal simultaneous equations empirical application for instructors who want an example that goes beyond the typical textbook two-equation model. Among its many strengths are the following.

- It falls within the classic simultaneous equations paradigm of supply and demand for an agricultural commodity, in which each equation stands on its own as a ceteris paribus behavioral relation.

- It is of some historical importance, in terms of being seminal at the time and influential in subsequent work.

- It is based on a well-documented real-world data set of modest size that is easily disseminated to and used by students.

- It yields interesting and intuitive results that are readily interpretable in terms of elementary economic theory.

- It provides scope for a range of interesting hypothesis tests, confidence intervals, and the use of Wald test routines to obtain standard errors for nonlinear coefficient expressions.

- It provides a vehicle for discussing a variety of minor but nevertheless important practical issues, such as the use of log transformations, the interpretation of dummy coefficients, and the role of lagged variables in identification, estimation, and dynamics.

- Although it uses time series data over 20 years, conventional SEM analysis can reasonably be regarded as not being contaminated by a failure to treat nonstationarity.

Finally, Suits' watermelon model is an empirical application that pays tribute to the early development of the SEM, one of the few econometric methodologies that advances fundamentally beyond statistics proper.

Notes

[1]These are the issues that Suits refers to in his footnote 2 by way of explanation for his conclusion (p. 241) that "...wartime policy clearly reduced the supply of melons significantly, the size of the parameter corresponding to a reduction of 30 percent." The correct interpretation of dummy coefficients in log-lin regressions is well known among empirical labor economists, because it so often arises in that context. Curiously, it is seldom discussed in econometrics textbooks.

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Table 1: Wold's Reconstruction of Suits' Data

year	$\log q$ (Q)	$\log h$ (X)	$\log p$ (P)	$\log p^{\mathrm{c}}$ (C)	$\log p^{\mathrm{v}}$ (T)	$\log w$ (W)	$\log n$ (N)	$\log(y/n)$ (Y/N)	$\log p^{\mathrm{f}}$ (F)
1930	1.932	1.902	2.068	0.976	0.367	1.462	2.090	2.781	1.101
1931	1.892	1.882	2.004	0.753	1.184	1.362	2.093	2.712	1.106
1932	1.826	1.761	1.897	0.814	1.124	1.230	2.096	2.591	1.129
1933	1.751	1.741	1.968	1.007	0.993	1.204	2.099	2.561	1.149
1934	1.779	1.778	2.017	1.092	0.641	1.267	2.102	2.614	1.140
1935	1.822	1.805	1.982	1.045	0.822	1.290	2.104	2.662	1.177
1936	1.796	1.793	2.107	1.092	0.824	1.301	2.108	2.713	1.174
1937	1.851	1.835	2.025	0.925	0.903	1.352	2.110	2.741	1.175
1938	1.850	1.825	2.037	0.934	1.322	1.342	2.113	2.704	1.196
1939	1.800	1.797	2.075	0.959	0.739	1.352	2.117	2.731	1.169
1940	1.887	1.860	2.037	0.995	1.101	1.362	2.121	2.760	1.196
1941	1.809	1.809	2.127	1.231	1.373	1.431	2.125	2.843	1.198
1942	1.752	1.752	2.380	1.280	1.691	1.531	2.130	2.940	1.192
1943	1.686	1.686	2.648	1.298	1.582	1.648	2.136	2.990	1.148
1944	1.850	1.850	2.566	1.317	1.038	1.732	2.141	3.025	1.144
1945	1.863	1.863	2.613	1.353	0.805	1.792	2.146	3.031	1.171
1946	1.908	1.906	2.584	1.514	1.490	1.851	2.150	3.051	1.170
1947	1.909	1.879	2.504	1.504	1.503	1.881	2.159	3.069	1.246
1948	1.868	1.868	2.590	1.483	1.729	1.903	2.166	3.107	1.341
1949	1.894	1.893	2.494	1.456	1.187	1.892	2.174	3.101	1.376
1950	1.916	1.879	2.509	1.603	0.818	1.898	2.181	3.133	1.398
1951	1.921	1.917	2.567	1.579	missing	missing	2.189	3.166	1.419

Source: Wold (1958, Table 1). The original Suits-Wold variable symbols are given in parentheses. All variables are expressed as common (base 10) logarithms. Wold also provided a cost of living index L (in his words, "the only new item") that he used to deflate watermelon prices and disposable income (although not, curiously, the other nominal variables) in his recursive system. Suits apparently did not deflate his nominal values, perhaps because the price level was fairly stable over this period, being little different at the end of World War II from what it had been in 1930.

Table 2: Suits' Watermelon Market: Original and Modern Results

| Equation | | | | Modern results[b] | | | | | |
| | | | | Single equation | | | | System estimation | |
Variable	Coefficient	Suits[a]	OLS	2SLS	LIML	GMM	3SLS	FIML
Crop supply:								
$\log p_{t-1}$	α_2	0.587	0.5792	0.5792	0.5792	0.5810	0.5793	0.5789
		(0.156)	(0.0032)	(0.0032)	(0.0032)	(0.0012)	(0.0027)	(0.0141)
$\log p^{\mathrm{c}}_{t-1}$	α_3	−0.320	−0.3209	−0.3209	−0.3209	−0.3227	−0.3212	−0.3209
		(0.095)	(0.0034)	(0.0034)	(0.0034)	(0.0017)	(0.0027)	(0.0111)
$\log p^{\mathrm{v}}_{t-1}$	α_4	−0.141	−0.1238	−0.1238	−0.1238	−0.1238	−0.1237	−0.1236
		(0.238)	(0.0008)	(0.0008)	(0.0008)	(0.0002)	(0.0007)	(0.0039)
Harvest supply:								
$\log(p_t/w_t)$	β_2	0.237	0.1080	0.1254	0.1328	0.0925	0.1211	0.1558
		(0.110)	(0.0569)	(0.0611)	(0.0613)	(0.0157)	(0.0562)	(0.1938)
$\log q_t$	β_3	1.205	1.0461	1.0674	1.0765	1.0429	1.0617	1.1004
		(0.114)	(0.0896)	(0.0936)	(0.0939)	(0.0195)	(0.0862)	(0.3946)
Demand:								
$\log(h_t/n_t)$	γ_2	−1.110	−0.8520	−0.9086	−1.1782	−1.4787	−0.8891	−1.0185
		(0.246)	(0.2687)	(0.2776)	(0.2897)	(0.1224)	(0.2476)	(0.6703)
$\log(y_t/n_t)$	γ_3	1.530	1.5635	1.5634	1.5631	1.3993	1.5563	1.5616
		(0.088)	(0.0908)	(0.0909)	(0.0949)	(0.0587)	(0.0807)	(0.3213)
$\log p^{\mathrm{f}}_t$	γ_4	−0.682	−0.8236	−0.8195	−0.8002	−0.6458	−0.8604	−0.8604
		(0.183)	(0.2212)	(0.2216)	(0.2312)	(0.0910)	(0.1950)	(1.6299)

[a] Suits estimated crop supply over the years 1919–1951 but estimated his demand and harvest supply equations for 1930–51. Only his elasticities are reproduced here; owing to the data scalings he used, Suits' other coefficient estimates (for the equation intercepts and the two dummy variables shifting crop supply) are not equivalent to those obtained from Wold's reconstructed data.

[b] Estimated using EViews and TSP. Wold's (1958) reconstruction of Suits' data permits estimation for the sample period 1930–1950. In all estimations the instrument set comprises an intercept, the predetermined variable $\log p_{t-1}$, and the exogenous variables $\log w_t$, $\log n_t$, $\log y_t$, $\log p^{\mathrm{f}}_t$, d^{c}_t, d^{w}_t, $\log p^{\mathrm{c}}_{t-1}$, $\log p^{\mathrm{v}}_{t-1}$. The GMM and 3SLS results are iterative. The GMM results use a Newey–West heteroskedasticity-and-autocorrelation-robust covariance matrix estimator based on a Bartlett kernel.

Table 3: Use of Wald Test Routines to Compute Standard Errors for Nonlinear Expressions

	Expression	2SLS	LIML	GMM	3SLS	FIML
own-price elasticity of demand	$1/\gamma_2$	-1.101	-0.849	-0.676	-1.125	-0.982
		(0.336)	(0.209)	(0.056)	(0.313)	(0.646)
income elasticity of demand	$-\gamma_3/\gamma_2$	1.721	1.327	0.946	1.750	1.533
		(0.536)	(0.336)	(0.098)	(0.496)	(1.115)
freight cross-price elasticity of demand	$-\gamma_4/\gamma_2$	-0.902	-0.679	-0.437	-0.968	-0.845
		(0.384)	(0.269)	(0.079)	(0.360)	(1.841)
dynamic stability	$\phi = \gamma_2\beta_3\alpha_2/(1 - \gamma_2\beta_2)$				-0.494	-0.560
					(0.124)	(0.344)
half-life	$\log(1/2)/\log(|\phi|)$				0.982	1.196
					(0.349)	(1.268)

Table 4: Instrumental Variables Diagnostics

Equation Dependent variable	Crop supply $\log q$	Harvest supply $\log h$	Demand $\log p$
Instrument exogeneity:[a] (p-values)			
2SLS Sargan J test	0.319	0.719	0.110
GMM Hansen J test	0.612	0.865	0.714
Endogenous regressors:[b] (p-values)			
Hausman test using 2SLS residuals		0.707	0.425
Weak instruments:[c]			
Staiger-Stock first-stage F statistic	7680.08	24.06	168.46

[a] The null hypothesis is that the restrictions implied by the overidentifying instrument set are satisfied. Rejection indicates misspecification.

[b] The null hypothesis is that the regressors specified as endogenous are actually exogenous, implying that IV estimation is unnecessay. Rejection indicates endogeneity, suggesting that IV estimation is desirable. There is no Hausman test for crop supply because that equation has no endogenous regressors.

[c] These are the F statistics from OLS regressions of each endogenous variable ($\log q$, $\log h$, $\log p$) on the instrument set. A Staiger-Stock F value less than 10 is a rule-of-thumb indicator of weak instruments.