Hide and Seek Search: Why Angels Hide and Entrepreneurs Seek

Merwan Engineer[1], Paul Schure[1,*] and Dan Vo[2]

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[1] University of Victoria

[2] Hartwick College

[*] Corresponding author. University of Victoria, Department of Economics, PO Box 1700, Victoria, BC, Canada V8W 2Y2, Tel: +1 250 721 8535, Fax: +1 250 721 6214, Email: schure@uvic.ca. We thank Jean-Etienne De Bettignies, Thomas Hellmann, Ian King, Bernard Lebrun, Eran Manes, Guido Mantovani, Veikko Thiele and seminar participants at the Canadian Economics Association Conference, World Finance Conference, and the University of Victoria for comments. We gratefully acknowledge the Social Sciences and Humanities Research Council of Canada (SSHRC) for financial support. All errors are ours.

ABSTRACT

The angel capital market poses a puzzle for search theory. Angel investors ("angels") are often described in the literature as hiding from entrepreneurs that seek angel capital investment. Hiding behavior by angels forces entrepreneurs to engage in costly search for angels. In our model, engaging in costly search by the entrepreneur can become a credible signal of her productivity level. An equilibrium exists in which hiding by angels screens out low-productivity entrepreneurs who would inundate any visible angels. Interestingly, social surplus may increase despite the assumed costs for hiding and searching because it improves the quality of the matches between entrepreneurs and angels. Hide and seek search contrasts with traditional search theory where agents choose strategies to mitigate inherent physical and informational search frictions.

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1. INTRODUCTION

> "...the search is also extremely inconvenient for the seller, entrepreneur, because angel investors prize their privacy. For good reason, they make themselves extremely difficult to find. The entrepreneur has a difficult time indeed locating investors with discretionary net worth, the inclination to subject themselves to the high levels of risk associated with this type of investment and the skills necessary to evaluate and add value to these ventures."
>
> Benjamin and Margulis (2001 p. 15)

The angel capital market is usually described as highly inefficient because entrepreneurs have to arduously search for angel investors. The difficulty for entrepreneurs in finding angels is usually attributed to a lack of information about angels stemming from their deliberate strategy of making "themselves extremely difficult to find" (see the opening quote). According to Van Osnabrugge and Robinson (2000), angels would be swamped with hundreds of project proposals were information about them widely available. The implication we explore in this paper is that angels deliberately hide to avoid being swamped by low-productivity entrepreneurs. When angels hide, only high-productivity entrepreneurs find it profitable to seek them out.

In this paper, we develop a model of hide and seek search. Unlike the traditional search models, there are no natural spatial or informational impediments to angels seeking entrepreneurs. If angels do not hide, they encounter with probability 1 entrepreneurs of unknown productivity. The main feature of our model is that the search friction is deliberately induced by angels on entrepreneurs rather than the result of an exogenous feature of the environment. We model angels as incurring a cost to move to a "search hinterland" with the aim to evade all, but the most "fit" entrepreneurs. This catch-me-if-you-can hiding strategy is a way to screen entrepreneurs. Specifically, angels will incur a modest cost of hiding when only high-productivity entrepreneurs find it worthwhile to acquire a signal of their quality by incurring the costs of search. In a fully separating equilibrium all angels hide, all high-productivity entrepreneurs search, while no low-productivity entrepreneurs search. We show that the fully separating equilibrium exists, and may be unique, under reasonable conditions.

For simplicity, we examine a static model with a continuum of agents of each type where, consistent with the evidence, angel capital is scarce so that there are more entrepreneurs than angels and the average productivity of entrepreneurs is quite low. Initially, angels and entrepreneurs can costlessly locate each other in a visible capital market. However, an angel forming a firm with a randomly selected entrepreneur in the visible market suffers an expected loss. Angels and entrepreneurs also have recourse to enter a search market at a cost. Matches in the search market are generated according to a constant-returns-to-scale matching function as in Pissarides (2000).[1] The equilibrium depends on the relative numbers of angels, entrepreneurs, productivity levels, investment amount, and the costs to hide and seek. When high-productivity entrepreneurs are sufficiently productive, there is a fully-separating equilibrium where matches only occur in the search market. The visible market becomes active and interacts with the search market when the model is extended to include “super angels”, angels with the ability to directly screen entrepreneur type at low cost.

Search theory has become a standard framework to analyse the market for venture capital, which includes the angel segment (see the next section). In the fully separating equilibrium of our model high-productivity entrepreneurs are more likely to match with an angel than low-productivity entrepreneurs. This result is analogous to the positive assortative matching result of Lentz (2010) who, like us, considers a random matching environment. Our analysis highlights that in such a setting some agents may have the incentives to try worsen the search frictions. In several papers search frictions are said to be "endogenous" (e.g. Lagos (2000), Julien, Kennes and King (2000), Burdett, Shi, and Wright (2001), or Stevens (2007)), which appears to coincide with our main result. However, the agenda in this branch of the search literature is to derive the matching function from "fundamentals", while in our paper we assume away fundamentals which necessarily imply matching problems. The feature of deliberately introduced frictions is shared with Barry, Hatfield, and Kominers (2014) who show that the introduction of bargaining costs in a multilateral bargaining setting can, for some parties, lead to better outcomes than in a

[1] Cipollone and Giordani (2016) find support for a constant-returns-to-scale matching function based on longitudinal data on the angel capital markets of 17 developed nations.

frictionless Coasean bargaining setting. In our model deliberate frictions can even increase welfare as the associated ability to signal may improve the average quality of matches.

Whereas our analysis is motivated and framed in terms of the angel capital market, we believe it may well apply more broadly. Interestingly, the origin of the term "angel" refers to well-heeled individuals who financed Broadway theatre productions in the beginning of the 20th century.[2] Like the angel capital market, the entertainment financing market seems to be characterized by reclusive financers trying to hide from large numbers of people with ideas of variable quality. Indeed the theme of a number of movies is about the obstacles placed in front of writers trying to find an agent to promote their work as plays and movies.[3] Similarly, in the labour market the common practice of not postings jobs, but rather letting workers search for jobs may have a hide and seek aspect.

The paper proceeds as follows. Section 2 further motivates our search on-its-head approach by describing the roles of various financiers and entrepreneurs in the capital market. Section 3 describes the model, and Section 4 examines when the search fully-separating equilibrium is unique. In Section 5 the model is extended to endow a subset of angels, "super angels", with the ability to directly screen entrepreneurs. Section 6 concludes.

2. HIDING AND SEEKING IN THE ANGEL CAPITAL MARKET

The presence of search frictions and the associated inefficiencies in the angel capital market have been reported since the inception of the research on the angel capital market. Van Osnabrugge and Robinson (2000, p 46) summarize the early literature as follows:

> *… the informal venture capital market of business angel finance is quite inefficient … thanks to the fragmented nature of the market, imperfect channels of communication, and the invisibility of business angels (their preference for anonymity) (Harrison and Mason, 1992;*

[2] Benjamin and Margulis (2001) discuss the origins of the term "angel". Wetzel (1983) was the first to use the term "angel" to describe an individual who provides their own capital to support entrepreneurial ventures.

[3] Some movies include *The Lonely Lady*, *The Player*, *French Exit*, and *Pitch*. Meyers (2009) this is not merely a feature of the past. Similarly, Orrell (2010) describes the difficulty for authors in finding and landing a literary agent to help them find a publisher.

Freear, Sohl, and Wetzel, 1994). Indeed, if it becomes widely known that an individual has money to invest, then he or she may be besieged with hundreds of proposals per year, when his or her desire may be only for three or four… These inefficiencies impose high search costs on both investors and entrepreneurs (Mason and Rogers, 1996; Wetzel, 1987). In the informal market, therefore, entrepreneurs can find only limited guidance in locating business angel funding, and the majority of business angels tend to rely on random discovery of potential investment opportunities.

The quote pose a puzzle: Why do angels evade entrepreneurs in a way that generates considerable inefficiency by largely imposing "…high search costs on both investors and entrepreneurs"? The quote also suggests an answer to the puzzle: the market is predominated by large number of desperate entrepreneurs which impose costs on angels who are not evasive. Our model formalizes the strategy of hiding as a rational profit-maximizing response.

Is forcing greater search cost on entrepreneurs the best screening mechanism? In contrast to angel investors, venture capitalists are typically well-known entities in formal venture capital markets who directly screen entrepreneurs. Venture capitalists are usually distinguished from angels as having greater funds as well as a greater flexibility to take on and support different types of projects. They usually take on quite a few projects. Angels usually undertake only one or a few projects at a time (Van Osnabrugge and Robinson, 2000). Whereas angels are usually involved in pre-seed and seed funding, venture capitalist are predominately involved in funding for projects in latter stages of start-up firms (Madill, Haines and Riding (2005), Sapienza, Manigart and Vermeir (1996)). Though there is considerable heterogeneity and some overlap in the informal and formal venture capital markets (e.g. Hellmann, Schure and Vo (2015)), we keep with the standard view that that the informal venture capital market for angel funds is distinct.

Table 1: OECD estimates of the sizes of the angel and venture capital markets
Table 1 is a copy of Table 2.2 in OECD (2011, p45)

USD millions

	"Visible" angel market size (share of total market) in 2009	Estimated size of angel market in 2009	Total VC* market in 2009
United States	469 (3%)	17 700	18 275
Europe	383 (7%)	5 557	5 309
United Kingdom	74 (12%)	624	1 087
Canada	34 (9%)	388	393

*Note: VC market size includes VC investments in all stages: *i)* seed, *ii)* start-up, *iii)* early, *iv)* expansion, and *v)* later stage.

Source: OECD based on estimates by the Centre for Venture Research (CVR), EBAN (The European Trade Association for Business Angels, Seed Funds, and other Early Stage Market Players), and Canada's National Angel Capital Organisation (NACO). VC data based on industry statistics by EVCA/PEREP Analytics and PricewaterhouseCoopers/National Venture Capital Association MoneyTree Report and Canada's National Angel Capital Organization.

Table 1 shows that the angel capital market is roughly of the same size as the formal venture capital (VC) market.[4] The VC market is a visible market. In contrast, the table reveals that only a small fraction (12% or less) of the angel capital market is "visible". Visible angel funding includes super angels, angels with large investment portfolios that adopt a formal approach to investment selection and monitoring, akin to venture capitalists (e.g. Lerner et al. (2015). Visible angel funding also include visible angel consortiums and angel networks, which provide direction to entrepreneurs and often screen or pre-screen their funding applications (e.g. Wong (2010), Ibrahim (2013)).[5] While most angel networks receive government support (e.g. Prowse

[4] Other sources find even a relatively larger role for angels than found in Table 1. Riding (2008) finds that there are about 15,800 angels investing at least $1.9 billion annually in the entrepreneurial firms in Canada. In contrast, venture capital firms invest less than half that amount, at about $870 million annually. See Shane (2008) for similar information on the US Angel Capital Market. Madill, Haines, and Riding (2005) and many other studies note that business angel investors not only constitute an important source of financing, they also provide significant non-financial inputs to the growth and viability of the firms through, among other things, mentoring their industry experience and contacts. In this paper, we abstract from the other roles angel investors might play in startup firms.

[5] Interestingly, while angel networks provide entrepreneurs with a visible outlet for their funding applications, angels within such networks typically remain anonymous to the entrepreneurs (OECD, 2011). Perhaps angel networks provide the desirable combination of visibility to the entrepreneurs, yet anonymity to their angel members. A more nuanced view on visible angel consortiums and networks would be arguably to view them as representing

(1998), Schure and Dodaro (2015)), they are still not the norm in the angel capital market as Table 1 shows.

The majority of the theoretical literature on entrepreneurial finance focuses on the relationship between the entrepreneur/startup and the financier/capitalist.[6] In the last 15 years the standard model for the *market* for venture capital has become the Diamond-Mortensen-Pissaridis search framework (see e.g. Inderst and Muller (2004), Michelacci and Suarez (2004), Boadway, Secrieru, and Vigneault (2005), Sørensen (2007)). Hellmann and Thiele (2015)) use two interrelated search models to describe the formal venture capital as well as the informal venture capital/angel segment of the market.[7] In the search literature search is a necessary consequence of problems of assumed inherent physical and informational frictions. While such inherent frictions may indeed be a characteristic of both the formal and the angel segments of the market for venture capital, our paper focuses on the angel capital market because the evidence indicates that many angels deliberately cause (some of the) search frictions faced by entrepreneurs.

Our model is a stylized representation of angel capital market. We assume angels entirely avoid the search environment by default. If angels chose to stay visible they would operate in a market characterised by Akerlof (1970)'s market for lemons. In this market informed agents/entrepreneurs have no ability to produce a credible signal of their productivity. By hiding in the search market, angels enable entrepreneurs to produce costly signals in the form of search costs, as well as, possibly, a lower probability of a match with an angel. In a separating equilibrium, high-productivity entrepreneurs credibly signal their productivity. Indeed, entrepreneurs that search for hiding angels operate in a market described by the standard signalling model of Spence (1974). In an extension we investigate the implications of endowing some angels, "super angels", with the ability to screen entrepreneurs directly. Such angels will form the empirically observed visible segment of the angel capital market.

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search paths that are complementary to more traditional modes of search in the angel capital market, such as for example networking at local enterprise forum meetings.

[6] See Metrick and Yasuda (2011) and Da Rin, Hellmann and Puri (2013) for recent reviews of the venture capital literature.

[7] The closely related "market for ideas" has also been modelled as a search market (Silviera and Wright, 2010).

3. MODEL

Agents. There are two groups of risk neutral agents in the angel capital market: angels and entrepreneurs. There is a continuum of angels of mass 1. The total number of entrepreneurs is $E = E_H + E_L$, where E_H is the number of high-productivity entrepreneurs and E_L is the number of low-productivity entrepreneurs. For simplicity, we assume that high-productivity entrepreneurs outnumber angels, $E_H > 1$, so there is no absence of good ideas.

Entrepreneurs have no capital, but each has an idea for a project that requires an upfront investment of $K > 0$. Angels do not have an idea for a project, but do have sufficient capital to fund one. They always have the option to store their capital to earn a zero net return. We look at the case where angels always prefer to invest if they can be assured they are matched with a high-productivity entrepreneur.

Timing. The timing of actions and events is as follows:

- Stage 1: Entrepreneurs can costlessly locate angels in the visible market and apply for funding from them. Each angel agrees whether or not to form a firm with an applicant entrepreneur according to an exogenous sharing rule known to both parties.
- Stage 2 : Agents that did not form firms in Stage 1 choose whether or not to enter the search market.
- Stage 3: The angels and entrepreneurs in the search market are randomly matched according to a known matching function. Each matched angel-entrepreneur pairs agrees whether or not to form a firm according to an exogenous sharing rule.
- Stage 4: Firm returns are realized and shared according to the sharing rule.

The timing is consistent with angels being initially visible but having the recourse to hide. This timing is not essential in the basic model, and we get identical results if agents simultaneously choose which market to participate in. In the extended model the sequencing the markets matters and sequencing the visible market enables ready agreements before recourse to the search market.

Firms and contracts. A firm is the outcome of an agreement between an entrepreneur and an angel. In Stage 4, the firm produces a gross return that depends on the productivity level of the entrepreneur: $R_L > 0$ for a firm that is managed by a low-productivity entrepreneur, and $R_H > R_L$ with a high-productivity entrepreneur. Critically, the productivity level of an entrepreneur is private information, and the agreement between the angel and entrepreneur cannot be conditioned on entrepreneur type.[8] Further, we assume for simplicity that the proportion of lower-productivity entrepreneurs reduces the average productivity below the cost of capital.

Assumption 1: The average project generates a loss, $\left(\frac{E_H}{E}R_H + \frac{E_L}{E}R_L\right) - K < 0.$

Assumption 1, combined with asymmetric information implies a severe adverse selection problem, which necessitates that angels must find a way of selecting *H*-entrepreneurs.

If a firm is formed, the angel receives a fixed exogenous share of a firm's gross return, denoted σ, and the entrepreneur receives the remaining share $1-\sigma$. We assume that $1-\sigma > 0$ such that both high and low quality entrepreneurs have a dominant strategy to match as they receive strictly positive payoffs, i.e. $(1-\sigma)R_H > 0$ and $(1-\sigma)R_L > 0$, respectively. A minimum condition on the angel's share $0<\sigma<1$ is that they enjoy a positive payoff after they have paired with an *H*-entrepreneur, $\sigma R_H - K > 0$.[9] Combining this assumption with Assumption 1 implies that angels make a loss from financing a low quality entrepreneurs, $\sigma R_L - K < 0$.

Markets and Matches. Initially (Stage 1), each agent is in the visible market. For simplicity, we assume there are no costs for agents to match in the visible market. However, Assumption 1

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[8] The productivity levels could each be considered to be expected values. For example, suppose that the underlying project either returns 0 or $R > 0$, and the high quality entrepreneur has higher probability of realizing R. Then $R_H = \theta_H R$ and $R_L = \theta_L R$, where the probabilities satisfy $0 < \theta_L < \theta_H \leq 1$ and are private information. Here type differs according to ability to realize a given known project rather than the project being different *per se*.

When realized project returns are different, type might be inferable by the angel after the investment is made. However, the agreement can't be made contingent on this outcome if the outcome is not observable by a their party or enforceable in law.

[9] The share must also cover other costs as we discuss later on. A sufficiently large share that covers costs is consistent with sharing the surplus, a standard assumption in search models. Also, it is consistent with empirical evidence that angels and entrepreneurs strike simple contracts with interior shares (Wong (2010)).

In principle shares could depend on the environment, hiding or staying. Our generic results do not depend on variable bargaining power as long as both sides have positive shares. See Engineer and Shi (2001) for a discussion of how variable bargaining power can generate new results in search models.

implies that under asymmetric information no angel would form a firm with an entrepreneur of unknown productivity that is randomly sampled from the overall population of entrepreneurs.[10] Thus, in the basic model, the visible market fails.

In Stage 2, agents choose whether to stay in the failed visible market, the default, or incur a cost to enter the search market. By entering the search market, angels are deliberately becoming elusive and remote. This hiding behavior involves each angel incurring a non-negative cost $c_a \geq 0$. The measure of angels that hide is denoted by $a \leq 1$. By assumption entrepreneurs incur a positive cost, $c_e > 0$, if they enter the search market. The measure of entrepreneurs that searches is denoted by $e \leq E$, where $e = e_L + e_H$ describes the composition of entrepreneurs searching.

In Stage 3, angels and entrepreneurs in the search market are randomly paired according to a non-decreasing matching function $m(a, e)$ with upper bound $m(a, e) \leq \min[a, e]$ and lower bound $m(0, e) = m(a, 0) = 0$. As in Pissarides (2000) the matching function satisfies constant returns to scale, $\theta m = m(\theta a, \theta e)$ for $\theta > 0$, and in the interior, $0 < m(a, e) < \min[a , e]$, is increasing in both arguments $a > 0$ and $e > 0$. These assumptions on the matching function are in line with the evidence on the matching function of Cipollone and Giordani (2016), which is based on angel market data of 17 developed nations in 1996-2014. Together, $a > 0$ and $e > 0$ and the upper bound imply that the probability of a hiding angel matching is $p(a,e) = \frac{m(a,e)}{a} \epsilon (0,1]$ and the probability of a searching entrepreneur matching is $\frac{m(a,e)}{e} = p(a,e)\frac{a}{e}\epsilon(0,1]$.

Payoffs. *Ex ante* profits for agents that enter the search market in in Stage 2 depend on the populations searching angels, *H*-entrepreneurs and *L*-entrepreneurs as follows:

[10] Any pairing protocol that generates a random sample is consistent with the invisible market not starting firms. As entrepreneurs are identical except for their productivity levels, there is no basis for selection of *H*-entrepreneurs in Stage 1. In Section 5 we develop an extension of the model where the visible market becomes active.

$$\pi_A(a, e_H, e_L) = \frac{m(a,e)}{a}(\sigma R_H - K) - c_a \quad (1)$$
$$\pi_H(a, e_H, e_L) = \frac{m(a,e)}{e}(1-\sigma)R_H - c_e \quad (2)$$
$$\pi_L(a, e_H, e_L) = \frac{m(a,e)}{e}(1-\sigma)R_L - c_e \quad (3)$$

where the subscripts *A*, *H* and *L* respectively denote the representative angel, *H*-entrepreneur and *L*-entrepreneur. Recall that Assumption 1 implies that in the visible market all angels maximize profits at zero by rejecting any offers to form a firm. Thus, in Stage 2, each agent would enter the search market only if they made an expected profit of at least zero.

4. EQUILIBRIUM

Let us focus on the fully-separating equilibrium, in which all angels profitably hide (a = 1), all *H*-entrepreneurs profitably search (e_H=E_H), and no *L*-entrepreneurs search because it would be unprofitable (e_L=0, and hence also e=E_H). If it exists, the equilibrium payoffs are:

$$\pi_A(1, E_H, 0) = m(1, E_H)(\sigma R_H - K) - c_a > 0 \quad (1')$$
$$\pi_H(1, E_H, 0) = \frac{m(1,E_H)}{E_H}(1-\sigma)R_H - c_e > 0 \quad (2')$$
$$\pi_L(1, E_H, 0) = \frac{m(1,E_H)}{E_H}(1-\sigma)R_L - c_e \leq 0 \quad (3')$$

Here a match between a hiding angel and a searching entrepreneur involves a *H*-entrepreneur with probability 1. As matches would generate positive payoffs for searching agents, the surplus is positive, $\pi_A(1, E_H, 0) + \pi_H(1, E_H, 0)E_H = p(1, E_H)(R_H - K) - c_a - E_H c_e > 0$.

Recall that zero profits is the alternative for any agent who does not enter the search market under Assumption 1. As any one agent is small relative to a market, the choice for each agent is between the expected profits of search in equations (1') - (3') and a zero payoff. Thus, all agents are optimizing in the fully-separating equilibrium: angels and *H*-entrepreneurs enter the search market, and *L*-entrepreneurs stay unmatched in the visible market with zero profits.[11] The

[11] Equations (1')-(3') are consistent with a rule that agents agree to form a firm only if they are strictly better off by doing so. This simplification unambiguously gives the fully-separating equilibrium. See footnote 10 for the extension to include the case where agents may agree to form a firm when indifferent to doing so.

following proposition summarizes and states the conditions for existence of the fully separating equilibrium implied by (1') - (3') in terms of threshold productivity levels.

Proposition 1. *There exists a fully-separating Bayesian Nash equilibrium in which firms generate profits for both angels and entrepreneurs if and only if the L-entrepreneurs are sufficiently unproductive,* $R_L \leq \frac{c_e E_H}{m(1,E_H)(1-\sigma)}$*, and H-entrepreneurs are sufficiently productive,* $R_H > max\left[\frac{c_e E_H}{m(1,E_H)(1-\sigma)}, \frac{c_a}{m(1,E_H)\sigma} + \frac{K}{\sigma}\right]$.

These inequalities are satisfied for a wide range of parameter values. Consider a Cobb-Douglas matching function: $m(a, e) = \min[\mu(a)^{\alpha}(e)^{1-\alpha}, a, e]$, where $0 < \alpha < 1$ and $\mu > 0$ are constants. With specific values: $\alpha = ½$, $c_e = c_a = 1$, $\sigma = ½$, $K = 3$ and $E_H = 4$, an interior equilibrium exists for $R_L \leq \frac{4}{\mu}$ and $R_H > max\left[\frac{4}{\mu}, \frac{1}{\mu} + 6\right]$ provided that $\mu \leq ½$. If $\mu \geq ½$, the matching function is at the upper bound $m=a=1$ so that $m(1, E_H) = 1$. For example, if $\mu = ½$ then the equilibrium exists for $R_L \leq 8$ and $R_H > 8$; and if $\mu = \frac{2}{3}$ then for $R_L \leq 6$ and $R_H > 7.5$. As a benchmark for the rest of the paper, we assume equations (1')-(3') hold so that the fully-separating equilibrium in Proposition 1 exists.

Assumption 2. *Equations (1')-(3') hold.*

Besides the fully-separating equilibrium there also exists an autarky equilibrium in which no agents enter the search market: $a = e_H = e_L = 0$. No firms are formed and all agents have zero profits. A deviating angel that hides would not meet a searching entrepreneur, while attaining a negative payoff, $-c_a$. Similarly, a deviating entrepreneur would not meet a hiding angel and attain a negative payoff $-c_e$. The following proposition shows no other equilibria exist.

Proposition 2. *Only two Bayesian Nash equilibria exist under Assumption 2: the fully-separating equilibrium and the autarky equilibrium. The fully-separating equilibrium weakly Pareto dominates autarky.*

The proof is in the Appendix. In the fully-separating equilibrium angels and H-entrepreneurs earn positive profits. Thus, this equilibrium is a Pareto improvement over the autarky equilibrium, which by comparison represents a coordination failure.[12]

5. DIRECT SCREENING IN THE VISIBLE MARKET

In the basic model, no firms form in the visible market. However, Table 1 shows that a small fraction (12% for Britain) of the value of transactions take place in the visible angel market. Here we model visible angel market activity by endowing a fraction $\hat{\varphi} < 1$ of angels, "super angels", with a direct screening technology. Assume screening correctly identifies the entrepreneur sampled with probability $\beta \equiv \Pr(H\text{-signal}|H\text{-entrepreneur}) = \Pr(L\text{-signal}|L\text{-entrepreneur}) \geq 0.5$, and that it costs the super angel $c_a^s \geq 0$ for each entrepreneur sampled.

Recall, that all agents start in the visible market, where there are no transaction costs by assumption. Thus, it is a domannt strategy for all entrepreneurs to apply for funding to the visible super angels, who therefore screen from a random sample of the full population of entrepreneurs. A screened entrepreneur who generates an H-signal is actually an H-entrepreneur with probability $\mu_1^H \equiv \Pr(H\text{-entrepreneur}|H\text{-signal})$, where

$$\mu_1^H = \frac{\Pr(H\text{-signal}|H\text{-entrepreneur}) * \Pr(H\text{-entrepreneur})}{\Pr(H\text{-signal})} = \frac{\beta\left(\frac{E_H}{E}\right)}{\beta\left(\frac{E_H}{E}\right) + (1-\beta)\left(\frac{E_L}{E}\right)} = \frac{\beta E_H}{\beta E_H + (1-\beta)E_L}$$

For example, when the H-signal is perfectly informative, $\beta = 1$, the super angel's posterior belief is always correct, $\mu_1^H = 1$, and when the signal is non-informative, $\beta = 0.5$ then $\mu_1^H = \frac{E_H}{E}$.

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[12] The autarky equilibrium is robust, in part because the two sides of the market moving simultaneously after Stage 1. Instead, if angels moved before entrepreneurs, then the autarky equilibrium would be eliminated if we allowed a positive measure of angels, $a^h > 0$, to deviate from autarky. If $a^h > 0$ angels were to profitably hide, then (following the logic to case $0 < e < E_H$ in the Proof of Proposition 2) the optimal response in the subgame would be that $e_H^s \geq a^h E_H > 0$ H-entrepreneurs deviated by searching. Deviating angels would attain strictly positive profits as $p(a^h, e_H^s) = \frac{m(a^h, e_H^s)}{a^h} = m(1, e_H^s/a^h) \geq m(1, E_H) > 0$, while deviating H-entrepreneurs would remain at zero profits. Thus, if angels moved first in Stage 2 the fully-separating equilibrium would be the only stable equilibrium.

Super angels that choose to screen continue to sample entrepreneurs until they obtain an H-signal and therefore incur an expected direct screening cost of $\frac{c_a^s}{\Pr(H\text{-signal})} = \frac{c_a^s E}{\beta E_H + (1-\beta)E_L}$. This cost enters into the expected profit of a screening super angel as follows:

$$\pi_A^s(\beta, c_a^s) = \sigma(\mu_1^H R_H + (1-\mu_1^H)R_L) - \frac{c_a^s E}{(\beta E_H + (1-\beta)E_L)} - K \tag{4}$$

Here profits are increasing in the accuracy of the signal, β and decreasing in its cost, c_a^s. A super angel only screens if she cannot do better by either not forming a firm or hiding:

$$\pi_A^s(\beta, c_a^s) > \max[0, \pi_A(a, e_H, e_L)] \tag{5}$$

For $\beta \to 1$, screening profits approach $\pi_A^s = \sigma R_H - \frac{c_a^s E}{E_H} - \mathrm{K}$. Hence, if there are relatively few H-entrepreneurs, condition (5) is violated even with a low screening cost c_a^s. However, if $\beta \to 1$ and in addition $c_a^s \to 0$, then the payoffs from screening are strictly greater than the maximum feasible payoff for a super angel that chooses to search, because trivially $\pi_A(a, e_H, e_L) \leq \sigma R_H - \mathrm{K} - c_a$. In summary, super angels screen for β sufficiently large and c_a^s sufficiently small.

Consider an equilibrium where (5) holds for $a = 1, e_H = E_H$, and $e_L = 0$. Then a fraction $0 < \varphi \leq \hat{\varphi}$ of the super angels prefer screening over hiding, and we also know from Assumption 2 that hiding is preferred over not hiding and receiving zero payoff. As angels continue to screen until they receive an H-signal, φ super angels are matched. This leaves 1-φ angels and $E - \varphi$ entrepreneurs unmatched, among which $(E_H - \varphi\mu_1^H)$ H-entrepreneurs. What does the equilibrium behaviour of the unmatched agents now look like? Observe that screening reduces the relative proportion of unmatched H-entrepreneurs in the visible market. Hence Assumption 1 continues to hold and the (1-φ) unmatched angels will not start a firm with a randomly selected remaining entrepreneur in the visible market. But also observe that direct screening by super angels has increased the relative proportion of unmatched high-productivity entrepreneurs to unmatched angels to $E_H'' \equiv \frac{E_H - \varphi\mu_1^H}{1-\varphi} > E_H$. This feature alone alters the search analysis.

Let us focus on the *screening and fully-separating search equilibrium*, in which all remaining (1-φ) angels and all *H*-entrepreneurs profitably enter the search market, while none of the unmatched *L*-entrepreneurs do. The profit conditions for this equilibrium are analogous to equations $(1') - (3')$):

$$\pi_A(1-\varphi, E_H - \varphi\mu_1^H, 0) = \frac{m(1-\varphi, E_H - \varphi\mu_1^H)}{1-\varphi}(\sigma R_H - K) - c_a > 0 \tag{1''}$$

$$\pi_H(1-\varphi, E_H - \varphi\mu_1^H, 0) = \frac{m(1-\varphi, E_H - \varphi\mu_1^H)}{E_H - \varphi\mu_1^H}(1-\sigma)R_H - c_e > 0 \tag{2''}$$

$$\pi_L(1-\varphi, E_H - \varphi\mu_1^H, 0) = \frac{m(1-\varphi, E_H - \varphi\mu_1^H)}{E_H - \varphi\mu_1^H}(1-\sigma)R_L - c_e \leq 0 \tag{3''}$$

The property of a homogenous-of-degree-1 matching function can be employed to show that the probability that a hiding angel matches is larger than in the fully-separating equilibrium:

$$\frac{m(1-\varphi, E_H - \varphi\mu_1^H)}{1-\varphi} = m(1, E_H'') > m(1, E_H)$$

It follows that equation $(1')$ implies $(1'')$, meaning that all hiding angels (be they super angels or not) attain a higher payoff with the introduction of super angels, $\pi_A(1-\varphi, E_H - \varphi\mu_1^H, 0) > \pi_A(1, E_H, 0) > 0$. This also shows that if the screening and fully-separating search equilibrium exists, then the fully-separating equilibrium cannot exist. Further, the equilibrium conditions (5) and $(1'')$ require $\pi_A^s(\beta, c_a^s) \geq \pi_A(1-\varphi, E_H - \varphi\mu_1^H, 0) > 0$. As $lim_{\varphi\to 0}\pi_A(1-\varphi, E_H - \varphi\mu_1^H, 0) \to \pi_A(1, E_H, 0)$, we have the following Bayesian Nash equilibrium.

Proposition 3. *Assumption 1 holds. Then a screening and fully-separating search equilibrium exists if and only if:*

(i) Screening by super angels is sufficiently informative and inexpensive $\pi_A^s(\beta, c_a^s) > max[0, \pi_A(1, E_H, 0)]$ *;*

(ii) Hiding by non-super angels is profitable when H-entrepreneurs that did not match with a super angel search, $\pi_A(1-\varphi, E_H(\varphi), 0) > 0$*;*

(iii) L-entrepreneurs are sufficiently unproductive, $R_L \leq \frac{E_H''(\varphi)}{m(1, E_H''(\varphi))}\frac{c_e}{(1-\sigma)}$ *, and*

H-entrepreneurs are sufficiently productive, $R_H > \max\left[\frac{E_H''(\varphi)}{m(1, E_H''(\varphi))}\frac{c_e}{(1-\sigma)}, \frac{c_a}{m(1, E_H''(\varphi))\sigma} + \frac{K}{\sigma}\right]$;

where $\varphi = \hat{\varphi}$ when $\pi_A^s(\beta, c_a^s) \geq \pi_A(1-\hat{\varphi}, E_H(\hat{\varphi}),0)$, and $\varphi < \hat{\varphi}$ is given by $\pi_A^s(\beta, c_a^s) = \pi_A(1-\varphi, E_H(\varphi),0)$ when $\pi_A^s(\beta, c_a^s) < \pi_A(1-\hat{\varphi}, E_H(\hat{\varphi}),0)$.

The productivity bounds are endogenous, as $0 < \varphi \leq \hat{\varphi}$ is endogenous and $E_H'' \equiv \frac{E_H - \varphi\mu_1^H}{1-\varphi}$. Compared with Proposition 1, the upper bound on R_L is greater whereas the lower bound on R_H is potentially less or greater. Thus, endowing some angels with the ability to screen may or may not enlarge the domain demarcated by the necessary condition (ii) in Proposition 3 for which an equilibrium with hiding and seeking exists. If the introduction of super angels reduces the domain, then on the subdomain that is lost there is a loss in overall surplus as condition (i) of Proposition 3 rules out the existence of a (beneficial) fully separating equilibrium. In this case the ability to screen imposes a negative externality on other angels. By contrast, if the screening-ability of some angels implies the domain emarcated by (ii) becomes larger, then super angels imply a positive externality on all agents since the alternative is autarky. Finally, note that Proposition 3 carries over to the case in which $\pi_A(1, E_H, 0) < 0$ as long as $\pi_A^s \geq \pi_A(1-\varphi, E_H - \varphi\mu_1^H, 0) > 0$. As $\pi_A(1, E_H, 0) < 0$ violates $(1')$, a viable search market arises due to the presence of super angels.

6. CONCLUSION

Entrepreneurs seeking angels in the angel capital market appears to be a classic search story. There are scattered agents on both sides of the market who do not know much about each other. The evidence cited in Table 1 reveals that at least 88% of angel capital investment is not visible in leading countries. However, the literature suggests that the search problem in the angel capital market does not spring from the standard spatial, information and coordination impediments. Rather angels face a market that is too "thick" and would rather face fewer, higher productivity entrepreneurs. Angels may willfully erect barriers to matching, which in turn result in the appearance of a classic search environment. The barriers erected by angels create search costs (or induce greater search costs) for entrepreneurs that seek funding.

Our analysis explains and analyzes this hide-and-seek phenomenon in the angel capital market. In our basic model, angels hide away in a search market where only high-productivity entrepreneurs find it worthwhile to seek them out. Thus, the novelty of our paper is that search environment is the result of a deliberate choice, rather than exogenous. The presence of a search market adds value to some or all the agents because it enables costly search to becomes a credible signal of quality.

In an extension of the basic model, we show that a equilibrium with hiding and seeking does not preclude that direct screening mechanisms may be active at the same time, as seems to also be the case in the angel capital market. Super angels and angel groups and networks are often visible reference points in the angel capital market and they usually have formal screening methods to select or pre-select young companies. These institutions still represent a minority of the angel capital. Such visible angels may be in competition for deals with VCs more often than "casual angels" (Hellmann et al. (2015). Possibly therefore the "visible angels" of our model reflect VCs as well as super angels and angel groups. In this sense our model may explain the phenomenon of co-existence of VCs and angels, however note this discussion is incomplete as angels and VCs differ in terms of other dimensions besides the way in which they select projects.[13] Further exploration of the scope of the hiding-and-seeking mechanism, and the interaction with possible visible segments of markets, is left for future research.

[13] Notably in terms of the financial contracts, as is shown by the evidence of Wong (2010).

7. REFERENCES

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APPENDIX

Proof to Proposition 1. The existence of the equilibria is shown in the text. For $a = 0$ the autarky equilibrium is unique as Assumption 1 implies that angels are worse off if they finance a firm in the visible market. Below we prove that for $a > 0$ the fully-separating equilibrium is unique.

It is useful to first show that optimization by entrepreneurs implies: (a) if $e_L > 0$ then $e_H = E_H$, and (b) if $e_H < E_H$ then $e_L = 0$. As for (a), if $e_L > 0$, then optimization by L-entrepreneurs implies they earn non-negative profits, $\frac{m(a,e)}{e}(1-\sigma)R_L - c_e \geq 0$. As $R_H > R_L$, then $\frac{m(a,e)}{e}(1-\sigma)R_H - c_e > 0$ and it is profitable for all H-entrepreneurs to search, $e_H = E_H$. As for (b), if $e_H < E_H$, then $\frac{m(a,e)}{e}(1-\sigma)R_H - c_e \leq 0$ and $\frac{m(a,e)}{e}(1-\sigma)R_L - c_e < 0$; searching is unprofitable for L-entrepreneurs, so $e_L = 0$.

We now consider the full range of e_H and e_L to identify all possible equilibria. Consider $e = E_H \neq e_H$, i.e. $e_H < E_H$ and $e_L > 0$. This contradicts (b) above.

Consider $e > E_H$. Then by (a) $e_H = E_H$ and $e_L > 0$. Optimization by L-entrepreneurs implies non-negative profits of search, $R_L \geq \frac{c_e e}{m(a,e)(1-\sigma)}$. The fully-separating equilibrium equation (3') requires $R_L \leq \frac{c_e E_H}{m(1,E_H)(1-\sigma)}$. These conditions on R_L imply

$$\frac{E_H}{e}m(a,e) = m\left(\frac{E_H a}{e}, E_H\right) \geq m(1, E_H)$$

If the proposed solution is at an interior, $m(1, E_H) < 1$, so must be the proposed alternative, $m\left(\frac{E_H a}{e}, E_H\right)$ as $\frac{E_H a}{e} < 1$ and $m\left(\frac{E_H a}{e}, E_H\right) < m(1, E_H)$ which gives a contraction. If the proposed solution is at the boundary $m(1, E_H) = 1$, it exceeds the proposed alternative, $m\left(\frac{E_H a}{e}, E_H\right) \leq \frac{E_H a}{e} < 1$, and again we get a contradiction.

Consider $e = 0$. This cannot be an equilibrium since hiding angels make negative profits.

Consider $0 < e < E_H$. Then (b) requires $e = e_H < E_H$. Optimization by H-entrepreneurs makes them indifferent to searching and earning zero profits, $R_H = \frac{c_e e}{m(a,e)(1-\sigma)}$, or not searching and being in autarky. Not searching results in autarky as entrepreneurs are unable to match with possible visible angels; Assumption 1 implies $\sigma\left(\frac{E_H}{E_L+E_H}R_H + \frac{E_L}{E_L+E_H}R_L\right) - K < 0$ which rules out visible angels making profits through pooling, $\sigma\left(\frac{E_H-e_H}{E_L+E_H}R_H+\frac{E_L}{E_L+E_H}R_L\right)-K<0$. But (2’) requires $R_H>\frac{c_e\,E_H}{m(1,\,E_H)(1-\sigma)}$. Comparing the conditions for R_H implies

$$m(1,\ E_H)>\frac{E_H}{e}\,m(a,e)=m\left(a\frac{E_H}{e},\ E_H\right) \qquad (*)$$

If $\frac{E_H}{e}\geq 1$, then the above is false (both for $m(1,\ E_H)$ interior and at the boundary). If $a\frac{E_H}{e}<1$, then $a<\frac{e}{E_H}<1$ and optimization by angels requires that they are indifferent between hiding and autarky $m(a,e)(\sigma R_H-K)=ac_a$ or $m\left(1,\frac{e}{a}\right)(\sigma R_H-K)=c_a$.[14] Recall, (1’) specifies that angels make profits in equilibrium, $m(1,E_H)\ (\sigma R_H-K)>c_a$. Substituting for c_a gives

$$m(1,E_H)>m(1,\frac{e}{a}) \qquad (**)$$

which implies $a>\frac{e}{E_H}$. This contradicts the above $a<\frac{e}{E_H}<1$.[15]

Finally, consider $a<1$ and $e=e_H=E_H$. Here $a<\frac{e}{E_H}=1$ so (**) applies which gives the contradiction $a>\frac{e}{E_H}$.

Thus, with $a>0$ the fully-separating equilibrium, where $a=1$ and $e=e_H=E_H$, is unique.

[14] Propositions 1 and 2 hold for $c_a=0$. With $c_a=0$, $m\left(1,\frac{e}{a}\right)(\sigma R_H-K)>c_a$ which contradicts the requirement that angels be indifferent. However, autarky can now take the form where $a>0$ angels enter the search market even though they know no entrepreneurs will do so.

[15] If, instead of assuming positive firm profits in the fully-separating equilibrium, we assume non-negative profits $\pi_A(1,E_H,0)\geq 0$ and $\pi_H(1,E_H,0)\geq 0$, then (*) and (**) both hold with a weak inequality implying $a\leq\frac{e}{E_H}<1$ and $a\geq\frac{e}{E_H}$. Thus, there are knife-edge equilibria in which an equal fraction of angels hide and H-entrepreneurs search, $a=\frac{e_H}{E_H}\in(0,1)$. Firms have zero profits in equilibrium, so that $m(1,E_H)(\sigma R_H-K)=c_a$ and $R_H=\frac{c_e\,E_H}{m(1,\,E_H)(1-\sigma)}$. Substituting for $m(1,E_H)$ gives the improbable restriction $R_H=\frac{c_e\,E_H K}{c_e\,E_H\sigma-c_a(1-\sigma)}$. These equilibria are not eliminated when a measure of angels move first since a best response is for the same ratio of H-entrepreneurs to search.