Tax Competition and the Efficiency of "Benefit-Related" Business Taxes

Elisabeth Gugl[#] George R. Zodrow*

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Abstract

We consider a model in which business public services must be financed with either a source-based tax on mobile capital, such as a property tax, or a tax on production, such as an origin-based Value Added Tax, and assess which of the two tax instruments is more efficient. Many taxes on business tax mobile inputs such as retail sales taxes or property taxes and their inefficiency has been noted in the literature, but calls from experts to broaden the tax base to an origin-based VAT have not been analyzed theoretically. In line with the intuition of such calls, we find that the production tax is less inefficient than the capital tax under some conditions. A user fee on the public business input would be efficient and by taxing production, a tax on this public input is included in the tax base as well as a tax on immobile labor. We find that if there is room for significant substitution between capital and the public service (e.g. a substitution elasticity greater than 1 in the case of a CES production function), a broader tax base that includes the contribution of the public service to output mimics more closely user charges on the public service. Both a capital tax and a production tax are inefficient, but the degree of inefficiency is lower under a production tax than a capital tax if the production function is log submodular in capital and the public service.

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[#] Department of Economics University of Victoria

P.O. Box 1700 STN-CSC Canada – Victoria, BC, V8W 2Y2 egugl@uvic.ca

* Rice University Department of Economics, 6100 Main Street, MS-22, Houston TX 77005, USA – zodrow@rice.edu

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1. Introduction

At the state and local levels in the United States and at the provincial level in Canada, retail sales taxes often apply tax to business inputs, even though in principle they are supposed to be limited to final consumer goods. However, eliminating such taxes on business can provoke strong opposition from the electorate. For example, British Columbia in 2010 moved from a retail sales tax that was known to tax business inputs due to problems in its administration to a "pure" consumption tax — the Harmonized Sales Tax (HST), which combined the provincial retail sales tax and the federal Goods and Services Tax into a single value-added tax. This reform was projected to decrease tax payments from business by CAN $730 million, according to The Independent Panel Report [26] and critics of the HST emphasized its shift away from taxing business (CBC News [3]). Premier Gordon Campbell introduced the HST shortly after winning office despite denying any intent to do so during his campaign; as a result, strong opposition to replacement of the provincial sales tax with the HST was also sparked by the way the provincial government had introduced the change. The opposition was so fierce that continuation of the HST was the subject of a mail-in referendum, in which 55% of voters opposed the HST. As a result, British Columbia had to make the costly transition to re-establishing its provincial retail sales tax.

In addition to taxing retail sales of business inputs, local taxes in the United States and Canada often include property taxes on business capital as well. Both forms of business input taxation may distort a wide variety of decisions, including those regarding capital accumulation and allocation and the level of public services provided, as stressed in the tax competition literature.

In light of this situation, several prominent public finance experts have argued that, in the absence of explicit benefit taxes or user charges, broad-based taxes on local production, such as an origin-based value added tax (VAT), are an attractive option (Bird [1], Hines [9]). They stress that such taxes serve as a proxy for user charges — that is, they are relatively efficient "benefit-related" taxes.

In this paper, we provide what we believe is the first attempt to systematically analyze the assertion that a production-based tax can be viewed as a proxy for a benefit tax, and is thus preferable to the often-used alternative of taxing business inputs, especially capital. To be sure, there are many papers analyzing the efficiency of capital taxation when firms receive a public service and jurisdictions compete with each other for mobile capital; see, e.g., Zodrow and Mieszkowski (ZM) [33], Oates and Schwab [21], [22], Noiset [20], Sinn [24], Keen and Marchand [11], Bayindir-Upmann [2], Richter [23], Matsumoto [14], [13], and Dhillon, Wooders and Zissimos [4]. However, these papers focus on the inefficiency of capital taxation alone, and do not consider a tax on local production, or compare such a production tax to a capital tax.

Our results provide some support for the idea that a production-based tax may be viewed as a "benefit-related" tax, although it does not in general substitute perfectly for an explicit user charge. Our analysis reveals that the log modularity properties of the production function play a major role in determining the relative efficiency properties of production-based taxes and capital taxes, an issue that has thus far not been examined in models of interjurisdictional tax competition.

In our analysis, we assume that output is produced with a constant returns to scale (CRS) production function using labor, capital, and business public services as inputs. The log modularity properties of the production function with respect to capital and business public services reflect how the elasticity of output with respect to increases in business public services varies at different levels of the amount of capital employed in production. Specifically, if the production function is log modular (log submodular, log supermodular), then the elasticity of output with respect to business public services is constant (decreases, increases) as the capital employed in production increases. Note that all three possibilities are consistent with a CRS production function; in particular, a linearly homogenous Cobb-Douglas production function is log modular, while a constant elasticity of substitution (CES) production function is log submodular (log supermodular) if the elasticity of substitution is greater than (less than) one.

In the special case of a production function that is log modular in capital and the public service, a production tax is effectively a benefit tax, and is thus analogous to a user charge for public services that ensures an efficient level of public service provision. In the same vein, if the production function is log submodular in capital and the public service (e.g., a CES production function with an elasticity of substitution greater than one), a production tax will be inefficient and lead to underprovision of the public service, but it will result in less underprovision and thus be less inefficient than a capital tax. However, although we can show that the production tax leads to overprovision of public services in the case of a log supermodular production function (e.g., a CES production function with an elasticity of substitution smaller than 1), the ambiguity of the effect of a capital tax on public services in this case implies that a ranking of the relative efficiency properties of the two taxes is impossible without further restrictions on the production technology. In a companion paper (Gugl and Zodrow [7]), we provide a wide variety of simulations using the CES production function and find that efficiency gains in the instances in which capital taxes are more efficient are modest.

There is also a vast literature analyzing the efficiency of destination-based vs. origin-based value-added taxes (VATs). In a closed economy, there is no difference between a uniform tax on all consumption and a similar tax on production. In contrast, as economies become increasingly more open, the distinction between a tax on local production and a tax on local consumption becomes important in terms of efficiency (Mintz and Tulkens [19], Kanbur and Keen [10], Lockwood [12], Haufler and Pflueger [8]). In tax competition models in which firms are perfectly competitive, a tax on local consumption such as a destination-based VAT is efficient when countries are too small to affect world prices (Lockwood [12]). Haufler and Pflueger [8] investigate the difference between a destination-based and an origin-based VAT in several settings of international duopoly and find that only the former is efficient when competition between countries is imperfect. In contrast, McLure [18] notes that the growing importance of electronic commerce implies that a destination-based VAT is increasingly difficult to administer, providing an argument in favor of an origin-based VAT.

Despite the vast literature on different forms of VATs, an origin-based VAT has not been analyzed in a model of tax competition in which public services are provided to firms. To the best of our knowledge, this paper, along with Gugl and Zodrow [7], is the first to examine this issue.

In the next two sections, we present the model and discuss our results on the production tax. We then contrast these with the well known results on capital taxation in section 4. Section 5 compares the efficiency properties of a capital tax and a production tax. Section 6 concludes.

2. The Model

Our model follows the ZM [33] framework.[1][2] A federation or union consists of N jurisdictions, each with the same number of residents who are immobile across jurisdictions. All residents have identical preferences and endowments. Individuals work where they live, provide a fixed amount of labor, and obtain utility from consumption of an aggregate composite good, X. The labor supply of each jurisdiction, L, is therefore fixed.[3] People own an equal share of the union's capital stock $\bar{K}$ which is fixed in total supply at $N\bar{K}$.

Labor (L) and capital (K) are the private inputs in the production of X. In addition, the local government provides a fully congestible business public service B that is used directly in the production of the consumption good. Each jurisdiction produces X with a technology

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1 Gugl and Zodrow [7] provide further discussion of the assumptions in the model.

2 See also Wilson [28]. For reviews of the tax competition literature, see Wilson [29], Wilson and Wildasin [30], and Zodrow [31], [32].

3 The fixed factor can also be thought of as a combination of labor and land, as assumed in Zodrow and Mieszkowski (ZM) [33].

characterized by constant returns to scale (CRS) in the two private inputs and one public input. We assume throughout the paper that the production function is strictly concave in capital and the public service. The consumption good is assumed to be tradable and is taken as the numeraire. We assume that the number of firms is fixed in each jurisdiction (or equivalently, given CRS, that there is a single representative firm) and focus therefore on the aggregate production function in each jurisdiction, given by $F(B_i, K_i, L)$, where $F_B, F_K, F_L > 0, F_{BB}, F_{KK}, F_{LL} < 0, F_{BK}, F_{BL}, F_{KL} > 0$.[4] Note that our assumptions thus far imply

<u>Property 1</u>: *A proportional increase in B and K causes F_K and F_B, respectively to decrease, i.e. $F_{KK}K + F_{KB}B < F_K$ and $F_{BK}K + F_{BB}B < F_B$.*

The government can costlessly transform the consumption good into the public service, so the unit cost of *B* is also equal to one.[5] For the economy as a whole, it must therefore be true that

$$\sum_{i=1}^{N} F(B_i, K_i, L) = X + B. \qquad (1)$$

Before we analyze the decisions of the governments of the local jurisdictions, we derive the efficient amount of B in this economy and state the conditions that lead to efficient local production of X in each jurisdiction.

2.1. Efficiency

4 See, e.g., Zodrow and Mieszkowski [32], Bayindir-Upmann [2], Keen and Marchand [11], Dhillon et al. [4] for the same assumptions.

5 We follow most of the literature in assuming constant marginal costs for the public service (Oates and Schwab [21], [22], Sinn [24], Bayindir-Upmann [2], Keen and Marc- hand [11], Richter [23], and Matsumoto [14]). Two alternative approaches, which Matsomoto [14] points out are equivalent, would be to assume either an imperfectly congestible public input and a constant marginal cost of producing that public input, or a perfectly congestible public input (i.e., our publicly provided private service) and decreasing marginal costs of producing the public service.

To create a benchmark, we begin by calculating the efficient amount of business public services B if all inputs can move freely across jurisdictions.Given CRS in the production of X, the aggregate production function is

$$F\left(B,N\overline{K},NL\right)=NF(B/N,\overline{K},L) \qquad (2)$$

This function implies that production is independent of where labor is located. Although we assume labor is fixed and identically distributed across jurisdictions, for now we neglect this constraint to determine the allocation of labor, capital, and business public services that is efficient without jurisdiction-specific constraints. Since the total amounts of capital and labor are fixed in the whole economy, and consumers care only about the consumption good X, the sole choice variable in determining efficiency is how much of the business public service should be produced to maximize X. That is,

$$\max_B F\left(B,N\overline{K},NL\right)-B$$

The first order condition is

$$F_B-1=0 \qquad (3)$$

which simply indicates that the efficient aggregate amount of business public services, which we denote as $B*$, occurs where the marginal product of such services equals the marginal rate of transformation of 1.

Lemma 1: *Given identical local jurisdictions with fixed labor supply L, the efficient level of the business public service in a jurisdiction is* $B*/N$ *and the optimal amount of capital is* $\overline{K}$.

Proof: Condition (3) states that it is efficient to divert units from the consumption good into business public services until the marginal product of the business public service equals the marginal rate of transformation of one. Given CRS, we achieve the same efficient allocation by distributing $B*$ and capital to each jurisdiction in equal shares and then have them produce $F\left(B*/N,\overline{K},L\right)$.

In analyzing the impact of financing business public services either with a capital tax or a tax on production, we compare the marginal productivity of the public service under either of the two taxes with the efficient marginal productivity of one. If the marginal productivity of the public service is smaller (larger) than one, there is over (under) provision because, in equilibrium, the amount of public service that each jurisdiction provides is larger (smaller) than the efficient amount. We analyze next the financing of business public services with a production tax.

3. Production Tax

Assume that local jurisdictions tax local production to finance the provision of the public service to firms. The amount of capital a local jurisdiction might attract depends on the public service level and the production tax rate denoted by t_i, hence K_i is a function of the policy mix B_i, t_i. We derive below the perceived change in capital in a jurisdiction as it increases its tax on production to finance an increase in its level of the business public service. For now we simply note that capital is perceived to change with changes in the policy mix.

The local government's budget constraint is given by

$$B_i = t_i F\left(B_i, K_i\left(B_i, t_i\right), L\right) + H \ , \qquad (4)$$

where H is a fixed amount of the public service that is financed externally. We will focus on the jurisdiction's problem when $H = 0$, but it is useful to consider the case where $H > 0$ when interpreting the Lagrange multiplier λ below.

Firms maximize profits, given factor prices, the tax on production, and the level of local business public services provided. They demand both labor and capital, but since labor is fixed in each jurisdiction, the local labor market will clear at L. Since capital is perfectly mobile across jurisdictions, the after-tax rate of return to capital, r, is the same in every jurisdiction.

We assume the local government acts to maximize the welfare of its representative resident, who owns all of the fixed input, and that any profits earned from production due to the provision of business public services will accrue to that resident as well. Thus the profits in any given jurisdiction enter the local government's social welfare function together with locally earned wages and the capital income $r\bar{K}$ earned by its residents. To simplify the notation below, we suppress the subscript i in analyzing a single local government's optimization problem, which is to choose the production tax rate and the level of business public services to

$$\max_{t,B}\left[(1-t)F\left(B,K\left(B,t\right),L\right)-rK\left(B,t\right)\right]+r\bar{K}$$
$$s.t.\quad B=tF\left(B,K\left(B,t\right),L\right)+H\ ,$$

where the term in square brackets is the sum of wage income and profits generated in the jurisdiction.

The Lagrange function for the government's optimization problem is

$$\max_{t,B}\left[(1-t)F\left(B,K\left(B,t\right),L\right)-rK\left(B,t\right)\right]+r\bar{K}+\lambda\left[H+tF\left(B,K\left(B,t\right),L\right)-B\right]$$

and the first order conditions with respect to t and B are

$$-F\left(B,K\left(B,t\right),L\right)+\lambda\left(F\left(B,K\left(B,t\right),L\right)+tF_K\frac{dK}{dt}\right)=0 \qquad (5)$$

$$\left(1-t\right)F_B+\lambda\left(tF_B+tF_K\frac{dK}{dB}-1\right)=0 \qquad (6)$$

Note that at the optimal production tax and business public service level, an increase in H and hence less reliance on self-financing of the business public service leads to an increase in residents' income/consumption equal to λ.

For the moment, consider the hypothetical case in which capital is fixed in a jurisdiction. In this case dK/dt and dK/dB are zero and the first order conditions become

$$-F\left(B,K\left(B,t\right),L\right)+\lambda\left(F\left(B,K\left(B,t\right),L\right)\right)=0$$
$$\left(1-t\right)F_B+\lambda\left(tF_B-1\right)=0$$

This implies that the local jurisdiction provides business public services such that $F_B = 1$. It also implies that $\lambda = 1$ — that is, if local jurisdictions faced a "soft" budget constraint and could be given one unit of public service for free, then, given that the Lagrange multiplier is 1, residents' income would go up by 1. We summarize our result in Proposition 1.

Proposition 1: *If local jurisdictions believe that capital does not react to changes in the production tax and in the level of business public services, they provide the efficient level of public services.*

By comparison, if capital reacts to changes in the public service level and the production tax, then dK/dt and dK/dB are derived as follows.

The jurisdiction takes as given the firm's profit maximization condition

$$(1-t)F_K\left(B,K,L\right)=r\ . \qquad (7)$$

Differentiating (7) with respect to the production tax t and capital yields

$$\left(1-t\right)F_{KK}dK - F_K dt = 0\ ,$$

and solving for dK/dt yields

$$\frac{dK}{dt} = \frac{F_K}{(1-t)F_{KK}} < 0\ . \qquad (8)$$

Differentiating (7) with respect to public service and capital yields

$$F_{KB}dB + F_{KK}dK = 0\ ,$$

and solving for dK/dB yields

$$\frac{dK}{dB} = \frac{-F_{KB}}{F_{KK}} > 0 \qquad (9)$$

Thus, from the local government's perspective, an increase in the production tax without an increase in public services would drive out capital, and an increase in the public service without an increase in the production tax would attract capital to the jurisdiction. These two perceived distortions created by mobile capital need to be considered in setting the optimal production tax rate and the associated level of business public services.Note that the two distortions are perceived in the sense that, in equilibrium, each jurisdiction follows the same policies and thus adopts the same production tax rate, so that each jurisdiction receives and equal share of the total (mobile) national capital stock..

Substituting from (8) and (9) into the first order conditions (5-6) yields

$$-F\left(B,K\left(B,t\right),L\right)+\lambda\left(F\left(B,K\left(B,t\right),L\right)+tF_K\frac{F_K}{\left(1-t\right)F_{KK}}\right)=0$$

$$\left(1-t\right)F_B+\lambda\left(tF_B+tF_K\frac{-F_{KB}}{F_{KK}}-1\right)=0$$

It is clear from the first equation that if the local government were to receive a free unit of the public service, this would no longer increase residents' income by one but by more than one, as

$$\lambda=\frac{F\left(B,K\left(B,t\right),L\right)}{F\left(B,K\left(B,t\right),L\right)+tF_K\frac{F_K}{\left(1-t\right)F_{KK}}}>1 \qquad (10)$$

Intuitively, λ is the increase in residents' income if the self-financing constraint is relaxed by one unit at the optimal tax rate and public service level. So if the local government is given one unit of externally-financed business public service, this marginal benefit should both lower the production tax rate and increase the public service level — analogous to the case where an increase in a person's fixed income by $1 leads to the same increase in marginal utility per dollar spent on each good. Hence (10) measures the percentage of the increase in residents' income in terms of the decrease in marginal revenue due to a drop in t.

From the second equation, the Lagrange multiplier must also equal

$$\lambda=\frac{\left(1-t\right)F_B}{1-tF_B+tF_K\frac{F_{KB}}{F_{KK}}} \qquad (11)$$

Given that for an interior solution $\lambda>1$,

$$\left(1-t\right)F_B>1-tF_B+tF_K\frac{F_{KB}}{F_{KK}}$$

which implies that the increase in residents' income from increasing the public service exceeds the local government's cost of providing it, once the additional revenues from increasing productivity are taken into account. This net marginal cost is given by

$$1 - tF_B + tF_K \frac{F_{KB}}{F_{KK}} > 0 .$$

At the optimal amount of the public service, the marginal revenue due to an increase in the public service is smaller than its marginal resource cost of one. This is expected in the context of an interior solution because the production tax must be increased to finance the additional unit of the public service; the additional unit never "pays for itself" by increasing output enough to generate the required revenues. Indeed, Property 1, CRS in the production of X, and the self-financing constraint together guarantee that this inequality holds. To see this, note that multiplying the inequality by $FKF_{KK} < 0$ yields

$$FK(tF_B F_{KK} - tF_K F_{KB} - F_{KK}) > 0$$
$$F_{KK}K\left(BF_B - F\right) - BF_{KB}F_K K > 0$$

Both terms in the difference are positive, but the first term is larger than the second because by CRS it can be written as $-F_{KK}K(KF_K + LF_L)$ and by Property 1 $-F_{KK}K > F_{KB}B$.

To sum up, depending on which first order condition we consider, a free unit of the public service should lead to a combination of a production tax decrease and a public service increase, where the percentage increase in marginal benefits is identical for the two policy instruments.

Setting the right hand sides of (10) and (11) equal yields

$$t = \frac{F_{KK}F\left(1 - F_B\right)}{F_K\left(F_K F_B - FF_{KB}\right)}$$

Note that by Young's Theorem, $F_{KB} = F_{BK}$. This allows us to express the optimal production tax rate using the elasticities $\varepsilon_{F_K,K} = \frac{-F_{KK}K}{F_K}$, $\varepsilon_{F_B,K} = F_{BK}K / F_B$ and $\varepsilon_K = F_K K / F$.

$$t = \frac{\left(1 - F_B\right)\varepsilon_{F_K,K}}{F_B\left(\varepsilon_{F_B,K} - \varepsilon_K\right)} \qquad (12)$$

where $\varepsilon_{F_K,K}$ is the elasticity of the marginal productivity of capital with respect to an increase in capital, $\varepsilon_{F_B,K}$ is the elasticity of the marginal productivity of the public service with respect to an increase in capital, and ε_K is the elasticity of production of the consumption good with respect to capital. Whether the production tax rate leads to over or underprovision therefore depends on the difference between $\varepsilon_{F_B,K}$ and ε_K. The numerator of (12) is positive if $F_B < 1$, i.e., if there is overprovision of public services, an interior solution for the production tax rate must be characterized by a positive difference between $\varepsilon_{F_B,K}$ and ε_K. We summarize our finding below.

Proposition 2a: *With a positive production tax rate, underprovision (overprovision) of business public services occurs if at* $F_B > (<)\, 1$, $\varepsilon_{F_B,K} < (>)\, \varepsilon_K$.

In the case in which production functions are log sub or log supermodular, $\varepsilon_{F_B,K}$ and ε_K exhibit the same relationship to each other, regardless of the amounts of B, K, and L.

Lemma 1: *A production function is log sub (super) modular if and only if* $\varepsilon_{F_B,K} < (>) \varepsilon_K$.

The proof of Lemma 1 is provided in the appendix, which includes a detailed discussion of log sub (super) modularity.

For a CES production function given by $F(B,K,L) = \left[aB^{\alpha} + cK^{\alpha} + (1-a-c)L^{\alpha}\right]^{1/\alpha}$ where $\alpha \in (-\infty,1)$, $0 < a,c < 1$, $a + c < 1$ the two elasticities are given by

$$\varepsilon_{F_B,K} = \frac{c(1-\alpha)K^{\alpha}}{aB^{\alpha} + cK^{\alpha} + (1-a-c)L^{\alpha}}$$

$$\varepsilon_K = \frac{cK^{\alpha}}{aB^{\alpha} + cK^{\alpha} + (1-a-c)L^{\alpha}}$$

Hence for $\alpha \in (0,1)$ we have log submodularity, i.e. $\varepsilon_{F_B,K} < \varepsilon_K$ and for $\alpha < 0$ we have log supermodularity, i.e. $\varepsilon_{F_B,K} > \varepsilon_K$.

The next result follows immediately from Lemma 1 and Proposition 2a.

Corollary 1: *A production tax is inefficient whenever the production function is log sub- or log supermodular in capital and the public service. Log submodularity (supermodularity) leads to underprovision (overprovision) of business public services.*

Note that (12) cannot be used to assess the interior solution if the difference between $\varepsilon_{F_B,K}$ and ε_K is zero. Also note that a log modular production function would generate a product of zero in the numerator. Instead, we take a different approach to assess this case and ask what would happen when a *budget-balanced increase* in the tax rate and public services is considered, in which case both budget balance and the firm's profit maximization condition must hold. Totally differentiating both equations with respect to the production tax, capital, and the public service, yields

$$dB = tF_B dB + Fdt + tF_K dK$$
$$(1-t)\left(F_{KB}dB + F_{KK}dK\right) - F_K dt = 0$$

Solving for dK/dt

$$\frac{dK}{dt} = \frac{\left(1-tF_B\right)F_K - (1-t)F_{KB}F}{(1-t)\left(F_{KB}tF_K + \left(1-tF_B\right)F_{KK}\right)} \ .$$

Suppose jurisdictions choose the efficient amount of business public services $(F_B = 1)$. In this case dK/dt becomes

$$\frac{dK}{dt} = \frac{F_K - F_{KB}F}{\left(F_{KB}tF_K + (1-t)F_{KK}\right)} \ .$$

Jurisdictions will set the business public service level efficiently only if they are assured that no capital flows in or out at the tax rate that leads to $F_B = 1$. Also note that in the case where $F_B = 1$,

$$\frac{dK}{dt} = \frac{F_B F_K - F_{KB} F}{\left(F_{KB} t F_K + (1-t) F_{KK}\right)} .$$

Thus the numerator is equal to zero if at $F_B = 1$, $F_B F_K = F_{KB} F$, or equivalently if $\varepsilon_{F_B, K} = \varepsilon_K$. We summarize our result in the next proposition.

Proposition 2b): *With a positive production tax rate, efficient provision of business public services occurs if at* $F_B = 1$, $\varepsilon_{F_B, K} = \varepsilon_K$.

Lemma 2: *The equality* $\varepsilon_{F_B, K} = \varepsilon_K$ *is satisfied always when the production function is log modular in* B *and* K.

Let $F(B,K,L) = B^{\alpha} K^{\beta} L^{1-\alpha-\beta}$ where $0 < \alpha, \beta < 1$ and $\alpha + \beta < 1$. The Cobb-Douglas function is log modular, since $\varepsilon_{F_B, K} = \beta = \varepsilon_K$.

Corollary 2: *If a production function is log modular, a production tax is efficient.*

The intuition underlying these results is as follows. The government of the jurisdiction considering an increase in the production tax balances two factors. First, an increase in the tax will reduce profits, and thus tend to drive capital out of the jurisdiction. Second, the increase in public services financed with the tax will increase the productivity of capital (generating more output at any given level of capital), and thus attract capital to the jurisdiction, which will in turn increase the productivity of local business public services The case of log modularity corresponds to a production function for which these two effects are exactly offsetting. In this case, the increase in the capital stock attracted by the increase in public services generates an "average" increase in the productivity of public services, that is, one equal to the increase in output due to the same increase in capital. By comparison, in the case of a log supermodular production function, the production tax increase has an additional benefit: the production function is such that, as the capital stock increases in response to the provision of additional public services, the productivity of public services increases more than proportionately. This additional benefit implies that the total marginal benefit to capital of the additional tax financed public services outweighs the cost of paying the additional tax, and the production tax attracts capital to the jurisdiction. This ability to attract capital is sufficient to cause the government of the jurisdiction to overprovide business public services in order to attract additional capital (it does not take into account the cost to other jurisdictions that are losing capital as a result of their policy). These results are reversed in the case of a log submodular production function, as the productivity of public services increases less than proportionately (relative to the increase in output) as additional capital is attracted to the jurisdiction with tax financed public services, in which case the tax effect outweighs the productivity effect, and capital is driven out of the jurisdiction.

4. Capital Tax

Many authors have analyzed capital taxes using the original ZM framework.[6] Our analysis of capital taxation follows the approach used above to analyze the production tax to facilitate comparisons of the effects of the capital tax to those of the production tax.

Suppose that local governments impose a tax on capital at rate τ to finance business public services. Analogous to the case studied in the previous section, the jurisdiction solves the problem

$$\max_{\tau,B}\left[F\left(B,K\left(B,\tau\right),L\right)-(r+\tau)K\left(B,\tau\right)\right]+r\bar{K}+\mu\left[H+\tau K\left(B,\tau\right)-B\right]$$

where the first term in square brackets is the sum of wage income and profits generated in the jurisdiction and the second term in square brackets captures the jurisdiction's budget constraint. The first order conditions are

[6] For a detailed literature review see Gugl and Zodrow [7].

$$-K+\mu\left(K+\tau\frac{dK}{d\tau}\right)=0 \qquad (13)$$

$$F_B+\mu\left(\tau\frac{dK}{dB}-1\right)=0 \qquad (14)$$

As in the previous section, if the capital stock is unchanged in the jurisdiction, $F_B=1$. The jurisdiction takes the firm's profit maximization condition as given when deriving its perceived change in capital due to changes in the capital tax rate and the public service level. The firm's condition for profit maximization under the capital tax is

$$F_K\left(B,K,L\right)=r+\tau \qquad (15)$$

Differentiating (15) with respect to the capital tax τ and capital yields

$$F_{KK}dK-d\tau=0$$

and solving for $dK/d\tau$ yields

$$\frac{dK}{d\tau}=\frac{1}{F_{KK}}<0. \qquad (16)$$

The reaction of capital to an increase in the level of public services is the same under both taxes and is given by (9).

With these effects in mind, substituting (16) and (9) into the first order conditions (13-14) yields

$$-K+\mu\left(K+\frac{\tau}{F_{KK}}\right)=0$$

$$F_B+\mu\left(\tau\frac{-F_{KB}}{F_{KK}}-1\right)=0$$

It is clear from the first equation that if the local government were to receive a free unit of the public service, this would no longer increase residents' income by 1 but more than one, as

$$\mu=\frac{K}{K+\frac{\tau}{F_{KK}}}>1 \qquad (17)$$

Note that (17) is positive if and only if $-KF_{KK} > \tau$.

From the first order condition with respect to B,

$$\mu = \frac{F_B}{1+\frac{\tau F_{KB}}{F_{KK}}} > 1 \qquad (18)$$

Note that (18) is positive if and only if $F_{KK} + F_{KB}\tau < 0$. This condition is met in our model because it follows from Property 1 and the budget constraint.

Equating (17) and (18),

$$\tau = \frac{-F_{KK}K\left(1-F_B\right)}{F_{KB}K - F_B}.$$

We express the optimal capital tax in terms of the elasticity of the marginal productivity of the public service with respect to an increase in capital, $\varepsilon_{F_B,K} = F_{BK}K / F_B$, and the elasticity of the marginal productivity of the capital with respect to an increase in capital, $\varepsilon_{F_K,K} = -F_{KK}K / F_K$

$$\tau = \frac{\left(1-F_B\right)\varepsilon_{F_K,K}F_K}{F_B\left(\varepsilon_{F_B,K}-1\right)} \qquad (19)$$

<u>Proposition 3a</u>: With a positive capital tax, underprovision (overprovision) of business public services occurs if $F_B > (<) 1$, $\varepsilon_{F_B,K} < (>) 1$.

Proposition 3a states that if we deal with production functions that exhibit $\varepsilon_{F_B,K} < 1$ always, under a capital tax we will have underprovision of public services. Such is the case for a Cobb-Douglas production function; specifically, for $F\left(B,K,L\right) = B^{\alpha}K^{\beta}L^{1-\alpha-\beta}$ where $0 < \alpha, \beta < 1$ and $\alpha + \beta < 1$, then $\varepsilon_{F_B,K} = \beta < 1$. Recall that for the CES production function

$$\varepsilon_{F_B,K} = \frac{c(1-\alpha)K^{\alpha}}{\left[aB^{\alpha} + cK^{\alpha} + \left(1-a-c\right)L^{\alpha}\right]}.$$

This implies that for $\alpha \in [0,1)$, that is, for a CES production function with a substitution elasticity greater than one, we can rule out that $\varepsilon_{F_B,K} = 1$. Moreover, we know that for $\alpha \in [0,1)$ $\varepsilon_{F_B,K} < 1$, and hence we must have underprovision. For $\alpha < 0$, however, it is possible that for a particular combination of a, c, α, B, K and L, $\varepsilon_{F_B,K} > 1$, $\varepsilon_{F_B,K} < 1$ or even $\varepsilon_{F_B,K} = 1$.

In order to examine the case of efficient provision, consider again the perceived outflow of capital after taking into account budget balance

$$\frac{dK}{d\tau} = \frac{1 - F_{KB}K}{F_{KK} + \tau F_{KB}} .$$

Suppose that $\varepsilon_{F_B,K} = 1$, which implies that $F_B = F_{BK}K$. At $F_B = 1$ with $\varepsilon_{F_B,K} = 1$ we must have $F_{BK}K = 1$ and hence $dK / d\tau = 0$.

Proposition 3b: With a positive capital tax, efficient provision of business public services occurs if and only if at $F_B = 1$, $\varepsilon_{K,F_B} = 1$.

The analysis of the CES production function above demonstrates that for $\alpha < 0$, it is possible that at a particular combination of a, c, α, B, K and L, $\varepsilon_{F_B,K} = 1$. Thus in the case of a CES production function with a substitution elasticity smaller than one it is possible that the parameters are such that the capital tax is efficient.

Propositions 3a and 3b relate to the existing literature on the efficient provision of business public services with a capital tax. Matsumoto [13] notes (p. 471): “If the number of firms is constant in each jurisdiction (normalized to one), […] $[F_B]$ may be below one because the sign of $[F_{KB}K - F_B]$ is indeterminate under linear homogeneity with respect to all inputs. This argument corresponds to the Noiset [20] result of potential overprovision in the [ZM] model where the variability of the number of firms is not explicitly considered.” Dhillon et al. [4] construct a model with two production inputs (capital and public services only), and assume $F_{KK} + F_{KB}\tau < 0$ as do ZM. They develop alternative conditions that guarantee existence of a unique interior solution for the optimal capital tax, and then show that overprovision, efficient provision or underprovision of the public service can occur depending on whether the production function is only locally or globally strictly concave in capital and the public service.

5. Comparison between the Production Tax and Capital Tax

We summarize below the various equilibrium outcomes under the the two tax regimes.

	efficient provision $F_B = 1$	**underprovision** $F_B > 1$	**overprovision** $F_B < 1$
production tax	$\varepsilon_{K,F_B} = \varepsilon_K$	$\varepsilon_{K,F_B} < \varepsilon_K$	$\varepsilon_{K,F_B} > \varepsilon_K$
capital tax	$\varepsilon_{K,F_B} = 1$	$\varepsilon_{K,F_B} < 1$	$\varepsilon_{K,F_B} > 1$

5.1 Class of Log (Sub/Super) Modular Production Functions and Performance under the Production and Capital Taxes

Corollaries 1 and 2 identify the properties of the production function that guarantee either efficient or over (under)provision under the production tax. We thus consider log modular (for which $\varepsilon_{F_B,K} = \varepsilon_K$ always holds) and log submodular (for which $\varepsilon_{F_B,K} < \varepsilon_K$ always holds) production functions below and investigate the efficiency properties of the capital tax under the same technologies. In order to do this it is important to note how these properties relate to the question of whether $\varepsilon_{F_B,K} < 1$.

<u>Proposition 4</u>: *If $F(B,K,L)$ is log submodular or log modular and strictly concave in (B,K), then $\varepsilon_{F_B,K} < 1$.*

Proof: For a strictly concave function in K, $F_K K < F$ and hence $\varepsilon_K < 1$. By Lemmas 1 and 2, a log (sub)modular production function in (B,K) exhibits $\varepsilon_{F_B,K} = (<)\varepsilon_K$. Together these two properties imply the desired result. Note that the converse of Proposition 4 is not true; even if $\varepsilon_K < 1$ holds, no restriction is imposed on the log modularity property of $F(B,K,L)$ in (B,K).

Given Proposition 4, it is immediately clear that

<u>Proposition 5</u>: *If $F(B,K,L)$ is log modular, then the production tax leads to efficient provision while the capital tax leads to underprovision of business public services and hence the production tax is more efficient than the capital tax.*

In the case of log submodular production functions we get underprovision under both tax regimes. Yet, it is possible to compare the two equilibria.

Proposition 6: *Suppose that the production function is log submodular in* (B,K) *and that interior solutions to both the optimal capital tax and the optimal production tax problems exist. Under these conditions, the production tax is unambiguously more efficient than the capital tax.*

Proof: Suppose that each jurisdiction choses the capital tax optimally and the economy is in equilibrium. Then, $\tau K = B^\tau$. Total output is the same in all jurisdictions (given symmetry and a fixed national aggregate capital stock) and thus all derivatives are also identical. If (19), the optimality condition for the capital tax, holds, the optimality condition for the production tax does not hold simultaneously. That is, the level of the production tax necessary to yield the public service level under the capital tax is lower than the production tax equivalent under the capital tax equilibrium. So at the same level of B, we would need a lower level of the production tax. This means we can increase B by using the optimal production tax. Since both tax equilibria lead to underprovision, the production tax must result in less under provision than the capital tax.

$$t^\tau F\left(B^\tau, K, L\right) = B^\tau$$

$$t^\tau = \frac{B^\tau}{F\left(B^\tau, K, L\right)}$$

Recall that the optimal production tax for given B is

$$t = \frac{\left(1 - F_B\right)\varepsilon_{F_K,K}}{F_B\left(\varepsilon_{F_B,K} - \varepsilon_K\right)} \qquad (12)$$

So if $t^\tau > t$, then we know that the capital tax leads to more underprovision than the production tax and hence is less efficient.

$$t(B^\tau) = \frac{\left(1 - F^\tau{}_B\right)\varepsilon^\tau{}_{F_K,K}}{F^\tau{}_B\left(\varepsilon^\tau{}_{F_B,K} - \varepsilon^\tau{}_K\right)}, \qquad t^\tau = \frac{\left(1 - F^\tau{}_B\right)\varepsilon^\tau{}_{F_K,K} F^\tau{}_K K}{F^\tau{}_B\left(\varepsilon^\tau{}_{F_B,K} - 1\right)F^\tau}$$

$$t^\tau > t(B^\tau) \Leftrightarrow -F^\tau{}_K\left(\varepsilon^\tau{}_{K,F_B} - \varepsilon^\tau{}_K\right) < -\left(\varepsilon^\tau{}_{K,F_B} - 1\right)\frac{F^\tau}{K}$$

The last inequality holds because both sides are positive and the right hand side yields a bigger value than the left hand side due to strict concavity of $F(B,K,L)$ in K.[7]

Our results thus far validate the conjecture that in the case of log submodular production functions, a broad-based tax on local production is more efficient than a tax on capital. However, there is no analog to Proposition 6 in the case of log supermodular production functions — although there are prominent examples of such functions. As noted previously, the family of CES production functions includes examples of all three log modularity properties, depending on whether the elasticity of substitution between capital and public services/labor is greater than, equal to, or smaller than one. In Gugl and Zodrow [7], we examine more closely this class of production functions in the context of tax competition in the provision of business public services and the relative efficiency of capital taxation and production taxation.

The basic intuition underlying the result that the production tax is often more efficient than the capital tax is that the capital tax taxes a narrower tax base. If there is a very strong complementarity between the capital and the public service (as assumed in Sinn [24]), then the capital tax is more likely to approximate a benefit tax than in the case were capital and public service allow for a higher degree of substitutability. However, if there is room for significant substitution between capital and the public service, a broader tax base that includes the

[7] Note that this proof is valid even if there are multiple candidates for the optimal tax rates. The provision rule under each tax regime — equations (12) and (19), respectively — indicates how a jurisdiction should choose the optimal tax rate, given K and B. In the case in which the rule does not provide a unique tax rate, jurisdictions will choose the tax rate from all the candidates provided by the provision rule that leads to the highest residents' income, given K. Residents' income as a function of the public service level, which is calculated from the tax rate and, given K, is well-behaved and strictly concave in the public service level with its maximum at the efficient provision level of the public service.

To compare the levels of underprovision under a capital tax and a production tax when the production function is log submodular, we assume the jurisdiction has adopted the optimal capital tax and the corresponding level of public service, and then examine whether the jurisdiction would find it desirable to increase the level of public service provision using a production tax. We find that this is the case but yet we could be moving toward a suboptimal production tax rate if (12) does not yield a unique solution. However, given the assumption that we have an interior solution under both tax regimes, the optimal production tax must be larger than any suboptimal production tax rate, so that switching to a production tax equilibrium increases the level of public service provided and results in less underprovision and thus less inefficiency than under the capital tax equilibrium.

contribution of the public service to output mimics more closely user charges on the public service.[8]

6. Conclusion

Can a production tax, such as an origin-based value-added tax, approximate a benefit tax for public services provided to businesses, as suggested by Bird [1] and Hines [9]. And how does a source-based capital tax such as the property tax compare to a production tax as a proxy for a benefit tax? Using the Zodrow-Mieszkowski [33] model of interjurisdictional tax competition, we find that a production tax more closely approximates a benefit tax than does a capital tax in many instances. In particular, although a production tax is efficient only when the production function is log modular in the public service and capital, it is *less* inefficient than a capital tax in the case of log submodular production technologies.

Our results have some interesting implications for potential reforms of state/provincial tax systems. Our analysis suggests that a production tax may be a viable business tax alternative to the provincial retail sales tax, which is typically characterized by the taxation of business inputs similar to that which occurs under our capital tax. By comparison, at least under certain circumstances in our admittedly highly stylized model, a production tax is less distortionary than the capital tax portion of a retail sales tax.

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Appendix: Log (Sub/Super) Modularity

Before discussing log (sub/super) modularity, we first review (sub/super) modularity and then point out the differences between these two concepts.

To define (sub/super) modularity of B and K, consider any two input vectors with the same amount of labor but different amounts of B and K, with one input vector containing strictly less of B and K than the other. Denote the smaller amounts of inputs by a lower bar and the larger amounts by an upper bar. By definition, modularity holds if and only if

$$F\left(\underline{B},\underline{K}\right)+F\left(\overline{B},\overline{K}\right)=F\left(\underline{B},\overline{K}\right)+F\left(\overline{B},\underline{K}\right).....(M1)$$

For modularity, more of one input contributes to an increase of output in the same way regardless of how much of the output has already been produced. Another way of thinking about the implications of modularity is by choosing the input combinations on the RHS of equation (M1) so that

$F\left(\underline{B},\overline{K}\right)=F\left(\overline{B},\underline{K}\right)=Y_0$ (see Figure 1).

In this case, we can think of modularity as comparing the arithmetic mean of $F\left(\underline{B},\underline{K}\right)$ and $F\left(\overline{B},\overline{K}\right)$ with the output associated with an isoquant that lies between the $F\left(\underline{B},\underline{K}\right)$ isoquant and the $F\left(\overline{B},\overline{K}\right)$ isoquant. With modularity, the arithmetic mean of $F\left(\underline{B},\underline{K}\right)$ and $F\left(\overline{B},\overline{K}\right)$ must equal Y_0. For super (sub) modularity, the arithmetic mean of $F\left(\underline{B},\underline{K}\right)$ and $F\left(\overline{B},\overline{K}\right)$ must be greater (smaller) than Y_0. Thus in the case of submodularity, in order to produce a total of Y_0 with the arithmetic mean of $F\left(\underline{B},\underline{K}\right)$ and another output level generated with higher amounts of both goods, output level $F\left(\overline{B},\overline{K}\right)$ is not large enough and hence larger amounts of the inputs than $\left(\overline{B},\overline{K}\right)$ are necessary to achieve an arithmetic mean of Y_0. By comparison, see Figure 1 for a graphical representation of supermodularity.

An example of a production function that is modular in B and K is any production function for which the functions built around inputs B and K enter additively, i.e. $F\left(B,K,L\right)=g(B)h(L)+i(K)j(L)+m(B)+o(K)$ where all the individual functions are increasing. A function of the form $F\left(B,K,L\right)=g(B)h(K)i(L)+j(L)$ is supermodular.

Log modularity, in contrast, is not concerned about comparing absolute marginal contributions, but relative marginal contribution. That is, log modularity in B and K holds if and only if

$F\left(\underline{B},\underline{K}\right)F\left(\overline{B},\overline{K}\right)=F\left(\underline{B},\overline{K}\right)F\left(\overline{B},\underline{K}\right).....(LM1)$

Analogous to the discussion of (sub/super) modularity, another way to think about the implications of log modularity is by choosing the input combinations on the RHS of equation (LM1) so that $F\left(\underline{B},\overline{K}\right)=F\left(\overline{B},\underline{K}\right)=Y_0$ (see Figure 1).

In this case we can think of log modularity as comparing the geometric mean of $F\left(\underline{B},\underline{K}\right)$ and $F\left(\overline{B},\overline{K}\right)$ with the output associated with an isoquant that lies between the isoquants associated with $F\left(\underline{B},\underline{K}\right)$ and $F\left(\overline{B},\overline{K}\right)$. For log modularity the geometric mean of $F\left(\underline{B},\underline{K}\right)$ and $F\left(\overline{B},\overline{K}\right)$ must equal Y_0. For log super (sub) modularity the geometric mean of $F\left(\underline{B},\underline{K}\right)$ and $F\left(\overline{B},\overline{K}\right)$ must be greater (smaller) than Y_0. For example, in the case of log submodularity, in order to reach Y_0 with the geometric mean of the output associated with $F\left(\underline{B},\underline{K}\right)$ and another output level generated with higher amounts of both goods, producing output level $F\left(\overline{B},\overline{K}\right)$ is not high enough so that larger amounts than $\left(\overline{B},\overline{K}\right)$ are necessary to achieve a geometric mean of Y_0. See Figures 1 and 2 for a graphical representation.

An example of a log modular production function in B and K is any production function for which the functions built around all inputs enter multiplicatively, e.g. $F\left(B,K,L\right)=g(B)h(K)i(L)$ where all the individual functions are increasing. Note that such functions are simultaneously supermodular in B and K. An example of a log supermodular function is $F\left(B,K,L\right)=g(B)h(K)i(L)+j(L)$ which is simultaneously super modular in B and K. An example of a log submodular function is $F\left(B,K,L\right)=g(B)+h(K)+i(L)$ which is simultaneously modular in B and K. However, log submodular functions can also be simultaneously supermodular like $F\left(B,K,L\right)=\left(h(B)+i(K)+j(L)\right)^{\alpha}$ where all functions are increasing and $\alpha>0$.

Proof of Lemma 1: Expressing Log (Sub/Super) Modularity in terms of elasticities

Note that log modularity can be expressed as

$$\frac{F\left(\underline{B},\overline{K}\right)}{F\left(\underline{B},\underline{K}\right)}=\frac{F\left(\overline{B},\overline{K}\right)}{F\left(\overline{B},\underline{K}\right)}$$

Hence a production function needs to exhibit the same percentage change in output due to an increase in K no matter at which level of B this percentage change this is evaluated. Using derivatives to capture this idea

$$\frac{\partial\varepsilon_K}{\partial B}=0$$

where $\varepsilon_K=F_K\frac{K}{F}$

Hence $\frac{\partial\varepsilon_K}{\partial B}=0$ if and only if

$$\frac{\partial\left(F_K\frac{K}{F}\right)}{\partial B}=0$$

$\Leftrightarrow$

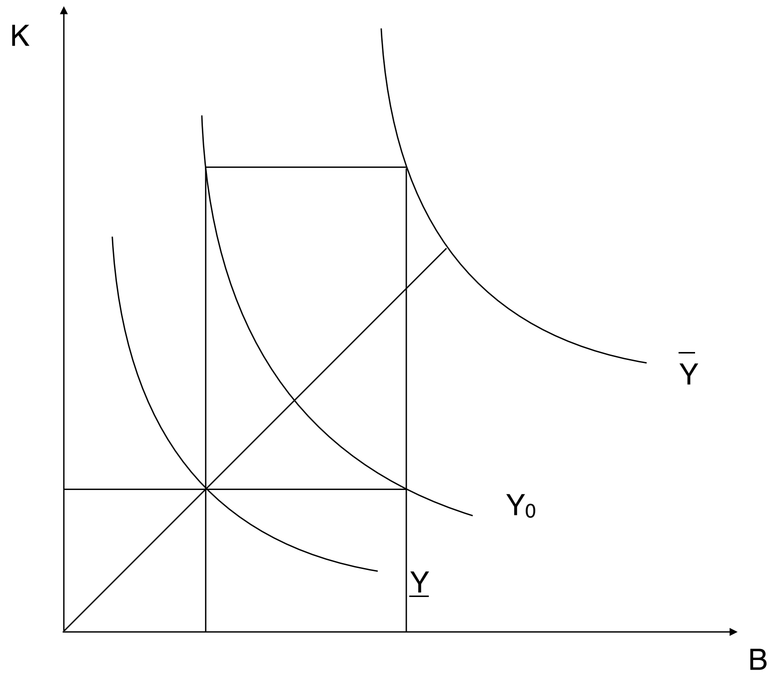
$$F_{KB}\frac{K}{F_B}-F_K\frac{K}{F}=0$$

Since $F_{KB}=F_{BK}$ by Young's Theorem,

$$\varepsilon_{F_B,K}-\varepsilon_K=0$$

Similarly, log sub modularity requires $\varepsilon_{F_B,K}<\varepsilon_K$ and log super modularity requires $\varepsilon_{F_B,K}>\varepsilon_K$.

Figure 1

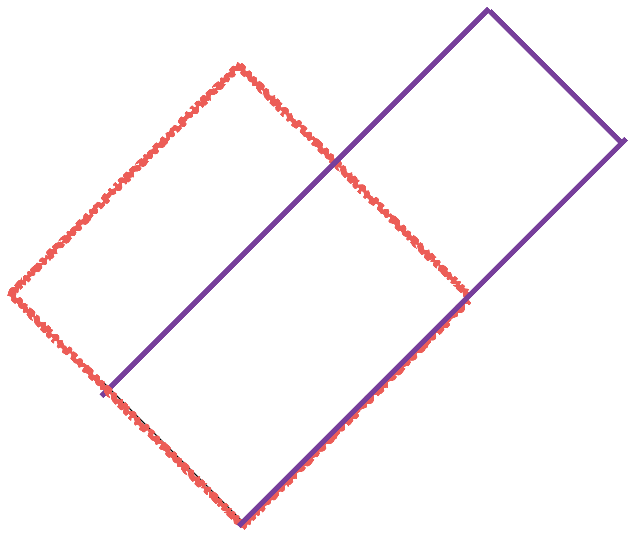


$F\left(\underline{B},\underline{K}\right)=\underline{Y},F\left(\overline{B},\underline{K}\right)=F\left(\underline{B},\overline{K}\right)=Y_0,F\left(\overline{B},\overline{K}\right)=\overline{Y}$

The figure depicts a CRS production function. Hence the distance on the 45 degree line from the origin to its intersection with an isoquant can be used to measure the output associated with the isoquant. Comparing the distances on the 45 degree line, it is clear that the arithmetic mean of $\underline{Y}$ and $\overline{Y}$ is greater than Y_0. If this relationship holds for any $\underline{B},\overline{B},\underline{K},\overline{K}$ then the production function is supermodular.

For the geometric mean, we would need to compare the areas created by $F\left(\underline{B},\overline{K}\right)F\left(\overline{B},\underline{K}\right)$ and $F\left(\underline{B},\underline{K}\right)F\left(\overline{B},\overline{K}\right)$. Theses areas measured in units on the 45 axis correspond to the areas shown in Figure 2 below. The area of the square $F\left(\underline{B},\overline{K}\right)F\left(\overline{B},\underline{K}\right)$ and the area of the rectangle is $F\left(\underline{B},\underline{K}\right)F\left(\overline{B},\overline{K}\right)$. The former is larger than the latter. If this relationship holds for any $\underline{B},\overline{B},\underline{K},\overline{K}$ then the production function is log sub modular.

Figure 2



Note that for CRS production functions, as he degree of complementarity is varied, everything else being the same, log supermodularity becomes more likely. This relationship can be seen for the CES production function as a lower substitution elasticity (smaller than 1) yields log supermodularity and a higher substitution elasticity (greater than 1) yields log submodularity.