Sequential Investment with Technical Innovation and Heterogeneous Site Quality

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Abstract

We study the optimal sequence of investment when technology improves over time and the productivity of deployed capital differs by site quality. Our perspective is that of a price- and technology-taking individual or firm. We begin with a model where the price of output produced with the technology is a known constant and technology improves according to a known differential equation. We specify an optimization problem that allows for the solution of the optimal date of initial investment and the dates for optimal replacement. We then develop models where output price evolves according to geometric Brownian motion and technology evolves deterministically or stochastically, with up-jumps (breakthroughs). The possibility of breakthroughs will further delay initial investment compared to the model where technology evolves deterministically. Our analysis is relevant for the initial investment in renewable energy (wind or solar) and determining when and where to replace capital that is inefficient relative to current technology.

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1 Introduction

The question of when to adopt a new technology, whose future efficiency is likely to improve, is a fundamental economic question. In addition to the optimal timing of initial investment, optimal replacement, when more efficient capital is available, is also of critical importance. These questions arise when considering investments in information technology,[1] renewable energy,[2] and other forms of durable capital that continue to improve over time.

Initial investment and replacement can be viewed as a series of real options (Mcdonald and Siegel, 1986; Dixit and Pindyck, 1994; Trigeorgis, 1996). Options to initially invest or upgrade to more efficient capital arise when a firm is faced with one or more sources of uncertainty. Sendstad and Chronopoulos (2016) identify three sources of uncertainty which can affect the value of options and thus the timing of adoption or replacement: (1) stochastic output price, (2) evolving technology with periodic breakthroughs, and (3) public policies which might be introduced or terminated. In addition to these three sources of uncertainty, several authors have shown that the evolving structure of an industry can influence strategic adoption. In non-competitive industries, there may be an incentive for preemptive investment, which might be followed by a war of attrition, Huisman and Kort (2004).

The current paper differs from the above contributions in two important respects. First, we allow technology to evolve according to a differential equation, but with the possibility of periodic up-jumps (breakthroughs) in efficiency. Second, we allow for heterogeneity in the productivity of deployed capital based on location or site quality. If an individual or firm owns several sites of differing quality, which sites should be developed first, high or low quality sites? This second feature of our research is especially relevant for the timing of investments in renewable energy, such as solar panels or wind turbines where site quality can vary widely. In our deterministic model, we identify a sufficient condition for high quality sites to be saved for more mature technology.

A plausible form for the evolution of technical efficiency has the gap between maximum efficiency and current efficiency proportionally closing over time. When this deterministic growth in efficiency is paired with output price evolving as geometric Brownian motion (GBM), we can derive a closed-form expression for a separating barrier in efficiency-price space. The optimal time to initially adopt or upgrade will

[1] See e.g. Schwartz and Zozaya-Gorostiza (2003); Pendharkar (2010); Milliou and Petrakis (2011); Kauffman et al. (2015).

[2] See e.g. Boomsma and Linnerud (2015); Adkins and Paxson (2016); Sendstad and Chronopoulos (2016).

be a random variable, determined by the time it takes for a stochastically evolving efficiency-price realization to reach the separating barrier. We then add the possibility of Poisson up-jumps to the evolution of efficiency and re-derive the separating barrier. A numerical example, loosely based on the evolving efficiency of wind turbines and the annual price of electricity in the US, is analyzed to quantitatively assess the shift in the separating barrier when Poisson up-jumps are added to the evolution of efficiency.

The remainder of this paper is organized as follows. In the next section we present a deterministic model where output price is constant and the efficiency gap proportionally closes over time. The deterministic model allows us to address the question of which quality sites to develop first. In Section 3, the deterministic model of efficiency is now paired with output price evolving as GBM. In Section 4 we then add the Poisson up-jump process to the evolution of efficiency and re-derive the separating barrier. In Section 5 we calibrate both stochastic models to the evolution of efficiency in wind turbines and average annual electricity prices in the US. We can then quantitatively identify the extent to which the Poisson up-jump process causes the separating barrier to shift in efficiency-price space. Section 6 concludes.

2 The Deterministic Model

Let $X = X(t)$ denote the efficiency of a capital item (perhaps a solar panel or wind turbine) at instant t, $\infty > t \geq 0$. We assume efficiency is increasing over time according to a first-order differential equation $dX/dt = X'(t) = f(X(t))$. We will consider special cases where this differential equation has an analytic solution given by $X(t) = F(t)$ and where $X(t)$, $X'(t)$, and $X''(t)$ are all continuous. We further assume that there is a maximum, practical efficiency for this capital item, denoted by $\bar{X}$, and that $X(t) \to \bar{X}$ as $t \to \infty$. Finally, we assume that current efficiency is less than the practical maximum, so that $X(0) < \bar{X}$. Henceforth, we will simply refer to this particular capital item as "capital."

While the efficiency of capital at instant t is available to all potential investors, the productivity of deployed capital may differ based on site quality. Suppose there is a continuum for site quality, Q, on the unit interval, so that $Q \in (0, 1)$. If $Q_i > Q_j$, then site i has higher quality, and greater production potential, than site j for any efficiency $X(t)$.

For a particular site of quality Q, let τ_k denote the date that capital of efficiency $X(\tau_k)$ was installed at that site. Then, $X(\tau_1)$ is the efficiency of capital initially installed at site Q and $X(\tau_2)$ would be the efficiency of capital replacing $X(\tau_1)$, where

$X(\tau_2) > X(\tau_1)$ when $\tau_2 > \tau_1$.

Suppose that $P > 0$ is the constant per unit price for output produced using capital of efficiency $X(\tau_k)$ at a site of quality Q. Instantaneous revenue would be given by $PQX(\tau_k)$.[3] Let C_k denote the cost of purchase and installation of capital of efficiency $X(\tau_k)$. If $X(\tau_k)$ is replacing $X(\tau_{k-1})$, C_k will also included the net present value of maintaining the technology adopted at τ_{k-1}. This cost would probably vary by site, but we will ignore this possibility in the present paper. Finally, let $\delta > 0$ denote the instantaneous discount rate. Then, the owner of site Q wants to maximize the net present value (discounted cash flow) by determining the optimal time to install and replace capital. Specifically, the owner of site Q wishes to

$$\begin{aligned}\underset{\tau_1,\tau_2,\tau_3,\ldots}{\text{Max}}\ N &= -\sum_{k=1}^{\infty} C_k e^{-\delta\tau_k} + \int_{\tau_1}^{\tau_2} PQX(\tau_1)e^{-\delta t}dt + \int_{\tau_2}^{\tau_3} PQX(\tau_2)e^{-\delta t}dt + \ldots \\ &= -\sum_{k=1}^{\infty} C_k e^{-\delta\tau_k} + \left[\frac{1}{\delta}PQX(\tau_1)\right]\left[e^{-\delta\tau_1} - e^{-\delta\tau_2}\right] \\ &\quad + \left[\frac{1}{\delta}PQX(\tau_2)\right]\left[e^{-\delta\tau_2} - e^{-\delta\tau_3}\right] + \ldots\end{aligned}$$

The optimality condition for deploying capital of efficiency $X(\tau_k)$ at site Q requires

$$\delta C_k - PQX(\tau_k) + \left[\frac{1}{\delta}PQX'(\tau_k)\right]\left[1 - e^{-\delta(\tau_{k+1}-\tau_k)}\right] = 0 \tag{1}$$

In general, optimality condition (1) requires that one know all the future optimal replacement times, $\tau^*_{k+1}, \tau^*_{k+2}, \ldots$ before one can determine τ^*_k.[4] If, however, $\tau^*_{k+1} >> \tau^*_k$, so that $e^{-\delta(\tau^*_{k+1}-\tau^*_k)} \approx 0$, then the optimality condition for τ_k simply requires

$$\delta C_k - PQX(\tau_k) + \frac{1}{\delta}PQX'(\tau_k) = 0 \tag{2}$$

For initial investment, when $\tau_2 >> \tau_1$, Equation (2) requires

$$PQX(\tau_1) = \delta C_1 + \frac{1}{\delta}PQX'(\tau_1) \tag{3}$$

[3] We could define $p \equiv PQ$ as quality-adjusted price, where lower site quality implies a lower effective price. The comparative statics of adoption time, τ_k, as discussed below, with respect to Q or p will be the same and we prefer to retain Q explicitly.

[4] One might select a distant date, say τ_K and solve Equation (1) recursively backward in time to determine τ_1. Alternatively, one could numerically maximize N for initial investment and a finite number of replacements. We use this approach, solving for τ_1, τ_2, τ_3, τ_4, and τ_5 in Section 5.

Equation (3) says that at the optimal time to initially invest, the instantaneous revenue from $X(\tau_1)$ must equal the interest cost on purchase and installation, C_1 plus the present value of the marginally more efficient technology which would be available if one waited the small time increment, dt.

Should high quality sites be developed first or saved for more mature technology? For initial investment, when $\tau_2 >> \tau_1$, if we divide all the terms in Equation (3) by PQ and take a total derivative you can show that

$$\frac{d\tau_1}{dQ} = \frac{\delta C_1}{PQ^2[X''(\tau_1)/\delta - X'(\tau_1)]} \tag{4}$$

We see from Equation (4) that the sign of $d\tau_1/dQ$ depends on the signs and magnitudes of the first and second derivatives of $X(t)$ evaluated at τ_1 and the discount rate, δ. If $X''(\tau_1) > \delta X'(\tau_1) > 0$, then $d\tau_1/dQ > 0$, implying one should save high quality sites for more efficient technology. If $X''(\tau_1) < \delta X'(\tau_1)$, then $d\tau_1/dQ < 0$, and the best sites are developed first. With $X'(\tau_1) > 0$, a sufficient condition for high quality sites to be initially saved for more mature technology is $X''(\tau_1)/[\delta X'(\tau_1)] > 1$.

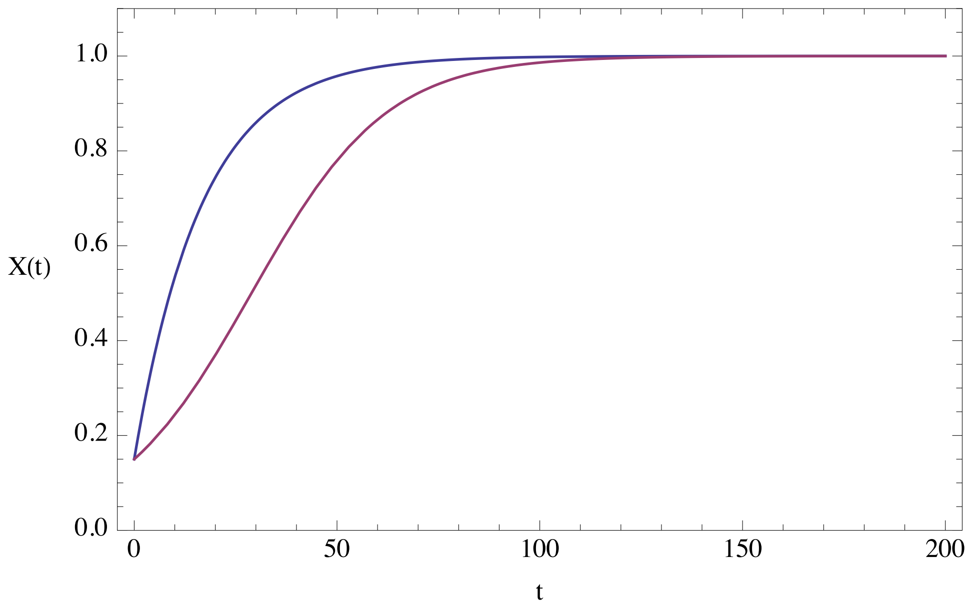
We will consider two candidate differential equations that might be suitable for describing the evolution of efficiency. The first candidate takes the form $X'(t) = \eta_1[\bar{X} - X(t)]$. With $\eta_1 > 0$, this equation implies that the efficiency gap, $(\bar{X} - X(t))$, is closed proportionally over time. This candidate has $X''(t)/[\delta X'(t)] = -\eta_1/\delta < 0$, so one would always develop higher quality sites first.

The second candidate is the logistic form where $X'(t) = \eta_2 X(t)[1 - X(t)/\bar{X}]$. $\eta_2 > 0$ is the rate of technical progress when $X(t)$ is very small relative to the practical maximum, $\bar{X}$. For the logistic function $X''(t)/[\delta X'(t)] > 1$ if $(\eta_2 - \delta)/(2\eta_2) > X(t)/\bar{X}$. If current technology is very primitive (inefficient), so that $(\eta_2 - \delta)/(2\eta_2) > X(0)/\bar{X}$, one would save high quality sites for more mature (efficient) capital.

These two candidate differential equations have analytic solutions for $X(t) = F(t)$ which will prove useful in our numerical analysis of the timing of wind turbine deployment on sites of different quality. The first candidate, $X'(t) = \eta_1[\bar{X} - X(t)]$, can be integrated directly to yield $X(t) = \bar{X} - (\bar{X} - X(0))e^{-\eta_1 t}$. The logistic equation can be integrated by separation of variables to yield $X(t) = \bar{X}/(1 + \alpha e^{-\eta_2 t})$, where $\alpha = (\bar{X} - X(0))/X(0)$. Figure 1 shows a plot of $X(t) = \bar{X} - [\bar{X} - X(0)]e^{-\eta_1 t}$ and $X(t) = \bar{X}/(1 + \alpha e^{-\eta_2 t})$ when $X(0) = 0.15$, $\bar{X} = 1$, and $\eta_1 = \eta_2 = 0.06$. The logistic function exhibits the familiar sigmoid shape while the proportional function is strictly concave in its approach to $\bar{X}$.

If $\tau_2^* >> \tau_1^*$, one can solve Equation (3), for the technology threshold $X(\tau_1^*)$ for initial investment. When $X'(t) = \eta_1[\bar{X} - X(t)]$ the critical technology threshold is

Figure 1: Technological Progress: Proportional (Blue) and Logistic (Red).



$$X(\tau_1^*) = \frac{\delta^2 C_1 + PQ\eta_1 \bar{X}}{(\delta + \eta_1)PQ} \tag{5}$$

When $X'(t) = \eta_2 X(t)[1 - X(t)/\bar{X}]$, the critical technology threshold for initial adoption is

$$X(\tau_1^*) = \frac{(\eta_2 - \delta)\bar{X}}{2\eta_2} + \sqrt{\left[\frac{(\eta_2 - \delta)\bar{X}}{2\eta_2}\right]^2 + \frac{\delta^2 \bar{X} C_1}{PQ\eta_2}} \tag{6}$$

One can then solve for the optimal time to initially adopt, τ_1^*, based on the technology thresholds, Equations (5) or (6), and the analytic solutions for $X(t)$. For Equation (5) we know that $X(\tau_1^*) = \bar{X} - (\bar{X} - X(0))e^{-\eta_1 \tau_k^*}$. Given the right-hand-side of Equation (5) we can solve for τ_1^* yielding

$$\tau_1^* = (1/\eta_1) \ln \left[\frac{(\delta + \eta_1)PQ(\bar{X} - X(0))}{PQ\delta\bar{X} - \delta^2 C_1}\right] \tag{7}$$

Similarly, for Equation (6), we know that $X(\tau_1^*) = \bar{X}/(1 + \alpha e^{-\eta_2 \tau_1^*})$. Solving for τ_1^* for this technology yields

$$\tau_1^* = (1/\eta_2) \ln \left[\frac{X(\tau_1^*)(\bar{X} - X(0))}{X(0)(\bar{X} - X(\tau_1^*))}\right] \tag{8}$$

We will report numerical results when maximizing N for initial investment, τ_1^*, and replacement dates in Section 5 when we evaluate the timing of initial investment in wind turbines in Altamont Pass in California in the early 1980s assuming $X'(t) = \eta[\bar{X} - X(t)]$.

3 Stochastic Price and the Separating Barrier

The deterministic model, with constant price, is now modified so that price evolves as GBM. Capital efficiency is assumed to evolve deterministically according to $X'(t) = \eta[\bar{X} - X]$. Taken together they imply a system where

$$dP = \mu P dt + \sigma P dz$$
$$dX = \eta[\bar{X} - X]dt$$

where $\mu \geq 0$ is the price drift rate, $\sigma > 0$ is the price standard deviation rate, $dz = \epsilon(t)\sqrt{dt}$ is the increment of a Wiener process, with $\epsilon(t) \sim N(0,1)$, and where $\eta > 0$ is the rate at which the efficiency gap is closed. The Bellman Equation, while optimally waiting to make an initial investment, requires

$$\delta V(P,X) = \mu P V_P(\bullet) + (\sigma^2/2)P^2 V_{PP}(\bullet) + \eta[\bar{X} - X]V_X(\bullet) \tag{9}$$

or

$$(\sigma^2/2)P^2 V_{PP}(\bullet) + \mu P V_P(\bullet) + \eta[\bar{X} - X]V_X(\bullet) - \delta V(P,X) = 0 \tag{10}$$

The value function that satisfies the above Bellman Equation takes the form

$$V(P,X) = AP^{\beta}\eta[\bar{X} - X] \tag{11}$$

where A is a positive constant and where

$$\beta = 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2(\delta + \eta)/\sigma^2}. \tag{12}$$

Note that $V(P,X)$ is the option value of site Q. Option value increases with higher price, P, and decreases as current efficiency approaches the practical maximum. As technology improves the option value of waiting to invest declines. Also note, in Equation (12), the proportional rate of closure in the efficiency gap, η, augments the discount rate.

Suppose it is optimal to make an initial investment at $t = \tau$, when price is $P(\tau)$

and efficiency is $X(\tau)$. If this initial investment is operated for the rest of time, it has an expected present value of

$$V(P(\tau), X(\tau)) = \frac{P(\tau)QX(\tau)}{(\delta - \mu)} \tag{13}$$

The value-matching condition requires

$$AP(\tau)^{\beta}\eta[\bar{X} - X(\tau)] = \frac{P(\tau)QX(\tau)}{(\delta - \mu)} - C \tag{14}$$

while the smooth-pasting condition requires

$$\beta AP(\tau)^{\beta-1}\eta[\bar{X} - X(\tau)] = \frac{QX(\tau)}{(\delta - \mu)} \tag{15}$$

Multiplying both sides of the smooth-pasting condition by $P(\tau)/\beta$ implies

$$AP(\tau)^{\beta}\eta[\bar{X} - X(\tau)] = \frac{P(\tau)QX(\tau)}{\beta(\delta - \mu)} = \frac{P(\tau)QX(\tau)}{(\delta - \mu)} - C \tag{16}$$

The second equality implies that the separating barrier in $[X - P]$ takes the form

$$P(\tau) = \frac{(\delta - \mu)\beta C}{(\beta - 1)QX(\tau)} \tag{17}$$

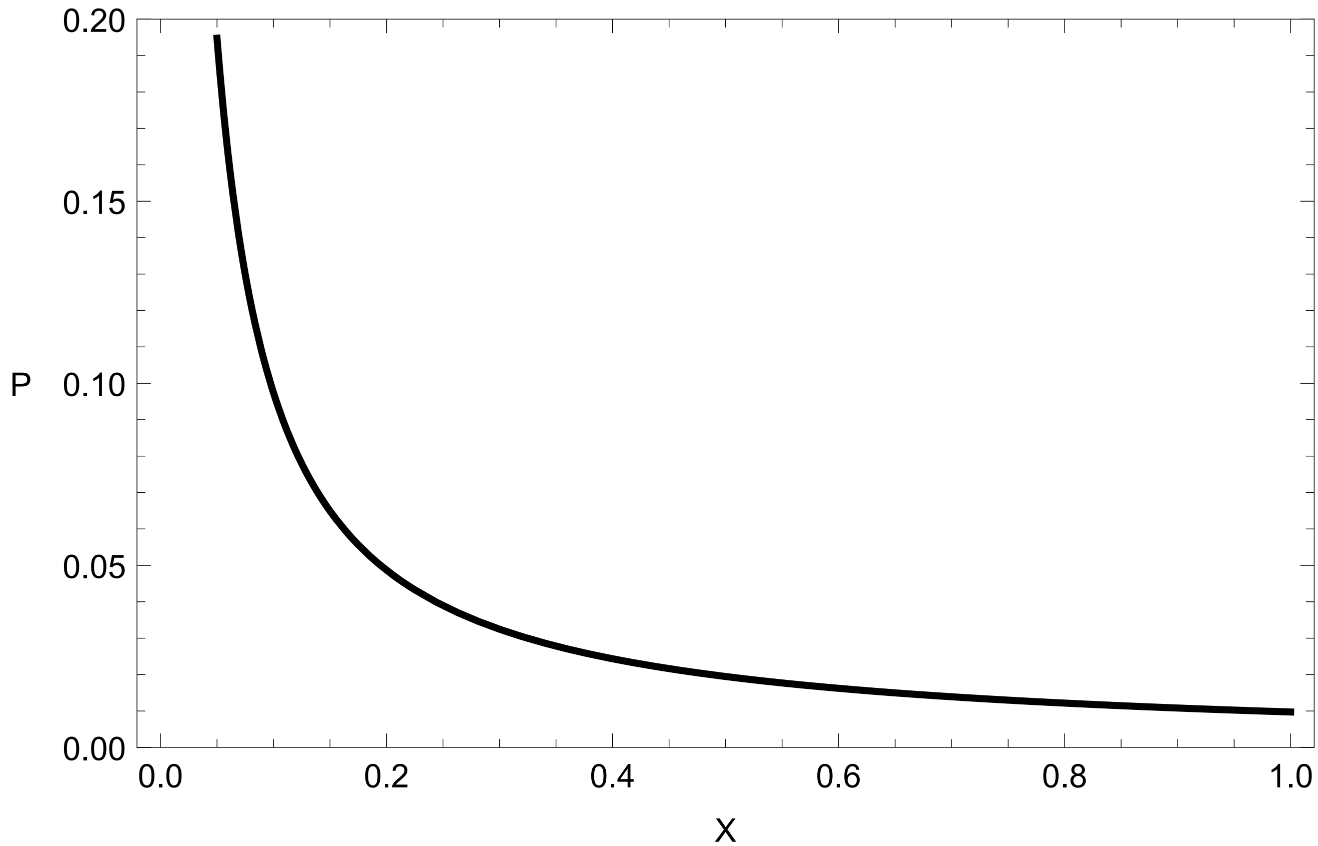
This separating barrier is asymptotic to the X and the P axes, shifts upward with an increase in C, and shifts downward with an increase in Q. In Figure 2 we show this barrier for $\eta = 0.0198$, $\delta = 0.0614$, $C = \$0.394/kWh$, $Q = 1$, $\mu = 0.053219$, and $\sigma^2 = 0.066768$. The values for η, δ, μ, and σ^2 imply $\beta = 1.4948$. We will discuss the genesis of these parameter values in Section 5.

4 Stochastic Price and Technological Breakthroughs

We now add a Poisson jump process to the evolution of efficiency to analyze the effect of technological breakthroughs. The stochastic system for this model takes the form

$$dP = \mu P dt + \sigma P dz$$
$$dX = \eta[\bar{X} - X]dt + [\bar{X} - X]dq_X = [\bar{X} - X][\eta dt + dq_X]$$
$$dq_X = \begin{cases} 0 & \text{with probability } 1 - \lambda_X dt \\ U_X & \text{with probability } \lambda_X dt \end{cases}$$

Figure 2: The Separating Barrier in X-P Space.



where $1 > \lambda_X > 0$ is the probability of an up-jump in the interval $[t, t + dt]$ and $1 > U_X > 0$ is the proportion of the efficiency gap, $[\bar{X} - X]$, which is closed if a jump occurs in that interval. The Bellman Equation while optimally waiting to make an initial investment now requires

$$(\sigma^2/2)P^2 V_{PP}(\bullet) + \mu P V_P(\bullet) + \eta[\bar{X} - X]V_X(\bullet) - \delta V(P, X) \\ + \lambda_X[V(P, X) - V(P, X + U_X[\bar{X} - X])] = 0, \qquad (18)$$

where the last term on the left-hand-side of Equation (18) arises from adding the jump process to the evolution of technology. Note that the term $\lambda_X[V(P, X) - V(P, X + U_X[\bar{X} - X])] < 0$ is the expected opportunity cost of investing now when there is a chance for a breakthrough in efficiency in the interval $[t, t + dt]$.

Perhaps surprisingly, the value function which satisfies Equation (18) is given by Equation (11) where $V(P, X) = AP^\beta \eta[\bar{X} - X]$. With this value function, one can show that

$$\lambda_X \left[V(P, X) - V(P, X + U_X[\bar{X} - X])\right] = \lambda_X U_X A P^\beta \eta \left[\bar{X} - X\right] \qquad (19)$$

which changes the positive root of the fundamental quadratic so that now

$$\beta = 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2(\delta + \eta - \lambda_X U_X)/\sigma^2} \qquad (20)$$

The equation for the separating barrier in $[X-P]$ space is unchanged, see Equation(17), but the barrier shifts upward because of the change in the value of β. With $(\delta + \eta - \lambda_X U_X) < (\delta + \eta)$ the value of β given by Equation (20) will be smaller than the value of β given by Equation (12). The upward shift in the separating barrier with the potential for Poisson up-jumps in efficiency makes intuitive sense since that possibility creates a further incentive to delay investment.

5 California in the 1980s

In this section we estimate the initial efficiency and the parameters for the deterministic and stochastic models for California in the 1980s. We then conduct numerical analysis to determine the quantitative significance of tax credits and the avoided cost policy on optimal efficiency and the date of initial adoption of wind technology. In the deterministic model we use the differential equation where $X'(t) = \eta[\bar{X} - X(t)]$. In the stochastic models we plot the separating barrier when $dP = \mu P dt + \sigma P dz$ and

$dX = \eta[\bar{X} - X]dt$ and when $dP = \mu P dt + \sigma P dz$ and $dX = [\bar{X} - X][\eta dt + dq_X]$ where

$$dq_X = \begin{cases} 0 & \text{with probability } 1 - \lambda_X dt \\ U_X & \text{with probability } \lambda_X dt \end{cases}$$

We present evidence, based on our deterministic model, that the adoption of low efficiency, unreliable turbines in California in the early 1980s was not optimal if investors wished to maximize discounted expected net revenue. The early adoption in Altamont Pass was not only influenced by tax credits and the high, feed-in tariff under avoided cost, but also by the fact that investors rightly viewed these credits and the feed-in tariff as ephemeral. This view would be consistent with what Sendstad and Chronopoulos (2016) refer to as “policy uncertainty”.

5.1 Estimating η, X_0, λ_X, and U_X

In Table 1 below we list the average turbine rating (kWh) in 1980, 1985, 1992, 1997, 2002, 2007, and 2010 as reported in Smith (1987) and Lantz et al. (2012). The first turbines deployed in Altamont Pass in the early 1980s had a nameplate capacity of approximately 50 kWh. By 2010, nameplate capacity for a single turbine, with rotor diameter of 100 meters, had increased to 3,000 kWh. We assume that the nameplate capacity of a turbine in 2010 is 90% of the practical maximum, our $\bar{X} = 1$. We can then convert the earlier ratings (R_t) into our efficiency index by the formula $X_t = 0.9R_t/3000$, yielding the X_t values in the third column of Table 1. This assumption implies that the 50 kWh turbine in 1980 would have had an efficiency index of $X_0 = 0.0150$.

Table 1: Turbine Rating and Efficiency in Selected Years.

Year	R_t (kWh)	X_t
1980	50	0.0150
1985	75	0.0225
1992	300	0.0900
1997	750	0.2250
2002	1,500	0.4500
2007	1,800	0.5400
2010	3,000	0.9000

Sources: Smith (1987) and Lantz et al. (2012)

We then treat the efficiency index numbers in 1985, 1992, 1997, 2002, 2007, and 2010 as the "truth," and solve for the Maximum Likelihood estimates for η, λ_X, and U_X, which minimize the mean sum of squared deviations for those years when

$$X_{t+1} = X_t + \eta(1 - X_t) + J_t U_X, \tag{21}$$

for $t = 1980, 1981, \ldots, 2009$. This is done repeatedly after generating standard uniform random variates, $z_t \sim U(0, 1)$, where $J_t = 1$ if $z_t < \lambda_X$ and $J_t = 0$ if $z_t \geq \lambda_X$. The best-fit parameter values were $\eta = 0.0198$, $\lambda_X = 0.2478$, and $U_X = 0.0500$.

5.2 Estimating μ, σ, and δ

Average annual prices for electricity in California and the US do not exhibit the price spikes observed in higher frequency (daily) price data. Geometric Brownian motion provides a plausible model for the evolution of average annual electricity prices. In Figure 2 we show 100 realizations when $\mu = 0.053219$ and $\sigma = 0.066768$. These are the estimates for μ and σ in California from 1960 through 1980. The red realization in Figure 2 plots the actual average price during this period. We will use these estimates for μ and σ when computing the separating barriers for our two stochastic models.

The early 1980s were marked by high inflationary expectations and high prime rates of interest. The average annual prime rate from 1980 through 1986 was 0.0614 which we adopt as the discount rate, δ, for wind farm investors in Altamont Pass.

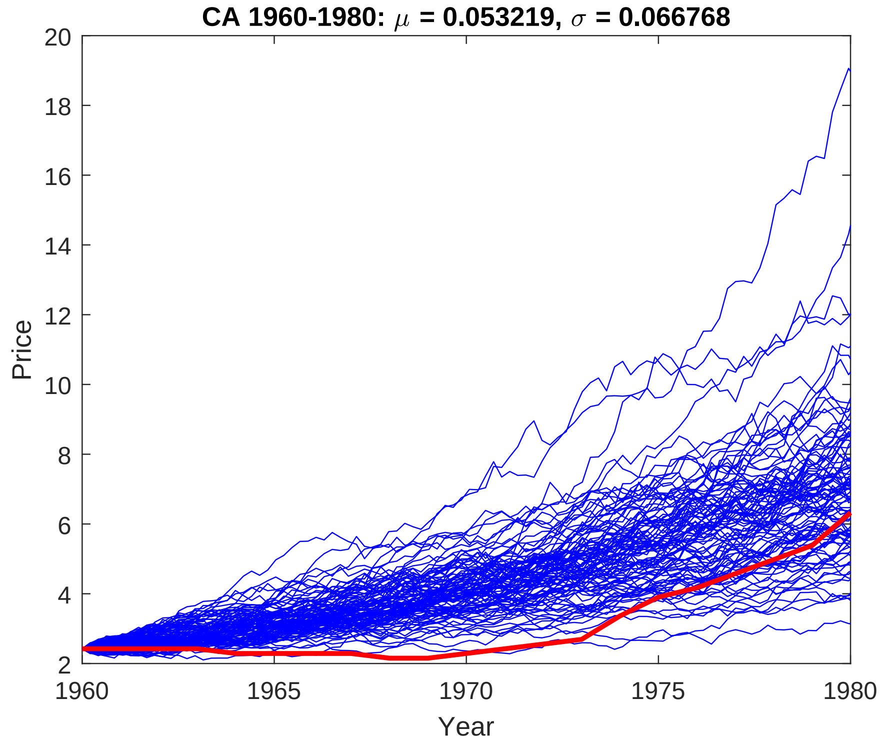
5.3 P and C_k

According to Smith (1987), by 1986 the total nameplate capacity in the Altamont Pass in California was estimated at 642,000 kWh in the form of approximately 7,000 turbines. Many of the turbines installed in 1981-1982 had been replaced with larger, more reliable models. The average turbine had a nameplate capacity of 92 kWh but was only generating electricity at a capacity factor that averaged 0.1326. The annual expected electricity production from such a turbine would be

$$E = (92 \text{ kW/hour})(24 \text{ hours/day})(365 \text{ days/year})(0.1326) = 106{,}865 \text{ kW/year}$$

In our notation, P is the annual revenue from sales of electricity. We calculate P based on the formula $P = pE$, where $p = \{\$0.08/kWh, \$0, 16/kWh, \$0.24/kWh\}$. The value $p = \$0.24/kWh$ corresponds to the high, avoided-cost, tariff paid by PG&E when

Figure 3: Actual price and GBM realizations based on US data for the period 1960-1985.



it was operating all of its plants at close to maximum capacity.

The average cost of purchase and installation for a 92 kWh turbine in the 1980s was approximately \$140,000 before tax credits and accelerated depreciation. The Crude Oil Windfall Profits Tax Act of 1980 (PL 96-23) allowed for a 15% Energy Tax Credit over and above the standard 10% investment tax credit. The Economic Recovery Tax Act of 1981 (PL 97-34) allowed for the complete depreciation of a wind turbine over five years. In addition to these federal incentives, the State of California allowed a 25% tax credit to wind farm partnerships. This California State credit was set to expire at the end of 1986. Smith (1987, p.150) notes that investors taking advantage of all of these tax savings would effectively reduce the cost of purchase and installation by 70% in the first year of operation. With a year-one combined investment credit of $TC = 0.7$, the cost of purchase and installation of our 92 kWh turbine would be reduced to $C_1 = (1 - 0.7) \cdot \$140,000 = \$42,000$.

Because of the of the expiration date for the California State tax credit, along the tenuous nature of the federal tax credits, we assume that the purchase and installation costs for subsequent turbines would increase to $C_k = \$200,000$ for $k = 2, 3, 4, 5$. Our base-case set of parameters, with $p = \$0.24/kWh$, is summarize in Table 2.

Table 2: Parameters for California in the 1980s.

Parameter	Value
$X(0)$	0.0150
$\bar{X}$	1.0000
η	0.0198
λ_X	0.2478
U_X	0.0500
μ	0.0532
σ	0.0668
δ	0.0614
P	\$25,647
Q	1
Tax Credit (TC)	0.70
C_1	\$42,000
C_k	\$200,000

5.4 Maximizing N when Choosing τ_1, τ_2, τ_3, τ_4, and τ_5

Table 3 reports the values of τ_1^* and τ_2^* when we used numerical optimization to find the values of $\tau_1, ..., \tau_5$ that maximize the net present value of a site, N, without assuming that $\tau_2 >> \tau_1$. Technology evolves according to $X'(t) = \eta[\bar{X} - X(t)]$. We then change a single parameter (shown in the first column of Table 3) and resolve for τ_1, τ_2, τ_3, τ_4, and τ_5. In Table 3 we only report τ_1^*, τ_2^*, $X(\tau_1^*)$, and N^*.

With initial efficiency in 1980 at $X(0) = 0.015$, a tax credit of $TC = 0.7$, and an avoided-cost- feed-in tariff of \$0.24/kWh (implying $P = \$25,647$), it is optimal to wait $\tau_1^* = 18$ years until efficiency has improved to $X(\tau_1^*) = 0.32$. When the feed-in tariff is reduced to $p = \$0.16/kWh$, implying $P = \$17,098$, it becomes optimal to wait 22 years until $X(\tau_1^*) = 0.36$. A further reduction to $p = \$0.08/kWh$ ($P = \$8,549$) increases the wait-time to initial investment to $\tau_1^* = 31$ when efficiency reaches $X(\tau_1^*) = 0.47$.

Reductions in the tax credit, TC, increase τ_1^*, and the required level of efficiency. A reduction in the discount rate to $\delta = 0.03$ implies greater patience, with investors waiting 21 years before initial investment, but with replacement at $\tau_2^* = 48$. Also, when $\delta = 0.03$ the discounted net present value of the optimal sequence of investments increases to $N^* = \$170,471$. An increase in the discount rate to $\delta = 0.09$ reduces τ_1^* to 17 years and lowers N^* to \$9,208.

If the quality of the site declines from $Q = 1$ to $Q = 0.5$, τ_1^* increases to 25 years. When η is increased to $\eta = 0.04$ technology is improving more rapidly and τ_1^* declines

Table 3: Optimal Timing of an Initial Investment in a Wind Turbine

Scenario	τ_1^*	τ_2^*	$X(\tau_1^*)$	$N^*(\$)$	$e^{-\delta(\tau_2-\tau_1)}$
Base-Case	18	75	0.32	29,256	0.03117
$P = \$17,098$	22	354	0.36	15,299	0.00000
$P = \$8,549$	31	542	0.47	3,431	0.00000
$TC = 0.35$	26	97	0.41	16,408	0.01225
$TC = 0.1$	31	441	0.47	10,293	0.00000
$\delta = 0.03$	21	48	0.35	170,471	0.45378
$\delta = 0.09$	17	243	0.30	9,208	0.00000
$Q = 0.5$	25	468	0.40	8,933	0.00000
$\eta = 0.4$	14	62	0.44	59,921	0.05327
$C_k = \$400,000$	19	492	0.32	29,052	0.00000

Notes: Other parameter values in base-case are: $X(0) = 0.015$, $\bar{X} = 1$, $\delta = 0.0614$, $\eta_1 = 0.0198$, $C = \$140,000$, $TC = 0.7$, $C_1 = (1 - TC) * C$, $C_k = \$200,000$, for $k = 2, 3, 4, 5$, $Q = 1$, and $P = \$25,647$. We used the following initial guess in the optimization: $\tau_1^* = 20$, $\tau_2^* = 40$, $\tau_3^* = 60$, $\tau_4^* = 80$ and $\tau_5^* = 100$.

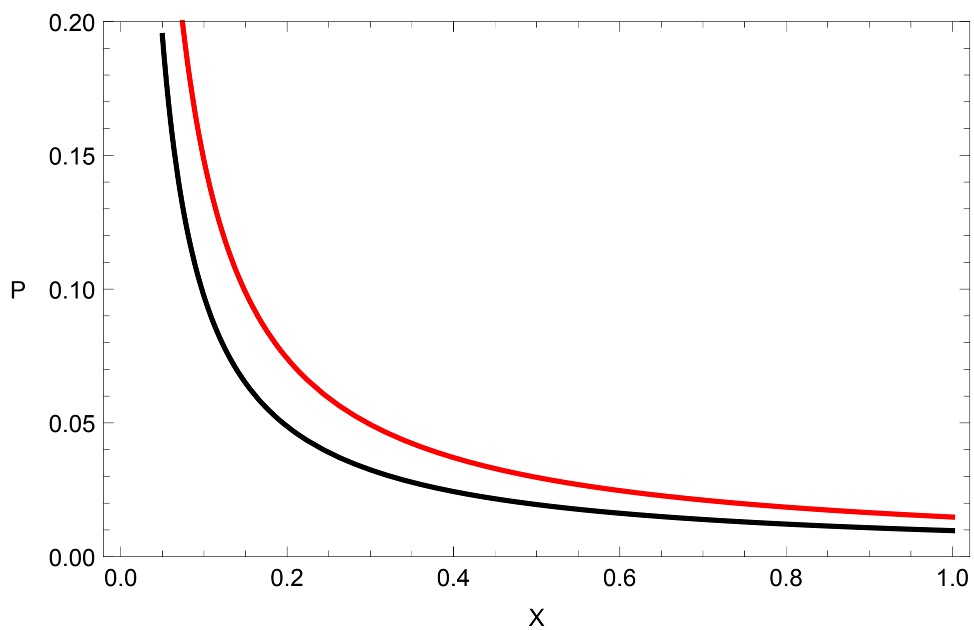
to 14 years with an efficiency threshold of $X(\tau_1^*) = 0.44$. If the future purchase and installation costs of turbines were expected to increase to $C_k = \$400,000$, $k = 2, 3, 4, 5$, the optimal time for initial adoption only increases slightly to $\tau_1^* = 19$ but replacement is pushed into the distant future with $\tau_2^* = 492$.

In Table 3 we also report the term $e^{-\delta(\tau_2-\tau_1)}$. If $e^{-\delta(\tau_2-\tau_1)} \leq 10^{-5}$, then Equations (5) and (7) will yield the same values for $X(\tau_1^*)$ and τ_1^* as those obtained when maximizing N by choosing τ_1, τ_2, τ_3, τ_4, and τ_5.

5.5 The Separating Barriers

To plot the separating barriers, with and without the efficiency up-jump process, we need to specify C to be the levelized cost of energy for the wind turbines installed in Altamont Pass in the 1980s. Lantz et al. (2012), in their Figure 3, show a DEA(1999) estimate of \$250/MWh, or \$0.25/kWh, indexed in 2010 dollars. If we assume that the levelized cost for our 92 kWh turbine, with tax credits, is \$42,000, producing 106,685 kW/year we get a levelized cost of $C = \$0.394/kWh$. When $\eta = 0.0198$, $\delta = 0.0614$, $C = \$0.394/kWh$, $Q = 1$, $\mu = 0.053219$, $\sigma = 0.066768$, $U_X = 0.0500$, and $\lambda_X = 0.2478$ we plot the separating barriers in $X - P$ space in Figure 4 below. When efficiency evolves with up-jumps, the separating barrier increases above the separating barrier without possible up-jumps. The possibility of jumps in efficiency would further delay the initial investment in a wind turbine. Some reference points are informative. On the lower, no-jump, barrier the price (feed-in tariff) would need to be \$0.1948/kWh when $X(t) = 0.05$, \$0.0974/kWh when $X(t) = 0.10$, but only \$0.0103/kWh when $X(t) = 0.95$. For the upper, up-jump, barrier the feed-in tariff would need to be \$0.2963/kWh when $X(t) = 0.05$, \$0.1481/kWh when $X(t) = 0.10$, and \$0.0156/kWh when $X(t) = 0.95$. Without the possibility of efficiency up-jumps the value of β from Equation (12) is $\beta = 1.4948$. With the possibility of up-jumps the value of β from Equation (20) is $\beta = 1.2781$.

Figure 4: The Separating Barriers: With Up-Jumps (red) and Without (black).



6 Conclusions

We have developed deterministic and stochastic models to determine optimal sequential investment when technical efficiency improves over time. In the deterministic model one must compute the optimal dates for replacement before one can determine the optimal date for initial adoption. If, however, $\tau_2^* >> \tau_1^*$ we can derive expressions for the optimal efficiency threshold and optimal initial date of adoption when the differential equation describing the evolution of technology has an analytic solution.

In both deterministic and stochastic models we allowed for site quality, as measured by our index Q, to potentially influence the optimal timing of initial investment. In our deterministic model, when $\tau_2^* >> \tau_1^*$, we identified a sufficient condition,

$X''(\tau_k)/[\delta X'(\tau_k)] > 1$, for saving higher quality sites for more mature technology.

The stochastic models, with the price of output evolving as geometric Brownian motion, result in separating barriers in efficiency-price space. The optimal time to adopt is a random variable determined by the arrival time of a particular realization, $[X, P]$ at the separating barrier. We allowed efficiency to evolve deterministically or with Poisson up-jumps, which can be viewed as allowing for periodic, technical breakthroughs. Allowing for Poisson up-jumps in technical efficiency causes the separating barrier to shift upward in efficiency-price space because β, the positive root of the fundamental quadratic, decreases. As one might have conjectured, the possibility of up-jumps in turbine efficiency would further delay initial investment.

We calibrated the deterministic and stochastic models to determine the technology thresholds and separating barriers as they might have existed when California was siting small, first-generation wind turbines in Altamont Pass. Our estimate for the technical efficiency of wind turbines in the early 1980s was very low, $X(1980) \approx 0.015$. This estimate was based on the fact that turbines sited in 1980 had a nameplate capacity of approximately 50 kWh and were prone to failure. It was also based on the assumption that the 3,000 kWh turbines available in 2010 had reached 90% of our practical maximum efficiency, $\bar{X} = 1$.

Starting from such a low level of efficiency, even with a feed-in tariff of \$0.24/kWh and tax incentives that reduced the cost of purchase and installation by 70%, we found it optimal to wait 18 years before investing, at which time efficiency would have reached $X(1998) = 0.32$. This would seem to imply that the investment by small wind-farm partnerships siting 50 kWh turbines in Altamont Pass in the 1980's was premature and thus sub-optimal. It is possible that subsequent turbines became more efficient because of the experience (and failure) of the initial 50 kWh turbines installed in Altamont Pass and that society was made better off from "learning by doing." Empirically testing such a hypothesis is beyond the scope of the present paper.

Sendstad and Chronopoulos (2016) argue that the possible termination of tax credits and a reduction in avoided-cost, feed-in tariffs paid by the larger regulated utilities, would create an incentive to invest while they were still available. Another explanation might be that investors at that time thought the available turbines were approximately 30% efficient relative to an unknown practical maximum. They did not have the knowledge that more reliable 300kWh turbines would be available in 1992. An optimal wait time of 18 years is not inconsistent with the timing of wind-turbine investment in Texas, which now has the largest nameplate capacity of wind turbines in the United States.

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