Structural Change and Lag Length in VAR Models

by

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Abstract: This paper investigates the relationship between changes in policy rules and the estimated lag length in empirical models. Using simulated data from a standard macroeconomic model generated under both learning and rational expectations, the paper first establishes a theoretical link between estimated lag lengths and policy parameters and contrasts the results under learning and rational expectations. Next, the paper shows that theoretical predictions regarding changes in policy parameters and changes in estimated lag lengths are present in data for the U.S. Finally, the paper investigates potential explanations for the empirical results.

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1. Introduction

To what extent do the dynamics of theoretical and empirical models of the macroeconomy change when there is a change in macroeconomic policy? This paper addresses this question by looking at one aspect of macroeconomic dynamics, lag length in empirical models, and its relationship to changes in the parameters of a monetary policy rule for the federal funds rate.

Motivation for such an inquiry comes from the developing literature on structural change arising from shifts in the monetary policy rule over time. Theoretical and empirical investigation into shifts in monetary policy rules has a long history, but the recent literature is motivated largely by Clarida, Gali, and Gertler's (2000) finding that policy changed from one that generates indeterminacy in the 1970's to a policy consistent with determinacy in the 1980's. This generated considerable debate concerning whether policy prior to the 1980's indeed led to indeterminacy, and whether structural breaks in policy parameters have occurred at all. Papers such as Clarida, Gali, and Gertler (2000), Cogley and Sargent (2002), Boivon and Giannoni (2003), and Boivon (2004) support the idea that there have been important shifts in monetary policy parameters. Some papers, such as Sims and Zha (2004), argue that allowing for changes in the variance of structural disturbances over time can potentially overturn the result that there have been monetary policy shifts, but even they allow for and find support for parameter shifts over time.[1] Thus, though work in this area is still developing, strong empirical evidence for changes in the parameters of the monetary policy rule over time exists, and theoretical work shows that such changes can have important effects on how variables in the model

[1] Sims and Zha (2004) discuss recent empirical work in this area and notes that Bernanke and Mihov (1998) conclude there is little evidence for major shifts in policy regimes over time.

respond dynamically to shocks. This paper investigates another aspect of changes in policy regimes not previously recognized. This paper shows that changes in the policy rule, which affects the dynamics of the system, can affect estimated lag lengths in empirical models used to investigate monetary policy even when the underlying theoretical model has a constant lag structure across policy regimes.[2]

Additional motivation for examining the relationship between estimated lag lengths and policy changes is contained in Figures 1 and 2. Figures 1a, 1b, 1c, and 1d plot the lag length as estimated by either the Akaike Information Criterion (AIC) or a Likelihood Ratio Test (LR) along with the smoothing parameters from an estimated policy rule for the federal funds rate from February 1970 through December 2002.[3] Figures 1a and 1b show the estimated coefficient on the first lag of the federal funds rate in the policy equation while Figures 1c and 1d show the sum of the two smoothing coefficients.[4] The graphs suggest an association between changes in the smoothing parameters and estimated lag lengths, with lag length estimates appearing to lengthen when the sum of the smoothing parameters is increased. The association becomes even more evident in Figures 2a, 2b, 2c, and 2d which correspond to Figure 1 but plots a shorter time period, 1997-2002, the last six years in the sample. This paper investigates the relationship between monetary policy parameters and estimated lag lengths suggested in these graphs both theoretically and empirically.

[2] A recent paper by Kim and McMillin (2002) shows that lag structure is important in assessing monetary policy effects.

[3] The estimation methodology is discussed in detail in section 4 of the paper. Briefly, rolling ten year window regression is used to generate the parameter estimates and the lag length estimates. The lag lengths are estimated using a VAR model involving the monthly values of industrial production, inflation, and the federal funds rate. The smoothing parameters are the two coefficients on the lags of the federal funds rate in equation (14) in section 4.

[4] The coefficient on the second lag is the difference between the sum shown in Figures 1c and 1d, and the first coefficient shown in Figures 1a and 1b.

The paper begins with an investigation of whether standard theoretical macroeconomic models can account for changes in the estimated lag length in VAR models due to changes in the parameters of the policy rule. There are four reasons to expect that a change in policy can affect estimated lag lengths in empirical VAR models: changes in policy bringing about changes in persistence which causes changes in estimated lag lengths, estimation of learning equations bringing variable numbers of lagged values into the solution causing estimated lag lengths to change, changes in parameters causing changes in the gain and hence changes in the estimated learning equation, and policy parameters consistent with indeterminacy.

The first of these, changes in policy causing changes in the persistence of output, inflation, interest rates and other variables is discussed, for example, by Orphanides and Williams (2003).[5] An increase in persistence can cause estimated lag lengths to increase even though lag lengths in the theoretical model generating the data are unchanged.

To demonstrate that increases in persistence can increase estimated lag lengths even though the theoretical lag lengths are unchanged, an AR(2) model is simulated under various degrees of persistence, and the simulated data is used to estimate lag lengths using AIC and LR tests with a 5% level of significance. Figure 3 presents the results. In particular, the simulation varies the values of the autoregressive parameters, which are assumed to be equal, from .01 to .49 so that their sum varies from .02 through .98.[6] For each set of parameter values, the model is simulated 10,000 times and AIC and

[5] Here, and throughout the paper, the term persistence is used as in the discussion of serial persistence on page 15 of Orphanides and Williams (2003). That is, it is defined as the expected time required for a variable to return to its long-run or target value after a shock. Thus, for example, in Orphanides and Williams a variable following an AR(1) becomes more persistent if the autocorrelation coefficient increases.

[6] The AR(2) model includes a constant term with a value 1.0 in the simulations.

LR tests are used to determine the optimal lag length. The mean value over all 10,000 simulations for that set of parameter values is calculated and this constitutes one point on the graph.[7]

Two features of Figure 3 are noteworthy. First, estimated lag lengths increase as persistence increases up until the sum of the AR coefficients equals approximately .4 and then level off and increase only slightly if at all after that.[8] Second, when the AR coefficients are very small, the LR test produces lag lengths shorter than the those from the AIC whereas when the coefficients are larger, i.e. when they sum to a value of approximately .32 or more, the lag lengths for the LR test are larger than those from the AIC.[9]

The second reason that estimated lag lengths might vary with changes in policy parameters arises in learning models. In learning models, parameter values are estimated by economic agents using a particular window of data, i.e. a particular gain.[10] These regressions bring lagged values of variables into the solutions. In models such as Evans and Ramey (2003) the gain varies through time according to the frequency of policy changes. For example, in the learning regressions, frequent changes in policy parameters can cause agents to place more weight on more recent data, and less weight on data that is more distant, thus changing the gain. As the gain changes, the number of lagged

[7] For example, the first point on the graph, when the sum of the AR parameters is .02, is the average estimated lag length over the 10,000 simulations of an AR(2) model. The next point, .04, is also the average estimated lag length over 10,000 simulations of the AR(2) model, and so on.

[8] Because these are averages, the lag lengths need not be integer values.

[9] This is also true when an AR(1) rather than an AR(2) is simulated for values of the AR parameter between .01 and .99, i.e. the LR lag lengths are below the AIC lag lengths when persistence is low and above when persistence is high. The patterns do differ however. The LR pattern for an AR(1) is very similar to the pattern for the AR(2) in Figure 13, but the pattern for the AIC differs. For an AR(1) it is much flatter, nearly horizontal, than for an AR(2).

[10] See Orphanides and Williams (2003) for a discussion of the equivalence between a fixed gain under a Kalman filtering approach and a fixed window length under a rolling regression approach. The gain, which measures the sensitivity of the estimates to recent data points, is inversely related to the window length.

variables in the learning regressions changes which can bring about a change in estimated lag lengths.

Third, changes in the policy parameters themselves rather than the frequency of policy changes may also change the gain or window size during the learning transition. When policy parameters change, past data are less informative which can temporarily increase the optimal gain (decrease the window size). As the gain increases, the learning regressions also change potentially reducing estimated lag lengths.

Fourth, policy might be consistent with indeterminacy and hence sunspot equilibria during some time period. It can be shown that the solution under indeterminacy can contain more lags than the solution for parameter ranges with a determinate solution so that variation between these parameter ranges might cause variation in both estimated and theoretical lag lengths. In addition, there may be multiple solutions with different persistence characteristics leading to different estimated lag lengths. The focus in the current paper is on the impact of parameter changes under the assumption that the model remains determinate. Indeterminacy raises many issues beyond the scope of this paper, such as instability under learning, the selection of the equilibrium, and whether indeterminacy has actually occurred,[11] and thus the indeterminacy issue is left as a subject for further investigation.

The first of these potential explanations for variation in estimated lag lengths, changes in persistence, applies to both rational expectations and learning models. However, the second two apply only to models with learning. It is expected, and verified below, that variations in estimated lag lengths for data generated from learning models

[11] The paper by Sims and Zha (2004) discusses this issue and argues that allowing for stochastic volatility and relaxing very strong identifying restrictions overturns the Clarida, Gali, and Gertler (2000) indeterminacy result.

are larger than variations from data generated from models with rational expectations because learning models are subject to each of the first three reasons for variations in lag length while rational expectations models are subject to only the first reason.

To determine whether standard theoretical macroeconomic models can account for changes in the estimated lag length in VAR models due to changes in the parameters of the policy rule, two versions of a macroeconomic model very similar to the models used in Clarida, Gali, and Gertler (1999) and Honkapohja and Mitra (2003), one with rational explanations and one with learning, are simulated. Standard statistics for the specification of the lag length are then used to document how changes in estimated lag lengths vary with changes in the policy rule for the federal funds rate. The results show a clear association between the parameters of the policy rule and the estimated lag length with longer lag lengths associated with policies that place more weight on output deviations from target, less weight on inflation deviations from target, a higher value for the smoothing parameter, and, in the case of learning, an increase in the gain parameter. Furthermore, the sensitivity of lag length to changes in the parameters of the policy rule is greater when learning is present in the model than when expectations are formed rationally.

To establish that estimated lag length varies with the values of the parameters of the policy rule, the model is simulated under different parameter values, but during any given simulation the parameters are constant. Changes in the parameters during a particular simulation, that is, simulation of unanticipated changes in the parameters of the policy rule are also conducted. These results show that lag lengths respond more strongly to changes in smoothing than to changes in other policy parameters, and that the changes under learning are much more persistent and more subject to over or under shooting than the changes under rational expectations.

Finally, the paper asks whether actual data reveal tendencies identified from the simulations of the theoretical model and whether there is any evidence, based upon how lag lengths change with the parameters, on whether learning or rational expectations models are more consistent with the empirical evidence regarding changes in lag length. A regression of estimated lag lengths on estimated policy parameters, using a fixed window size, reveals support for the theoretical predictions. There is a strong significant positive relationship between smoothing and lag lengths, and a significant negative relationship between the coefficient on inflation and lag lengths, and no evidence that the parameter on output gap affects lag lengths. The paper concludes with an examination of other potential explanations for the empirical results.

2. Simulations under Rational Expectations and Learning:

The simulations are based upon a two lag version of the model discussed in Gali, Gertler, and Clarida (1999) and in Honkapohja and Mitra (2003).[12] The basic model used is

$$\tilde{y}_t = b_0 + \eta_1 y_{t-1} + \eta_2 y_{t-2} + (1-\eta_1-\eta_2)E_t y_{t+1} - \phi(i_t - E_t \pi_{t+1}) + g_t \quad (1)$$

$$\tilde{y}_t = y_t - \bar{y}_t \quad (2)$$

$$\bar{y}_t = (1-.95)*\ln(100) + .95\bar{y}_{t-1} + e_t \quad (3)$$

$$\pi_t = c_0 + \beta[\xi_1 \pi_{t-1} + \xi_2 \pi_{t-2} + (1-\xi_1-\xi_2)E_t \pi_{t+1}] + \lambda \tilde{y}_t + u_t \quad (4)$$

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[12] The model is solved using programs generously provided by Bennett T. McCallum on his web site at http://wpweb2k.gsia.cmu.edu/faculty/mccallum/research.html.

$$i_t = (1-\alpha_{i1}-\alpha_{i2})(\alpha_0 + \alpha_\pi \pi_t + \alpha_{\tilde{y}} \tilde{y}_t) + \alpha_{i1} i_{t-1} + \alpha_{i2} i_{t-2} + w_t \quad (5)$$

$$g_t = \theta_g g_{t-1} + \varepsilon_{gt}, \qquad \varepsilon_{gt} \sim N(0,1) \quad (6)$$

$$u_t = \theta_u u_{t-1} + \varepsilon_{ut}, \qquad \varepsilon_{ut} \sim N(0,1) \quad (7)$$

$$w_t = \varepsilon_{wt}, \qquad \varepsilon_{wt} \sim N(0,1) \quad (8)$$

The model is discussed in detail in Clarida, Gali, and Gertler (1999). Briefly, the first equation is an IS curve where the output gap at time t, $\tilde{y}_t$, depends negatively upon the real interest rate and positively upon expected future output since it is assumed that $\eta_1 + \eta_2 \leq 1$. The presence of lagged output on the right hand side allows for endogenous persistence and, as noted by Clarida, Gali, and Gertler, the primary justification for these terms is empirical though they can be motivated by the presence of some type of adjustment costs. The second equation defines the output gap as the deviation of output from the target level of output $\bar{y}_t$ which is assumed to be the natural rate of output. The dependence of current output on expected future output arises from consumption smoothing, and the presence of the real interest rate arises from intertemporal consumption decisions. The term g_t captures demand shocks and shifts the IS curve. The third equation defines how the target level of output, the natural rate, evolves over time. It is assumed that the process is an AR(1).[13]

The fourth equation is the aggregate supply curve, derived from a model of staggered nominal price setting, where inflation depends positively upon the output gap

[13] The model describes the evolution of the log of output. The third equation is normalized so that the unconditional mean of the level of output is 100. This is because the measure of output used in the empirical section is the industrial production index which is 100 in the base year.

and positively upon expected future inflation. This differs from the standard Phillips curve formulation in that expected future rather than expected current inflation affects current inflation, a difference that has important implications. As with the IS curve formulation, the presence of lagged inflation on the right-hand side is motivated mainly by empirical considerations, but as noted by Clarida, Gali, and Gertler, this is not entirely acceptable.[14] By assumption, $\xi_1 + \xi_2 \leq 1$. The term u_t captures marginal cost or markup shocks and brings about shifts in the relationship between inflation and the output gap.

The next equation, equation (5), describes the policy rule for the federal funds rate. The specification is a standard Taylor rule augmented by lagged interest rate terms to capture interest rate smoothing. Two lags of the interest rate are included. In the empirical section below, monthly data is used. With monthly data, the second lag of the federal funds rate is highly significant in the estimated policy equation. Thus, a two lag specification is adopted in the simulations.[15] It is assumed that $\alpha_{i1} + \alpha_{i2} \leq 1$ and that $\alpha_\pi \geq 1$. In this specification, the federal funds rate is increased when inflation or output rises above target with the strength of the response depending upon the values of the parameters of the policy rule.[16]

The final three equations specify the process followed by shocks to the IS, AS, and policy rules with shocks to the IS and AS curves assumed, as is standard, to follow a first-order autoregressive process and the shock to the policy rule to be white noise.

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[14] Further discussion of the reasons for including lagged or inertial terms in the AS and IS equations can be found in Woodford (2003) and Evans and McGough [2004]. Evans and McGough also reference additional papers where inertia is explicitly modeled. A recent paper by Smets (2003) fits a theoretical model in which lags appear to fit European data.

[15] Orphanides (2003) discusses interest rate smoothing in detail.

[16] A forward looking version of the policy rule was also examined. Because the results of the theoretical investigation do not vary substantially when a forward looking policy rule is introduced, the simpler formulation of the policy rule (5) is used.

Two versions of this model are simulated, a rational expectations (RE) version and a least-squares learning model (LE) where agents must estimate the values of the parameters of the policy rule. The two versions of the model differ according to how expectations are formed. The RE solution is obtained by first writing the model in the form

$$AE_t x_{t+1} = Bx_t + Cz_t \tag{9}$$

where $x_t = \left[y_t^T, k_t^T \right]^T$ is a vector of endogenous variables with y_t a vector of non-predetermined variables and k_t a vector of predetermined variables. Letting $n_x = n_y + n_k$ be the length of the vectors x_t, y_t, and k_t, the dimension of the matrix A is ($n_x x n_x$) as is the dimension of the matrix B. The dimension of the matrix C is ($n_x x n_y$). The vector z_t contains exogenous variables and is assumed to follow a vector AR(1) process

$$z_t = \Phi z_{t-1} + w_t \tag{10}$$

where the matrix Φ has dimension ($n_y x n_y$). The solution to this is the Markov decision rule

$$y_t = Mk_t + Nz_t \tag{11}$$

$$k_{t+1} = Pk_t + Qz_t \tag{12}$$

where the matrices M, N, P, and Q have dimensions ($n_y x n_k$), ($n_y x n_y$), ($n_k x n_k$), and ($n_k x n_y$). When a solution exists, it is derived using the techniques described in

McCallum (1998, 1999) and in Klein (2000).[17] The Markov decision rule is used to simulate data for the RE model.[18]

To simulate the model under learning, expectations of π_{t+1} and $\tilde{y}_{t+1}$ are generated by regressions of π_{t+1} and $\tilde{y}_{t+1}$ on g_t, u_t, w_t, π_{t-1}, $\tilde{y}_{t-1}$, and i_{t-1}.[19] It is assumed that there is a constant Kalman gain so that agents in the model use 120 observations in the regressions that determine the expectations.[20] From these regressions, forecasts of π_{t+1} and $\tilde{y}_{t+1}$ can be obtained from data on g_t, u_t, w_t, π_{t-1}, $\tilde{y}_{t-1}$, and i_{t-1} which agents in the model are assumed to know. With expectations determined through these regressions, the model described in equations (1) through (8) is used to simulate data.[21]

Both models yield time series for $\tilde{y}_t$, π_t, and i_t. Because most empirical investigations use the log level or growth rate of output rather than the deviation of output from trend, equation (2) is used to generate values for y_t from the simulated values of $\tilde{y}_t$. The 500 observations for these three variables are simulated 1,000 times for each set of parameter values. For a given simulation, data on y_t, π_t, and i_t are used to construct a three variable VAR model and to conduct tests of lag length.

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[17] See "Software for RE Analysis" by Bennett T. McCallum August 23, 2001 (Revised 02-17-04) at http://wpweb2k.gsia.cmu.edu/faculty/mccallum/Software%20for%20RE%20Analysis.pdf.
[18] In the simulations, the starting values are the unconditional means and 2,000 observations are generated. Only the last 500 observations are used in the analysis described below.
[19] In accordance with much of the learning literature, current exogenous variables but not current endogenous variables are assumed to be in the information set when expectations are formed.
[20] That is, at each point in time, agents use the current and previous 119 observations in forming expectations. Sensitivity to the choice of the window length (i.e. the gain parameter) is examined below.
[21] The starting values for the LE model are the RE solutions. The simulations proceed by first generating 1,000 values for the RE model. Then, using the same stochastic shocks for observations 1001 through 2001, both RE and LE data are generated, the RE data from the Markov decision rule stated in the text and the LE data from using least-squares rolling window (fixed gain) regressions to generate expectations. To minimize the dependence of the LE model on the RE starting values, only observations 1,501 - 2,000 are used in the analysis, i.e. observations 1,501-2,000 are generated under both RE and LE so that the only difference is the solution technique, the underlying structural shocks are identical.

In the theoretical model, the lag length of the RE solution is invariant to changes in policy parameters. However, estimated lag lengths do change when the policy parameters change. A rolling regression approach is used to estimate lag lengths. Each simulation generates 500 observations. The procedure is to first do AIC and LR lag length tests for lag lengths from 1 to 12 using observations 1-120. Then, roll the sample forward one time period and use observations 2-121 and repeat the tests. Continue until observations 481-500 are used. This gives a series of values for the optimal lag length for each window. Lag lengths are indexed by the end of the sample and the window size. An overall measure of the optimal lag length for each simulation is generated by averaging over the optimal lag length for each window. This process is then repeated 1,000 times and the average lag length over all 1,000 simulations is constructed. Summarizing, for a particular simulation of 500 observations and a particular set of parameter values, the rolling window approach yields a set of estimates for lag length, one for each window, which are averaged into a single measure of lag length. This is repeated 1,000 times and the average lag length over all 1,000 observations is calculated.

For a given set of parameter values for the simulated model, the expected value of the lag length at each point in time is identical and this rolling window approach does not add anything over and above simply estimating the model over all 500 observations and testing for the optimal lag length.[22] However, if there are structural breaks arising from unexpected changes in the parameters of the policy rule, as examined later in the paper, then this approach can identify changes in lag length that occur as a result of the

[22] If the number of simulations becomes large, the two approaches converge to the same optimal lag length. Similarly, if the number of observations generated for each simulation becomes large, the two approaches converge to the same value.

structural break.[23] A single measure of lag length over all 500 observations will not reveal lag length changes arising from unanticipated structural breaks.

The baseline simulations use the following values for the parameters:

Table 1

Parameter Values in the Simulations

IS Curve	Aggregate Supply	Policy Rule
$b_0 = 0$	$c_0 = 0$	$\alpha_0 = 0$
$\eta_1 = .25$	$\beta = .99$	$\alpha_\pi = 1.5$
$\eta_2 = .25$	$\xi_1 = .25$	$\alpha_y = .5$
$\phi = 1$	$\xi_2 = .25$	$\alpha_{i1} = .2$
$\theta_g = .4$	$\lambda = .3$	$\alpha_{i2} = .2$
	$\theta_u = .4$	

The parameter values are from various sources. Clarida, Gali, and Gertler (1999) use $\phi = 1$, $\beta = .99$, and $\lambda = .3$ and also let the constant terms be zero. The values of $\theta_g = .4$ and $\theta_u = .4$ are from Evans and Honkapohja (2003). The classic Taylor rule is represented with the values of $\alpha_\pi = 1.5$ and $\alpha_y = .5$, and the values of the smoothing parameters $\alpha_{i1} = .2$ and $\alpha_{i2} = .2$ are lower than the one lag estimates of .75-.91 in Orphanides (2003).[24] Larger values are investigated below. Less guidance is available for choosing values for the η's and ξ's and .5 is chosen for the values of $\eta_1 + \eta_2$ and $\xi_1 + \xi_2$. Equal values are chosen for convenience. The baseline window size for the

[23] While there is no theoretical reason to necessitate that the length of the rolling window in the learning model be the same as the length in the regressions used to determine optimal lag length, it seems reasonable to suppose that the same information set is used.

[24] Values on the low end are chosen to lessen the influence of this choice on the lag length estimates obtained from the simulated data and to reduce the chance of unstable paths arising in the learning models as the parameters of the model are varied.

regressions used to generate expected future values in the IS and AS equations is 120 observations for agents in the learning version of the model.

Figures 4a and 4b show how the estimated lag length in a VAR model including y_t, π_t, and i_t varies as the value for the sum of the smoothing parameters, $\alpha_{i1} + \alpha_{i2}$, varies from 0 to .8 in increments of .1 with all other parameters held at their baseline values.[25] Figure 4a is based upon the AIC while Figure 4b is derived from the LR test. The two figures exhibit similar patterns for the lag length selection as the smoothing parameters vary upward from zero, though the lag length is uniformly longer when the LR test is used. Initially, there is a slight decline in the lag length reaching a minimum when the two parameters sum to around .3 followed by an increase in the estimated lag lengths thereafter. In addition, as the degree of smoothing increases, there is an increasing separation between the lag lengths estimated from the data simulated under RE and LE, with the LE estimates increasing more rapidly than the RE estimates. Since many estimates of the smoothing parameter in one lag models, e.g. in Orphanides (2003) are in the range above .3, an increase in smoothing in this range increases the estimated lag length. Thus, any policy change that increases smoothing is expected to increase estimated lag lengths.[26, 27]

[25] Recall that the two parameters are assumed to be equal so that if the sum is .6, the parameter value for each lag is .3. The range was limited to .8 because beyond .8 there is a high likelihood of unstable paths emerging in the learning model. This happens because the models are locally but not globally stable and beyond .8 the likelihood of stochastic shocks occurring that are large enough to push the estimates from the regressions used to form expectations into the unstable range is high.

[26] It is only the estimated lag lengths that increase, not the lag lengths in the theoretical model. Changes in theoretical lag lengths may arise in more general models where the gain depends upon the frequency of parameter changes or for other reasons, but theoretical lag lengths do not change in this paper.

[27] In many of the simulation results, the differences in the estimated lag lengths shown in the figures are distinct, but not always large in magnitude. Experimentation with other parameter values and different stochastic structures (e.g. assumed variances of shocks) affect the magnitude of this difference. Because the simulations take a considerable amount of computer time (several days of computer time are required to

Figures 5a and 5b examine the relationship between changes in the parameter governing the strength of the response of the interest rate to deviations of inflation from target, α_π, and the average estimated lag length in the three variable VAR model. The parameter is allowed to vary between 1.0 and 3.5 in increments of .25.[28] Once again, the pattern is consistent across the AIC and LR tests and, as before, the LR test selects longer lags than the AIC. Initially, when α_π is equal to 1.0, the lags are longest and the distance between the RE and LE outcomes is the greatest. As α_π increases, the estimated lag lengths decline, with the LE lag lengths declining more quickly than the RE lag lengths until eventually leveling off and becoming nearly equal. Thus, according to these simulations, a policy change that reduces α_π will increase the estimated lag length with the LE lag length increasing more than the RE lag length.[29]

The next parameter examined is α_y. Figures 6a and 6b show how changes in the parameter governing the strength of the policy response to deviations of output from target, α_y, affect estimated lag lengths. This parameter varies between 0 and 5.0. Yet again, the pattern is consistent across the AIC and LR tests and, as before, and generally for the LR test versus the AIC, the LR test selects longer lags than the AIC. Initially, as α_y increases from zero, all else equal to baseline values, the estimated lag lengths fall

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generate each of the eleven graphs showing the simulation results), the model was not reparameterized to accentuate the differences in estimated lag lengths.

[28] When α_π is less than 1.0 there isn't a unique stable equilibrium.

[29] It is also possible to plot how the variances of y_t, π_t, and i_t change as the degree of smoothing, $\alpha_{i1} + \alpha_{i2}$, the policy parameter for inflation, α_π, the policy parameter for the output gap, α_y, and the window size vary. Two interesting results are first that the variance of the individual series for output, inflation, and the interest rate are not always lower under RE. There are cases where the LE model produces smaller variances. Second, the variance of output is minimized is at approximately α_π=1.5. Thus, the traditional Taylor rule value of 1.5 results in the output variance being nearly minimized with all other parameters at the baseline values given above.

reaching a minimum when α_y is approximately .5, then increasing until α_y equals approximately 2.0. When α_y increases above 2.0, the lag lengths decline when the AIC is used, and level off when the LR test is used. Interestingly, the point where estimated lag lengths are minimized, .5, coincides with the value of α_y usually assumed for the Taylor rule. The estimated lag lengths under the LE model are always larger than for the RE model. There is some indication that the difference in the estimated lag lengths for the LE and RE models increase as α_y increases as seen in previous diagrams, but this is not nearly as evident as for the previous two parameters examined and appears to be confined to the initial period when α_y increases from zero.

The final parameter examined is the gain.[30] Changes in this parameter should only affect the LE lag estimates, changes in the gain play no role in the RE simulations. Figures 7a and 7b show how the estimated lag length for the VAR model varies as the window size varies from 55 to 240.[31] Theoretically, there shouldn't be any change in the estimated lag length for the RE model as the window size changes. In all graphs the estimated lag length is nearly horizontal indicating that the 1,000 simulations used to obtain the average lag length for each value of the gain are precisely estimated.[32] The effect of changes in the window size on the estimated lag length is consistent across the

[30] Unlike the other parameters examined, this parameter is not affected directly by policy changes. Though the theoretical results are not fully worked out, it seems reasonable for unanticipated changes in policy to affect the optimal gain indirectly since observations prior to the policy change are less informative than observations after the change. If this is the case, then intuitively the gain should increase when there is a change in one of the policy parameters.

[31] The values are 55, 60, 80, 120, and 240. Values less than 55 caused quite a bit of instability in the LE model due to imprecise estimates of parameters, which are sometimes in the unstable range, and imprecise expectations. Values larger than 240 had little effect on the estimated lag length as adding more observations has very little affect on the precision of the parameter estimates once this many observations are used (and given the assumed baseline values of all other parameters).

[32] Averaging each of the estimates under RE across the values of the gain gives an even more precise estimate of the lag length as this is the result of averaging 5,000 independent simulations, 1,000 for each of the five values of the gain.

AIC and LR tests. In both cases the lag length declines as the window is increased eventually leveling off and maintaining a persistent deviation from the RE lag estimates. In all cases the estimated lag length is larger under LE, and is larger for the LR test.

3. Unexpected Changes in Policy Parameters

The previous simulations have constant parameters in any particular set of generated data. An additional set of simulations is conducted where this is no longer the case. In these simulations, there is an unanticipated change in policy at the midpoint of the sample and rolling window regressions as described above are used to see how the estimated lag lengths change as a result of the unanticipated change in the policy parameter.

Figure 8 shows how the lag length changes when there is an unanticipated change in the smoothness parameters, $\alpha_{i1} + \alpha_{i2}$, at the midpoint of the sample from .4 to .6 with all other parameters, including the window size, held constant.[33] The results are in accordance with the results in Figures 4a and 4b. These figures show that the difference in the estimated lag length should be larger with the LE model and should increase as the smoothing parameters increase. Initially, prior to the change in the smoothing parameters, there is a difference in the estimated lag lengths equivalent to the difference shown in Figures 4a and 4b with the lag length estimated to be longer under the LE model. In the time period where the change occurs (recall that it is assumed that agents know all exogenous variables dated t and earlier in the theoretical model) the estimated lag length begins increasing and continues increasing with the spread between the LE and

[33] If, as in Evans and Ramey (2003), the gain increases (window size decreases) as a result of the change in parameter, then the lag length changes shown in the following graphs would be magnified.

RE results increasing until the estimated lag lengths stabilize at higher values. Note two things, however, regarding the LE model. First, there is overshooting. Initially the lag length increases above the level at which it eventually stabilizes, the value shown in Figures 4a and 4b. Second, the LE model takes much longer to stabilize after a shock than the RE model. In the RE model the transition period is equal to the size of the estimation window the indicating immediate response to the shock.[34] In the LE model the main effects take place within this period, but also persist beyond.[35]

Figure 9 repeats the exercise in Figure 8 but varies the parameter on inflation, α_π, rather than the smoothness parameter. The value is unexpectedly reduced from 1.5 down to 1.1. Figures 5a and 5b predict that, after adjusting to the shock the lag length should increase, though the increase is small with the LR test and RE data. Figure 9 shows this effect, but also shows that the LE model takes a considerable amount of time to adjust to the unanticipated shock.

The longer transition period under LE is in evidence again in Figure 10 where the parameter on the output gap, α_y , is varied from .5 to 1.5. As in Figures 6a and 6b, the

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[34] Recall that the estimation window used to determine lag length and to estimate parameter values can be different from the window used by agents in the LE model to form expectations, though they are assumed to be the same here. When there is a change in a coefficient in the policy rule, the estimation window can contain observations from both regimes. For example, suppose there is a regime change at observation 500 and there are 120 observations in the estimation window. As the window is rolled through the data, the data will be from the first regime up to the window containing observations 380-499. The next window will contain observations 381-500 and have one observation from the new regime, one from the old. This continues until observations 500-620, which contains one observation from the old regime and the rest from the new, and thereafter the window only contains observations from the new regime. Thus, there are 120 time periods when observations from the two regimes are mixed and this is the transition period shown in Figures 8-10 for the RE estimates.

[35] For this reason the sample size is doubled in the unanticipated shock diagrams in Figures 5-7. In the previous diagrams, the sample size is 500, whereas here it is 1,000. The index in the graph from 1,001 to 2,000 is because 1,000 observations are generated prior to drawing the sample to avoid dependence upon initial values. More precisely, 4,000 observations are generated. The first 2,000 are eliminated to avoid dependence on starting values. Then, starting at the RE values for observation 2,000, observations 2,001 through 4,000 are generated under both RE and LE. The first 1,000 of these are eliminated to avoid dependence of the LE results on the RE starting values and it is these observations that are indexed from 1,001-2,000.

lag length increases with little increase in the difference between the LE and RE results. Under LE the response after the parameter change is to increase, reach a peak, and then fall below the eventual long-run value. Thus, in all cases the transition dynamics under LE are much longer than under RE.

4. Empirical Results:

This section examines data for the U.S. economy to determine if the types of relationships between the parameter values of the policy rule and lag length implied by the theoretical model are present in U.S. data. Monthly data for the federal funds rate, the inflation rate, and the log of industrial production from 1959:01 through 2002:12 are analyzed as described below.[36] In addition, monthly data on real potential output are obtained from Congressional Budget Office's estimate of potential output, which is used to construct an index for potential IP.[37] This is accomplished by regressing industrial production on the log of real potential output and using the residual as an estimate of the output gap, and then converting the gap to a percentage deviation following the formula in Taylor (1993). Figure 11 shows the CBO estimates of the negative of the monthly unemployment gap along with the estimated gap implied by the estimates of potential IP used here.[38] The two series are very similar which should reduce any concerns over the measure of potential industrial production used below.

The next step is to use rolling regressions to obtain a series of changes in the values of the policy parameters over time and changes in the estimated optimal lag length

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[36] The data are obtained from the FRED II data set available on the St Louis Fed web site.
[37] The monthly data are interpolated from the CBO's quarterly data which itself is an interpolation of annual estimates.
[38] The unemployment gap is the actual unemployment rate minus the NAIRU unemployment rate from the CBO.

according to AIC and LR tests.[39] To obtain changes in the parameter values in the policy equation, the following regression, a Taylor rule augmented by smoothing that is identical to the policy rule used in the theoretical model, is estimated using rolling window regression

$$i_t = a_0 + a_\pi \pi_t + a_{\tilde{y}} \tilde{y}_t + a_{i1} i_{t-1} + a_{i2} i_{t-2} + w_t \qquad (13)$$

where the right-hand side variables are inflation, the output gap, and lags of the federal funds rate. As noted above, a two-lag specification is adopted because with the monthly data used here, the second lag of the interest rate is highly significant. Consistent with the theoretical section, the window is assumed to be 120 observations. The parameter values at each step are saved and indexed by the last observation in the window of data used in the estimation. This gives five series of parameter values, one for each parameter in equation (13).

These, however, are not the parameters needed for the regressions below. Recall the policy rule used in the theoretical model, equation (5)

$$i_t = (1 - \alpha_{i1} - \alpha_{i2})(\alpha_0 + \alpha_\pi \pi_t + \alpha_{\tilde{y}} \tilde{y}_t) + \alpha_{i1} i_{t-1} + \alpha_{i2} i_{t-2} + w_t . \qquad (14)$$

The parameters from equation (13) must be transformed to match those from equation (14). The transformations are

$$\hat{\alpha}_0 = \hat{a}_0 / (1 - \hat{a}_{i1} - \hat{a}_{i2}) \qquad (15)$$

$$\hat{\alpha}_\pi = \hat{a}_\pi / (1 - \hat{a}_{i1} - \hat{a}_{i2}) \qquad (16)$$

$$\hat{\alpha}_{\tilde{y}} = \hat{a}_{\tilde{y}} / (1 - \hat{a}_{i1} - \hat{a}_{i2}) . \qquad (17)$$

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[39] It is possible that a change in any parameter in the model will change the estimated lag length. The focus here is on the parameters of the policy rule because policy changes are very often the focus of theoretical research and because changes in policy parameters are examined in the simulations of the theoretical model in previous sections of the paper.

These parameters, along with the sum $\hat{a}_{i1} + \hat{a}_{i2}$ which is an estimate of $\hat{\alpha}_{i1} + \hat{\alpha}_{i2}$, are used in the regressions that follow.

To obtain the optimal lag length for each sample period, again use rolling regression, but this time using a VAR model comprised of the three variables output (not the output gap, just as in the theoretical section), inflation, and the federal funds rate.[40] For each window of 120 observations, two estimates of lag length are used, the Akaike Information Criterion (AIC) and the likelihood ratio test (LR), both of which are standard in the empirical literature. The optimal lag lengths for each sample period for each of the two methods of determining lag length are calculated and saved. The estimated lag lengths are indexed by the last time period in the window. This yields two time series of values for the optimal lag length, one from the AIC and one from the LR test.[41]

The simulations predict the following relationships. First, an increase in the sum of the two smoothing parameters results in increased lag lengths so long as the sum is not too small, i.e. beyond around .3, a value smaller than typical estimates of this parameter. Second, an increase in the coefficient on inflation in the policy rule decreases lag lengths. Third, the relationship between changes in the parameter governing the response of the federal funds rate to changes in the output gap depends upon the value of the parameter at the time of the change. Over some ranges, when the parameter is very small or very large, the relationship is negative, whereas at intermediate values the relationship is positive. Overall, there is no strong prediction. However, at the .5 value implied by the Taylor rule, any movement of the parameter in either direction causes an increase in the optimal lag length. Fourth, changes in lag lengths are generally larger under the LE

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[40] This is a standard baseline model in the empirical monetary policy and income causality literature. For example, see the discussion on page 20 of the recent paper by Bernanke, Boivin, and Eliasz (2004).
[41] The sample period follows Taylor (1993) and others and is 1987:1 to 2002:12.

model than under the RE model, and the effects more persistent when the changes are unanticipated, though this cannot be easily distinguished empirically. However, long periods of adjustment following a parameter change are suggestive of a LE interpretation of the data. Finally, the results from the unanticipated shocks in Figures 8 through 10 imply that lag lengths are more responsive to changes in the smoothing parameter than to changes in the other parameters examined. Thus, this parameter is predicted to have the strongest effects on lag lengths.

To examine these hypotheses empirically, the series for the lag lengths is regressed on the series of parameters of the policy rule, i.e. the lag length as determined by either the AIC or the LR test is regressed upon the series of the sum of the two smoothing parameters, the series of parameter values for the response to inflation, and the series of parameter values for the response to the output gap. The t-statistics from these regressions are shown in table 2.[42]

[42] The right-hand side variables in the regressions are estimates of the true policy parameters and are therefore generated regressors. Standard correction techniques do not incorporate the use of rolling windows to generate parameter estimates, so such corrections are unavailable. For this reason, the t-statistics may overstate the true level of significance. Though the degree of the overstatement is unknown, the significance level of the smoothing parameter is generally very high indicating a strong likelihood that the results would remain significant after correction. Monte Carlo techniques might be applied, but the results make this appear unnecessary.

Table 2

Parameter	Model A AIC	Model A LR	Model B AIC	Model B LR
Lagged dep.	0.91	23.5[a]		
$\hat{\alpha}_0$	-0.68	-0.96	-0.74	-2.89[a]
$\hat{\alpha}_{i1}+\hat{\alpha}_{i2}$	3.83[a]	0.41	4.43[a]	4.14[a]
$\hat{\alpha}_{\pi}$	-0.60	-0.67	-0.69	-2.80[a]
$\hat{\alpha}_{\tilde{y}}$	-0.12	-0.90	-0.10	0.65

[a]Significant at the 1% level, [b]significant at the 5% level, [c]significant at the 10% level. Standard errors for Model A are corrected for heteroskedasticity and Model B for heterskedasticity and first order autocorrelation using the Newey-West procedure.

There are two models in the table, Model A and Model B. These differ according to whether the lagged value of the dependent variable, i.e. the lagged value of the estimated optimal number of lags, appears on the right hand side of the estimated model. Because the rolling window regressions add and drop a single observation from the sample at each step, it is likely that the optimal lag estimates are serially correlated. In addition, it is possible that the residuals are heteroskedastic. The two models present two approaches to this problem. Model B uses the Newey-West correction for both heteroskedasticity and first-order autocorrelation. Model A, which includes the lagged value of the dependent variable, is corrected for heteroskedasticity only. Under each heading in the table there are two sets of results, one for lag lengths determined using the AIC and one for the LR lag length tests.[43]

The results in the table, when significant, strongly support the theoretical predictions. First consider the results for the sum of the smoothing parameters, $\hat{a}_{i1} + \hat{a}_{i2}$, the variable identified in the theoretical section as having the largest impact on estimated lag lengths. In three of the four cases, the LR test for model A being the one exception, the sum of the smoothing coefficients is positive and significant at greater than the 1% level.

According to the theoretical investigation, an increase in the parameter on inflation in the policy rule, a_{π}, should cause estimated lag lengths to fall. It is also shown in Figure 9 that an unanticipated change in this parameter does not have a large impact on lag lengths, though they do change some. The results in the table support this interpretation. In only one case, the LR test for Model B, is the parameter significant and negative as predicted. In this case it is significant at greater than the 1% level. In all other cases the parameter is insignificant, with t-statistics that are less than one in magnitude.

Finally, consider the parameter on the output gap, a_y. The theoretical prediction for the output gap parameter depends upon the value of the parameter before and after the change, but as noted above at the value implied by the Taylor rule, a change in the parameter in either direction will result in an increase in lag length because the estimated lag length was smallest at this value. It was also noted above that the lag lengths did not

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[43] In the simulations, the relationship between the output gap policy parameter, α_y, and the estimated lag length is non-linear as shown in Figure 5. Adding the squared value of α_y to the regressions does not change the results appreciably.

respond strongly to unanticipated changes in this parameter. In all four cases in the table the parameter is insignificant.

Overall, the results accord with the theoretical predictions. The theoretical model implied that the smoothing parameters have the largest impact on lag lengths and that the impact is positive. The empirical results generally support this prediction. The theoretical model also implied that the inflation parameter is negatively related to lag lengths, and in the case where this parameter is significant, it is in agreement with this prediction. The theoretical model also shows that the relationship between the parameter on the output gap and lag lengths is ambiguous overall, though positive in the neighborhood of the .5 value used by Taylor. The results from the unanticipated shock implied that even near the Taylor value of .5 the effect on lag lengths is present, but not large. In the empirical results, this parameter is insignificant in all cases.

In the discussion of the simulation of the theoretical model and the calculation of estimated lag lengths under RE and LE, several differences between the RE and LE results were noted. In general, lag lengths are longer under LE, there is a longer adjustment period after unanticipated changes in policy parameters under LE, and under LE there is often under or over shooting of the estimated lag length immediately following an unanticipated change in a policy parameter. Unfortunately, these differences are difficult to distinguish empirically. Because the lag length under RE is not known with certainty, it is difficult to determine if the estimated lag lengths are longer than expected under the RE assumption. Similarly, because the time required to recover from a shock under RE is not known, it is difficult to determine that adjustment is longer than expected under RE. However, overshooting after an unanticipated shock, e.g.

after an unanticipated change in the smoothing parameter as in Figure 8, is a feature of LE models but not RE models. The evidence for overshooting is mixed. Figures 1a and 2a, which show the results for the AIC, do offer some evidence for overshooting in the neighborhood of policy changes. For example, Orphanides (2003) breaks the sample he examines into two parts, one that ends in the second quarter of 1979 and a second that begins in the third quarter of 1982 to allow for structural change in policy. In Figure 1a there are spikes in the lag length estimates in the 1979-1982 time period that can be interpreted as overshooting. However, this is confined to the AIC results, the results using the LR test do not show such a spike.[44] This may be because LR tests are not as sensitive as the AIC to such changes in parameters.

5. Other Interpretations of the Empirical Results

This section discusses additional reasons why estimated lag lengths in empirical models might vary with the changes in the parameters of the policy rule.

In a recent paper, Orphanides (2003) examines the evolution of monetary policy over time using a generalized Taylor rule as a means of categorizing different policy regimes. To examine the changes in monetary policy regimes over time empirically, Orphanides (2003) uses a form of the Taylor rule that encompasses several popular

[44] There is a distinction between the RE and LE results. Under LE, lag lengths vary with the gain but under RE they do not. Thus, if it can be established that endogenous variations in the gain are associated with variations in lag length, then this would provide support for LE models. Implementation of this test requires an estimate of the gain at each point in time, or in subsamples, to compare to estimates of lag length. Estimation of a time-varying optimal gain is beyond the estimation strategy used here, the gain is assumed not estimated, so direct implementation of this test is not possible. However, as noted below, it is expected that theoretical models will show that the optimal gain lengthens (i.e. the window size shortens) when there are changes in policy. Estimation of the optimal time-varying gain is a subject of further research.

variants of the rule as special cases. Using quarterly real-time data[45] on the federal funds rate, and year ahead forecasts of the output gap and inflation from two different sources, Greenbook forecasts and the Survey of Professional Forecasters (SPF), and three different sample periods, the full sample, a sample with data prior to 1979:3, and a sample that begins in 1982:3, Orphanides finds that this model describes policy decisions with a large degree of consistency over the sub-samples and across the two sets of forecasts.

There is a difference in the estimates across the sub-samples. The results obtained by Orphanides imply that policy responded more forcefully to the level of the output gap rather than the growth rate of output during the mid 1960's to the late 1970's, a period he calls the Great Inflation, and has responded less forcefully to the output gap relative to inflation after the Great Inflation, i.e. after the 1970's. Orphanides argues that these changes likely made an important contribution to the improvement in macroeconomic stability over the last few decades.

To the extent that improved stability is consistent with shorter lag lengths in estimated models, these results imply that over time the coefficient on the inflation component of the policy rule has increased and the coefficient on the output coefficient has decreased at the same time the economy has become more stable. Thus, this implies that a decrease in the gap coefficient and an increase in the inflation coefficient potentially correspond to a fall in estimated lag lengths due to increased stability. This is consistent with the results of table 2, but since there is only one significant outcome out of eight for these two parameters, the inflation coefficient from Model B for the LR test

[45] Real-time data is the data actually available to policy makers when decisions are made, e.g. data on variable such as GDP and the CPI free of subsequent revisions, and prevailing views on the sign and magnitude of the output gap.

is significantly negative, the results do not provide overwhelming support for this interpretation.

A recent paper by Orphanides and Williams (2003) provides another potential explanation for changes in estimated lag lengths over time. This paper examines the role that incomplete information about the structure of the economy, particularly policy rules, plays in the dynamics of the system. In the model, agents rely upon adaptive learning mechanisms to form expectations and update their knowledge regarding the structure of the economy. This paper shows that as policy makers place more weight on inflation in determining optimal policy, the economy will take less time to return to equilibrium in response to shocks. This result holds under perfect information and under learning, though persistence increases when learning is present. The quicker return to equilibrium can be reflected in shorter lags in the system or a decrease in the coefficient measuring persistence. Thus, this implies that as the coefficient on inflation is increased, persistence decreases, which is in agreement with the results above to the extent that reduced persistence is reflected in shorter estimated lag lengths.

A recent paper by Evans and Ramey (2003) provides one more potential explanation for changes in the lag length of the system describing the economy over time. In their paper, changes in the policy rule may, or may not, cause changes in the forecast rule used by agents, and the paper gives the exact conditions when the forecast rule will change. In the cases where the forecast rule does change, there is also a change in the stochastic process followed by variables in the model. In particular, as the frequency of policy changes increases, the optimal gain increases (the window size decreases) and inflation becomes more persistent which could be reflected as longer lag lengths in empirical models. The frequency of policy changes is not addressed by the empirical specification used in the current paper. However, the main result to note here is that endogenous gain, i.e. endogenous window size, is another channel through which estimated lag lengths may change. For example, if the gain is endogenous, changes in policy parameters can temporarily increase the gain if agents know or suspect that a policy change has occurred, thereby potentially temporarily increasing estimated lag length. In terms of the investigation in this paper, this is a combination of the effects of parameter changes which can increase persistence and increase the gain, both of which can increase estimated lag lengths.

6. Conclusion

This paper investigates the relationship between changes in the policy rule for the federal funds rate and the estimated lag length in typical VAR models. The first part of the paper establishes a theoretical link between lag lengths and policy parameters. Using a popular prototype macroeconomic model of the type discussed by Gali, Gertler, and Clarida (1999) and used more recently by Honkapohja and Mitra (2003), the model is solved under both learning and rational expectations for various values of the policy parameters. For each set of policy parameters, the model is used to generate data on the output gap, inflation, and the interest rate from which data on output, inflation, and the interest rate are derived. These data are analyzed using a VAR model and the optimal number of lags is calculated under both the AIC and LR test. Using this procedure, the paper establishes that an increase in interest rate smoothing results in increased lag lengths at usual parameterizations, an increase in the coefficient on inflation in the policy rule decreases lag lengths but the effect is not large, the relationship between changes in the parameter governing the response of the federal funds rate to changes in the output gap is positive if the parameter is initially at the value suggested by the Taylor rule, but again the effect is not large, an increase in the window size (i.e. a reduction in gain) decreases lag lengths, changes in lag lengths are generally larger and more persistent under the LE assumption than under the RE assumption and sometimes exhibit over or under shooting, and the largest observed responses of lag lengths occurs in response to changes in the smoothing parameter.

Having established a theoretical link between policy parameters and estimated lag lengths, the paper next uses actual rather than simulated data to determine if the predicted relationships are present in U.S. data. A regression of changes in lag lengths on changes in parameters as estimated with a rolling window (fixed gain) reveals support for the theoretical predictions. There is a significant positive relationship between smoothing and lag lengths, a significant negative relationship between the coefficient on inflation and lag lengths when it exists, and no effect on lag lengths from changing the parameter on the output gap. Finally, alternative explanations for the empirical results are investigated.

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Lags Estimated by the AIC and the
First Smoothing Parameter

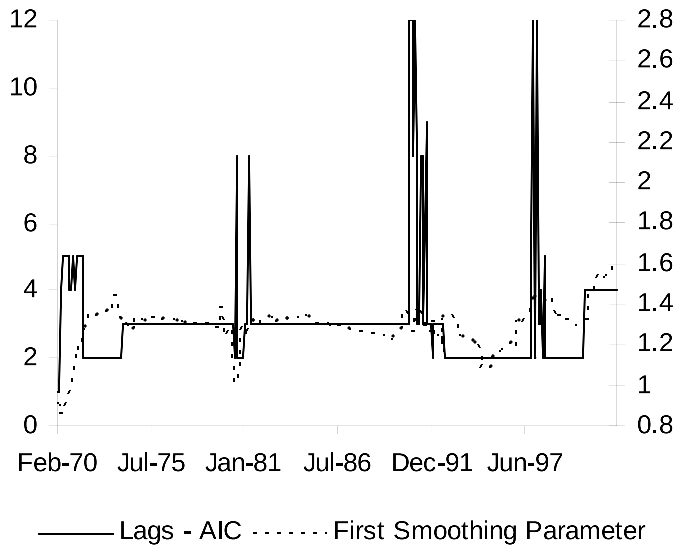

Figure 1a

Lags Estimated by the LR Test and the
First Smoothing Parameter

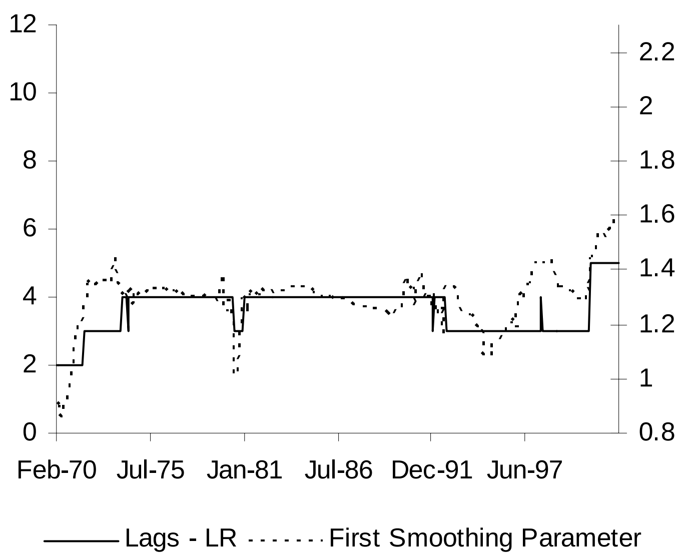

Figure 1b

Lags Estimated by the AIC and the
Sum of the Smoothing Parameters

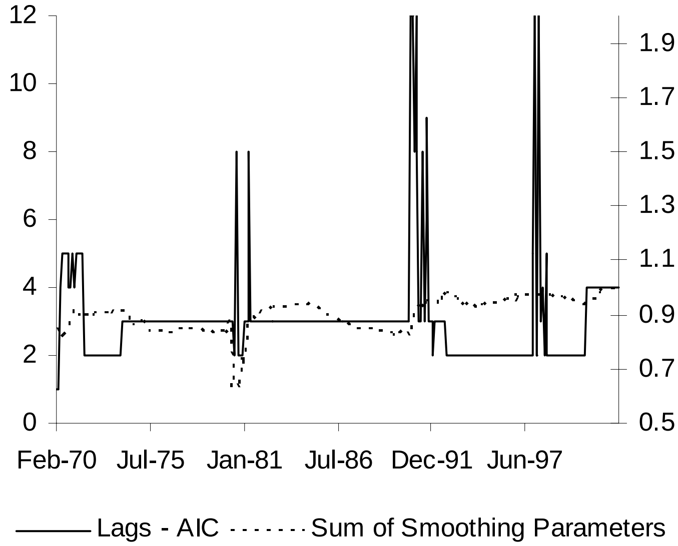

Figure 1c

Lags Estimated by the LR Test and the
Sum of the Smoothing Parameters

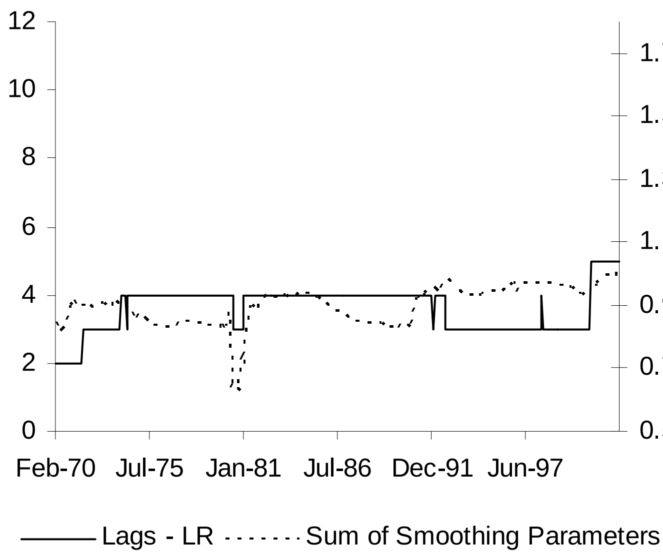

Figure 1d

Lags Estimated by the AIC and the
First Smoothing Parameter

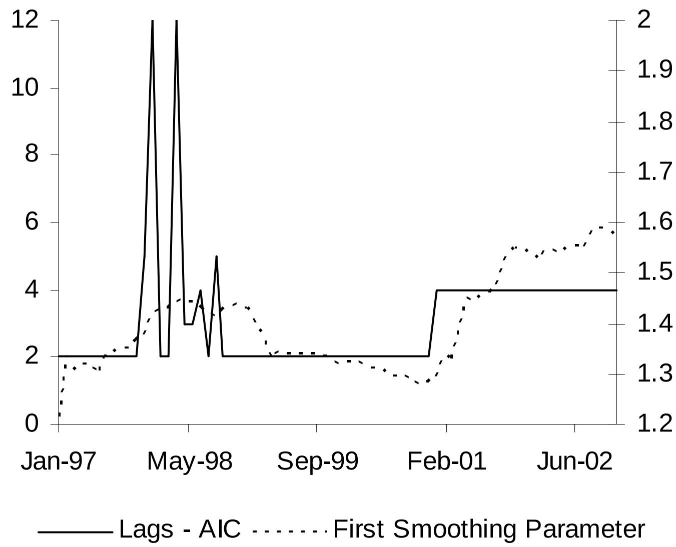

Figure 2a

Lags Estimated by the LR Test and the
First Smoothing Parameter

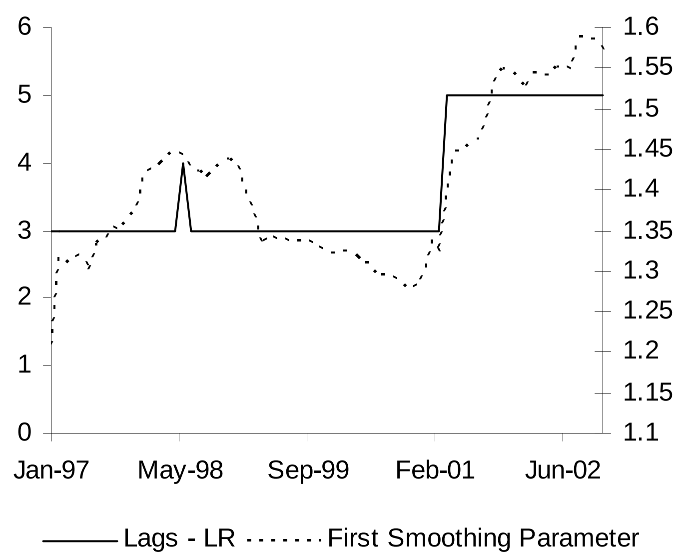

Figure 2b

Lags Estimated by the AIC and the
Sum of the Smoothing Parameters

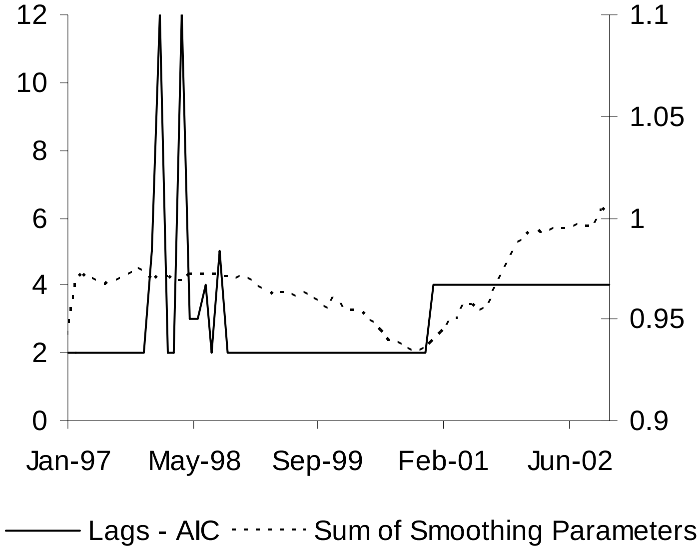

Figure 2c

Lags Estimated by the LR Test and the
Sum of the Smoothing Parameters

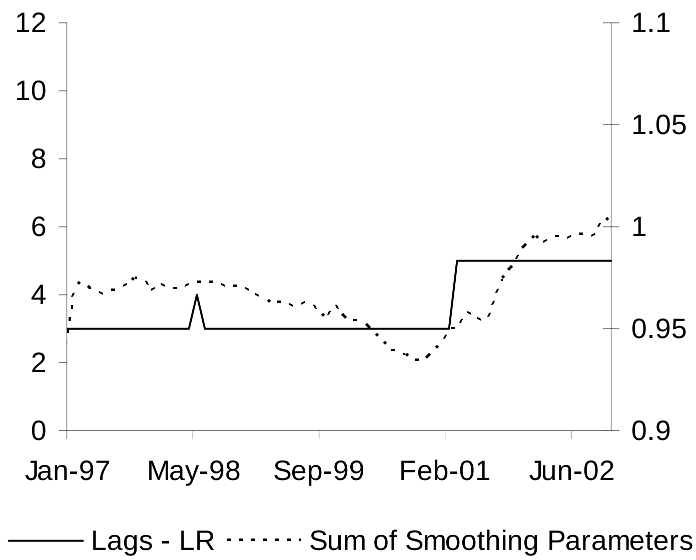

Figure 2d

Estimated Lag Lengths and the Parameters of an AR(2)

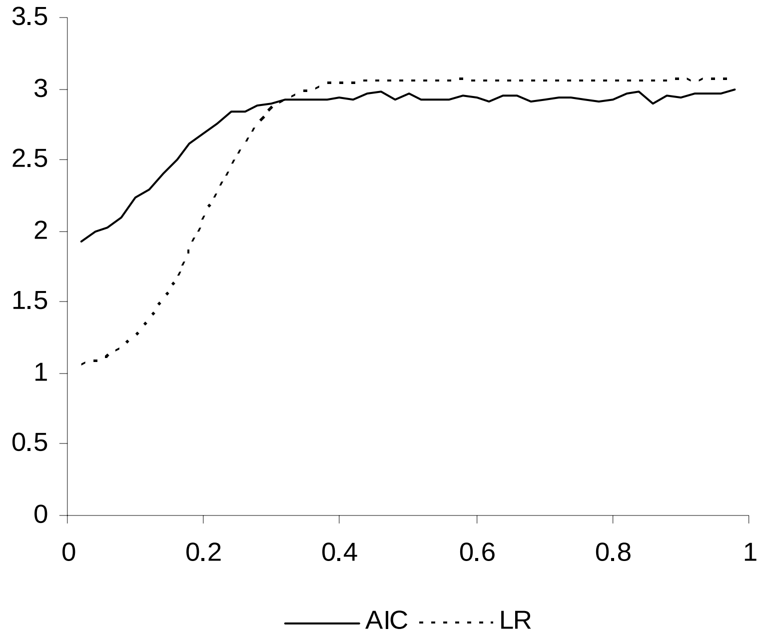

Figure 3

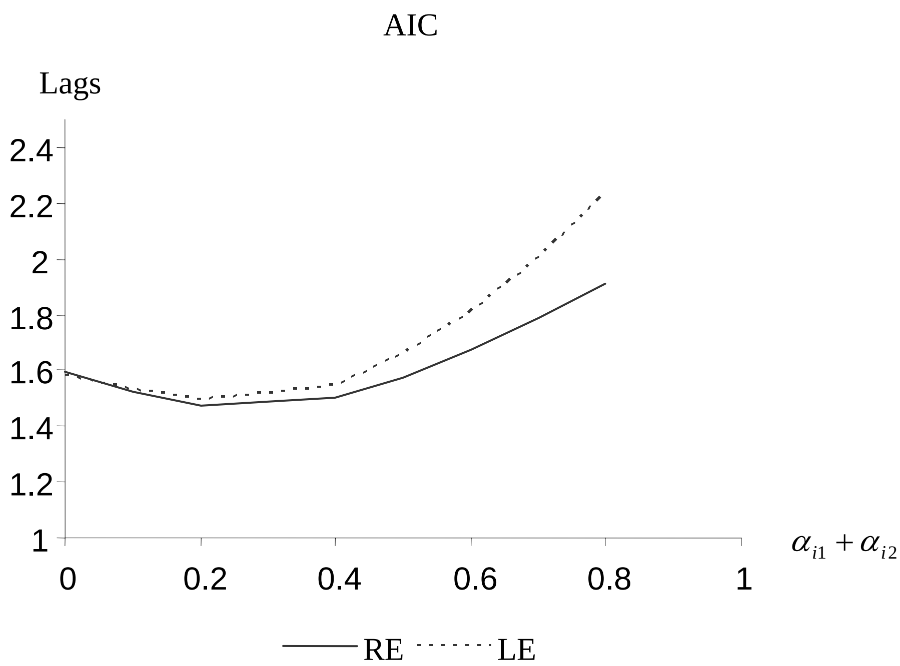

Figure 4a

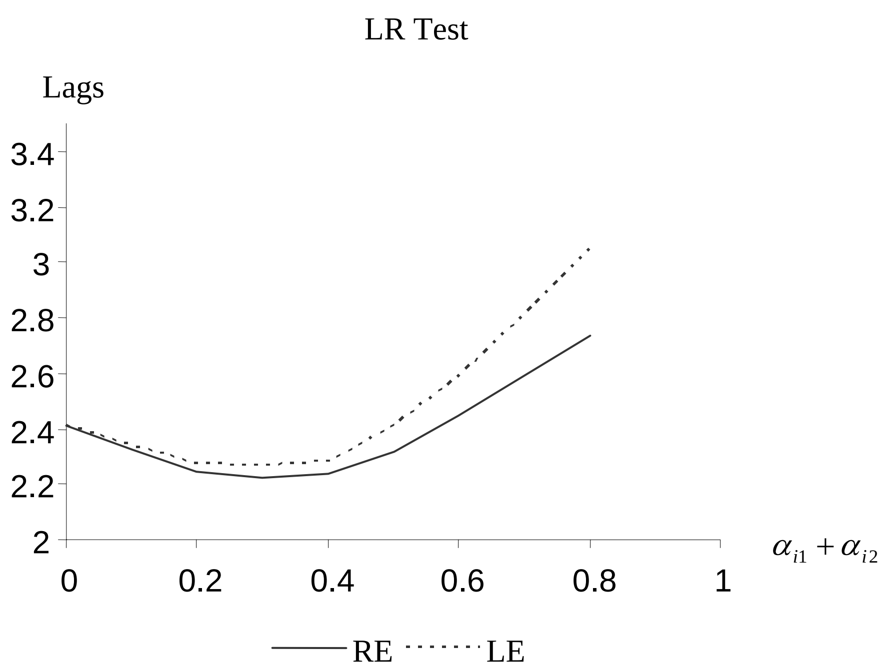

Figure 4b

AIC

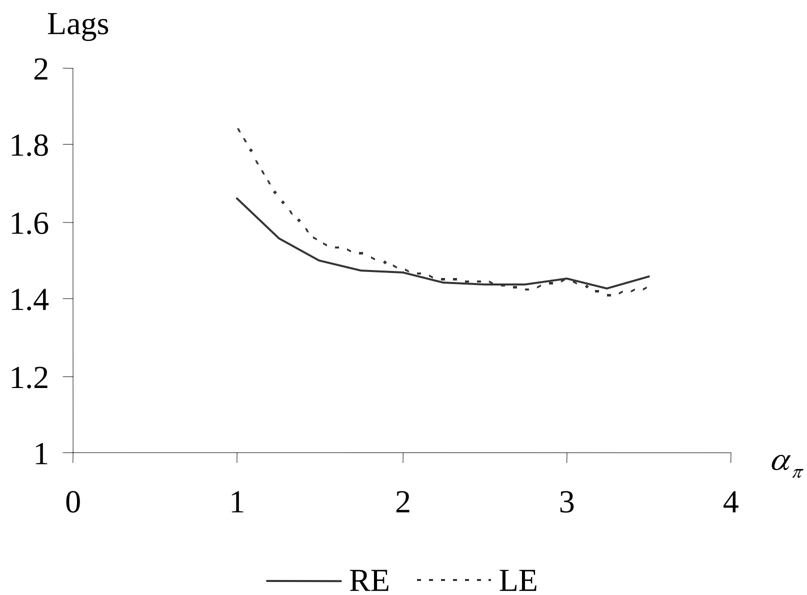

Figure 5a

LR Test

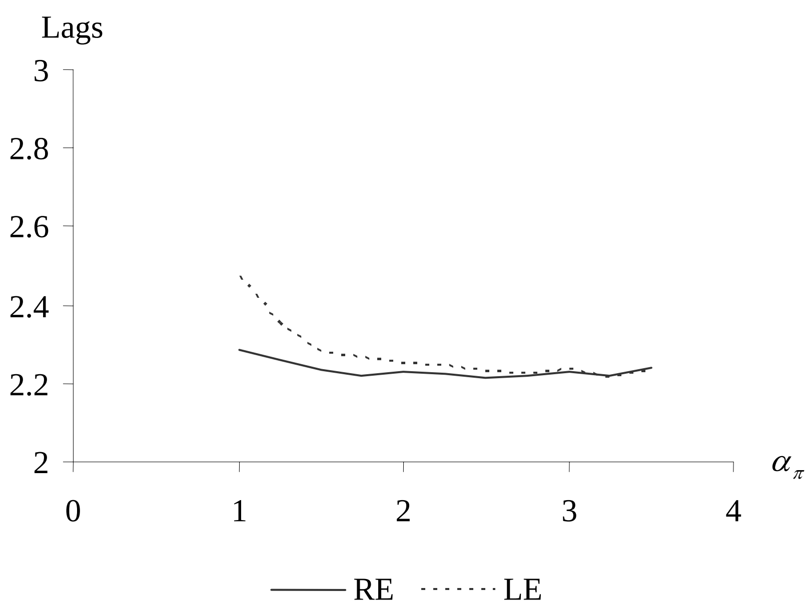

Figure 5b

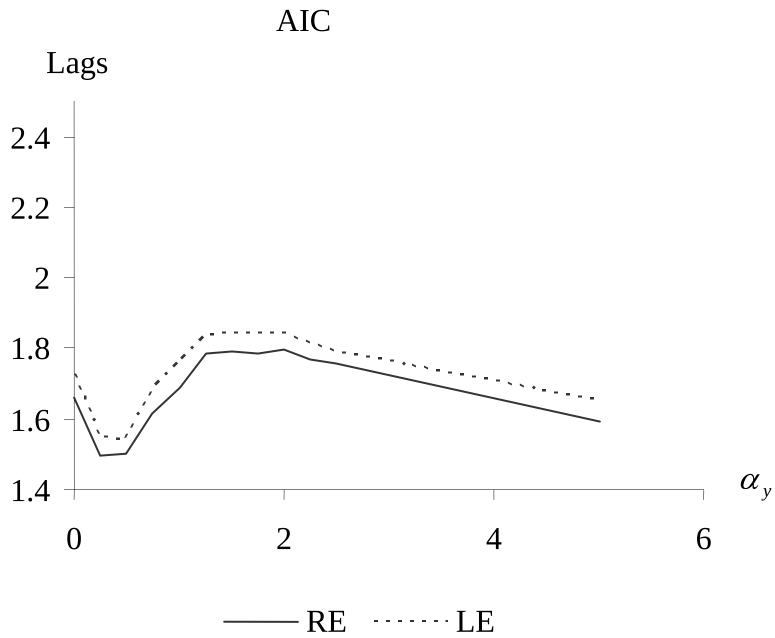

Figure 6a

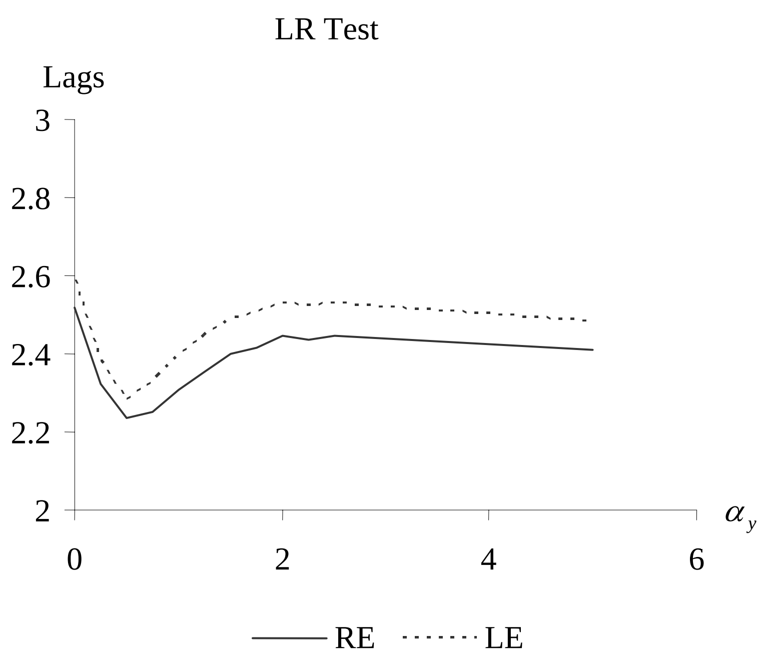

Figure 6b

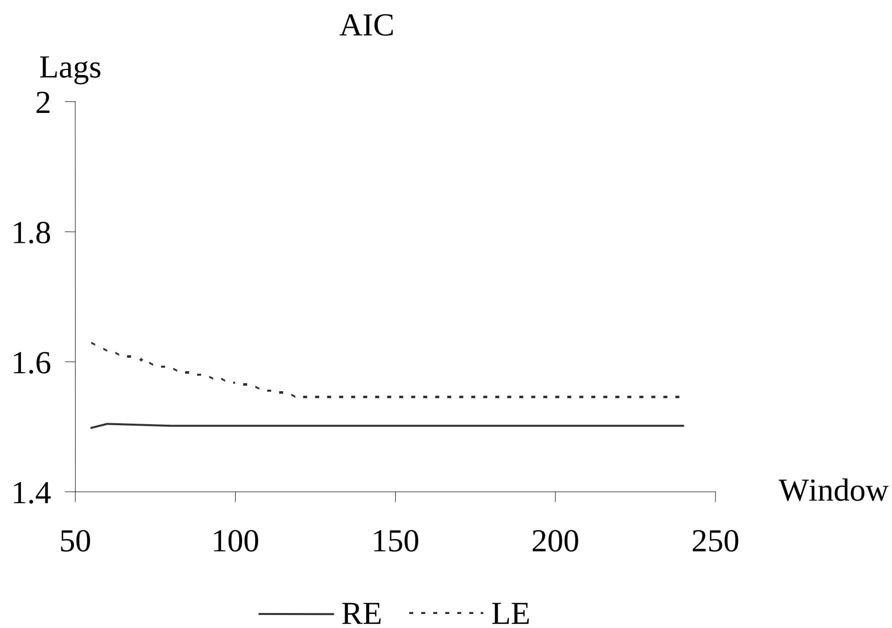

Figure 7a

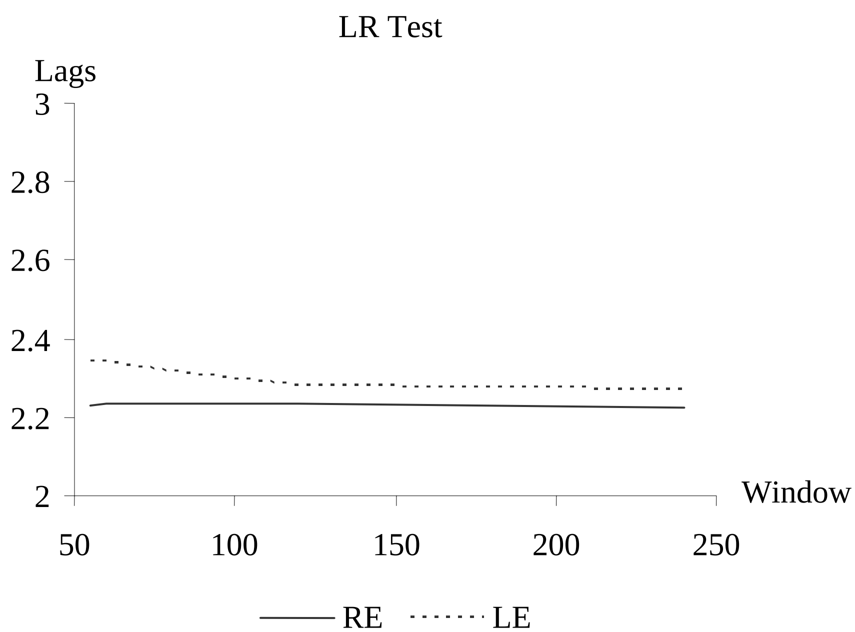

Figure 7b

Shock to $\alpha_{i1} + \alpha_{i2}$ (from .4 to .6)

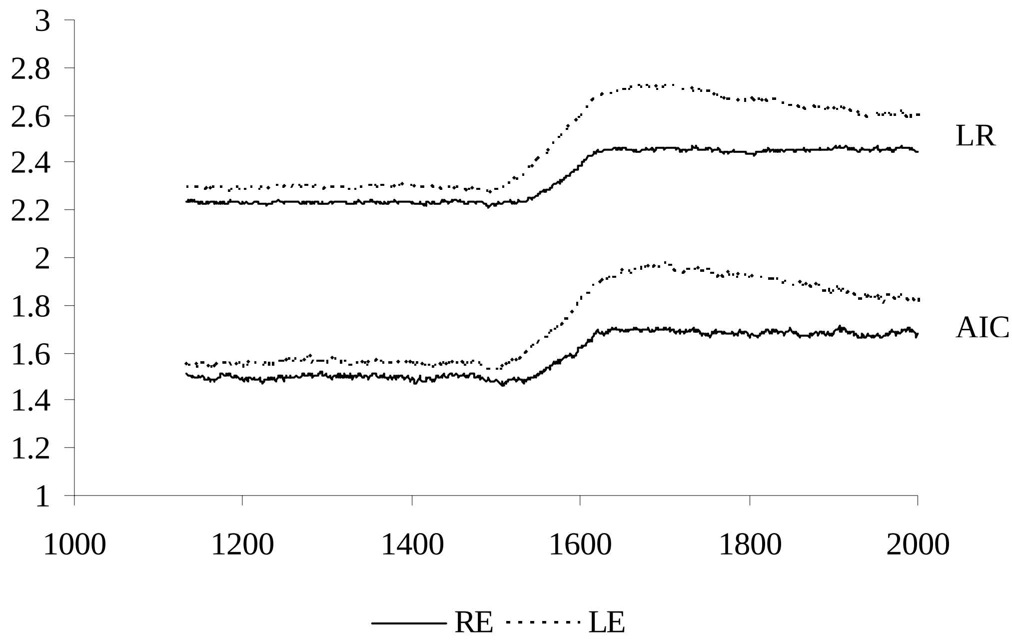

Figure 8

Shock to α_{π} (from 1.5 to 1.1)

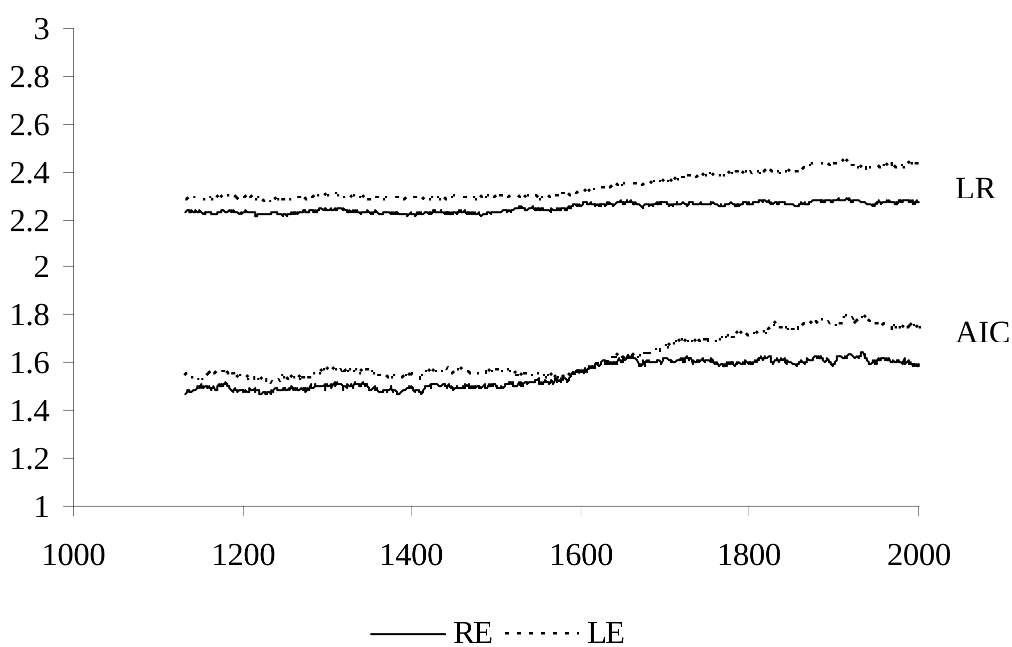

Figure 9

Shock to α_y (from .5 to 1.5)

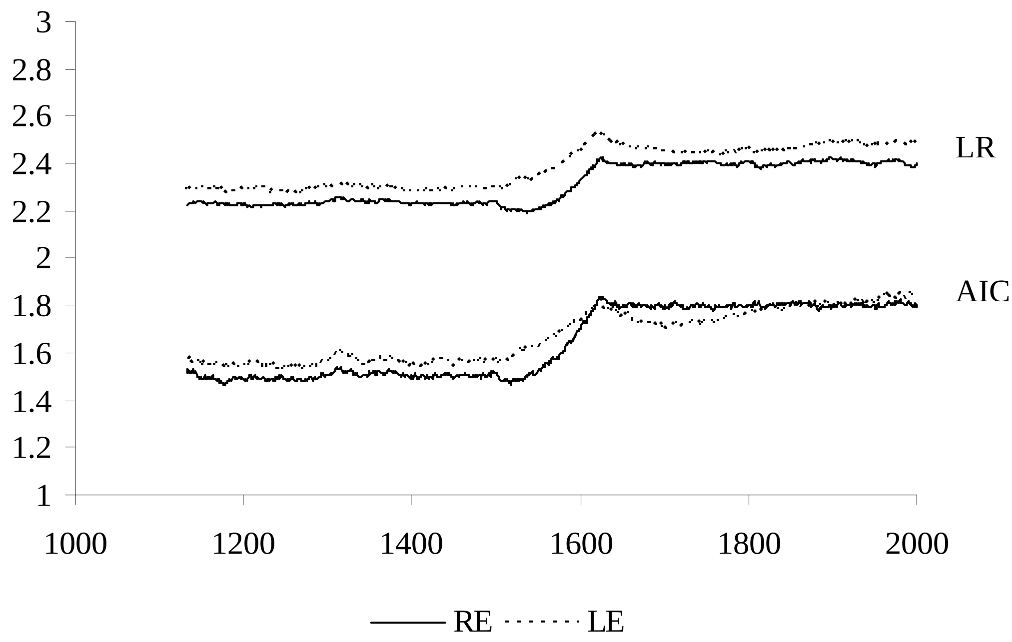

Figure 10

Output Gap as Measured by IP and UN
Relative to their Natural Rates

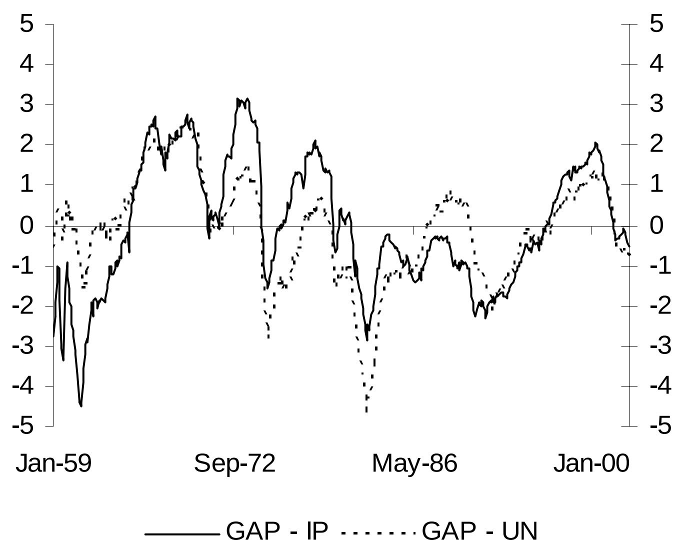

Figure 11