

Staggered Labor Contracts and Monetary Policy

Robert Amano
Stephen Murchison
Malik Shukayev*

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ABSTRACT

There is a growing body of research exploring the importance of time-dependent nominal wage contracts for monetary policy analysis. The typical form of these contracts specifies a nominal wage over its duration while leaving hours to be demand determined. Erceg, Henderson and Levin (2000) and Christiano Eichenbaum and Evans (2005) conclude that adding this type of labour contract to a standard sticky-price model is crucial for understanding the real macroeconomic effects of monetary policy. Barro (1977) and Hall (2005), however, emphasized that in the presence of monetary shocks, the aforementioned wage contracts imply an implausibly high degree of inefficiency in labour allocation. The inefficiency arises due to the “allocative” role of wages in such contracts: hours are demand determined at the realized real wage. In reality, many labour contracts specify both nominal wages and hours, or subject to institutionally imposed constraints on hours of work. In this paper we modify staggered labour contracts to specify (i.e. fix) both nominal wages and hours. Despite this additional rigidity, we find that the real effects of monetary shocks are drastically diminished. In fact, in absence of other nominal frictions, monetary shocks are neutral. Moreover, even in the presence of sticky prices, monetary shocks are nearly neutral. Further, the welfare cost of modified labour contracts relative to a flexible wages/flexible hours benchmark is rather small. Thus, the main finding of this paper is that labor contracts that specify both wages and hours serve as an effective buffer for the real economy against monetary shocks, even with sticky goods prices. This finding stresses importance of modeling details about how nominal frictions are introduced.

*Amano, Murchison and Shukayev: Research Department, Bank of Canada, 234 Wellington Street, Ottawa, Ontario, K1A 0G9, Canada. Opinions expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of Canada or its staff. Any errors or omissions are ours.

1. Introduction

There is a growing body of research emphasizing the importance of nominal wage friction for monetary policy analysis. Erceg et al. (2000) demonstrate that in a model with sticky goods prices, staggered nominal wage contracts are necessary to generate a meaningful policy trade off between volatilities of output and inflation. Levin et al. (2005) and Amano et al. (2007) show that in a model with both sticky nominal goods prices and sticky wages, optimal monetary policy is determined predominantly by the wage rigidity and is much less responsive to sticky goods prices. In a positive analysis framework, Christiano et al (2005) conclude that nominal wage rigidities are also crucial for building models that are able to match macroeconomic responses to monetary shocks. In fact, a version of their model with the nominal wage frictions alone is capable of closely matching impulse responses to monetary shocks constructed from the US macroeconomic data.

However, other researchers have noted some problems with the way sticky-wage literature treats labour contracts. More specifically, with the standard assumption of sticky wage models that requires workers to supply any number of hours demanded by firms at the pre-fixed contractual nominal wage. Barro(1977) and Hall(2005) pointed out that this assumption implies an implausible degree of inefficiency of labour contracts in the face of monetary shocks. Workers would often find themselves off their supply curve, working longer or shorter hours than desired at the realized real wage rate. The authors argued that there must be ways to protect efficiency of long-lasting labour contracts from monetary shocks, and that the realized real wages do not necessarily have much "allocative" consequences for the actual hours worked over the life of those contracts.

Another reflection of the inefficiency involved in the standard wage contracts was

noted by Amano et al (2007). They show that in the absence of a full, quarter-to-quarter indexation of nominal wages,¹ a standard sticky wage model implies a quantitatively large, exploitable relationship between a fully anticipated (and modest) trend inflation and output (or employment). This goes against one of the main tenets of monetary economics that in the long run there is no such exploitable relation between output and expected inflation.

This paper proposes a simple and intuitive modification to the standard sticky-wage model. It assumes that labour contracts specify (i.e. fix) not only the nominal wage rate, but also the hours supplied over their duration. Despite that additional rigidity, we find that the modified labour contracts serve as an effective shield for the efficiency of labour hours from expected or unexpected inflation. Furthermore these contracts buffer the real side of the economy from monetary shocks. In fact, in the absence of other nominal rigidities, monetary shocks are completely neutral. While with sticky goods prices in addition to modified labour contracts, monetary shocks are nearly neutral. Thus, the proposed modification not only impacts the labour market side of the model economy, but also drastically diminishes the real effects of nominal price rigidities in the goods market.

The intuition for these neutrality results of the modified model is quite simple and does not depend much on modeling details. For that reason, we state it here before describing the model. It is seen clearly from the comparison of the effects of monetary policy shocks in the modified model relative to those of the standard model. Let us start from the case in which staggered wage (labour) contracts is the only source of nominal rigidities. For concreteness, suppose there is a monetary shock that leads to higher aggregate price level. This unexpected price increase reduces the real wage rate of the workers who are locked in pre-set nominal

¹As Taylor (1999) notes, nominal wage contracts are typically adjusted once a year.

wage contracts. In the standard model, firms respond to falling real wages by making workers work longer hours. This leads to inefficient dispersion of hours across workers and to changes in output, consumption and investment. In short, monetary shocks have a large effect on real variables and welfare.

Now, in the modified model, an unexpected price level increase also reduces the real wages of pre-set labour contracts, but it does not affect pre-set contract hours. The only hours that could be affected are the ones that are being reset in the current period. Those contracts, however, are fully flexible and thus take the current price level increase in account. As a result, monetary shocks have no effect on labour hours, aggregate output, consumption, investment, or welfare.²

In the case with sticky goods prices in addition to staggered wage contracts, intuition is only slightly more subtle. In the standard model an unexpected price level increase erodes both pre-set wages and pre-set goods prices. Firms with lower relative price of their products have to produce more to satisfy higher demand for their less expensive output. These firms can increase output relatively cheaply by hiring more of less expensive hours from the workers whose real wages have been eroded by the price level increase. The result is a large and costly output and hours dispersion, as well as large effects on real output, consumption, and welfare.

On the contrary, in the modified model, the only labour hours that could adjust in response to higher demand are the hours of the newly renegotiated labour contracts. But those contracts are fully flexible in the current period. Thus, as firms with lower relative

²This perfect neutrality result is somewhat dependent on another standard feature of the sticky-wage model that assumes existence of a representative household, each member of which has the same level of consumption irrespective of his/her wage. It would be interesting to see how much our results change if one allows for heterogeneity in consumption induced by real wage differences.

prices try to increase output, they have to hire "expensive" extra hours at the fully flexible real wages. This makes output increases costly to firms, and consequently, the dispersion of output and hours smaller. General equilibrium requires that the aggregate price level increase that caused all these adjustments must have been smaller than would be the case in the standard model. Thus, a monetary disturbance has a much smaller effect on prices and on real variables in the modified model than in the standard model.

In absence of a full quarter-to-quarter indexation of nominal wages, exactly the same line of reasoning as above will also hold for a fully expected price increase due to trend inflation. In the standard model, that is what causing the exploitable relation between trend inflation and output noted by Amano et al (2007). In the modified model there is no such relationship: trend inflation has a negligible effects on output and employment.

All of the described properties of the modified contracts would not be very useful if they came at a high cost. There is of course an efficiency loss of having completely inflexible labour hours in pre-set contracts. Those hours would not be able to adjust immediately to real shocks (e.g. productivity shocks) changing the demand for labor. We quantify this welfare loss in a model with modified contracts relative to a benchmark with fully flexible wages and hours. Robustly in the parameter space, the welfare loss was found to be quite small. In all the plausible cases, it is orders of magnitudes smaller than the welfare loss implied by the standard model.

It is important to emphasize that we do not think of our modified contracts as being literally true. Despite many labour contracts being subject to institutional constraints on hours of work, there is also a lot of flexibility. Overtime hours do exist. For some types of salaried workers, like university professors, the statutory hours of work may have little

actual meaning. What is important for our results though, is that workers are not forced to work longer or shorter hours in response to fully observable (expected or not) inflation shocks. There seem to be effective safeguards against that in real life. For example, the legal requirement on higher overtime pay may serve as a deterrent for longer hours of work in response to inflation spikes, while salaries stated in annual terms may reduce incentives of firms to reduce hours in response to negative price level shocks. Further, the short-run neutrality results stated in this paper do not make our model a good monetary model. They do, however suggest that contractual constraints on hours of work may be a simple and inexpensive mechanism that protects efficiency of labour exchange in the face of monetary shocks. Our results also stress that modeling details about wage contracting have important consequences not only for the labour market, but also for the real effects of other nominal rigidities, such as sticky prices.

Section 2 below briefly reviews empirical studies of labour/wage contracts, as well as uses simple demand-supply diagrams to argue that modified contracts make sense. Section 3 sketches a modified sticky-wages/sticky-prices model and highlights the proposed modification to the standard labour contracts. Section 4 presents results, and Section 5 concludes. Some derivations are relegated to appendices.

2. Literature review and simple demand-supply analysis

Before we delve into model details let us briefly review some of the papers that look at micro-data of labour contracts as well as use simple demand-supply diagrams to illustrate that modified contracts might very well be more efficient than the standard ones.

The empirical literature that seems relevant to our focus on wage/hours contracts can

be (crudely) subdivided into two topics: 1) papers estimating the real wage elasticity of labour supply; and 2) micro data studies looking at the patterns of nominal wage adjustments. The main conclusion of the early literature on the labour supply elasticity (see Pencavel (1986) for a review) was that it varies between different population groups. However the labour hours of full-time workers, particularly males, were found to be nearly completely unresponsive to (uncompensated) real wage changes. This, taken at its face value, suggests that real wage changes induced by inflation and nominal wage contracts should also have little effect on hours of full-time workers.

A more direct approach would be to look at the actual patterns of adjustment in matched wage/hours pairs. Unfortunately, much of empirical literature on wage adjustments focused exclusively on wages, particularly on whether they are downward inflexible. Some of the papers that do look at hours as well are: 1) Kahn and Lang (1991) and Osberg and Phipps (1993) looking at Canadian data; 2) Lundberg (1985) and Dickens and Lundberg (1985) with US data; and 3) Heckel, Bihan, Montornes (2007) analyzing French data. The main conclusions from these studies relevant for our discussion are the following three. First, there is a substantial clustering of actual hours at the institutionally imposed upper bounds on regular working hours. It is particularly evident from the French data when the working hours were reduced from 39 to 35 hours per week, in 2000. Second, there are some evidence that labour contracts do in fact come as tied wage-hours pairs: workers with longer hours receive higher wages (controlling for other factors). Third, Taylor-type staggered wage contracts, with (typically) annual indexation are quite prevalent (see also a review in Taylor (1999) on this last point).

Guided by these findings, our modified model assumes that staggered labour contracts

fix both nominal wages and hours for the duration of the contracts (four quarters) in a staggered fashion. This is the only difference with the standard sticky-wages model which assumes that only wages are fixed, while hours are demand determined. To clarify intuition, Figures 1 and 2 use simple supply-demand diagrams to show the effect of price and labour demand increases on hours worked. Each figure shows a firm's downward sloping demand for working hours of a hypothetical worker, as well as an upward sloping supply curve of the same worker.

On Figure 1 we can see the effect on hours worked of an increase in the price level. As the price level rises from P_0 to P_1 (e.g. due to monetary shock), the real wage implied by a fixed nominal wage rate W , falls. In the standard model the workers are *required* to supply more hours to satisfy the firm's demand at the realized real wage. So hours worked increase along the demand curve, from the original level L_0 to L_1 . Generally speaking there is nothing that says that this outcome has to be the case. In principle, hours after the price increase could as well be supply determined, or be at any other intermediate value between the labour supply and the labour demand curves. These different values of hours cannot be ranked without additional structure, because a firm would prefer hours that's are closer to the demand curve, while workers would prefer hours closer to their supply curve. In our modified model realized real wages have no allocative implications for working hours, because those are fixed at L_0 by the same contract that fixes nominal wages. The assumption of fixed hours is no more restrictive than the standard one, that forces an increase in hours to meet demand. Moreover, the fixed hours outcome is exactly what an efficient response to price level changes would be in the world with fully flexible nominal wages (and hours). In the fully flexible world, nominal wages would increase proportionally to the price level to keep

real wages constant, and to keep hours (optimally) unchanged at L_0 . Thus, our modification has the advantage of replicating the efficient response of hours to price level changes.

Figure 2 depicts a response of hours to a labour demand increase. If the firm's demand for hours increased, due to productivity or general demand shocks, then the efficient (flexible wages/ flexible hours) response to that would be to increase hours from L_0 to L^{opt} . Assuming that the price-level does not change, the standard model will force hours to overshoot to L_1 instead.³ In the modified model, hours will stay fixed at L_0 . That is certainly a source of inefficiency of the modified model relative to the flexible wages/ flexible hours world. The degree of inefficiency involved needs to be quantified in a calibrated model.

3. Model

The standard model with both nominal wage and nominal price rigidities is well known, and described succinctly in Erceg et al. (2000). For this reasons we will be brief in describing the features of the modified model that remain unchanged from the ones in the standard model. The essence of the proposed modification of labour contracts will be highlighted in the description of the labour market.

A. Firms and price setting

Final Good Production

The final good, Y_t , is produced by assembling a continuum of intermediate goods Y_{jt} for $j \in [0, 1]$ that are imperfect substitutes with a constant elasticity of substitution ε . The

³Depending on the nature of the shock, there would be further adjustments in hours either down or up, as the price level adjusts. This however does not change the general point that the response of hours would overshoot the efficient response.

production function is constant-returns to scale and is given by

$$(1) \quad Y_t \equiv \left[\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The final-good sector is competitive; profit maximization leads to the following input-demand function for the intermediate good j :

$$(2) \quad Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\varepsilon} Y_t,$$

which represents the economy-wide demand for good j as a function of its relative price P_{jt}/P_t and of aggregate output Y_t . Imposing the zero-profit condition in the sector provides the final-good price index P_t :

$$(3) \quad P_t = \left(\int_0^1 P_{jt}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}.$$

Intermediate Good Production

After the current period productivity shock z_t is realized, a representative intermediate-good producing firm j , rents capital K_{jt}^d and labour L_{jt}^d to produce Y_{jt} units of the intermediate good j with a Cobb-Douglas production function

$$(4) \quad Y_{jt} = \exp(z_t) \left(A_t L_{jt}^d \right)^\alpha \left(K_{jt}^d \right)^{1-\alpha}.$$

The stochastic productivity shock, z_t , follows an AR1 process with normally distributed innovations

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \quad \varepsilon_t^z \sim N(0, \sigma_z^2),$$

while the economy-wide level of labour-augmenting technology, A_t , grows every period at the rate $g \geq 1$

$$A_t = g^t A_0, \quad A_0 = 1.$$

This intermediate (-good producing) firm j maximizes the expected discounted sum of future (real) profits

$$(5) \quad \max \mathbf{E}_t \sum_{\tau=0}^{\infty} (\beta^\tau \Lambda_{t+\tau} / \Lambda_t) \left[\frac{P_{j,t+\tau}}{P_{t+\tau}} Y_{j,t+\tau} - \frac{W_{t+\tau}}{P_{t+\tau}} L_{j,t+\tau}^d - q_{t+\tau} K_{j,t+\tau}^d \right],$$

subject to the demand for product j (2), the production function (4), and a timing restriction on its price adjustment described next.

The producers of intermediate goods set prices according to Taylor-style staggered nominal contracts of fixed duration.⁴ Specifically, firms set the price of their good for J quarters and price setting is staggered so that every quarter, a fraction $1/J$ of firms is resetting prices. Further, the cohorts are fixed, so the same fraction $1/J$ of firms reset prices every J quarters together.

Each quarter intermediate firms pay (nominal) dividends

$$(6) \quad D_{jt} = P_{jt} Y_{jt} - W_t L_{jt}^d - P_t q_t K_{jt}^d,$$

to the households, whose marginal utility of income, Λ_t , shows in the stochastic discount factor $\beta^k \Lambda_{t+k} / \Lambda_t$ of future profits in (5).

The maximization problem thus consists of choosing capital and labour inputs each quarter, as well as, prices, every J quarters, in order to maximize (5) subject to the economy-wide demand for product j (2) and the production function (4).

Substituting the demand constraint (2) into the (5) we can write the Lagrangian for the intermediate firm's problem as follows

$$\max \mathbf{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left\{ \begin{array}{l} \left(\frac{P_{j,t+\tau}}{P_{t+\tau}} \right)^{1-\varepsilon} Y_{t+\tau} - \frac{W_{t+\tau}}{P_{t+\tau}} L_{j,t+\tau}^d - q_{t+\tau} K_{j,t+\tau}^d \\ -s_{j,t+\tau} \left[\left(\frac{P_{j,t+\tau}}{P_{t+\tau}} \right)^{-\varepsilon} Y_{t+\tau} - \exp(z_{t+\tau}) \left(A_{t+\tau} L_{j,t+\tau}^d \right)^\alpha \left(K_{j,t+\tau}^d \right)^{1-\alpha} \right] \end{array} \right\},$$

⁴Having Calvo-type pattern of price adjustment will not change our results.

where the lagrange multiplier $s_{j,t+\tau}$ has an interpretation of the real marginal cost of output.

The first-order conditions for labour and capital inputs are:

$$(7) \quad \frac{W_t}{P_t} \frac{1}{A_t} = s_{jt} \frac{\alpha Y_{jt}}{A_t L_{jt}^d}$$

$$(8) \quad q_t = s_{jt} \frac{(1 - \alpha) Y_{jt}}{K_{jt}^d}.$$

Since all firms face the same factor price-ratio, the optimal capital labour ratio is constant across firms

$$\frac{K_{jt}^d}{L_{jt}^d} = \frac{1 - \alpha}{\alpha} \frac{W_t/P_t}{q_t}.$$

It follows then, that the marginal output cost is also common across firms:

$$(9) \quad s_t = \frac{1}{\exp(z_t)} \left(\frac{1}{\alpha} \frac{W_t}{P_t A_t} \right)^\alpha \left(\frac{q_t}{1 - \alpha} \right)^{1-\alpha}.$$

The first-order optimality conditions for the firms that are resetting prices in quarter t , yield the following optimal reset price:

$$(10) \quad P_{jt}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbf{E}_t \sum_{\tau=0}^{J-1} \beta^\tau \Lambda_{t+\tau} P_{t+\tau}^\varepsilon Y_{t+\tau} s_{t+\tau}}{\mathbf{E}_t \sum_{\tau=0}^{J-1} \beta^\tau \Lambda_{t+\tau} P_{t+\tau}^{\varepsilon-1} Y_{t+\tau}}.$$

Since the Taylor pricing structure allocates firms within fixed cohorts through time, firms resetting prices all behave identically. We can therefore omit the j subscript from the optimal price and write P_t^* . In equilibrium, there are now only J different prices in the economy and, following the definition in (3), the aggregate price index P_t becomes:

$$(11) \quad P_t = \left(\frac{1}{J} \sum_{\tau=0}^{J-1} P_{t-\tau}^* \right)^{\frac{1}{1-\varepsilon}},$$

where $P_{t-\tau}^*$ is the optimal price of the $1/J$ portion of firms who reset their price τ periods ago.

B. Labour market and wage setting

Composite labour

Following much of the sticky-wage literature, we assume that workers sell their services to Labour Aggregators. These labour aggregator firms assemble composite labour from differentiated, individual-specific labour hours according to the following aggregation function:

$$(12) \quad L_t \equiv \left[\int_0^1 H_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

where θ represents a constant elasticity of substitution.

Labour aggregators in turn sell this composite labour to firms, at the economy-wide price (the aggregate wage) W_t . Labour aggregators are price takers in both their input and output markets. Each unit of differentiated labour H_{it} costs these aggregators W_{it} . Wages, W_{it} , are determined as part of the household's optimization problem described below, and will be reset in a staggered fashion. As for hours, H_{it} , this is where we modify the standard sticky price model.

In the standard model nominal wages are set for I quarters, while hours are free to adjust to the labor aggregators' demand. As a result, labour aggregators chose a cost-minimizing quantity of hours on a quarter-by-quarter basis:

$$(13) \quad H_{i,t+s} = \left(\frac{W_{it}}{W_{t+s}} \right)^{-\theta} L_{t+s},$$

$$(14) \quad s = 0, 1, 2, \dots, I-1,$$

where W_{it} is the nominal wage rate set s quarters ago, W_{t+s} is the current aggregate wage

rate, and L_{t+s} is the current aggregate demand for labour from the intermediate firms.

We modify that by assuming that labor contracts specify both nominal wages and hours to be supplied over the duration of the contract. Thus, the labor aggregator can only optimize over currently reset contracts. For those, currently reset contracts, they choose hours based on the following profit-maximization problem:

$$\max_{H_{i,t+s}} \mathbf{E}_t \sum_{s=0}^{\infty} (\beta^s \Lambda_{t+s} / \Lambda_t) \left[W_{t+s} L_{t+s} - \int_0^1 W_{i,t+s} H_{i,t+s} di \right],$$

subject to $L_{t+s} = \left[\int_0^1 H_{i,t+s}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$. The currently reset wages and hours will be in effect for I periods, so we can write the following first-order condition for the choice of $H_{i,t}$:

$$\mathbf{E}_t \sum_{s=0}^{I-1} (\beta^s \Lambda_{t+s} / \Lambda_t) \left[W_{t+s} L_{t+s}^{\frac{1}{\theta}} - W_{i,t} H_{i,t}^{\frac{1}{\theta}} \right] = 0.$$

Solving it for $H_{i,t}$ we obtain the demand function

$$(15) \quad H_{i,t} = \left[\frac{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \Lambda_{t+s} W_{t+s} L_{t+s}^{\frac{1}{\theta}}}{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \Lambda_{t+s}} \frac{1}{W_{i,t}} \right]^{\theta}.$$

For the contracts that remain unchanged, both nominal wages and hours remain fixed at the levels set when they were locked in those labor contracts.

Household Optimization

Following again the sticky-wage literature we assume existence of a multi-agent, infinitely-lived representative household. Each member i ($i \in [0, 1]$) of this extended household has the same level of consumption C_t , nominal bonds B_t and capital holdings K_t , but supply individual-specific labour hours H_{it} to the labour aggregators.

The representative household's problem is

$$\max_{C_t, B_t, W_{it}, H_{it}} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \gamma \int_0^1 \frac{H_{it}^{1+v}}{1+v} di \right)$$

subject to the budget constraint

$$(16) \quad C_t + I_t + \frac{B_t}{P_t R_t} = \frac{1}{P_t} \left[\int_0^1 W_{it} H_{it} di + \int_0^1 D_{jt} dj + B_{t-1} + P_t q_t K_t \right],$$

the law of motion for capital

$$K_t = (1 - \delta) K_{t-1} + I_t,$$

the demand for labor, given by (15), and the timing restriction that both wages *and hours* are reset once in I quarters. In the *standard* sticky wage literature, once the nominal wage W_{it} is set, the labor hours in the subsequent $I - 1$ periods are determined from the static labour demand condition (13).

The first-order conditions of this problem are:

$$\begin{aligned} \Lambda_t &= C_t^{-1} \\ \frac{1}{R_t} &= \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}} \right] \\ 1 &= \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} (1 - \delta + q_{t+1}) \right] \\ (17) \quad \mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \left(\theta \gamma \left(\frac{\Upsilon_t}{W_{it}} \right)^{\theta(1+v)} W_{it}^{-1} + (1 - \theta) \Lambda_{t+s} \left(\frac{\Upsilon_t}{W_{it}} \right)^\theta \frac{1}{P_{t+s}} \right) &= 0, \end{aligned}$$

where Λ_t is the Lagrange multiplier on the budget constraint (16), R_t is the nominal interest rate paid on bond holdings, π_{t+1} is the gross inflation rate $\frac{P_{t+1}}{P_t}$, q_t is the real return on capital holdings, and Υ_t stands for

$$\Upsilon_t = \frac{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \Lambda_{t+s} W_{t+s} L_{t+s}^{\frac{1}{\theta}}}{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \Lambda_{t+s}},$$

the first term inside the square brackets of the labour demand condition (15).

The first -order condition (17) can be restated as

$$(18) \quad W_{i,t}^* = \left(\frac{\theta\gamma}{\theta-1} \Upsilon_t^{\theta v} \frac{\sum_{s=0}^{I-1} \beta^s}{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \frac{\Lambda_{t+s}}{P_{t+s}}} \right)^{\frac{1}{1+\theta v}}.$$

Since wage/hours-setting cohorts are fixed through time, all wages (and hours) being reset are set equal in a given cohort. Thus, individual subscript i on the optimal wage W_{it}^* , and optimal hours H_{it} can be eliminated. There are now only I different wage/hours pairs in the economy, and we can simply write W_{t-s}^* , H_{t-s}^* for the period- t wage/hours pair applying to the $1/I$ portion of differentiated labour, that had their labour contract reset s periods ago.

For future reference, let's also state the analog of the equation (18) in the standard sticky-wage model

$$(19) \quad W_{it}^* = \left(\frac{\theta\gamma}{\theta-1} \frac{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s (W_{t+s}^\theta L_{t+s})^{1+v}}{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s W_{t+s}^\theta L_{t+s} \frac{\Lambda_{t+s}}{P_{t+s}}} \right)^{\frac{1}{1+\theta v}}$$

as well as the aggregate wage equation in the standard model

$$(20) \quad W_t = \left(\frac{1}{I} \sum_{s=0}^{I-1} W_{t-s}^* \right)^{\frac{1}{1-\theta}},$$

derived from the Labour aggregator's zero profit condition.

C. Monetary Policy

Monetary policy follows a Taylor Rule:

$$(21) \quad \frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^\omega \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^b \left(\frac{Y_t}{\bar{Y}_t} \right)^d \right]^{1-\omega} \exp(u_t).$$

where \bar{R} , $\bar{\pi}$, \bar{Y}_t are the balanced growth path values of the nominal interest rate, inflation, and output, and u_t is a source of monetary shocks in the model. We assume that u_t follows an AR 1 process with normally distributed innovations

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \quad \varepsilon_t^u \sim N(0, \sigma_u^2).$$

There is no government taxation or spending in the model.

D. Market Clearing and Equilibrium

For the labour market to clear, total supply of the composite labour produced by labour aggregators must equal total demand arising from intermediate-good producing firms:

$$(22) \quad L_t = \int_0^1 L_{jt}^d dj$$

Similarly for capital market, intermediate firms must rent the total amount of capital carried over from the previous period by the households

$$(23) \quad K_{t-1} = \int_0^1 K_{jt}^d dj.$$

Finally, for the goods market we have

$$(24) \quad C_t + I_t = Y_t$$

The equilibrium of this economy consists of allocations and prices such that households, labour aggregators, final-good producing firms and intermediate-good producing firms optimize, the monetary policy rule (21) is satisfied, and all markets clear.

We focus on cohort-symmetric equilibria in which all resetting, intermediate-good producing firms choose the same price P_t^* for the goods they produce. As described above, this implies that only J different prices coexist in equilibrium at any time. It also implies that the firms within each price-setting cohort are characterized by identical demand for their product (so we can write $Y_{\tau t}$, $\tau = 0, \dots, J - 1$) and also by identical input demands ($L_{\tau,t}^d$, $K_{\tau,t}^d$ $\tau = 0, \dots, J - 1$).

This symmetry extends to wage/hours choices. All wages W_{it}^* reset in a given period are equal and in equilibrium, the economy is characterized by I different wages (W_{st} , $s = 0, \dots, I - 1$) and by I different quantities of labor hours supplied ($H_{s,t}$, $s = 0, \dots, I - 1$).

Data Transformations

A deterministic trend in the level of aggregate technology implies that, in the absence of shocks, aggregate output Y_t , consumption C_t , investment I_t , real wage W_t/P_t and real dividends D_t/P_t all grow at the same rate g as the labour-augmenting technology. We transform these variables to induce stationarity:

$$(25) \quad \begin{aligned} y_t &\equiv \frac{Y_t}{A_t}; \quad c_t \equiv \frac{C_t}{A_t}; \quad i_t \equiv \frac{I_t}{A_t}; \\ w_t &\equiv \frac{W_t}{P_t} \frac{1}{A_t}, \quad d_t \equiv \frac{D_t}{P_t} \frac{1}{A_t}. \end{aligned}$$

Marginal utility of real income Λ_t is another aggregate variable that needs to be transformed.

Given the first-order condition $\Lambda_t = C_t^{-1}$ it is clear how to do that:

$$\lambda_t \equiv \Lambda_t A_t = c_t^{-1}.$$

At the level of intermediate firms stationarity is achieved in the same way:

$$(26) \quad y_{\tau t} \equiv \frac{Y_{\tau t}}{A_t}, \quad k_{\tau t} \equiv \frac{K_{\tau t}^d}{A_t}, \quad \tau = 0, \dots, J-1.$$

Finally, trend inflation means that newly-set prices P_t^* , reset wages W_t^* and the price index P_t are growing over time. We detrend these nominal variables as follows:

$$(27) \quad p_t^* \equiv \frac{P_t^*}{P_t}, \quad w_t^* \equiv \frac{W_t^*}{W_t}, \quad p_t = \frac{P_t}{\bar{\pi}^t}.$$

With these transformed variables we can modify equations (10), (11), (18) and (15) as follows:

$$(28) \quad p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbf{E}_t \sum_{\tau=0}^{J-1} (\beta \pi^\varepsilon)^\tau \lambda_{t+\tau} \left(\frac{p_{t+\tau}}{p_t} \right)^\varepsilon y_{t+\tau} s_{t+\tau}}{\mathbf{E}_t \sum_{\tau=0}^{J-1} (\beta \pi^{\varepsilon-1})^\tau \lambda_{t+\tau} \left(\frac{p_{t+\tau}}{p_t} \right)^{\varepsilon-1} y_{t+\tau}};$$

$$(29) \quad 1 = \frac{1}{J} \sum_{\tau=0}^{J-1} \pi^{(\varepsilon-1)\tau} \left(\frac{p_{t-\tau}^*}{p_t} \right)^{1-\varepsilon};$$

$$(30) \quad w_t^* = \frac{\theta\gamma}{\theta-1} (L_t^*)^v \frac{\sum_{s=0}^{I-1} \beta^s}{\mathbf{E}_t \sum_{s=0}^{I-1} \left(\frac{\beta}{g\pi}\right)^s \lambda_{t+s} \frac{p_t}{p_{t+s}}};$$

$$(31) \quad L_t^* = \left[\frac{\mathbf{E}_t \sum_{s=0}^{I-1} (\beta\pi)^s \lambda_{t+s} w_{t+s} \frac{p_{t+s}}{p_t} L_{t+s}^{\frac{1}{\theta}}}{\mathbf{E}_t \sum_{s=0}^{I-1} \left(\frac{\beta}{g}\right)^s \lambda_{t+s}} \frac{1}{w_t^*} \right]^\theta$$

For the standard sticky wage model the detrended equation for the reset wage is

$$(32) \quad w_t^* = \left(\frac{\theta\gamma}{\theta-1} \frac{\mathbf{E}_t \sum_{s=0}^{I-1} \left(\beta(\pi g)^{\theta(1+v)}\right)^s \left(w_{t+s}^\theta \left(\frac{p_{t+s}}{p_t}\right)^\theta L_{t+s}\right)^{1+v}}{\mathbf{E}_t \sum_{s=0}^{I-1} \left(\beta(\pi g)^{\theta-1}\right)^s w_{t+s}^\theta \left(\frac{p_{t+s}}{p_t}\right)^{\theta-1} L_{t+s} \lambda_{t+s}} \right)^{\frac{1}{1+\theta v}}$$

while the detrended version of the demand for hours (13) is

$$H_{i,t+s} = (\bar{\pi}g)^{s\theta} \left(w_t^* \frac{w_t}{w_{t+s}} \frac{p_t}{p_{t+s}} \right)^{-\theta} L_{t+s}.$$

4. Parametrization and results

A. Benchmark parameter values

There is a substantial uncertainty about many of the model parameters. However, after an extensive sensitivity analysis we found that the results of the paper are quite insensitive to variation in those parameters. In particular, the near neutrality of monetary shocks seem to be extremely robust across the parameter space. Thus, one should think of most benchmark parameter values we set below, as simply midpoints for the ranges of values considered in our sensitivity analysis. Nevertheless, the parameter values we pick are as follows:

Parameter	Benchmark value
Annualized time discount factor, β^4	0.96
Weight on hours in utility function, γ	1
Inverse of Frisch elasticity of labour supply, v	1
Annualized net growth rate, $g^4 - 1$	0.02
Persistence of productivity shocks, ρ_z	0.95
St. Deviation of productivity shocks, σ_z	0.01
Labour share, α	0.64
Quarterly depreciation rate, δ	0.035
Elasticity of substitution for goods, ε	11
Elasticity of substitution for labour hours, θ	11
Number of quarters prices are fixed, J	3
Number of quarters wages/hours are fixed, I	4
Persistence of monetary shocks, ρ_u	0
St. Deviation of monetary shocks, σ_u	0.01
Weight on past interest rate in monetary rule, ω	0
Weight on inflation in monetary rule, b	2.5
Weight on output in monetary rule, d	0.1.

Some of the parameters above are quite conventional. For example, there seems to be little disagreement about the time discount factor, β , the labour share, α , or the long run labour productivity growth g . For other parameters there is less consensus, but no major debates either. The benchmark values of the depreciation rate δ , as well as values for per-

sistence ρ_z and for the standard deviation σ_z of productivity shocks, are taken as in much of the real business cycles literature. The duration of wage contracts, $I = 4$ also appears fairly standard to the literature on sticky wages (see Taylor (1999) for a review of evidence on that). The elasticities of substitution for both intermediate goods, ε , and for hours, θ , are set to yield the average mark-ups of roughly 10 percent in line with Basu (1996) and Basu and Fernald (1997). The most controversial parameters are the elasticity of labour supply $\left(\frac{1}{v}\right)$, and the duration of price contracts J . We set the former one at unity following evidence in Kimball and Shapiro (2003), as well as in Chang and Sun-Bin Kim (2005). The estimates of the average number of quarters prices stay fixed (J in our model) range quite widely, with some of the recent estimates (e.g. Bils and Klenov (2004)) pointing at 2-3 quarters. We pick the higher value of 3 to give a better chance for monetary shocks to affect real variables and welfare.

With regard to monetary policy rule, one of the main results of the paper is that it does not matter much for the modified model. However, it does matter a lot for the standard sticky-wage model. To make this point we first pick *a rule* (in the table above) and then try other values for ω , b and d to show that it matters in the standard model, but not in the modified model. Among those rules we include an optimized rule for the *standard* sticky wage model.

B. Long-run analysis results

In this section we compare the long-run predictions of the standard and modified models about the effects of trend inflation on output and on welfare. To accomplish that we find a steady state for detrended variables in three models: the standard sticky-wage model,

the modified labour contacts model and, as a benchmark, the model with fully flexible wages and hours. The appendix A shows how to compute steady-states in all three models.

Figure 3 depicts the output cost of trend inflation in both the standard and the modified models relative to the flexible wages/hours model, while Figure 4 shows the welfare cost of trend inflation in the same two models relative to the flexible model. As we can see, for the modified model the output and welfare costs are zero irrespectively of trend inflation. On the contrary, for the standard model there is a big effect of a fully anticipated trend inflation on output and welfare: an increase in the trend inflation rate from 2 to 4 percents leads to nearly 1 percent permanent loss in output, and a permanent welfare loss equivalent to 0.6 percent of consumption. These big output and welfare effects of a fully anticipated inflation go against one of the main tenets of monetary economics, suggesting that in the long-run there is no exploitable relationship between the levels of output and inflation. Further, as shown in Amano et al (2007) the welfare maximizing rate of trend inflation in the standard model is roughly equal to $\frac{1}{g}$, the inverse of the aggregate labour-productivity growth rate. It is approximately -2 percent annualized inflation rate. As we can see from Figure 3 this is actually the only common point between two curves. That is not surprising: in the standard model the number of hours supplied by each worker is a decreasing function of his/her productivity-adjusted real wage $\frac{W}{PA}$. Thus, if the price level P declines at the same rate as A grows, neither productivity-adjusted real wage, nor hours change over time, and the standard model is identical to the modified model! The fact that this rate of inflation happens to be the optimum of the standard model suggests that modified contracts may serve as an efficient protection for the real economy from the effects of anticipated inflation.

Another prediction of the steady-state analysis is that with sticky goods' prices, the

optimal trend inflation in the modified model is close to zero, and independent of the duration of labour contracts. This goes against the common finding of the standard sticky-wages/sticky prices literature that in a model with both frictions optimal monetary policy is determined predominantly by the sticky wages.

C. Short-run dynamic results

Let us now turn to the analysis of short-run dynamic responses to productivity and monetary shocks. In this section we consider two cases: Case 1 focuses on wage/hours friction and excludes any other nominal rigidities, such as sticky goods prices. Case 2 adds sticky prices.

Case 1: No price rigidities

As we already mentioned in the introduction, in the absence of any other rigidities, modified staggered wages do not create nominal frictions in the sense that monetary shocks remain neutral, as would be the case in a model with fully flexible wages and hours (and prices). It is possible to establish that result by examining the necessary equilibrium conditions. However, it might be simpler to go through the intuition instead. By assumption of the modified model, current period monetary shocks do not affect hours of pre-set labour contracts. The only hours they could affect are the ones in the currently reset contracts. However, those contracts are fully flexible now, and thus, take the current monetary shock in account. So, the monetary shock has no effect on labour hours. Also, since the capital stock is pre-determined, there is no effect on output. Finally, since all members of the representative household, irrespectively of their labour earnings, share the same level of consumption and investment, there is no effect on those variables either. Thus, no real variables, except the

real wages, are affected. Real wages do change, but have no allocative consequences.⁵

It is important to emphasize that, in the absence of price rigidities, changes in the monetary policy rule have no consequences for the modified model due to its complete monetary neutrality. This is not going to be as clear cut once we add price rigidities, but remains close to being true even then.

Case 2: Price rigidities in addition to staggered labour contracts

We use second-order approximation techniques to approximate the dynamic equations for the standard sticky-wage model and the modified labour contracts model. Once the models are solved, we perform a variance decomposition analysis to assess what fraction of the total variation in real and nominal variables is due to productivity, or monetary shocks. Further, we simulate these models and compute (second-order accurate) welfare losses of the standard and modified models relative to the flexible wages-flexible hours benchmark model.

The following table shows results for the benchmark set of parameters with the annualized inflation rate set at 2 percents. From the second and third columns of the table we can see that in the standard model 96 percent of the variation in consumption is caused by productivity shocks, with the rest accounted by monetary shocks. For other real variables, including welfare, monetary shocks cause a substantial fraction of variation that ranges from 43 percents for output to 89 percents for hours. Perhaps surprisingly, only 11 percent of variation in inflation is caused by monetary shocks.

⁵This last conclusion about the absence of allocative consequences of real wage changes obviously depends on the aggregation assumptions. It would be interesting to see how much monetary non-neutrality one could obtain by relaxing the assumption of perfect income pooling within households.

Model	Standard		Modified	
Shocks	productivity	monetary	productivity	monetary
Output	57	43	99	1
Consumption	96	4	99	1
Investment	30	70	98	2
Hours	11	89	43	57
Inflation	85	15	55	45
Welfare	37	63	99	1
Welfare Loss	1.5		0.1	

Things are quite different in the modified model. Monetary shocks account only for 1-2 percents of variation in output, consumption, investment and welfare. Hours are more affected, 57 percent of the variation in hours is due to monetary shocks.

So, why don't monetary shocks affect real variables in the modified model? It helps to start with the standard model first. In the standard model, an unanticipated increase in the price level due to a monetary shock, reduces the relative prices of intermediate goods producers, whose nominal prices were set in the previous quarters. In response, those producers *must* increase their output to meet higher demand for their cheaper products.⁶ To accomplish that, they must hire more labour. "Luckily" there is also an abundant supply of inexpensive labour hours from the workers who find their pre-set nominal wages eroded by the same price level increase, and who *must* increase their hours to meet higher demand for their less expensive services. As a result, the economy ends up with large and costly

⁶That is of course due to the same monopolistic competition assumption, but in the product market, where it seems to make much more sense than in the labour market.

dispersions of intermediate goods outputs and labour hours.

Similarly in the modified model, an unanticipated increase in the price level due to a monetary shock forces intermediate goods producers with lower relative prices to increase their output by hiring more labour. However, the only hours that could increase in response to that, are the hours in the labour contracts that are being reset in the current period. Those hours are expensive, because their wages are fully flexible, and the suppliers of those hours must be paid a premium to justify increasing marginal disutility of longer hours. As a result, the output response to a monetary shock must be small, which requires that the price increase created by this monetary shock must also have been smaller than would be the case in the standard model. So, in effect, modified contracts serve as the source of real rigidity which buffers the real side of the economy from monetary shocks.

Certainly, modified contracts must also affect the responsiveness of the economy to real shocks. This is a source of inefficiency that needs to be quantified. The last row of the table above shows that for the benchmark set of parameters the welfare loss of the modified model is 0.1 percents of the steady-state consumption in the flexible labour market model. That is 15 times smaller than the welfare loss of the standard sticky-wage model equal to 1.5 percents of the steady-state consumption (of the flexible model).

The results presented above were computed for an arbitrary monetary policy (Taylor) rule. What happens if we change the rule? As it turned out, changes in the monetary rule have very little effect on the predictions of the modified model, but a large effect on the welfare implications of the standard sticky-wage model. To make this point we searched over the values of coefficients ω, b, d in (21) to find the optimal Taylor rule for the *standard* model.

Simulation results with that rule are presented in the following table.⁷

Model	Standard		Modified	
Shocks	productivity	monetary	productivity	monetary
Output	99	1	99	1
Consumption	99	1	99	1
Investment	98	2	99	1
Hours	91	9	82	18
Inflation	99	1	97	3
Welfare	98	2	99	1
Welfare Loss	0.7		0.03	

As we can see from these results, in the standard model, the optimal Taylor rule reduces the impact of monetary shocks on real variables. Almost the entire variation in the real variables is now caused by productivity shocks. Thus, the optimal Taylor rule is trying to accomplish the same thing which modified contracts deliver irrespectively of the monetary rule: shield the real economy from monetary shocks. The last row of the table shows that even with the monetary rule optimized for the standard model, the welfare loss in that model is more than 20 times as large as the welfare loss in the modified model.

D. Sensitivity analysis

Finally, we conducted a thorough sensitivity analysis for all the parameters in the model and found that all of our results are extremely insensitive to variation in those parameters. In particular the near neutrality of monetary shocks is preserved almost uniformly

⁷The optimized rule has $\omega = 0$, $b = 30$, $d = 1$.

in the parameter space. A notable exception to that rule is: a very high degree of interest rate smoothing makes monetary shocks to account for a larger fraction of variation in real variables. For example, if $\omega = 0.9$, then monetary shocks account for a quarter of variation in output, and a third of variation in welfare. However, even with ω as high as 0.7, this fraction falls to five percents for output, and seven percents for welfare.

Further, the welfare losses of modified contracts, relative to the flexible labour market model, were never greater than 0.5 percents of steady state consumption, and normally much smaller than that. Moreover nearly uniformly in the parameter space, they were orders of magnitude smaller than the corresponding welfare losses of the standard model. The only exception to that rule was the case with very high Frisch elasticity of labour supply $\frac{1}{v}$. When Frisch elasticity is equal to 10 ($v = 0.1$), the welfare loss of the modified model, 0.23 percents of consumption, is greater than 0.15 in the standard model. However with elasticity as high as 2.5, ($v = 0.4$) the welfare losses are 0.17 percents for the modified model, and 0.5 percents for the standard model. The intuition for these results is simple: with very high Frisch elasticity of labour supply, agents are nearly indifferent to variation in their labour hours. So workers do not mind much working longer hours, when their real wage falls due to inflation. The values of the Frisch elasticity in excess of 2.5 seem to be out of the estimated range in the empirical literature on individual labour supply.

5. Conclusions

This paper proposes a simple and intuitive modification to the standard sticky-wage model which helps to address some of the model's shortcomings. In particular, modified labour contracts that specify both nominal wages and hours of work for the duration of

the contracts, rather than just nominal wages as in the standard model, serve as an effective shield for the efficiency of labour exchange in the face of monetary shocks. Thus, the proposed modification addresses Barro's (1977) critique of standard sticky wage models. Furthermore, modified contracts resolve another counterfactual prediction of the standard sticky wage model of an exploitable long-run trade off between inflation rate and output. Quantitatively, the welfare loss of inflexible hours in pre-set contracts appears to be quite small at less than 0.25 percent of steady-state consumption.

On the other hand, modified contracts have implications for monetary policy that seem quite extreme. In absence of other nominal rigidities, monetary shocks are neutral and do not affect hours, output or welfare. Even in the model with both sticky prices and modified labour contracts, monetary shocks are nearly neutral. These results suggest that some of the powerful macroeconomic effects of nominal rigidities, such as sticky prices, are quite sensitive to modeling details about labour market contracting.

6. Appendix: Computing steady-states

The goods market side is common across the three models we consider: the standard model, the modified model, and the flexible (wages/hours) model. The steady state version of the equations (29) yields p^* as a function of steady-state inflation $\bar{\pi}$ and model parameters ε and J :

$$(33) \quad p^* = \left(\frac{1}{J} \sum_{\tau=0}^{J-1} \bar{\pi}^{\tau(\varepsilon-1)} \right)^{\frac{1}{\varepsilon-1}} .$$

Substituting the value of p^* in the steady state version of the optimal pricing equation (28)

allows us to solve for the marginal cost s

$$(34) \quad p^* = s \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{\tau=0}^{J-1} \beta^\tau \pi^{\tau\varepsilon}}{\sum_{\tau=0}^{J-1} \beta^\tau \pi^{\tau(\varepsilon-1)}}.$$

Using (9) and the steady state rate of return to capital found from:

$$g = \beta(1 - \delta + q),$$

we can compute the aggregate wage rate from

$$s = \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{q}{1 - \alpha}\right)^{1-\alpha}.$$

From the steady-state (detrended) versions of the market-clearing condition for capital (23),

of the first-order condition (8) and of equation (2) we can find the capital-output ratio as

$$\frac{k}{y} = (1 - \alpha) \left(\frac{s}{q}\right) (p^*)^{-\varepsilon} \frac{1}{J} \sum_{\tau=0}^{J-1} \pi^{\tau\varepsilon}.$$

Similarly, from the market-clearing condition for labour (22), the first-order condition (7)

and equation (2) we can find the capital-output ratio as³²

$$\frac{L}{y} = \alpha \frac{s}{w} (p^*)^{-\varepsilon} \frac{1}{J} \sum_{\tau=0}^{J-1} \pi^{\tau\varepsilon}.$$

Finally, from the market-clearing condition (24) we obtain the steady-state consumption-output ratio:

$$\frac{c}{y} = 1 - (g + \delta - 1) \frac{k}{y}.$$

Knowing these three ratios, we can use equation (32) for the standard model, and equation

(30) for the modified model to find output as

$$(35) \quad y = \left[\frac{w}{\gamma \frac{c}{y} \left(\frac{L}{y}\right)^v \mu_w} \right]^{\frac{1}{1+v}}$$

where μ_w depends on the model.

In the standard model we have

$$(36) \quad \mu_w = \frac{\theta}{\theta - 1} \frac{\sum_{s=0}^{I-1} \beta^s (\pi g)^{s\theta(1+v)}}{(w^*)^{1+\theta v} \sum_{s=0}^{I-1} \beta^s (\pi g)^{s(\theta-1)}}$$

with

$$w^* = \left[\frac{1}{I} \sum_{s=0}^{I-1} (\pi g)^{s(\theta-1)} \right]^{\frac{1}{\theta-1}}$$

derived from the steady state version of (20).

In the modified contracts model

$$(37) \quad \mu_w = \frac{\theta}{\theta - 1} \frac{\sum_{s=0}^{I-1} \beta^s}{w^* \sum_{s=0}^{I-1} \beta^s (\pi g)^{-s}}$$

with

$$w^* = \frac{\sum_{s=0}^{I-1} (\beta \pi)^s}{\sum_{s=0}^{I-1} \left(\frac{\beta}{g}\right)^s}.$$

derived from the steady state version of (15).

Finally, setting $I = 1$ in either (36), or (37) and recognizing that $w^* = 1$ in the flexible wages/hours model we obtain $\mu_w = \frac{\theta}{\theta-1}$. This can be used in (35) to compute output for the flexible model.

7. Figures

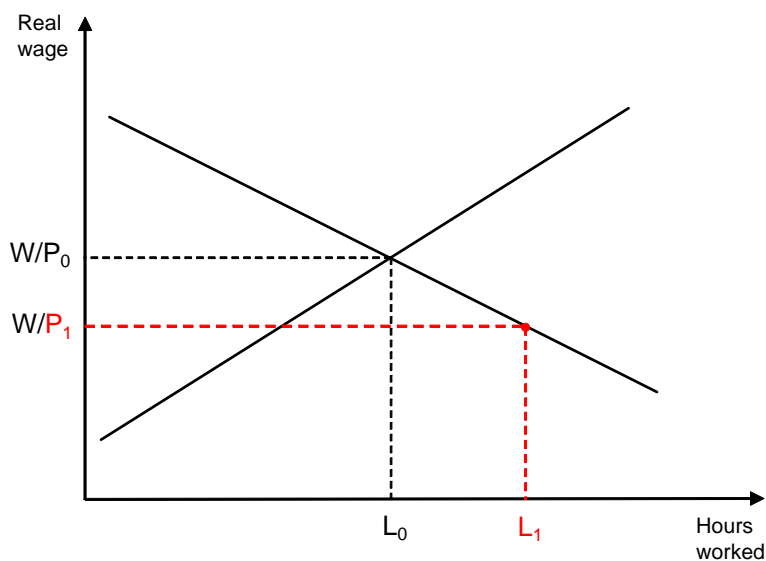


Figure 1: Hours response to a price level increase. Standard sticky-wage model forces hours to increase along the firm's demand curve in response to the decline in real wage. Modified model will have hours unchanged at L_0 .

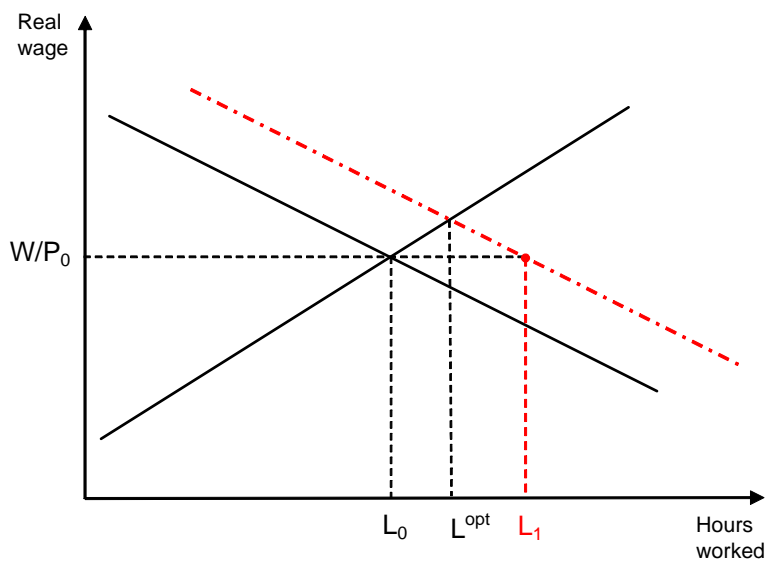


Figure 2: Hours response to a labour demand increase. Assuming price level does not change, standard sticky-wage model forces hours to increase to L_1 . Modified model will have hours unchanged at L_0 . The efficient (flexible wages/ flexible hours) response to that shift would be to increase hours to L^{opt} .

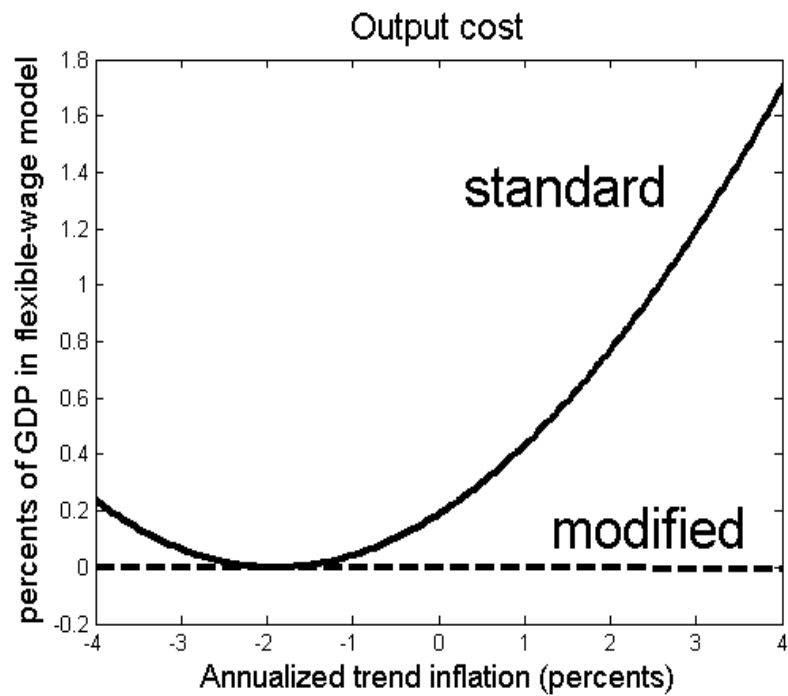


Figure 3: Output cost of trend inflation in the standard and modified models relative to the model with flexible wages and hours. Inflation is annualized in percents. Output cost is measured in percents of steady-state output of the flexible model.

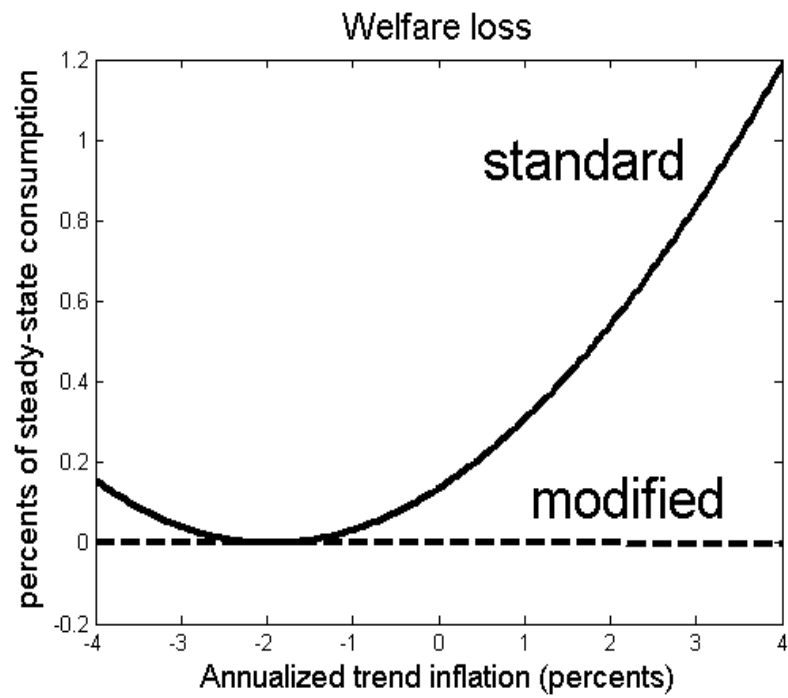


Figure 4: Welfare cost of trend inflation in the standard and modified models relative to the model with flexible wages and hours. Inflation is annualized in percents. Welfare cost is measured in percents of steady-state consumption of the flexible model.