THE UNIMPORTANCE OF THE SCITOVSKY REVERSAL PARADOX AND ITS IRRELEVANCE FOR COST-BENEFIT ANALYSIS

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ABSTRACT

The possibility of preference reversals using cost-benefit analysis has concerned economists since the early 1940s, when Scitovsky (1941) published his results. Lawyers and philosophers have used this possibility to attempt to discredit cost-benefit analysis. Scitovsky reversals will not occur when one position is first best, when there is no crossing of the production possibility curves, when actual compensation is paid, and when the Net Present Value (NPV) criterion rather than an extended compensation principle is used. In addition, there is no ambiguity when there is a distribution rule or choice that can be used to prefer one of two otherwise indifferent choices. Reversals can occur only when the potential principle is invoked for particular crossing of Scitovsky indifference maps. This particular configuration does not occur, for example, in the presence of technological change. For purposes of practical work, we suggest the reversal possibility be ignored. To our knowledge, the empirical calculations of cost benefit ratios do not encounter the paradox possibilities. As an example, the work on

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mechanization by Schmitz and Seckler (1970) illustrates clearly that, even in the absence of compensation, the reversal paradox problem did not arise. (These results are consistent with the theoretical discussion).

Classification codes: H43, K10

Keywords: reversals, technology, welfare gains

I. INTRODUCTION

The possibility of preference reversals using cost-benefit analysis has concerned economists since the early 1940s, when Scitovsky (1941) published his results. Lawyers and philosophers have fastened onto this possibility to attempt to discredit, sometimes entirely, the use of cost-benefit analysis (e.g., Coleman, 1980, and Markovits, 1993). This paper focuses on two issues: (1) Why does the Scitovsky paradox arise in the first place? and (2) What is the role of inferior goods?

II. SCITOVSKY REVERSALS

Coleman's (1980, 519f) uses the following example of reversibility to argue that Kaldor-Hicks tests are not a useful criterion for decision-making. Mr. A has two units of X and Ms B has one unit of Y. Mr. A prefers one unit of X and one of Y to two units of X. Ms B prefers one unit of Y and one unit of X to two units of Y. A project will result in B having two units of Y and A one unit of X. The project satisfies the potential compensation test, (PCT). The situation before and after the project is shown in Table 1. Note that the project supposes that one unit of X can be transformed into one unit of Y.

Table 1. Reversal Possibilities

[Insert Table 1 here]

The proposed project passes the KH test as, in the new state of the world, B could give one unit of Y to Mr. A, leaving him better off and Ms. B no worse off. However, in the status quo situation, Mr. A could give one unit of X to Ms. B leaving her better off than the proposed project and Mr. A no worse off than the proposed project. On this basis Coleman argues that cost-benefit analysis is not a useful basis for decision-making.

The example necessarily concerns second best situations. This means there will be states of the world that are Pareto superior to the status quo and to the proposed project. The full array of possibilities is shown in Table 2. To make the example more concrete suppose that the two goods are wheat and cotton and that one acre produces one unit of either wheat or cotton.

Table 2. Reversals and Pareto Superiority

[Insert Table 2 here]

When the compared states of the world are second best, one must ask why not consider Pareto superiority and select a first best situation? Trade (in this case, in either land or goods) will produce a first best situation. From a first best situation, reversibility is not possible. Second best situations assume implicitly that (1) trade is forbidden and (2) the government will not compensate the losers. If trade is not forbidden, a first best position will be achieved and there is no reversal. If the government requires compensation (as with a taking) then we end up in Pareto situation 3 (or differing only by epsilon) as a result of the project.

The Scitovsky paradox (reversals) is essentially a criticism of the compensation principle (i.e., hypothetical compensation). Its domain, however, is severely limited. The paradox does not arise under the Pareto principle (actual payment of compensation) because this will produce a first best situation. The paradox does not arise when one of the bundles being compared is of a first-best nature. Nor does the paradox arise when there is a rule rating the value of alternative income distributions. Finally, the reversal can only arise when the aggregate willingness to pay exceeds the aggregate willingness to accept, as is the case when there are inferior goods. The reversal arises only when compensation principles are applied to a comparison of second-best bundles and when the Scitovsky curves cross in a particular manner.

Consider Figure 1, where two goods, X_1 and X_2, are produced, given the production possibility surface (PPC). Consider a bundle of goods, x_1 and x_2, distributed between two individuals at point c_1, yielding a Scitovsky indifference curve of C_1. Note that C_1 is not tangent to PP. Compare this situation to a different bundle of goods x_3 and

x_4 distributed among the two individuals at c_2. Note that c_2 is not tangent to PP. In other words, the two bundles being compared are second-best bundles. Now, in moving from A to B, using the compensation test, the paradox arises because, at B, C_2' lies above C_1. However, at A, C_1' lies above C_2. This reversal is not possible if compensation were actually paid in the movement from A to B (i.e., adherence to the Pareto principle). In this case, C_2' actually lies above C_1, and therefore B is preferred to A. Likewise, if the starting point were B, A would be preferred to B under actual compensation, and there is no reversal paradox. With the use of benefit-cost analysis, the starting point would determine the outcome; there could be no reversal. This is different from the usual case in benefit cost analysis in which the starting point will remain the outcome where there are significant divergences between willingness-to-pay and willingness-to-accept measures (Zerbe, 2001).

If one of the bundles being compared is first-best, the paradox will not arise. As shown in Figure 1, the paradox cannot arise unless second-best bundles are being compared, and the Scitovsky curves cross in a particular manner. For example, unlike in comparing A and B, a comparison of D and A does not create a reversal problem, even though A and B represent second-best bundles, because the Scitovsky indifference curves do not cross in the required manner nor, for example, does a comparison of second-best bundles such as D and E. As a result, the range of bundles subject to the reversal problem in narrowed.

It is important to note that the reversal Paradox may or may not occur even for a given set of Scitovsky curves. Given C_1 and C_2 in Figure 1, the reversal does not occur when comparing B and E, but it does occur when comparing A and B. This is the case even though the intersection point of C_1 and C_2 does not depend on the bundles and the corresponding distributions under consideration. Why is this the case? When comparing D and E, sufficient surplus cannot be generated so that one bundle can be preferred to the other even in the presence of compensation, but when comparing A and B, this is no longer the case

In the preceding discussion, compensation is possible when comparing A and B because the surplus is generated when the distortion is removed, thus attaining a first-best solution. Consider further Figure 1 where we show that by picking a bundle of goods between the two bundles that led to the Paradox and comparing it with either of the bundles that led to the Paradox, one avoids the reversal problem. To see this, bundles x_3' and x_2' corresponds to the crossing of Scitovsky curves C_1 and C_2. Now if one adds an amount a of good X_1, it is possible to make at least one better off and no one worse off since a tangency can be achieved where C_3 is tangent to the PP at A′ (tangency is not required for A′ to be preferred to A). Alternatively, an amount b of good X_2 could be added to achieve utility level C_4 where no one is made worse off (Point B′). As a result, there exists a range of bundles of goods that lie between the so-called Reversal Bundles which, when compared to either of the reversal bundles, does not yield a paradox. Again, the possibilities for reversals are narrowed. In the same vein, one could always pick a bundle that would be preferred to the starting point.

Figure 1. Scitovsky Reversals and Second-Best Bundles

[Insert Figure 1 here]

III. REVERSALS AND TECHNOLOGICAL CHANGE

In the above discussion, comparisons were made for bundles, given a single production possibility surface. Can the Scitovsky paradox arise when comparing two first-best bundles on two different production surfaces where the production surfaces lie entirely outside each other? The answer is no. Also, in this context, the compensation principle is irrelevant, why? .

In Figure 2, there are two PP curves and two first-best bundles represented by points A and B (normal goods). PP_2 represents the production surface with technological change. Even though the Scitovsky curves C_1 and C_2 cross, they do not give rise to the Scitovsky paradox (see Figure 1 for the correct crossing). A is not preferred to B (i.e., one cannot rotate C_2 to be above C_1 at point A) using the compensation test nor is B preferred to A. These states are said to be non-comparable. There is no reversal paradox.

Figure 2. Expanded Production: First-best Bundles and Reversals

[Insert Figure 2here]

However, one could adhere to the Pareto principle to move from point B to point E, where E is a first-best bundle corresponding to C_3 (any such point and corresponding bundle within the cigar shape could come about only through lump sum income redistribution). Remember that at point E, one is comparing a different bundle of goods than one would be comparing at point A.

When the new production possibilities surface lies entirely outside the old, the paradox need not arise, even for second-best bundles. For example, in Figure 2, consider C_4 with the corresponding price P_1. Bundle E distributed in this manner is clearly a second-best bundle. Even so, with the compensation principle, E is preferred to B, and there is no reversal paradox.

Mishan (Mishan, 1981, p. 363) has shown that ambigious allocative rankings arise from the absence of a distributional rule:

> "whenever an unambiguous allocative ranking is not possible by means of compensation tests, owing to the interection of community indifference curves,. . . a comparison of two collections of goods Q_1 and Q_2 can be reduced to a comparison of two alternative distributions of a hypothetical collection Q_3. If, by reference to society's norms of distributional justice, these alternative distributions can be ranked, it follows that the original collections Q_1 and Q_2 can be ranked also…."

IV. Scitovsky Reversals and Cost-Benefit Analysis

We show that to obtain a reversal, the aggregate WTP must exceed the aggregate WTA for one of the moves between situations 1 and 2. This condition, that WTP < WTA, is usually associated with inferior goods. Yet we show that inferiority is not required for reversibility. The explanation must lie in the fact that WTP > WTA can arise in situations in which Scitovsky curves cross even when no inferior goods are involved.

Suppose two persons or groups, 1 and 2, and two states of the world, A and B. In Table 1, in which all figures are expressed as absolute numbers, the WTA of $150 for person 1 exceeds his or her WTP of $100, and the absolute value of the WTA for person 2 of $95 exceeds the absolute value of his or her WTP of $90. A move from state of the world A to state B is considered. Person 1 would pay $100 for the move. Person 2 would accept $95 to agree to the move so that the move would satisfy Kaldor/Hicks (KH) with a net present value (NPV) of $5.

Table 3. No Scitovsky Reversal Is Possible

[Insert Table 3 here]

For a move back from B to A, person 2 would be the winner and person 1 would be the loser. To find the value of a move back from B to A, it is useful to know that the WTP for person 2 for a move back is equal to the negative of the value of his or her WTA for the original move from A to B. And the WTA for person 2 for the move back is equal to the negative of the WTP value for the move from A to B. Thus, the value for a move back will be $90 minus $150 or a negative $60. There is no reversal.

Table 4 shows that a reversal can occur when the WTA < WTP, that is when one of the choices is inferior for one party. This is the case for the numbers in Table 4.

Table 4. Scitovsky Reversals When WTP > WTA

[Insert Table 4 here]

The value of a move from A to B will be the WTP for person 1, who would gain, and the WTA for person 2, who would lose. This would be $100 - $95, or $5, just as shown before. For a move back from B to A, the signs get reversed, as do the winners and losers. The KH value of a move back from B to A would be WTP for person 2, which is $90 minus the WTA for person 1, which is $80. Thus, the net gain for a move from B to

A is $10. If the move is inferior for one person (here person 1), this is necessary (but not sufficient) for reversal.

These examples generalize. Represent the values in Table 5 as abstract values a, b, c, and d, in which losses are listed as absolute values as follows:

Table 5. Scitovsky Reversals

[Insert Table 5 here]

KH is passed in a move from A to B if a > d. The reverse move from B to A is passed if c > b. Normality requires that b > a and d > c. By normality, d > c, so that a > c but a < b by normality. It is not possible for c > b, so no reversal occurs. Normality is a sufficient condition for no reversal.

With inferiority, a > d by assumption, a > b by inferiority, and c > d by inferiority. From this, we can make no conclusion about whether or not c is greater or less d; that is, c > = < b, so reversal could occur but it need not occur. This shows that the condition in which WTP > WTA is a necessary but not sufficient condition for a Scitovsky reversal. This condition that WTP > WTA is not limited to inferior goods.[1]

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[1] The following is our modification of a proof used b Robles to prove that inferiority is a necessity condition.

Definition: The compensating variation (CV) for a change from situation 1 to situation 2:

$$V^h(y_{h1}, a_1) = V^h(y_{h1} - CV_{h12}, a_2)$$

CVh12 is the amount of money *h* needs to give up in situation 2 (if *h* is better off in situation 2, $CV_{h12} > 0$), or the amount of money h must be paid to accept the move to situation 2 (if *h* is worse off in situation 2, $CV_{h12} < 0$), in both cases to achieve the same utility after the change. In the new situation, we ask what income change in the new situation is necessary to compensate (positively or negatively) the individual for the situation change?

Definition: The equivalent variation (EV) for a change from situation 1 to situation 2:

$$V^h(y_{h1} + EV_{h12}, a_1) = V^h(y_{h1}, a_2)$$

is the amount of money h can equivalently receive instead of getting to situation 2 (if *h* is better off in situation 2, $EV_{h12} > 0$), or the amount of money *h* needs to give up instead of getting to situation 2 (if *h* is worse off in situation 2, $EV_{h12} < 0$), to achieve the same utility as before the change. Thus, the base perspective is the old situation: what income change in the old situation is equivalent to the situation change in terms of utility? This concept is rather general and parameter *a* can be many different things.

We will apply this concept to a price decrease. The measures differ because of the income effect, substitution effect, or endowment effects, but not the sign $(CV > 0 \ (\ )\ EV > 0)$. Consider again the price decrease: For a normal good and for a change from p_0 to p_1,

$$CV_{01} < EV_{01}.$$

The *CV* from a decrease from p_0 to p_1 equals the (negative) *EV* from a change from p_1 to p_0:

$$CV_{01} = -EV_{10}$$

Thus another way to characterize normality is

$$CV_{01} < -CV_{10} (= EV_{01})$$

Hicks-Kaldor-Criterion

The Hicks-Kaldor-Criterion recommends a move from situation 1 to situation 2 if the gainers can compensate the losers and still be better off; that is, if the sum of all *CV*s is strictly positive:

$$CV_{12}^{NS} + CV_{12}^{S} > 0 \qquad (1)$$

Scitovsky-Paradox: The Hicks-Kaldor-Criterion can lead to cycles where a change from situation 1 to situation 2 is recommended, and again, a change back from situation 2 to situation 1 is recommended, too. To have a cycle we also need

$$CV_{21}^{NS} + CV_{21}^{S} > 0 \qquad (2)$$

Adding both inequalities, we get

$$\left(CV_{12}^{NS} + CV_{12}^{S}\right) + \left(CV_{21}^{NS} + CV_{21}^{S}\right) > 0 \qquad (3)$$

$$\left(CV_{12}^{NS} + CV_{21}^{NS}\right) + \left(CV_{12}^{S} + CV_{21}^{S}\right) > 0 \qquad (4)$$

A necessary but not a sufficient condition for a reversal is that the sum (3) of the two inequalities (1) and (2) is greater than zero. This condition can be met without both inequalities being greater than zero, so that is not a sufficient condition. For inequality (4) to hold, it must be that:

$$CV_{12}^{i} > -CV_{21}^{i}$$

which is the definition of inferiority. The normal definition of inferiority does not hold when Scitovsky curves cross. Whata this proof has shown instead, as we show above, is that for a reversal, the WTP > WTA for some move, not that inferiority is a necessary condition for a reversal. Consider the following

Assumptions: Two goods *x* and *y*, two consumers *a* and *b*, lump-sum income redistribution, linear PPs before and after the project. After project values are denoted with a hat. Let,

V. THE ABSENCE OF COMPELLING REASON FOR THE POTENTIAL COMPENSATION TEST

In a typical cost-benefit analysis, one is comparing only points A and B and not the whole range of compensation possibilities. This is why the difference in outcome can arise between the demonstration that inferior goods are required and the earlier example in which reversals occur even though the goods are normal. This raises the question of why the potential compensation test (PCT) should be used at all as a criterion for cost-

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$U^a = x^a + \sqrt{y^a}$; $p \cdot x^a + y^a = m^a$

$U^b = y^b + \sqrt{x^b}$; $p \cdot x^b + y^b = m^b$

Substituting optimizing conditions, $U^a = m^a / p$ and $U^b = m^b$.

The PPC is $p \cdot X + Y = M$, with $X = x^a + x^b$, $Y = y^a + y^b$, and $M = m^a + m^b$.

The UPC is $U^b = M - p \cdot U^a$.

Assume $M = 1$, $p = 1$, and $U^a = m^a = 1/4$, $U^b = m^b = 3/4$ before the project while, $\hat{M} = 1.5$, $\hat{p} = 2$, and $\hat{m}^a = 5/4$, $\hat{m}^b = 1/4$ after the project, which implies an after project utility distribution $\hat{U}^a = 5/8$, $\hat{U}^b = 1/4$.

After the project, income can be distributed so as to increase utilities of both *a* and *b* over the pre-project distribution (1/4, 3/4). (Say, $\hat{m}^a = .6$, $\hat{m}^b = .9$, which yields utility distribution $\hat{U}^a = .3 > .25$, $\hat{U}^b = .3 > 1/4$).

Thus, we have a Scitovsky reversal and in a situation where there are no inferior goods.

The reason the proof of the necessity of inferior goods and the above example give different results is as follows. In the proof, the comparisons of the WTP and the WTA for two positions, the actual positions to be occupied before and as a result of the project. In the above example, the positions compared are the before and the after positions, not with each other, but with the actual positions after a hypothetical repositioning. That is, the original position is compared with a hypothetical position as is the project position. If in benefit-cost analysis we are comparing actual positions, the requirement that inferior goods must exist as a necessary condition for a reversal holds. This raises the question of why compare hypothetical positions?

benefit analysis. Since the actual costs of compensation are not normally considered, the use of potential compensation furnishes no useful policy information.

The rationale for the original adoption of the PCT was to avoid interpersonal comparisons of utility. This rationale is, however, false. Such comparisons are an unavoidable consequence of any decision criterion. As Chipman and Moore (1978, p548) note that "judged in relation to its basic objective of enabling economists to make welfare prescriptions without having to make value judgments and, in particular interpersonal comparisons of utility, the New Welfare Economics must be considered a failure." Both the undesirability and impossibility of value-free measures are now widely recognized (Sen, 1995, Hammond, 1985).

The use of a cost-benefit standard of positive net benefits does not in fact guarantee that the PCT is passed whether or not the value of moral sentiments are included as goods. (Milgrom) Rather the rationale for using results of CBA are its acceptability and the fact that it use makes it more likely that at the end of the day all will gain from its use. This rationale suggests simply a positive net benefits criterion, not a PCT criterion.

IV. CONCLUSION

Scitovsky reversals will not occur when one position is first best and when actual compensation is paid. In a cost-benefit context no reversal will occur when compensation is confined to the distributions indicated by the WTP and WTA unless inferior goods are involved. Reversals can occur only when the range of compensation possibilities is invoked for a particular crossing of Scitovsky indifference maps. In the use of the compensation tests, the costs of actual compensation, which can be quite high (Zerbe, 2001), are not discussed. The inclusion of the actual costs of compensation would narrow materially the possibility of any reversals, even if compensation is not made. For purposes of practical work, we suggest the reversal possibility be ignored. Inferior goods are unimportant in project analyses. To our knowledge, the empirical calculations of cost-benefit ratios do not encounter the paradox possibilities. As an example, the work on mechanization by Schmitz and Seckler (1970), in which there are losers and gainers, clearly illustrates that, even in the absence of compensation, the reversal paradox problem did not arise. (These results are consistent with the theoretical discussion above.)

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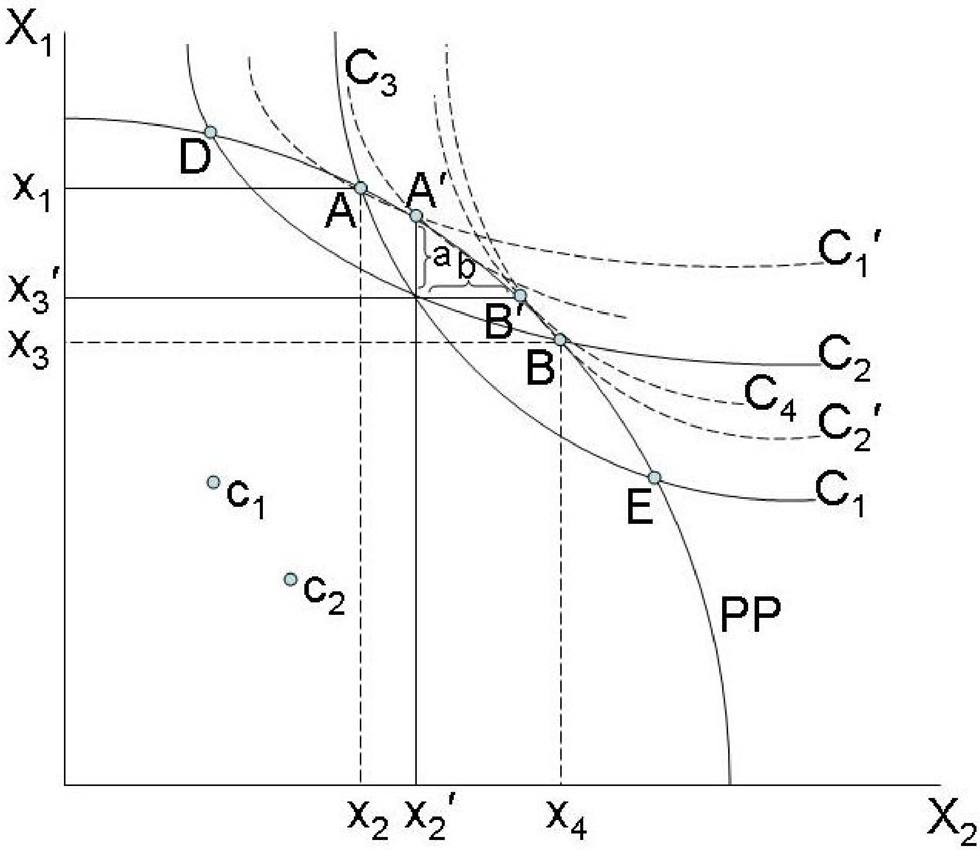

Figure 1. Scitovsky Reversals and Second-Best Bundles

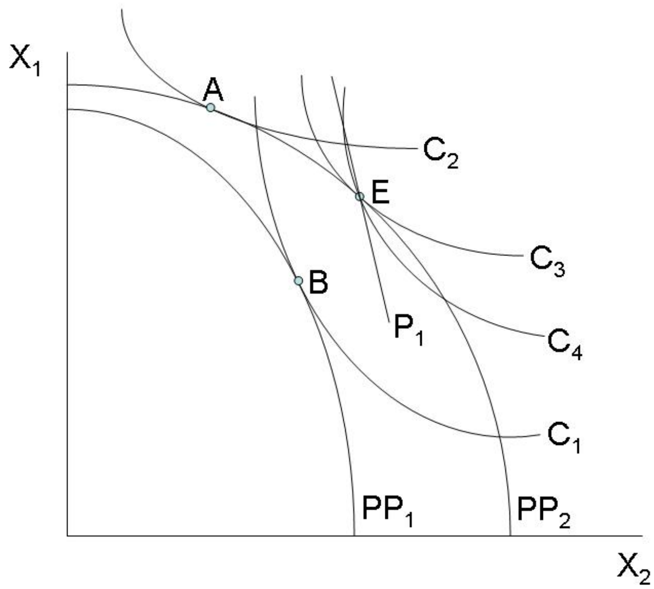

Figure 2. Expanded Production: First-best Bundles and Reversals

Table 1. Reversal Possibilities

	Status Quo		Proposed Project	
	X	Y	X	Y
Mr. A	2	0	1	0
Ms. B	0	1	0	2

Table 2. Reversals and Pareto Superiority

	1 Status Quo		2 Proposed Project		3 Pareto Superior Project to Status Quo		4 Pareto Superior Project to Proposed Project	
	Wheat	Cotton	Wheat	Cotton	Wheat	Cotton	Wheat	Cotton
Mr. A	2	0	1	0	1	1	1	0
Ms B	0	1	0	2	0	1	1	1

Table 3. No Scitovsky Reversal Is Possible

	WTP	WTA	NPV
Person 1	$100	$150	
Person 2	($90)	($95)	
Value of a Move from A to B			$5
Value of a Move from B to A			$60

Table 4. Scitovsky Reversals with Inferior Goods

	WTP	WTA	NPV
Person 1	$100	$80	
Person 2	($90)	($95)	
Value of a Move from A to B			$5
Value of a Move from B to A			$10

Table 5. Scitovsky Reversals Are Not Possible for Normal Goods

	WTP	WTA	NPV
Person 1	a	b	
Person 2	c	d	
Value of a Move from A to B			a-d
Value of a Move from B to A			c-b