Redistribution and Entrepreneurship with Schumpeterian Growth[1]

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Abstract

Income redistribution is a form of social insurance, and hence can promote risk-taking activities. We examine this idea in an R&D-driven growth model with stochastic innovations and risk-averse agents, where the growth rate is determined by the occupational choice of agents between entrepreneurship and employment. We address two broad questions. Since redistribution provides insurance for entrepreneurs, how does redistribution affect the relationship between growth and inequality? Since innovations create intertemporal spillovers, how does the size of spillovers matter for redistribution? We find that redistribution reduces inequality, raises growth and social welfare, and can be Pareto-improving, and we show the relationship between the size of spillovers and optimal tax rates.

JEL Classification*:* H21-O3 - O4

Key words*:* growth, innovation, optimal taxation, occupational choice

1 Introduction

The revival of interest in the relationship between inequality and growth has led economists to raise new questions about the effects of redistribution. The traditional incentive argument that redistribution reduces physical capital accumulation has been emphasized in models such as those of Alesina and Rodrik (1994) and Persson and Tabellini (1994). However, a number of authors have stressed alternative mechanisms that may reverse these results. In the presence of imperfect capital markets, redistribution can be growth enhancing either through an 'opportunity creation effect' that allows more agents to invest in education, as in Galor and Zeira (1993), or through an 'incentive mechanism' in the presence of moral hazard that increases work-effort, as in Aghion and Bolton (1997).[1] Even when credit is readily available, redistributive taxation that is used to finance public education expenditures may raise the return to private educational investments, increase the average level of education in an economy and promote growth; see Saint-Paul and Verdier (1993).

A possible reading of this literature is that redistribution fosters growth only in developing countries, where credit markets are highly imperfect and/or growth is driven by factor accumulation. Meanwhile, in industrial economies, which have well-functioning financial institutions, and where growth is due to private R&D activities, the above mechanisms do not apply and the reduction in the returns to entrepreneurs due to redistributive taxation is likely to hurt growth. Such argument misses the fact that industrial economies are characterized by a different type of market imperfection, namely the absence of private insurance for those who engage in risky income generating activities, as argued by Sinn (1995, 1996) and for which Bird (2001) provides cross-country empirical support.

In this paper, we examine the effects of redistribution on the occupational choice between employment and entrepreneurship, in the context of an R&D-driven growth model. We ask two related questions. Since redistribution reduces

[1] See Aghion, Caroli, and García-Peñalosa (1999) for a review, as well as Bertola, Foellmi, and Zweimüller (2006).

the income of successful innovators but provides insurance for entrepreneurs, what is the net effect on growth and inequality? Since innovations create intertemporal spillovers, what are the implications of considering entrepreneurship in a dynamic context for the choice of optimal income taxes?

There is plenty of evidence on the sizeable risks faced by entrepreneurs. There were on average 78,711 business failures per year in the United States from 1990 to 1997; 61.5 per cent of businesses exit within five years, and the founder of a private company faces a risk of about 10 per cent of losing all his/her investment in the first ten years (Dun and Bradstreet Corporation; Dunne et al., 1988; Moskowitz and Vissing-Jorgensen, 2002, respectively). The cross-sectional standard deviation of self-employment earnings is substantially higher than wages from paid employment (Hamilton, 2000).

This evidence contrasts with the standard approach in the growth literature, where, despite major steps to provide microfoundations for the innovation process, the role of risk-aversion and insurance has received little attention. We develop a discrete-time variant of the Schumpeterian growth model of Aghion and Howitt (1992), in which innovation is characterized by uncertainty and by the presence of intertemporal knowledge spillovers. To this framework we introduce risk-aversion. There are two sources of inequality—that between skilled and unskilled agents who are ex ante different, and that among skilled agents who choose different occupations and become different ex post. In particular, some skilled workers will choose to work in production for fixed wages, while others will become entrepreneurs/researchers and receive random profits. It is this endogenous choice of occupation that drives the growth process. The presence of unskilled workers plays an essential role in our analysis of optimal taxation, as it implies that a social planner will have equity considerations as well as the pure insurance ones derived from the riskiness of research.[2] We assume that the

[2] When agents are risk-neutral, the utilitarian welfare objective is equivalent to maximizing national income. In that case, as in Aghion and Howitt (1992), the distribution of payments to the fixed factor in the production function can be left unspecified. With risk-averse agents, however, optimal taxation inevitably must be explicit about the identity of the fixed factor. It is consistent with the literature to identify the immobile factor with unskilled labor, as we have done.

only policy tool available is an income tax that does discriminate across agents, which is in the tradition of the optimal tax literature; see, for example, Stern (1976). We explicitly model a linear income tax but it is clear our propositions also hold in the case of a progressive marginal tax schedule—e.g. where the source of redistributive revenues is a windfall tax levied only on the pure profits of monopoly.

The idea that redistribution can act as social insurance when private risk-pooling arrangements are absent was first noted by Eaton and Rosen (1980) and Varian (1980). Further studies by Kanbur (1981), Peck (1989), and Boadway, Marchand and Pestieau (1991) examined how redistribution affects the occupational choice between risky entrepreneurship and paid employment. A central concern in these papers is the implication of occupational choice for optimal taxation. The models used, however, are static as entry into entrepreneurship is assumed to have no impact on innovation or growth. As a result the social planner faces a tradeoff since greater redistribution reduces (ex post) inequality, but, because it provides more insurance against business failures, it encourages excessive risk taking (i.e. leads to too many entrepreneurs). Allowing for the dynamic aspect of entrepreneurship adds an important dimension to the optimal tax problem.

Several broad results emerge from our investigation. First, the social insurance effect of redistribution raises the amount of research undertaken and accelerates the growth rate, relative to the laissez-faire equilibrium.[3] Second, when the number of researchers increases, the number of skilled wage-earners falls, raising their equilibrium wage. This implies that skilled workers can experience an increase in their net incomes despite being net fiscal contributors. Since unskilled workers benefit directly from redistribution, it is possible for re-

[3] One can ask why Europe has relatively more social spending but lower entrepreneurship rates than the United States. There are two likely reasons for this. First, the generous discharge provisions in US bankruptcy law may encourage innovation. As Skeel (2001) puts it, "In the United States, bankruptcy has long served as a partial substitute for the more generous social protections provided by other nations." Second, the evidence from OECD countries provided by Ilmakunnas and Kanniainen (2001) suggests that "The Welfare State does not provide insurance to share the entrepreneur's risk of failure." Instead, the existing programs are designed to reduce the risk of workers.

distribution to simultaneously increase growth, reduce inequality, and be Pareto-improving. Third, we consider how an optimal linear income tax rate varies with intertemporal considerations, namely the degree of knowledge spillovers and the social discount rate. A stronger spillover implies a greater social gain from innovation and less inequality, but also a greater loss from monopoly pricing of intermediate goods. Numerical solutions indicate the net effect is, perhaps surprisingly, that the optimal tax rate is *decreasing* in the size of the spillover. The role of the social discount rate highlights the planner's balance between static losses and dynamic gains.

Caucutt, Imrohoroglu and Kumar (2002, 2003) are particularly close to our work. They examine the impact of tax progressivity on growth, when individuals invest in human capital, and find that greater progressivity can accelerate growth. Their work differs from ours in that they consider the risk associated with educational investments, and how progressivity may reduce this risk.[4] Also related is the work of Chou and Talmain (1996), who examine an R&D model in which redistribution can be both growth-enhancing and Pareto improving. Their mechanism relies on the elasticity of individual labor supplies. Redistribution impacts growth because an individual's wealth affects her consumption-leisure tradeoff and hence determines her labor supply. Under certain conditions, redistribution may raise the aggregate labor supply and hence the growth rate. Furthermore, since agents are infinitely lived, faster growth offsets the static loss imposed by redistribution on rich individuals, and a Pareto improvement may be possible. Our analysis differs from this approach in two crucial aspects. First, allowing for risk-aversion implies that redistribution affects innovation through occupational choice rather than through a 'scale effect.' Second, the Pareto improvement obtained by Chou and Talmain is due to the growth-effect offsetting the direct redistribution effect, and would not necessarily hold if individuals had finite lives. In our overlapping generations setup, we can determine the effect of redistribution on the welfare of a particular generation, independently of the

[4] However, some authors have argued that remaining unskilled actually entails a greater risk due to the greater probability of being unemployed, and that education is precisely a way to reduce this risk. See Gould, Moav, and Weinberg (2001).

weight given by the planner to future generations.

Lastly, our paper is related to the literature trying to better understand the determinants of innovation in the Schumpeterian growth model, such as Aghion, Dewatripont, and Rey (1999), Aghion, Harris, Howitt, and Vickers (2001), Martimort and Verdier (2004) and Zeira (2005). Although Martimort and Verdier have a very different focus than us—they examine the link between the internal organization of firms and innovation—their approach shares with our's the idea that the incentives to innovate are both causes and results of the growth process. In their paper, the rate of growth affects the structure of firms, and hence the incentives to innovate and the growth rate; in our context, growth affects the choice of optimal taxes which in turn will determine the number of entrepreneurs and hence the rate of innovation. Closely related is Zeira (2005) who considers the effect of risk-aversion in a model with patent races. He proposes two ways of promoting innovation, venture capital and the concentration of research in a large firm, and shows that both have the drawback of increasing duplication and can reduce the rate of growth despite an increase in R&D employment. Our setup proposes an alternative solution to deal with the under-provision of innovation caused by risk-aversion.

The paper is organised as follows. The next section presents the model. Section 3 describes the equilibrium. Section 4 analyzes the effects of redistribution on innovation and on the utilities of skilled and unskilled agents. The intertemporal social welfare function is derived and discussed in Section 5. Section 6 contains numerical simulations of the optimal tax rates, particularly to show the effects of the intertemporal spillover in the tradeoff between static welfare losses and dynamic welfare gains, resulting from fiscally-induced increases in entrepreneurship. Section 7 concludes.

2 The model

2.1 Population

The population consists of non-overlapping generations, each living for one period. Each generation is of size $N \in \mathbb{N}$, and consists of L unskilled and H skilled workers. Unskilled workers are employed in production. Skilled workers make an occupational choice at the beginning of their lives, choosing between being an entrepreneur or a manufacturing worker.

Researchers (also called 'entrepreneurs') undertake R&D in order to invent a higher quality intermediate good and obtain a patent for it. The number of entrepreneurs in period t is denoted R_t. The remaining $M_t = H - R_t$ skilled workers are hired for fixed wages to manufacture the intermediate good using the incumbent technology. Skilled workers have identical productivities when they are employed in production, but their productivity in research differs depending on whether or not they have 'research ability', as will be specified below. We suppose that a fraction z of skilled workers possess research ability and that this is private information, known only to the individual.

Given the assumptions made above, the timing is the following. At birth the type of the individual is revealed: skilled or unskilled (which is observable), with or without research ability (which is private information). Skilled individuals then make their occupational choices, all agents work, payoffs are received, and then consumption takes place at the end of the period. All individuals have identical utility functions, which depend only on consumption, C. The utility of someone born in period t is assumed to be given by

$$U(C_t) = (C_t)^{\alpha}, \quad 0 < \alpha < 1. \tag{1}$$

That is, agents are risk-averse.

2.2 Production technologies

We consider a small open economy that produces a single homogeneous final good and a single intermediate good. The final good is produced by a competitive sector according to the production function

$$Y_t = A_t x_t^{\theta} L^{1-\theta} \tag{2}$$

where $0 < \theta < 1$, A_t is the index of total factor productivity, which depends on the 'quality' or vintage of the intermediate good used, and x_t is the amount of intermediate good employed. The price of the final good is the numeraire.

Each innovation increases the value of A_t by a factor $\gamma > 1$, with $A_{t+1} = \gamma A_t$ if an innovation occurs in t and $A_{t+1} = A_t$ if no innovation occurs. We assume that innovations are non-drastic in the presentation of the model. The case of drastic innovations is very similar (see the appendix) and leaves our propositions unchanged.

There is a large number of risk-neutral firms that are willing to produce intermediate goods. After an innovation occurs, one of these firms purchases the patent for the new intermediate good and becomes a monopolist. The intermediate good is produced using skilled labor alone according to the linear technology

$$x_t = M_t, \tag{3}$$

and is not traded.

2.3 Research

Suppose that $\widetilde{R}_t$ individuals engage in R&D in order to invent the next vintage and that, of those, $R_t \leq \widetilde{R}_t$ have research ability. The probability of at least one entrepreneur discovering the 'next' quality of intermediate good during period t is assumed to be

$$\Pr(\text{at least one innovator}) = \lambda R_t \quad \text{where } 0 < \lambda < 1. \tag{4}$$

That is, the research undertaken by those without ability has no effect on the aggregate probability of innovation. A patent is awarded to the innovator, or, if there is simultaneous discovery, the monopoly is randomly granted to one of the successful innovators. We model this method of awarding the patent as a lottery among researchers. Conditional on an innovation occurring, the probability that the patent is awarded or not to a particular researcher is assumed to be, respectively,[5]

$$\begin{aligned} \Pr(\text{patent to a given entrepreneur}|\text{an innovation occurs}) &= 1/R_t \\ \Pr(\text{no patent to a given entrepreneur}|\text{an innovation occurs}) &= (R_t - 1)/R_t. \end{aligned}$$

These expressions together with (4) imply that the unconditional probability that a particular individual with research ability will obtain the patent is λ.[6] For an individual with no research ability who engages in research, the probability of obtaining the patent is zero. Note that we require $\lambda R_t \leq 1$ for the probability distribution to be defined. We hence assume $\lambda z H \leq 1$, that is, when all those who can do R&D do so, the probability of an innovation occurring is less than one.

A patent is infinitely lived, and enters in operation in the period following the discovery.[7] That is, a new intermediate good invented at t will start generating profits at $t+1$. The researcher who has been awarded the patent can sell it to an intermediate-good firm for a price V_{t+1}, to be determined below.

[5] This setup is also used in Cooper et al (2001) and Zeira (2005).

[6] We are assuming that λ is small and therefore the likelihood of ties (multiple innovators in the same period) is negligible. Our formulation of the probability that a particular entrepreneur wins the patent, given $R_t - 1$ rivals, is thus an approximation to the exact probability. Specifically, there could be ties between any number $2 < i < R_t$ of innovators, where in each case the chance a particular innovator obtains the patent is $1/i$. Thus the exact probability of a given entrepreneur winning the patent equals $\lambda \sum_{i=0}^{R_t - 1} \left(\frac{(R_t-1)!}{i!(R_t-1-i)!} \frac{\lambda^i (1-\lambda)^{R_t-1-i}}{1+i} \right) \approx \lambda$ when λ is small. For example, in our simulations of laissez-faire in Section 6 the exact probability is 1.83×10^{-4}— and the approximate probability λ is 2.00×10^{-4}.

[7] For an analysis of the impact of patent-design on growth see O'Donoghue and Zweimuller (2004).

2.4 Profits, wages, and the value of a patent

Since the final good sector is competitive, all factors are paid the marginal product. Let p_t denote the price of the intermediate good, and w_t the wage of skilled labor used to produce the intermediate good. Differentiating (2) to obtain the inverse demand function for intermediate goods, we can write the monopolist's problem as

$$\begin{aligned} \max_{x_t} \Pi &= p_t x_t - w_t x_t && (5) \\ \text{subject to } p_t &= A_t \theta x_t^{\theta-1} L^{1-\theta} && (6) \\ p_t &\leq \gamma w_t, && (7) \end{aligned}$$

where the second constraint ensures that only the latest quality of the good is used at each point in time. Non-drastic innovations imply that the monopolist charges the limit price $p = \gamma w_t$, which yields a level of intermediate good production $x_t = (A_t \theta / (\gamma w_t))^{\frac{1}{1-\theta}} L$. This expression, together with the market-clearing condition for skilled workers, $x_t = M_t$, gives the skilled wage w_t, the unskilled wage v_t, and the monopolist's profit Π_t as

$$\begin{aligned} w_t &= \frac{\theta}{\gamma} \frac{Y_t}{M_t} && (8) \\ v_t &= (1-\theta) \frac{Y_t}{L} && (9) \\ \Pi_t &= \theta \frac{\gamma - 1}{\gamma} Y_t. && (10) \end{aligned}$$

The value of an innovation is then determined by the familiar asset condition $rV_{t+1} = \Pi_{t+1} - \lambda R_{t+1} V_{t+1}$, where r is the (exogenously given) interest rate. This implies

$$V_{t+1} = \frac{\Pi_{t+1}}{r + \lambda R_{t+1}}, \quad (11)$$

indicating that the value of an innovation to a risk-neutral firm is equal to the stream of profits generated by the innovation, discounted by the interest rate plus the probability of being replaced if a new vintage is invented next period.[8] Note that $w_{t+1} = w_t$, $v_{t+1} = v_t$, and $\Pi_{t+1} = \Pi_t$ if an innovation does not occur at t. If there is an innovation, wages and profits increase by a factor of γ, that is $w_{t+1} = \gamma w_t$, etc.

2.5 Taxation

We consider a linear tax system having the general form $T(I) = -B + \tau I$, where I is individual income, τ the tax rate, and B a demogrant. An individual pays taxes on her income and receives a transfer B. A researcher who obtains a patent at t and sells it for V_{t+1} faces a tax bill of τV_{t+1}. We suppose that instead of the researcher paying the entire amount in the period in which she sells the innovation, the intermediate goods firm pays the researcher the *net* value of the innovation $(1-\tau)V_{t+1}$ and then pays the taxes due as a proportion of its profits each period. Assuming that the government sets a constant tax rate and holds a balanced budget in each period, its budget constraint is

$$NB_t = \tau \left(\Pi_t + w_t M_t + v_t L \right) = \tau Y_t. \tag{12}$$

Our assumption on the payment of the tax on profits is made in order to avoid the transfer B_t varying over time. Note that from the researcher's point of view the net value of the innovation is always $(1-\tau)V_{t+1}$ irrespective of when the tax is paid. If all the taxes were paid at t, then tax revenue would be $\tau \left(V_{t+1} + w_t M_t + v_t L \right)$ in the periods in which there is an innovation and $\tau \left(w_t M_t + v_t L \right)$ if there is none. As a result, with a constant tax rate, B_t would vary depending on whether or not an innovation has occurred. Our assumption avoids this.

[8] An alternative way to model the intermediate goods sector is to assume that patents last only one period, and that after that they can be produced by a large number of competitive firms. This implies that depending on whether or not there has been an innovation, the intermediate goods sector will produce under monopoly or perfect competition. We studied this case in an earlier version of the model and found equivalent conclusions; see García-Peñalosa and Wen (2004). This framework is also used in Cooper et al (2001) and Lambson and Phillips (2005).

3 Equilibrium

3.1 Occupational choice

An unskilled worker faces no occupational choice and obtains utility $U_{ut} = ((1-\tau)v_t + B_t)^{\alpha}$. That is,

$$U_{ut} = Y_t^{\alpha}\left((1-\tau)\frac{(1-\theta)}{L} + \frac{\tau}{N}\right)^{\alpha}. \tag{13}$$

Skilled workers make an occupational choice between working in manufacturing (m) or entrepreneurship (e). If an individual has no research ability, her income from engaging in R&D is simply the transfer B_t. For an individual with research ability, the expected utility of being an entrepreneur is

$$U_{et} = \lambda\left(C_{\pi t}\right)^{\alpha} + (1-\lambda)\left(B_t\right)^{\alpha}, \tag{14}$$

where $C_{\pi t} \equiv (1-\tau)V_{t+1} + B_t$ is the consumption of a period-t patent winner. Using the fact that, since there was an innovation at t, then $Y_{t+1} = \gamma Y_t$, we can express $C_{\pi t}$ as

$$C_{\pi t} = \left((1-\tau)\theta\frac{(\gamma-1)\,Y_t}{r+\lambda R_{t+1}} + \frac{\tau Y_t}{N}\right). \tag{15}$$

The utility of an entrepreneur is decreasing in R_{t+1}, because more future research shortens the expected life of an innovation and hence lowers the value of a patent.

All skilled workers obtain the same utility if they chose to work in production, $U_{mt} = ((1-\tau)w_t + B_t)^{\alpha}$, irrespective of their ability. This utility can be expressed as

$$U_{mt} = Y_t^{\alpha}\left((1-\tau)\frac{\theta}{\gamma M_t} + \frac{\tau}{N}\right)^{\alpha}. \tag{16}$$

The utility of a skilled worker is increasing in the number of entrepreneurs, as a reduction in employment in production raises the skilled wage.

Arbitrage determines the equilibrium allocation of skilled individuals across occupations. Assuming an internal solution exists (i.e., there is an R_t^* such that

$1 < R_t^* \leq zH$), it is determined from the equal-utilities condition, $U_{mt} = U_{et}$:

$$\left(\frac{\theta}{\gamma}\frac{1-\tau}{M_t} + \frac{\tau}{N}\right)^{\alpha} = \lambda\left((1-\tau)\theta\frac{(\gamma-1)}{r+\lambda R_{t+1}} + \frac{\tau}{N}\right)^{\alpha} + (1-\lambda)\left(\frac{\tau}{N}\right)^{\alpha} \tag{17}$$

where $M_t = H - R_t$, and Y_t^{α} has been factored out of both sides.[9]

3.2 Steady state equilibrium

Note from (17) that the occupational choice at period t depends on expectations about future research, as this determines the expected lifetime of a patent and hence its value. We assume perfect forecasting, and focus on the steady state equilibrium where, given the policy variable τ, we have $M = M_t = M_{t+1}$ and $R = R_t = R_{t+1}$ for all t.[10] The steady state equal-utilities condition determining occupational choices can then be written as

$$\left(\frac{\theta}{\gamma}\frac{1-\tau}{H-R} + \frac{\tau}{N}\right)^{\alpha} = \lambda\left((1-\tau)\theta\frac{(\gamma-1)}{r+\lambda R} + \frac{\tau}{N}\right)^{\alpha} + (1-\lambda)\left(\frac{\tau}{N}\right)^{\alpha} \tag{18}$$

which defines the equilibrium level of research, R^*, as a function of the tax rate and model parameters. Since the left-hand side of (18) is strictly increasing in R and right-hand side is strictly decreasing, there is a unique solution to the arbitrage condition.

It is clear from this expression that the effect of redistributive taxation on growth is due to the provision of insurance that encourages risk-taking. It can easily be checked from equation (18) that if agents were risk-neutral (i.e. $\alpha = 1$), redistribution would have no impact whatsoever on occupational choice, and hence would not affect the growth rate.

[9]Since M_t must be an integer, the equal-utilities condition may not hold exactly. A more precise, but awkward, statement of the equilibrium value of M_t is that, $U_{mt}(M_t) \geq U_{et}(R_{t+1})$ and $U_{mt}(M_t + 1) < U_{et}(R_{t+1})$.

[10]See Aghion and Howitt (1992, 1998) for a discussion of the role of expectations and the possibility of cycles.

3.3 Equilibrium insurance and patent purchase

Before we examine the properties of the equilibrium, we need to consider whether firms, which are risk-neutral, would provide private insurance to researchers. Suppose that an intermediate-goods firm hires one (or more) researcher and pays her a constant wage w_t^r, independently of whether or not she innovates. Private information on research abilities implies that the firm does not know whether a particular worker has research ability. In a pooling equilibrium, the firm will assume that the probability that a particular individual has research ability is z. Then, the wage it will be willing to pay is $w_t^r = z\lambda V_{t+1}$, which implies that the utility of being a hired researcher is

$$U_{rt} = Y_t^{\alpha}\left((1-\tau)z\lambda\theta\frac{(\gamma-1)}{r+\lambda R_{t+1}} + \frac{\tau}{N}\right)^{\alpha}, \tag{19}$$

which is independent of the individual's ability. Hence, if agents with high ability prefer to be hired researchers than to work in production, so would those with no ability. Since there is no possible way for individuals to signal their research ability, no separating equilibrium exists.

Now consider whether an individual with research ability prefers to be hired by the firm or to undertake research independently. From (14) and (15), $U_{rt} < U_{et}$ if and only if

$$\left((1-\tau)z\lambda\theta\frac{(\gamma-1)}{r+\lambda R_{t+1}} + \frac{\tau}{N}\right)^{\alpha} < \lambda\left((1-\tau)\theta\frac{(\gamma-1)}{r+\lambda R_{t+1}} + \frac{\tau}{N}\right)^{\alpha} + (1-\lambda)\left(\frac{\tau}{N}\right)^{\alpha}. \tag{20}$$

Note from (18) that, if this expression is satisfied then $U_{rt} < U_{mt}$, so that individuals with no ability will rather work in production than as hired researchers. In the absence of taxation, (20) becomes

$$z < \lambda^{(1-\alpha)/\alpha}. \tag{C1}$$

That is, no private insurance will be provided if the proportion of individuals with research ability is low enough, the probability of success high enough, or the degree of risk-aversion not too large. In the Appendix we show that even when there is positive taxation, (C1) implies that no skilled worker would accept the insurance proposed by the firm. In what follows we assume that (C1) holds. As a result, all insurance stems from redistribution by the government.

3.4 Laissez-faire equilibrium

We start by considering the laissez-faire equilibrium. In this case, there is no insurance, hence the entrepreneurs who fail to discover a new quality receive no income. The interior equilibrium for the number of researchers is then given by

$$R^{*} = H - \frac{r/\lambda + H}{1 + \lambda^{(1-\alpha)/\alpha}\gamma(\gamma - 1)}, \tag{21}$$

where $0 < R^{*} < zH$ if an interior solution exists.[11] The comparative statics are easily established.

Lemma 1 *The laissez-faire equilibrium value of R^{*} is increasing in H, λ, γ, and α, and decreasing in r.*

These are analogous to those in Aghion and Howitt (1992), except for the aspect of risk-aversion. As expected, greater risk-aversion (lower α) decreases entrepreneurship. Note that there is no effect of θ on the laissez-faire allocation of skilled workers to research. The reason for this is that both the skilled wage and the level of profits increase proportionally with θ.

[11] The first inequality is satisfied as long as $U_{et} > U_{mt}$ at $R = 0$, while the second requires $U_{et} < U_{mt}$ at $R = zH$. These inequalities imply

$$\lambda^{1/\alpha}\gamma\left(\gamma - 1\right) > r/H > \left(\lambda^{1/\alpha}\gamma\left(\gamma - 1\right)\left(1 - z\right) - \lambda z\right).$$

That is, the interest rate cannot be either too high nor too low. A high interest rate would reduce so much the value of an innovation that nobody would want to become an entrepreneur, while a very low interest rate would imply a very high V_{t+1}, and utility equalization across the two occupations would not be possible even when all able individuals become entrepreneurs.

3.5 Innovation and long-run growth

The change in output between periods t and $t+1$ can be written as

$$\ln\left(\frac{Y_{t+1}}{Y_t}\right) = \ln\left(\frac{A_{t+1}}{A_t}\right). \tag{22}$$

Because innovations are stochastic, so will be the realized growth rate. We hence define the growth rate g_t as the expected rate of technological change, $g_t = E(\ln A_{t+1} - \ln A_t)$. In steady state, there is a constant probability $q \equiv \lambda R$ that an innovation occurs, and the expected long-run growth rate is then

$$g = \lambda R \ln \gamma. \tag{23}$$

The long-run probability of innovation, and hence the growth rate, are increasing in the number of researchers. The central question we want to address is whether redistribution encourages or discourages economic growth through the impact of taxation on the probability of innovation.

4 Impact of redistribution

4.1 Redistribution and research

If we examine the right-hand side of (18) we can distinguish two effects of redistributive taxation on the utility of an entrepreneur. First, a higher marginal tax rate, holding constant the demogrant (represented by the term τ/N) reduces a successful entrepreneur's net income. This captures the traditional 'incentive' argument that taxes discourage entrepreneurship. However, with a proportional tax the net income of skilled wage earners is similarly affected and the incentive effect is mitigated. The incentive effect would be important if, instead of a proportional income tax, we imposed a tax only on windfall profits (e.g. taxing only V_{t+1}). Second, there is an insurance effect in the provision of a demogrant, which protects entrepreneurs to some extent against the cost of failure. Regardless of the marginal tax rate schedule, the insurance effect must dominate the incentive effect at low tax rates, or when the probability of winning the patent race is very small.[12]

Proposition 1 *The effects of redistribution on research can be characterized as follows:*

(i) For an economy in the laissez-faire equilibrium, the introduction of a small amount of redistribution ($\tau \to 0^+$) increases the number of researchers;

(ii) There exists a value $\overline{\lambda} > 0$ such that for all $\lambda \leq \overline{\lambda}$ the number of researchers is strictly increasing in the tax rate.

Proof: See appendix.

The first part of the proposition says that a small amount of redistribution always increases the number of researchers. This occurs because in the laissez-faire equilibrium unsuccessful entrepreneurs have no consumption.[13] Their marginal utility is then infinitely large, and the insurance effect always dominates any negative incentive effect. The second part says that a sufficiently low probability of success ensures that the insurance effect of redistribution always dominates incentive effects, making the level of research a monotonically increasing function of the tax rate. For higher values of λ, that is for $\lambda > \overline{\lambda}$, small tax rates increase research, but large rates might not.

To better understand these results it is convenient to represent (18) graphically, as in Figure 1. The curve labelled u_m is the left-hand side of (18), which is the utility of skilled workers divided by Y_t^{α}. Clearly u_m decreases with τ as long as the skilled wage is above average income,[14] implying that the introduc-

[12] The following proposition characterizes properties of our model that would continue to hold if the demogrant were financed with a profits tax only. In this sense, the results are general. In the case of our linear income tax, we find that the number of researchers in the limit as $\tau \to 1$ is $R = H - \frac{r/\lambda + H}{1+\gamma(\gamma-1)} > R^*$ (laissez-faire), while numerical simulations suggest that research is non-decreasing in the tax rate. However, we do not wish to stress these results because they are particular to a linear income tax and would cease to hold in the case of a windfall profits tax.

[13] In reality, entrepreneurs may have other forms of wealth to consume in the event of failure, but this merely underscores the point that, in the absence of social insurance, potential entrepreneurs may delay their entry into business until they have funds to fall back on.

[14] A sufficient condition for this is $L/H > (\gamma - \theta)/\theta$, implying that skilled workers are net fiscal contributors. We impose the stronger condition, $L/H >$

tion of a tax shifts u_m downwards. The curve labelled u_e is the right-hand side of (18), i.e. the utility of entrepreneurs divided by Y_t^α. The schedule u_e may shift upwards or downwards depending on whether the insurance effect or the incentive effect or dominates. Whenever a higher value of τ shifts u_e upwards the equilibrium value of R rises, as depicted. If u_e shifts downwards, that is, if the incentive effect is stronger, then the number of researchers may increase or decrease.

Figure 1

4.2 Wages, utility and the distribution of income

Redistribution also affects the utility of the current generation through general equilibrium changes in the wage rates of skilled and unskilled agents. To see this write the utility of a skilled individual working in manufacturing as $U_{mt} = ((1-\tau)w_t + \tau Y_t/N)^\alpha$. The direct impact of redistribution is to reduce her post-fisc income for a given wage and hence would tend to reduce her utility; but there is also an indirect impact due to the fact that, if the tax increases the number of researchers, this will reduce the supply of skilled labor in manufacturing and hence increase her wage and utility. The opposite occurs for an unskilled individual, with $U_{ut} = ((1-\tau)v_t + \tau Y_t/N)^\alpha$, as the direct impact would tend to increase her utility given v_t, while the reduction in the number of skilled workers in manufacturing would tend to reduce her wage and hence her utility. It is not possible to derive analytically when is it that taxation increases or reduces the utilities of these two groups, except if the tax rate is close to zero.

Proposition 2 *For an economy in the laissez-faire equilibrium, introducing a small amount of redistribution ($\tau \to 0^+$) increases (decreases) the wage and utility of skilled (unskilled) workers.*

Proof: See appendix.

$(\gamma - \gamma\theta)/\theta$, which ensures that skilled wages must exceed unskilled wages.

Proposition 2 implies that for tax rates close to zero, the general equilibrium wage effect dominates the direct effect of taxes and transfers for both types of workers. As a result the skilled are better off and the unskilled are worse off. This suggests the possibility that there may be some levels of redistributive taxation resulting in Pareto-improvements—i.e. a tax that increases the rate of innovation and raises the welfare of both skilled and unskilled workers in the current generation. We find that for relatively small tax rates, it is possible that the redistribution effect dominates for the unskilled and the gross-wage effect dominates for the skilled. As a result the utility of all individuals in the current generation is higher. We are unable to obtain analytical conditions for Pareto-improvements to occur, but a numerical example is provided in Section 6.

Propositions 1 and 2 also have implications for the effects of taxation on the distribution of income. From equation (11) a greater number of researchers will reduce the income of the innovator, which tends to reduce inequality. On the other hand, the general equilibrium changes in wages worsens inequality between skilled and unskilled workers. Thus for small tax rates, it is possible that the overall distribution of income becomes more unequal. This type of general equilibrium effect, which can partially offset the direct redistributive impact of taxation, has been previously identified both in the context of occupational choice with entrepreneurial risk, and in models with endogenous labor.[15] However, it is unlikely to dominate for non-marginal tax levels, and clearly full redistribution will reduce inequality relative to laissez-faire.

5 Social welfare and optimal tax rates

Consider a utilitarian welfare function, where the social planner weights the utilities of the various types of individuals by their population. Note that there are two possible states of the world, depending on whether or not an innovation has occurred at time t. We suppose that the planner weights these two

[15] See Sinn (1996) and García-Peñalosa and Turnovsky (2006).

states according to the probability of being in one or the other, that is λR and $(1-\lambda R)$. Social welfare at period t, denoted W_t, is then given by

$$W_t = LU_u + (H-R)U_m + \lambda R(C_{\pi t}^{\alpha} + (R-1)\,{B_t}^{\alpha}) + (1-\lambda R)\,R\cdot {B_t}^{\alpha}, \quad (24)$$

which, using the arbitrage condition and recalling that u_m and u_u are the output-adjusted utilities of skilled and unskilled workers, implies

$$W_t = \left(A_t M^{\theta} L^{1-\theta}\right)^{\alpha} (Lu_u + Hu_m)\,. \quad (25)$$

That is, although the planner cares about ex post inequality, social welfare at t is equivalent to the sum of expected utilities, weighted by population. All the terms in this expression are time-invariant except the technology index A_t, which increases by a factor γ with each innovation. Letting δ denote the social discount factor, the discounted value of expected social welfare over an infinite horizon can then be written as

$$W = \sum_{t=0}^{\infty} \delta^t \sum_{s=0}^{t} \left(\frac{t!}{s!(t-s)!}\right) q^s (1-q)^{t-s} \left(\gamma^{\alpha}\right)^s W_0, \quad (26)$$

where W_0 is welfare at time $t=0$. Using the binomial theorem, this becomes

$$W = \frac{\left(A_0 (H-R)^{\theta} L^{1-\theta}\right)^{\alpha}}{1-\delta\left[1+(\gamma^{\alpha}-1)\lambda R\right]} \left[Lu_u + Hu_m\right]. \quad (27)$$

This expression has an intuitive interpretation. The numerator of the first term in (27) is simply current output, which is then discounted by the social discount rate modified to take into account the expected rate of innovation. The number of researchers has a negative effect on welfare via the level of output and a positive effect because it increases the rate of innovation. The latter effect increases in importance the larger is δ. The second term in (27) captures pure redistributive considerations. Taxation has a direct fiscal effect on the utilities in this term, but also changes wages through changes in R.

5.1 The first-best

It is useful to consider the benchmark case of the first-best allocation of labor. Suppose the planner can (i) verify ability, (ii) allocate workers to sectors, and (iii) use lump-sum taxation in order to redistribute within a generation. Then the planner's problem is simply to chose the allocation of skilled workers that maximizes the expected discounted flow of output, that is

$$W^{FB} = \frac{\left(A_0(H-R)^{\theta}L^{1-\theta}\right)^{\alpha}}{1-\delta\left[1+(\gamma^{\alpha}-1)\lambda R\right]}, \tag{28}$$

and to use lump-sum taxation to deal with equity considerations. Maximizing (28) with respect to R implies a first-best first-best allocation of skilled labor given by

$$R^{FB} = H - \frac{\alpha\theta}{1-\alpha\theta}\,\frac{(1-\delta)/\delta\lambda-(\gamma^{\alpha}-1)H}{\gamma^{\alpha}-1} \tag{29}$$

The differences between this expression and research in laissez-faire, R^* as given by (21), reflect the three margins discussed by Aghion and Howitt (1992): the intertemporal spillover effect, the appropriability effect, and the business stealing effect. These effects are modified by risk-aversion.[16] A new effect due to risk-aversion is captured by the term $\lambda^{\frac{1-\alpha}{\alpha}}$ in R^*, which tends to lower R^* relative to R^{FB}.

Comparing R^{FB} and the laissez-faire solution R^*, one can see that the first-best is likely to exceed the laissez-faire level if δ is high (i.e. the planner cares a lot about dynamic gains), and if H is large or θ is small. If the number of skilled workers is large and/or the elasticity of output with respect to skilled manufacturing employment is small (θ small) the static output loss occurring when skilled workers move into research is low, hence the planner would chose a higher growth rate. The effect of all other variables is ambiguous. For example, a larger technological spillover increases both the laissez-faire and the first-best

[16] The presence of α in the terms of (29) are modifications of the standard effects. See Aghion and Howitt (1992) for a discussion of these.

level of research.

5.2 Welfare effects of taxation

Using (28), social welfare (27) can be expressed as $W(R,\tau) = W^{FB}[Lu_u + Hu_m]$, where the first term is the expected discounted flow of output and captures efficiency considerations, and the second term is the weighted sum of the utilities of the skilled and unskilled and captures equity considerations. Differentiating this expression we have

$$\frac{dW}{d\tau} = [Lu_u + Hu_m]\frac{dW^{FB}}{dR}\frac{dR}{d\tau} + W^{FB}\left[L\frac{\partial u_u}{\partial \tau} + H\frac{\partial u_m}{\partial \tau}\right] + W^{FB}H\frac{\partial u_m}{\partial R}\frac{dR}{d\tau}. \tag{30}$$

There are three effects of the tax rate on welfare. The first term captures efficiency considerations. More research has a positive growth effect, as it accelerates growth, but a negative level effect, since it reduces the levels of output. The net effect depends on the sign of $dW^{FB}/dR \times dR/d\tau$ which, in general, may be positive or negative, and is zero at R^{FB}. The second term captures redistributive considerations, with a higher tax redistributing income from the skilled to the unskilled, for given wages. The last term is the general equilibrium effect operating through the share of income received by skilled workers, which increases if the number of researchers rises. This last effect can also be viewed as the impact of social insurance on the utility of the skilled. In choosing the tax rate the social planner then needs to consider (i) the effect of τ on R, (ii) the static loss and dynamic gain of a higher R (as in any growth model), and (iii) the distributive impact of the tax for a given generation, adding to the standard static optimal taxation problem the fact that wages change with the tax.

Analytical solutions for the first-best tax are impossible to derive. However, we can establish the following:

Proposition 3 *If the first-best number of entrepreneurs exceeds the laissez-faire level, then introducing a small amount of redistribution ($\tau \to 0^+$) increases* *social welfare.*

Proof: See appendix.

Small tax rates increase social welfare for two reasons. The first is the efficiency gain. Since we are supposing that the first-best level of research is greater than the laissez-faire, the increase in research due to the introduction of the tax rate increases the discounted flow of output. The second is the redistributive effect. This effect is positive because of the very large increase in the utility of unsuccessful entrepreneurs who under laissez-faire have zero consumption.

6 Intertemporal spillovers and optimal tax rates

The effect of the intertemporal spillover on the optimal tax rate is complex. Intuitively, one would expect that stronger intertemporal spillovers, that is higher values of γ, would tend to increase the optimal tax rate, because a higher return to research should make the planner more willing to forgo current output in order to accelerate growth; and this could be achieved by providing more social insurance. However, this reasoning is incomplete, for two reasons. First, a higher γ increases both the level of research that maximizes W^{FB} and the laissez-faire level of research. Depending on which of these increases by more, the tax rate would tend up or down. The second reason is that a higher γ lowers inequality between skilled and unskilled workers (for a given R). In particular, a greater spillover increases the price charged by the monopolist and reduces intermediate-good output. The resulting decrease in the demand for skilled workers causes their wage to fall. Since inequality is thereby reduced the desired degree of redistribution is also lower. As a result of these considerations, the optimal tax rate can be declining in the size of the intertemporal spillover.

We use numerical simulations to find the net effect of γ on the optimal tax rate. The baseline parameter values for the simulations are shown in table 1. It is important to emphasize that the model is highly stylized, hence the examples we present are not intended to replicate growth rates, research employment, or any other feature of an actual economy. In general, they concern variables that are difficult or impossible to measure empirically (the probability of a research success, the magnitude of the quality improvement, etc.).

Tables 1 and 2 around here

Table 2 presents the solution for the baseline parameter values. The headings u_u and u_e denote the productivity-adjusted utilities of unskilled and skilled workers respectively, while ΔW is the percentage change in the discounted value of social welfare, relative to the laissez-faire equilibrium. *Gini* denotes the Gini coefficient of ex post income, which is derived in the appendix.[17] The first row reports the laissez-faire equilibrium. The economy exhibits a moderate research activity, with 3% of the labor force engaged in research. The growth rate has been annualized, under the assumption that each period corresponds to 10 years, about the lifetime of a research project. Our example implies an annual expected growth rate of 1.21 per cent. The expected utility of skilled workers is 28 per cent higher than that of unskilled workers, and the Gini coefficient is .29, in line with those observed in the more equal European economies. The second and third rows report the effect of introducing small tax rates of 1% and 10%, respectively. They illustrate Pareto improvements. In particular, with a 10% tax rate, both types of workers have greater utility than under laissez-faire, and the growth rate is higher. The fourth row gives the optimal tax rate.[18] At the optimal tax rate, research activity is increased to 5% of the labor force and growth is more than 0.6 of a percentage point higher than in laissez-faire. Under the optimal tax rate the current generation of skilled has a lower utility than without intervention due to the high degree of redistribution. Income

[17] In our economy, income inequality in a given period depends on whether or not there has been an innovation in the period. If there has been one, an individual receives the value of the patent and inequality is greater than if there were no innovation. The Gini coefficient reported in the tables is that obtained for the periods in which there has been an innovation. Inequality in the periods without innovation is lower but follows a similar pattern and is hence not reported.

[18] The optimal tax rate is high, but this is partly because we have assumed perfectly inelastic labor supplies.

inequality falls in all our examples, even for a small tax rate of 1%, indicating that the general equilibrium effects described above are weaker than the direct redistributive effect, except at very low tax rates.

Figure 2 shows the effect of the intertemporal spillover on the optimal tax rate, given our parameter values. We find that the optimal tax rate is a declining funciton of γ. To better understand this, Figure 3 depicts the highest tax rate that gives the skilled of the first generation the same utility as under laissez-faire, i.e. the highest Pareto improving tax rate, as a function of γ. The greater the value of γ, the lower this tax rate is. The reason is that a higher γ implies lower skilled wages, hence the increase in the skilled wage following a tax increase is small and the wage effect is less likely to offset the direct (negative) redistributive effect on skilled income. This suggests that the main reason why the optimal tax rate falls with γ is the reduced degree of inequality between the skilled and the unskilled, which calls for relatively less redistribution.

Figure 2 around here

7 Conclusion

We have examined a neglected implication of the Schumpeterian growth model; namely, that redistribution can increase the incentives for individuals to undertake R&D. Our point of departure is the fact that redistribution is a form of insurance. If agents are risk-averse, the social insurance effect is strong enough to offset the standard disincentive effects of taxation, resulting in faster growth and greater social welfare. Small amounts of redistribution actually worsen the inequalities between skilled and unskilled workers, because of general equilibrium effects on relative wages. Thus the welfare gain stems largely from the growth effect of more entrepreneurship.

An essential feature of the Schumpeterian growth model is that innovations have both a positive and a negative effect on social welfare, the latter being due to the fact that innovators will be monopolists and hence produce low output levels. We have shown how the social planner can balance these two considerations using a linear income tax. When the spillover is small, the benefits from growth are weak and the optimal tax rate is high to promote income equality. When the spillover is large, the optimal tax rate is relatively lower, despite the stimulus that social insurance can provide for entrepreneurship.

An important aspect of our analysis is that we have assumed finite lives. A large part of the literature on taxation and growth focuses on infinitely-lived agents. In this case, policies that increase the growth rate tend to also increase the welfare of all individuals, since the dynamic effect of faster growth offsets static losses. Our approach, in contrast, has examined the utility gains and losses of short-lived agents, and shown that even in this case Pareto improvements are possible. This occurs at levels of redistribution where the general equilibrium wage effect is strong enough to offset direct redistributive losses to the richest individuals; and, conversely for unskilled workers, the redistributive effect offsets the general equilibrium effect.

Our analysis has considered cash redistribution as the only possible policy the government can undertake. However, an important and well-known implication of the type of model we analyze is that R&D subsidies can affect the growth rate. Such subsidies have, however, been the object of substantial criticism from economists, because of the scope for diversion of expenditures and manipulation to which they are subject (Katz and Ordover, 1990). This manipulation can take place at the firm level, or even at the level of the government which can use them in order to engage in tax competition with other countries. By focusing on redistribution we are not arguing that R&D subsidies should not be used, but rather we have provided a possible alternative policy to foster growth, which could be used when R&D subsidies are deemed problematic and where redistributive equity is a part of the policy objective.

We have abstracted from considerations of individual labor effort. If effort were included in model the optimal tax rates would be much lower because of the incentive effects of income taxation, but the growth-enhancement argument for moderate levels of redistribution would continue to hold.

8 Appendix

This Appendix derives a number of the results and propositions in the text.

8.1 Drastic innovations

In the case of drastic innovations, the monopolist's problem is to maximize profits as given by

$$\max_{x_t} \Pi = A_t \theta x_t^{\theta} L^{1-\theta} - w_t x_t$$

The profit maximizing production of x_t is therefore, $x_t = \left(A_t \theta^2 / w_t\right)^{\frac{1}{1-\theta}} L$ (see Aghion and Howitt, 1998, for more details). This expression, together with the market-clearing condition for skilled workers, $x_t = M_t$, gives the skilled wage, w_t, and the monopolist's profit, Π_t, as

$$\begin{aligned} w_t &= \theta^2 \frac{Y_t}{M_t}, \\ \Pi_t &= (1-\theta)\theta Y_t. \end{aligned}$$

The unskilled wage, v_t, remains unchanged. It is straightforward to show that all our results hold in the case of drastic innovations. The laissez-faire number of researchers is given by

$$R^* = H - \frac{r/\lambda + H}{1 + \lambda^{(1-\alpha)/\alpha} \gamma (1-\theta)}. \tag{31}$$

8.2 Insurance with taxation

Whether or not the existence of the tax will induce agents to take insurance is complicated as the tax changes R and hence the production wage, the monopoly profits and the research wage. The insurance offered by the firm will not be taken by any skilled worker if

$$U_r < U_e = U_m$$

where the last equality is the equilibrium condition determining occupational choice. Using the expressions for utilities U_r and U_e, this inequality becomes

$$\left((1-\tau)z\lambda\theta\frac{(\gamma-1)}{r+\lambda R}+\frac{\tau}{N}\right)^{\alpha}<\left(\frac{\theta}{\gamma}\frac{1-\tau}{H-R}+\frac{\tau}{N}\right)^{\alpha} \tag{32}$$

i.e.

$$z<\frac{1}{\lambda}\frac{1}{\gamma(\gamma-1)}\frac{r+\lambda R}{H-R}. \tag{C4}$$

Note that we can rewrite the laissez-faire allocation as

$$\lambda^{(1-\alpha)/\alpha}=\frac{1}{\lambda}\frac{1}{\gamma(\gamma-1)}\frac{r+\lambda R^{*}}{H-R^{*}}. \tag{33}$$

Then if (C1) holds, i.e. if $z<\lambda^{(1-\alpha)/\alpha}$, this implies that (C4) is satisfied for any $R\geq R^{*}$. That is, any tax that increases the number of researchers implies that the certain wage proposed by firms will never be accepted by skilled workers.

8.3 The Gini Coefficient

When the population is divided into G groups the Gini coefficient is given by

$$Gini=\frac{1}{2Y}\sum_{i=1}^{G}\sum_{j=1}^{G}|Y_i-Y_j|\,n_in_j$$

where Y is aggregate income, Y_i and Y_j the post-tax incomes of individuals in group i,j and n_i and n_j the proportion of the populations in each group. When there is an innovation there are four types of agents and the Gini coefficient is given by

$$Gini(in)=\frac{1-\tau}{N}\left[\frac{\theta L}{\gamma}-(1-\theta)M+\theta\frac{\gamma-1}{\gamma}\left(\frac{N-1}{r+\lambda R}-(R-2)\right)+R-2\right].$$

When there is no innovation there are only three types of agents, as nobody receives monopoly profits, and hence

$$Gini(no-in) = \frac{1-\tau}{N}\left[\frac{\theta}{\gamma}(L+R) - (1-\theta)(M-R)\right].$$

8.4 Proofs of propositions

Proof of Proposition 1. Suppose an interior solution to the arbitrage equation exists.

(i) Evaluation of $dR/d\tau$ at $\tau = 0$.

Write the steady state equal-utilities condition (18) as

$$u_m(R,\tau) = u_e(R,\tau) \tag{34}$$

where

$$\begin{aligned} u_m(R,\tau) &\equiv \left(\frac{\theta}{\gamma}\frac{1-\tau}{H-R} + \frac{\tau}{N}\right)^{\alpha} \quad \text{and} \\ u_e(R,\tau) &\equiv \lambda\left(\theta\frac{(\gamma-1)(1-\tau)}{r+\lambda R} + \frac{\tau}{N}\right)^{\alpha} + (1-\lambda)\left(\frac{\tau}{N}\right)^{\alpha}. \end{aligned}$$

are the output-adjusted utilities. Totally differentiating (34) yields[19]

$$\frac{dR}{d\tau} = \frac{\partial u_e/\partial\tau - \partial u_m/\partial\tau}{\partial u_m/\partial R - \partial u_e/\partial R}. \tag{35}$$

Differentiating and setting $\tau = 0$ it is possible to show that the denominator of (35) is positive and finite, $\partial u_m/\partial\tau$ is finite, and $\partial u_e/\partial\tau$ tends to infinity as τ approaches zero. Thus $dR/d\tau$ approaches $+\infty$ as $\tau \to 0$.

(ii) Sign of $dR/d\tau$ for $\lambda < \overline{\lambda}$.

The sign of $dR/d\tau$ is given by (35). The denominator is positive and, under the assumption that skilled workers are net fiscal contributors, $\theta N > \gamma H$, we have $\partial u_m/\partial\tau < 0$. Then a sufficient condition for $dR/d\tau > 0$ is $\partial u_e/\partial\tau > 0$.

[19] The derivatives should be interpreted as so-called q-derivatives, which apply to intergervalued variables.

Differentiating we obtain

$$\frac{\partial u_e}{\partial \tau} \equiv \alpha\lambda \left(\frac{\theta(\gamma-1)(1-\tau)}{r+\lambda R} + \frac{\tau}{N} \right)^{\alpha-1} \left(\frac{1}{N} - \frac{\theta(\gamma-1)}{r+\lambda R} \right) + \alpha(1-\lambda) \left(\frac{\tau}{N} \right)^{\alpha} \frac{1}{\tau}. \tag{36}$$

Only cases in which the successful innovator is a net fiscal contributor are of interest, hence we assume

$$\frac{\theta(\gamma-1)}{r+\lambda z H} > \frac{1}{N}$$

which implies that the two terms in (36) may have opposite signs.

Now consider the second derivative of u_e

$$\frac{\partial^2 u_e}{\partial \tau^2} \equiv -\alpha(1-\alpha) \left[\lambda \left(\frac{\theta(\gamma-1)(1-\tau)}{r+\lambda R} + \frac{\tau}{N} \right)^{\alpha-2} \left(\frac{1}{N} - \frac{\theta(\gamma-1)}{r+\lambda R} \right)^2 + (1-\lambda) \frac{\tau^{\alpha-2}}{N^{\alpha}} \right] < 0, \tag{37}$$

implying that $\partial u_e / \partial \tau$ is decreasing in τ. Note also that

$$\left. \frac{\partial u_e}{\partial \tau} \right|_{\tau=0} \equiv \infty \quad \text{and} \quad \left. \frac{\partial u_e}{\partial \tau} \right|_{\tau=1} \equiv \frac{\alpha}{N^{\alpha}} \left[1 - \lambda N \frac{\theta(\gamma-1)}{r+\lambda R} \right]. \tag{38}$$

Using the value of $\lim_{\tau \to 1} R$ obtained above, we have

$$\left. \frac{\partial u_e}{\partial \tau} \right|_{\tau=1} \equiv \frac{\alpha}{N^{\alpha}} \left[1 - \frac{1+\gamma(\gamma-1)}{\gamma} \frac{\lambda \theta N}{r+\lambda H} \right]. \tag{39}$$

This derivative is decreasing in λ. Let $\overline{\lambda}$ be the critical value for which $\left. \frac{\partial u_e}{\partial \tau} \right|_{\tau=1} = 0$, defined by

$$\overline{\lambda} \equiv \frac{\gamma r}{(1+\gamma(\gamma-1))\theta N - \gamma H}. \tag{40}$$

For $\lambda \leq \overline{\lambda}$, we have $\partial u_e / \partial \tau \geq 0$ at $\tau = 1$, implying $\partial u_e / \partial \tau \geq 0$ and hence $dR/d\tau > 0$ for all τ. For $\lambda > \overline{\lambda}$, we have $\partial u_e / \partial \tau < 0$ at $\tau = 1$, implying that $u_e(\tau)$ is first increasing and then decreasing in the tax rate. Then, there exists a value $\overline{\tau}$ defined by

$$\left.\frac{\partial u_e(R(\tau),\tau)}{\partial \tau}\right|_{\tau=\overline{\tau}} \equiv 0 \tag{41}$$

such that $dR/d\tau > 0$ for all $\tau < \overline{\tau}$ but $dR/d\tau$ can be negative for higher tax rates. ■

Proof of Proposition 2. For the effect of taxes on equilibrium wages, differentiate (8) and (9) with respect to τ, using (3) and (2) to substitute $M = H - R$ for x in the production function, Y. The signs of the wage derivatives then follow immediately from proposition 1. For the effect of taxes on the equilibrium utilities of skilled workers, differentiate (16) and rearrange terms to obtain

$$\left.\frac{dU_m}{d\tau}\right|_{\tau=0} = \frac{\alpha Y^\alpha}{M^{1+\alpha}}\left(\frac{\theta}{\gamma}\right)^\alpha\left[(1-\theta)\frac{dR}{d\tau} - M\left(1-\frac{\gamma M}{\theta N}\right)\right] > \infty$$

Using proposition 1 for the sign of $dR/d\tau$ determines the sign of the derivative of utility. A similar calculation is applied to the utility of unskilled workers, equation (13), to show that $dU_u/d\tau < \infty$ at $\tau = 0$ and complete the proof. ■

Proof of Proposition 3. Differentiating welfare we have

$$\frac{dW}{d\tau} = W^{FB}\left[L\frac{\partial u_u}{\partial \tau} + H\frac{\partial u_m}{\partial \tau}\right] + \left([Lu_u + Hu_m]\frac{dW^{FB}}{dR} + W^{FB}H\frac{\partial u_m}{\partial R}\right)\frac{dR}{d\tau}, \tag{42}$$

For an infinitesimal tax rate $\partial u_u/\partial\tau$ and $\partial u_m/\partial\tau$ are finite while $dR/d\tau = \infty$ (from proposition 1). Hence the sign of the derivative is given by the sign of the term in brackets that multiplies $dR/d\tau$. Note that $\partial u_m/\partial R > 0$. Moreover, if $R^* < R^{FB}$ then $dW^{FB}/dR > 0$ since W^{FB} is strictly increasing and concave and attains its maximum at R^{FB}. Thus $dW/d\tau = \infty$ at $\tau = 0$. ■

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Table 1: Baseline Parameter Values

Production technology	$\theta = 0.3$	$\gamma = 2$
Research sector	$\lambda = 0.0002$	$r = 0.45$
Preferences	$\alpha = 0.9$	$\delta = 0.10$
Population	$H = 5,000$	$L = 25,000$

Table 2: Pareto-Improvements and Optimal Tax Rates

τ	R	g	u_u	u_e	ΔW	$Gini$
0	918	1.21	.445	.568	-	0.292
1.0	931	1.22	.445	.569	0.12	0.289
10.00	991	1.30	.450	.569	1.14	0.262
87.36	1484	1.89	.491	.516	7.54	0.036
98.61	1900	2.36	.482	.486	5.78	0.004

Figure 1
Equilibrium Number of Researchers

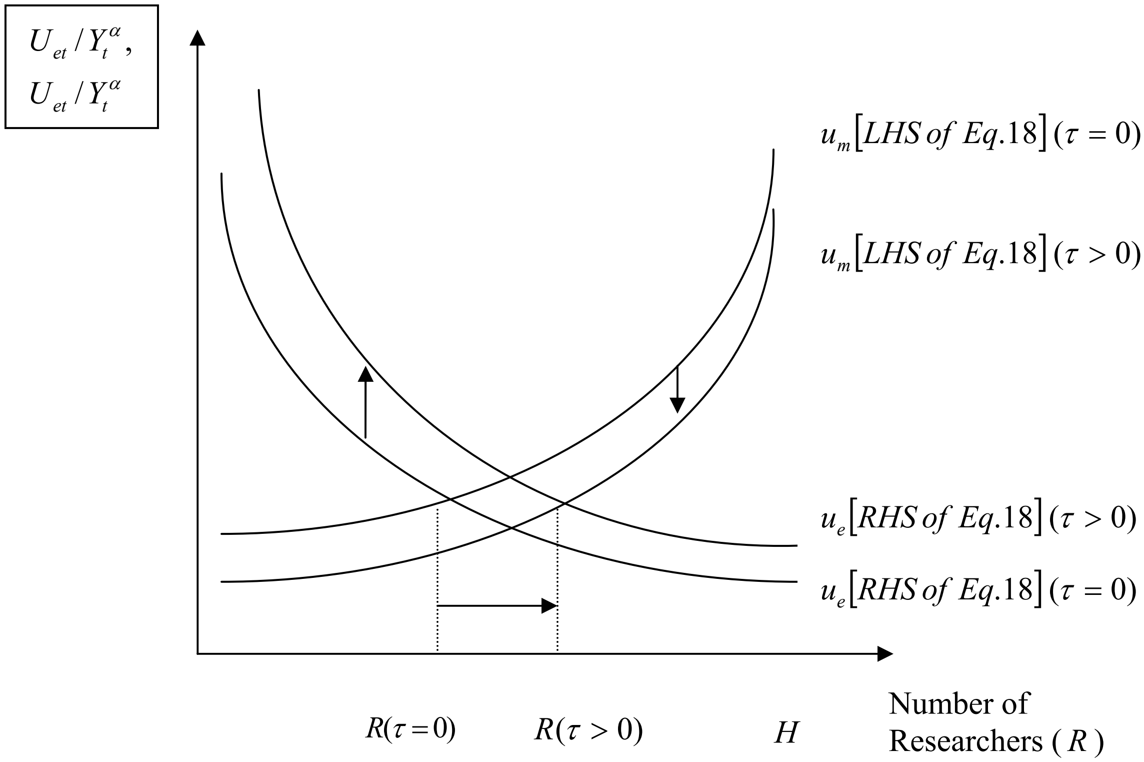

Figure 2

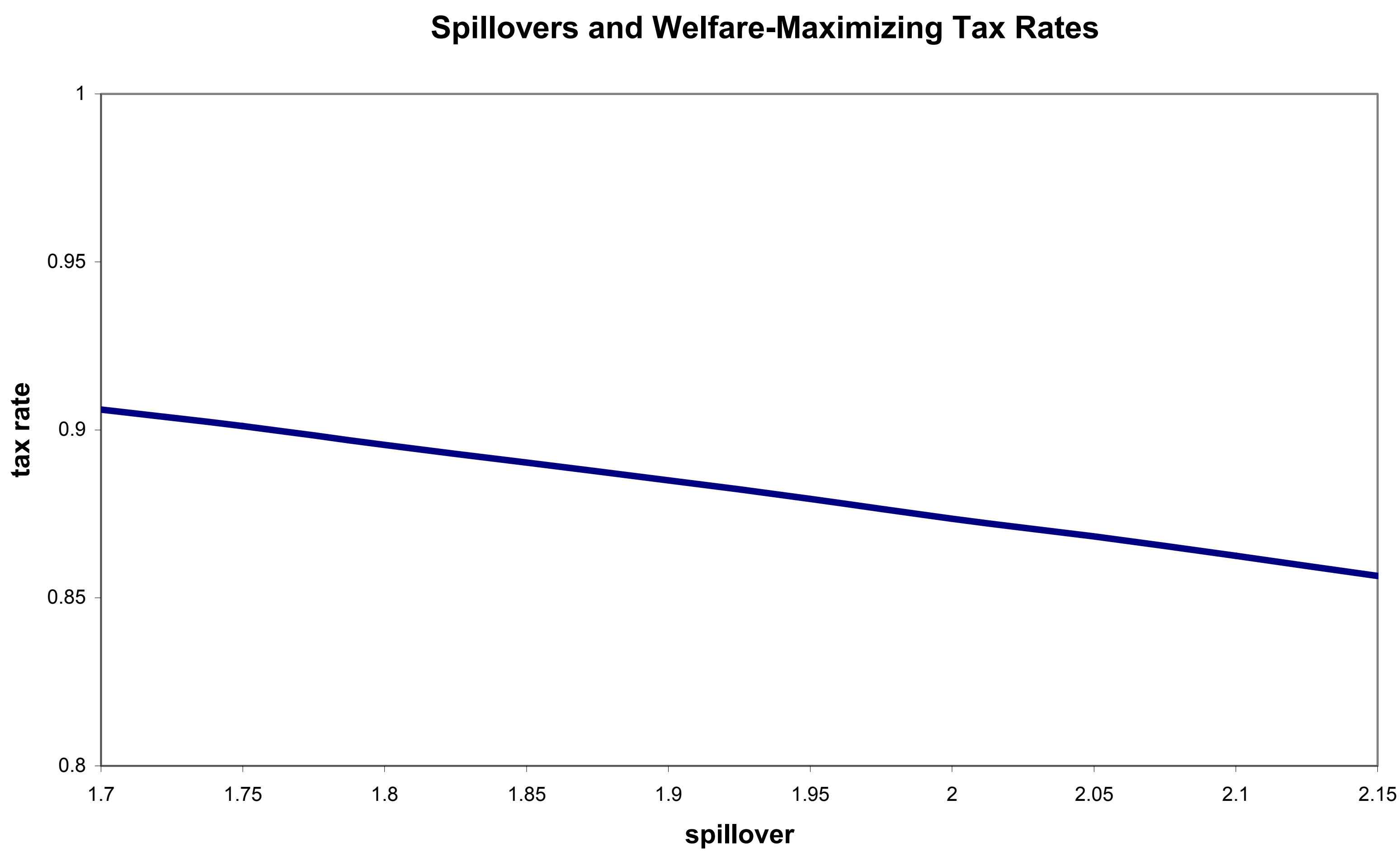
Spillovers and Welfare-Maximizing Tax Rates
1
0.95
0.9
0.85
0.8
tax rate
1.7
1.75
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
spillover

Figure 3

Spillovers and the Pareto-improving Tax Rate