Mergers, Managerial Efficiency, and Social Welfare

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Abstract

We examine the impact of a merger to monopoly in a Cournot duopoly framework where managers exert cost-reducing effort prior to choosing output. Absent agency costs, the merger has a well-known "Schumpeterian" effect leading to increased effort and lower costs. However, when agency costs are present, the Schumpeterian effect of the merger is dominated by increased agency costs, leading instead to lower effort, higher costs, and lower social welfare. We then discuss how our analysis might inform the evaluation of real mergers, with reference to the Superior Propane - ICG Propane merger in Canada.
1 Introduction

Among the major areas of antitrust enforcement, which include also detecting and punishing collusion and anticompetitive actions by dominant firms, merger review is perhaps the most challenging for modern enforcement agencies. One reason is simply the volume of work it generates: the evaluation of mergers typically absorbs a great share of agency resources by virtue of the frequency of mergers in concentrated industries. Other reasons derive from some policy uncertainty over the correct welfare standards to be applied. For example, mergers that limit competition resulting in harm to consumers may nevertheless raise total social surplus if they lead to productive efficiencies of sufficient magnitude. How to incorporate the effects of post-merger efficiencies, and to trade off consumer versus producer gains, has become a continuing challenge for a number of agencies.

The greatest challenges associated with merger review, however, derive from the prospective nature of the analysis. When it comes to collusion or abusive behavior by dominant firms, agencies can focus on actual harm done by the conduct in question. When it comes to merger review, however, the agencies must make predictions about the likely effects of the transaction on competition, prices and productive efficiency. This is no easy feat and work continues to help us better understand what considerations should go into this kind of analysis.

The potential benefits of mergers are well recognized, and are typically well-advertised by the merging parties. These can include improvements to productive efficiency through a better allocation of the resources of the merging firms. If the merger makes the combined operation a more competitive player in its markets, prices could even fall post-merger leading to improvements in allocative efficiency. Finally, merging firms will often argue that the transaction will enhance innovation and improve dynamic efficiency. Since the vast majority of mergers will have no effect on competition, the presumption is that the prospect of efficiency improvements is the force driving most mergers. Interestingly, however, a large body of research now suggests that a significant fraction of mergers are not successful ex post, even from the standpoint of the merging firms alone (i.e. ignoring potential harm to consumers).\footnote{See. e.g., Mueller (1997) and Scherer (2006) for some overviews and Ravenscraft and Scherer (1987) for a particularly important original study.}

While acknowledging the possibility of efficiencies, the point of merger review by an antitrust
enforcement agency is to block the creation of market power via mergers (or similar transactions shifting control of firms or their assets). Market power is disliked by economists because of its negative consequences for market efficiency. Modern textbook treatments of the social costs of market power (e.g. Church and Ware (2000, chapter 4)) typically identify three categories of inefficiencies associated with market power: (i) the allocative efficiency losses that come from prices raised further above marginal costs by profit-maximizing firms; (ii) the “wasteful” rent-seeking expenditures of firms looking to secure market power through non-market (i.e. generally government) means; and (iii) losses in productive efficiency associated with a reduction in competitive pressure to keep cost low – often referred to as “X-inefficiency”.

Interestingly, modern merger review typically focuses, in term of inefficiencies, almost exclusively on allocative inefficiencies.² Productive efficiency enters the picture only if it is determined that the merged firm will be more, not less, productive than the merging parties were pre-merger. Despite its popularity in textbooks, the possibility of X-inefficiencies created by a merger does not appear to be a consideration for the major antitrust agencies.³ This is perhaps due to the controversial nature of the concept of X-inefficiency since the concept was advanced

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²Scherer (1997, at pp 688-689) suggests that there may have been some implicit recognition of the possibilities of increased X-inefficiency post-merger in the U.S. antitrust community in the 1960s and 1970s.

³It is not that all competition agencies are unaware of the potential for X-inefficiency, at least a few agencies explicitly recognize this cost. For example in the 2004 Canadian Merger Guidelines we find:

**Loss of Productive Efficiency**

8.29 Mergers that prevent or lessen competition substantially can also reduce productive efficiency as resources are dissipated through x-inefficiency113 and other distortions. For instance, x-inefficiency may arise when firms, particularly in monopoly or near monopoly markets, are insulated from competitive market pressure to exert maximum efforts to be efficient. [Footnote 113: “X-inefficiency” typically refers to the difference between the maximum (or theoretical) productive efficiency achievable by a firm and actual productive efficiency attained.] [Footnote 114 omitted.]

Also, in a submission by the Australian Competition and Consumer Commission (ACCC) to the government’s Review of the Trade Practices Act in 2002 we find, under “8.2.3 Productive efficiency” (footnotes omitted):

*It has long been recognised within economic literature that competition plays an important role in deterring productive inefficiency. Smith observed in 1776 that ‘monopoly . . . is a great enemy to good management.’* 

Leibenstein postulated that the lack of incentives or competitive pressures may lead firms with market power to neglect productive efficiency and tolerate what he called X-inefficiency. X-inefficiency represents the gap between actual and minimum attainable supply costs. According to Leibenstein:

*With a given* set of human inputs purchased and … knowledge of production techniques available to the firm, a variety of outputs are possible. *If individuals can choose, to some degree the APQT bundles (Activity, Pace, Quantity of work, Time spent) they like, they are unlikely to choose a set of bundles that will maximise the value of output.*

*Although X-inefficiency can affect both monopolists and competitors alike, it will impose a far greater impact on a monopolist because a monopolist will have no discipline imposed by competition. The existence of X-inefficiency means that the price of market power to society is not just limited to the loss of allocative efficiency but also productive efficiency.*

Despite this recognition, we are not aware of merger cases in which X-inefficiency arguments were formally contemplated.
by Leibenstein (1966); a controversy based in part on the absence of formal modeling with optimizing agents in which such efficiencies would arise.4

Our view is that we should be open to consider bringing X-inefficiency into merger review in some circumstances. To make our point, in this paper we formalize a particular type of X-inefficiency: We suggest that mergers may exacerbate agency costs problems in the organization, resulting in weaker managerial incentives, lower managerial effort, higher production costs, and ultimately lower social welfare. We develop a model to examine the benefits and costs of mergers, in situations with and without agency costs; thus enabling us to disentangle “classic” effects of mergers from the X-inefficiency effects that result from changes in agency costs.

We propose a simple Cournot duopoly framework where managers exert (marginal) cost-reducing effort prior to choosing output; and start by examining the impact of a merger to monopoly on managerial effort.5 In the first-best scenario where agency costs associated with managerial effort can be circumvented, the merger is shown to have a well-known “Schumpeterian” effect leading to an increase in managerial effort and a decrease in costs of production. However - and this is a key result of the paper - in the second-best world, agency costs arise, and the Schumpeterian effect of the merger turns out to be dominated by an “increased agency” effect (the X-inefficiency). Indeed we find that in the second-best a merger leads to lower-powered incentives, to a decrease in managerial effort, and to higher costs of production.6

Building on this analysis, we then examine the impact of mergers from a social welfare point of view, and show that while in the first-best the Schumpeterian effect on managerial efficiency may lead to an increase in social welfare associated with the merger (dominating the usual negative output effect of mergers), in the second-best the decrease in managerial effort associated with the merger leads unequivocally to a drop in social welfare. Thus, a key implication of our model is that ignoring agency costs and its effects will lead to a systematic

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4See Leibenstein (1966, 1978) and Stigler (1976) on the initial debate on X-inefficiency. See also Frantz (1997) for a thorough discussion of the early theoretical and empirical work on X-inefficiency.

5As is well known (Salant, Switzer, and Reynolds, 1983; Kamien and Zang, 1990), a particularity of the n-firm Cournot model is that mergers may not always be profitable when \( n \geq 3 \); though this problem is alleviated when firms can delegate output decisions to agents (González-Maestre and López-Cuñat, 2001; Ziss, 2001). Our purpose here is to examine the consequences of a merger, rather than the merger decision itself; and accordingly we sidestep these issues altogether by focusing on Cournot duopoly, where disincentives to merge are absent.

6These results are broadly consistent with the recent empirical literature on product market competition, managerial incentives and productivity, which finds evidence that as the degree of competition weakens, the power of incentives decreases (Burgess and Metcalf, 2000; Cuñat and Guadalupe, 2005), productivity falls (Caves and Barton, 1990; Green and Mayes, 1991; Haskel, 1991; Caves et al., 1992; Nickell, 1996; Griffith, 2001; Syverson, 2004a, b); or both (Baggs and Bettignies, 2007).
overestimation of the net benefits of mergers. Indeed in our model this overestimate is assessed to represent 133% of the true net benefit.

Finally, we apply our theory to the *Superior Propane Case*, which involved the merger, in 1998, of Superior Propane and ICG Propane, the two largest marketers of propane in Canada at the time. This case is particularly interesting for our purpose for two reasons. First, with estimated merger synergy benefits of $29 million/year and deadweight loss of $25.5 million/year, the resulting net benefit of $3.5 million/year stands “well within the range of estimation error that is imposed by the limitations of data and econometric evidence in the case” (Mathewson and Winter, 2000). Second, agency concerns were entirely ignored in the net benefit estimations. Using data from the Canadian Competition Tribunal and from Superior Propane’s Annual Reports, we calculate the potential additional social welfare loss generated by X-inefficiency. We find that a 9% X-inefficiency-related decrease in gross margins would have produced additional harm of $30 million/year, higher than the synergy benefits or the initial deadweight loss estimation.\(^7\) Thus, even much smaller degrees of X-inefficiency would still tip the balance in favor of blocking the merger (in fact the merger was allowed to proceed).

Our model is closely related to the theoretical literature on product market competition and incentives,\(^8\) which for our purpose can be divided according to the way competition is modeled. In Hart (1983) and Scharfstein (1988), for example, an increase in competition is associated with a decrease in market price, and this in turn affects managerial incentives and effort. In Hermalin (1992) and Schmidt (1997), the main impact of competition on incentives takes place via a decrease in income (profits), and in the latter case via an increased threat of liquidation. Raith (2003) and Baggs and Bettignies (2007) model competition as the degree of substitutability between products.

However our model, being about mergers and about a change in market structure from

\(^7\)In a classic article, Williamson (1968) illustrated the social tradeoffs associated with mergers that both increase market power (leading to higher prices) and lower production costs. His analysis highlighted the relative importance of changes in productive efficiency relative to allocative efficiency – starting from a pre-merger competitive situation, small percentage reductions in production costs could more than make up for much larger percentage increases in price (with the associated allocative efficiency losses). By the same token then, we must acknowledge the potential importance of X-inefficiencies leading to higher production costs – again productive efficiency concerns could be much larger in magnitude than those associated with changes in allocative efficiency.

\(^8\)There is also related literature that examines the link between competition and innovation without necessarily focusing on agency and incentives. See for example, Tandon (1984), and the more recent work of Aghion et al. (2005) and Vives (2008).
duopoly to monopoly, is perhaps more closely related to papers where the degree of competition is related to the number of competitors in the industry. Several papers fall into that category: Farrell (1983) and Nalebuff and Stiglitz (1983) argue that managerial efficiency may be lower in monopoly than in industries with two or more firms, where efficiency-improving tournaments and relative-performance evaluations can be used. Chen and Chen (2005) compare managerial efforts in monopoly and competitive equilibria, both in a first-best scenario and in a classic second-best scenario where the agency problem arises from agent risk-aversion. They find that results depend crucially on the relationship between output and effort in the cost function. Our paper is perhaps most closely related to Martin’s (1993) work, which examines the effect of a change in $n$ on managerial effort in an $n$-firm linear Cournot oligopoly. However, the two approaches differ sharply in the way agency is modeled, and in the results generated. In our model the agency problem stems mainly from the agent limited liability, and is addressed with profit-contingent managerial compensation. In contrast, in Martin’s (1993) setup, the manager’s ability to shirk stems mainly from his private information about costs, and the agency problem is addressed in a cost information revelation game. Interestingly, unlike us he finds that a fall in $n$ increases managerial effort.

Our theoretical model makes two primary contributions to this literature. First, we focus on the impact of mergers - and the associated reduction in strategic behavior - on managerial efficiency, and explicitly identify the additional effects that arise when agency costs are present. In particular, we show that, and explain why, mergers have a positive impact on managerial effort in the first-best but a negative one in the second-best. Second, we examine how such findings affect the impact of mergers on social welfare, and use the Superior Propane Case to highlight the potential social costs of ignoring agency costs in merger reviews.

The paper is organized as follows: Section 2 describes the basic model structure. In sections 3 and 4, we derive the pre- and post-merger equilibria and examine the effects of a merger, in a first-best scenario where agency costs can be circumvented, and in a second-best scenario

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9 In that regard, our paper is more closely related to Baggs and Bettignies (2007), who also use a model with a wealth-constrained agent and profit-contingent compensation schemes. However, as mentioned above, they model competition as the degree of substitutability between products in a Hotelling market with neither entry nor exit; and thus cannot explicitly examine merger-related issues.

10 As well, interesting recent work examines agency conflicts and inter-division competition in multi-division merged firms. See, e.g. Wickelgren (2005), Fulghieri and Hodrick (2006), and Ziss (2007). In contrast, in our model two firms merge into a single division firm; indeed it is in part the reduction in strategic interaction associated with the merger which drives our results.
where they cannot, respectively. We examine the welfare implications of mergers in section 5. In section 6, we discuss the Superior Propane Case and examine our results within that context. Finally, section 7 concludes.

2 Basic Model Structure

2.1 Timing of the Game

Consider two firms, 1 and 2, poised to compete in output in a new product market where demand is still uncertain. With probability $\gamma = 1/2$, the good state of the world is realized: The new market does indeed emerge, characterized by inverse demand function $P = b - q_1 - q_2 + \varepsilon$, where $P$ is the market price, $q_1$ and $q_2$ represent outputs by firms 1 and 2, and $\varepsilon$ is a noise term - with mean 0 and support $[-m, m]$ - realized after quantities are set.\(^{11}\) With equal probability $1/2$, the bad state of the world is realized: The new product market does not materialize, with the result that the two competing firms produce nothing and exit the game. Each firm is composed of a principal and an agent, both of whom are risk-neutral. Throughout the paper, the principal-agent relationship captures the relationship between the firm’s owners and its manager, and we use the terms “agent” and “manager” interchangeably.\(^{12}\) For expositional convenience, throughout the paper we refer to the principal as female and to the agent as male.

Prior to learning about market demand and choosing output, agent $i$ exerts effort $e_i$ at a personal cost $K_i(e_i) = ke_i^2/4$. For simplicity, and without loss of generality, we posit that $k = 1$.\(^{13}\) This effort can be interpreted literally, as “hard work”, or as some sort of innovation effort. Either way, it benefits the firm by reducing its marginal cost of production $c_i = \bar{c} - e_i$ (with $2\bar{c} > b > \bar{c}$),\(^{14}\) should production take place.\(^{15}\) The marginal cost is observed by all

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\(^{11}\)We set $\gamma = 1/2$ for expositional convenience. Similar results obtain with any $\gamma \in (0, 1)$. As should become clear below (see section 4), the assumptions about the “market uncertainty” (i.e. the two states of the world) and about the “price uncertainty” (i.e. the noise term $\varepsilon$) are made to ensure that the agency problems have “teeth” in our framework.

\(^{12}\)The model could also capture the relationship the manager as principal and her subordinate as agent.

\(^{13}\)The results of the model are qualitatively the same for all $k > 8/9$ (a sufficient condition for second-order conditions to hold). See discussion in footnote 28.

\(^{14}\)This restriction on parameters $\bar{c}$ and $b$ ensures positive effort levels, outputs, and marginal costs of production, in equilibrium.

\(^{15}\)In our model, the technologies (costs of effort, marginal costs of production) are the same for both firms. For an interesting model of mergers with cost asymmetries \textit{ex ante}, see Tombak (2002)
parties, including the rival firm, prior to output being chosen.

At the beginning of the game, principal $i$ makes a take-it-or-leave-it contract offer to agent $i$.\footnote{For simplicity we assume that this offer is not observable to agent $j$.} We thus implicitly assume that there are many more agents than there are principals, and hence the latter have full bargaining power \textit{ex ante}. The agents are protected by limited liability and have zero initial wealth.

The timing of the game is thus very simple and can be summarized as follows:

\[ \text{[Insert figure 1 here.]} \]

Finally, throughout the paper we impose the restriction that $m \leq (6/11)(b - \bar{c})$, which is necessary for the concavity of the principal’s optimization program.

2.2 Contracts

We assume that profits are verifiable in a court of law and hence are contractible. In contrast, the marginal costs of production, though observable, may or may not be verifiable.\footnote{Whether outputs and revenues are verifiable or not is irrelevant in our setup, because it is never optimal for the principal to use a contract where pay is contingent on output or revenue. Indeed this would defeat the principal’s purpose to create incentives for the agent to exert cost-reducing effort.} This assumption is not unrealistic: marginal costs are notoriously difficult to estimate, let alone verify in court. We analyze both possibilities: The case where marginal costs are verifiable corresponds to our benchmark, first-best scenario in which agency costs can be circumvented. The case with non-verifiable marginal costs corresponds to the second-best scenario, in which agency costs are present.

Thus, depending on the verifiability of marginal costs, the optimal compensation scheme offered to the agent may be contingent on realized profits and/or marginal costs: $W_i = W_i(\Pi_i, c_i)$. Three conditions must hold for any contract to be feasible at date 0:

- Incentive compatibility (IC) constraint: Agent $i$ exerts effort $\hat{e}_i$, which maximizes his expected gross payoff $w_i$ - which is a function of expected profits $\pi_i/2$,\footnote{Expected profits $\pi_i/2$ can be expressed more explicitly as $\frac{1}{2}E\Pi_i(q_i(e_i), c_i(e_i), q_j(e_j), c_j(e_j))$.} and/or of marginal cost $c_i$ - minus his cost of effort:

  \[
  \hat{e}_i \in \arg \max \left[ w_i (\pi_i (c_i (e_i)) / 2, c_i (e_i)) - K_i (e_i) \right]. \tag{1}
  \]
Individual rationality (IR) constraint: For the agent to participate, his expected net payoff (given his optimal choice of effort) must be weakly larger than his (zero) reservation wage:

$$w_i (\pi_i (c_i (\hat{e}_i)) /2, c_j (\hat{e}_j)) - K_i (\hat{e}_i) \geq 0. \quad (2)$$

Wealth (W) constraint: The agent is protected by limited liability and has zero initial wealth; and thus his realized payoff must be non-negative in all states of the world:

$$W_i (\Pi_i (c_i (\hat{e}_i), c_j (\hat{e}_j))) \geq 0 \text{ for all realized values of profits } \Pi_i. \quad (3)$$

## 3 First-Best (Benchmark) Case: Verifiable Costs

### 3.1 Equilibrium Pre-Merger

Let us assume first that costs are verifiable. We call this the first-best scenario because - as verified below - under this condition agency problems can be circumvented.

**Contractual Form**

If principal $i$ wants to induce effort $e_i^*$ from agent $i$, at date 0 she offers him the following contract: The agent is to receive wage $W_i (c_i (e_i^*)) = K (e_i^*) = e_i^2 / 4$ at date 3 if the marginal cost is verified to be $c_i^* = c_i (e_i^*)$, and $W_i (c_i (e_i)) = 0$ otherwise.

The IC constraint (1), the IR constraint (2), and the W constraint (3) all hold (and bind) under the conditions of this contract. This contract will be accepted by the agent, and will induce him to exert the equilibrium effort $e_i^*$ chosen by the principal. Note also that the cost of eliciting effort to the principal simply equals the agent’s cost of effort: $W_i (c (e_i^*)) = K (e_i^*)$. There is no agency cost here.

**Backward Induction (Subgame Perfect Nash Equilibrium)**

At date 3, regardless of the state of the world, agent $i$ receives her compensation $W_i (c_i (e_i))$. As for principal $i$, in the good state of the world - if the new product market successfully emerged at date 2 - she receives the expected profit minus the payment made to the agent: $\pi_i (q_i, c_i (e_i), q_j, c_j (e_j)) - W_i (c_i (e_i))$. In the bad state, she receives nothing and her payoff is...
At date 2, if the good state is realized, after observing marginal costs $c_i(e_i)$ and $c_j(e_j)$, agent $i$ chooses output $q_i$. Since his expected payoff is unaffected by the output choice, we assume by convention that he selects the profit maximizing output (which incidentally also maximizes the principal's payoff):

$$\max_{q_i} \pi_i(q_i, c_i(e_i), q, c_j(e_j)),$$

where $\pi_i(q_i, c_i(e_i), q, c_j(e_j)) = [EP(q_i, q_j) - c_i(e_i)] q_i$. Taking the first-order conditions (FOC) with respect to output for $i = 1, 2$, and solving the resulting system of two equations yields the equilibrium output for firm $i$:

$$q_i = \frac{b - 2c_i(e_i) + c_j(e_j)}{3} = \frac{b - \bar{\epsilon} + 2e_i - e_j}{3}. \quad (5)$$

Substituting equilibrium outputs back into the inverse demand function, we obtain an expression for expected profits in the good state $\pi_i(e_i, e_j)$ for firm $i$, given efforts $e_i$ and $e_j$.

$$\pi_i(e_i, e_j) = \left[ \frac{b - \bar{\epsilon} + 2e_i - e_j}{3} \right] \left[ \frac{b - \bar{\epsilon} + 2e_i - e_j}{3} \right] = \left[ \frac{b - \bar{\epsilon} + 2e_i - e_j}{3} \right]^2, \quad (6)$$

where $[EP(q_i, q_j) - c_i(e_i)] = \left[ \frac{b - \bar{\epsilon} + 2e_i - e_j}{3} \right] = q_i$, is the equilibrium price-cost margin for firm $i$.

If the bad state of the world is realized, there is no demand for the product and agent $i$ optimally chooses not to produce any output.

At date 1, agent $i$ exerts effort $e_i^* = \bar{\epsilon} - e_i^*$, with $e_i^*$ determined in the initial contract offered by the principal at date 0.

At date 0, principal $i$ chooses the level of effort $e_i^*$ she wants the agent to exert at date 1, taking principal $j$’s contractual choice (and hence firm $j$’s marginal cost) as given. This equilibrium level of effort maximizes the principal’s expected net benefit: $e_i^* \in \arg \max \left[ \frac{1}{2} \pi_i(e_i, e_j) - K_i(e_i) \right]$. Taking the FOC,\(^\text{19}\) we obtain:

$$\frac{1}{2} \frac{\partial \pi_i(e_i^*, e_j)}{\partial e_i} = \frac{\partial K_i(e_i^*)}{\partial e_i}. \quad (7)$$

In other words, the principal chooses to induce an effort level such that, given rival effort $q_j$, the

\(^{19}\text{The second-order condition (SOC), } -1/9 < 0, \text{ can be easily be verified to hold.}\)
expected marginal benefit from an increase in effort equals the agent’s marginal cost of effort. Solving the system of two FOCs for principals $i$ and $j$, we get $e_i^* = e_j^* = e^* = \frac{4}{5} (b - \bar{c})$, yielding marginal costs $c_i^* = c_j^* = c^* = \bar{c} - \frac{4}{5} (b - \bar{c})$. Outputs are then determined from (5) in the good state, yielding $q_i^* = q_j^* = q^* = \frac{3}{5} (b - \bar{c})$, and are zero in the bad state. We summarize these results in the following lemma:

**Lemma 1** With verifiable marginal costs, the pre-merger subgame perfect Nash equilibrium (SPNE) is as follows: At date 0, both principals propose to pay their agents $w^* = \frac{8}{25} (b - \bar{c})^2$ at date 3 if marginal cost $c^* = \bar{c} - \frac{4}{5} (b - \bar{c})$ is verified; and zero otherwise. At date 1, agents exert pre-merger first-best effort $e^* = \frac{4}{5} (b - \bar{c})$, yielding marginal cost $c^* = \bar{c} - \frac{4}{5} (b - \bar{c})$. At date 2, both firms produce output $q^* = \frac{3}{5} (b - \bar{c})$, in the good state of the world; and zero otherwise. Finally, at date 3 the initial contracts between principals and their agent are honored.

**Proof.** See appendix. ■

### 3.2 Equilibrium Post-Merger

Suppose now that a merger takes place between the two firms. The marginal cost of production in the merged firm is $c_m = \bar{c} - \max (e_1, e_2)$, and hence it is optimal for the principal of the merged firm to retain only one of the two agents, in order to avoid replicating the costs of effort.\(^{20,21}\)

The contractual form is the same as in the pre-merger case: In order for the principal to elicit effort $e_m^*$ from the agent, at date 0 she commits to pay him $W_m (c_m (e_m^*)) = K (e_m^*) = e_m^{*2}/4$ at date 3 if the marginal cost is verified to be $c_m^* = c_m (e_m^*)$, and $W_m (c_m (e_m)) = 0$ otherwise.\(^{22}\)

As before we derive the equilibrium by backward induction:

At date 3, the agent receives her compensation $W_m (c_m (e_m^*))$. As for the principal, she receives an expected payoff $\pi_m (q_m, c_m (e_m^*)) - W_m (c_m (e_m^*))$ in the good state; and $-W_m (c_m (e_m^*))$
in the bad state.

At date 2, if the good state is realized, the agent chooses output $q_m$ to maximize the following program, taking $c_m(e_m)$ as given:

\[
\max_{q_m} \pi_m(q_m, c_m(e_m)),
\]

where $\pi_m(q_m, c_m(e_m)) = [EP(q_m) - c_m(e_m)]q_m$. Taking the FOC yields the following equilibrium output:

\[
q_m = \frac{b - c_m(e_m)}{2} = \frac{b - \bar{c} + e_m}{2}.
\]

Substituting equilibrium outputs back into the inverse demand function, we obtain the following expression for profits $\pi_m(e_m)$:

\[
\pi_m(e_m) = \left[ \frac{b - \bar{c} + e_m}{2} \right] \left[ \frac{b - \bar{c} + e_m}{2} \right] = \left[ \frac{b - \bar{c} + e_m}{2} \right]^2,
\]

where $[EP_m(q_m) - c_m(e_m)] = \frac{[b - \bar{c} + e_m]}{2} = q_m$, is the equilibrium price-cost margin.

If the bad state of the world is realized, optimal output is zero.

At date 1, the agent exerts effort $e^*_m = \bar{c} - c^*_m$, with $c^*_m$ determined in the initial contract.

At date 0, principal $i$ chooses the level of effort $e^*_m \in \arg \max \left[ \frac{1}{2} \pi_m(e_m) - K_m(e_m) \right]$ to elicit from the agent. The FOC can be expressed as:

\[
\frac{1}{2} \frac{\partial \pi_m(e^*_m)}{\partial e^*_m} = \frac{\partial K_m(e^*_m)}{\partial e^*_m},
\]

yielding effort $e^*_m = (b - \bar{c})$, marginal cost $c^*_m = \bar{c} - (b - \bar{c})$, and (in the good state) output $q^*_m = (b - \bar{c})$. Thus:

**Lemma 2** With verifiable marginal costs, the post-merger equilibrium is as follows: At date 0, the principal proposes to pay the agent $w^*_m = (b - \bar{c})^2/2$ at date 3 if marginal cost $c^*_m = 2\bar{c} - b$ is verified; and zero otherwise. At date 1, the agent exerts effort $e^*_m = (b - \bar{c})$, yielding marginal cost $c^*_m = 2\bar{c} - b$. At date 2, the firm produces $q^*_m = (b - \bar{c})$ in the good state of the world; and zero otherwise. Finally, at date 3 the initial contract is honored.
3.3 Mergers and Managerial Efficiency in the First-Best

The key result that emerges from lemmas 1 and 2 can be stated as follows:\textsuperscript{23}

**Proposition 1** In the first-best, the merger leads to an increase in managerial effort (i.e. the manager post-merger exerts more effort than either one of the two pre-merger managers) and to a decrease in the marginal cost of production.

**Proof.** Follows directly from above. ■

This result captures the Schumpeterian idea that market power increases a firm’s incentives to innovate, or as here its manager’s incentive to exert effort. To see the intuition behind this proposition, let us compare the effort FOCs (7) and (11): clearly the higher post-merger effort must be the result of a higher marginal product of effort: we must have $\frac{\partial \pi_m(e_m^*)}{\partial e_m} > \frac{\partial \pi_i(e_i^*, e_j^*)}{\partial e_i}$.

Note that in both pre- and post-merger cases the marginal product of effort can be written:

$$\frac{\partial q}{\partial e} = \frac{\partial q}{\partial e} (EP - c) + \frac{\partial (EP - c)}{\partial e} q. \tag{12}$$

Condition (12) can help us illustrate the key differences in effort choice between the pre- and post- situations. On the one hand, post-merger the monopolist maximizes profits over the entire market, while pre-merger each firm maximizes profits over its residual demand and ignores the rest of the market. As a result of this pre-merger externality, 1) individual output (as a function of effort) by each pre-merger duopolist is lower than the post-merger monopolist’s output: $q_m = (b - \bar{c} + e_m) / 2 > (b - \bar{c} + e_i) / 3 = q_i$.\textsuperscript{24} And 2) total industry output will fall following the merger, leading to a higher market price, and hence to a higher price-cost margin: $P_m - c_m = (b - \bar{c} + e_m) / 2 > (b - \bar{c} + e_i) / 3 = P_i - c_i$.

On the other hand, (5) and (9) suggest that the impact of an effort increase on output will fall with the merger: $\frac{\partial q_m}{\partial e_m} = 1/2 < 2/3 = \frac{\partial q_i}{\partial e_i}$. Under duopoly, when effort increases

\textsuperscript{23}A similar result was discussed previously by Tandon (1984), who showed that in a n-firm Cournot oligopoly with homogeneous goods and linear demand, innovation decreases with the number of competing firms. Tandon’s result was subsequently discussed as a special case of a more general Cournot model in Vives (2008).

\textsuperscript{24}In equilibrium $e_i = e_j$ and hence $q_i = (b - \bar{c} + 2e_i - e_j) / 3$ simplifies to $q_i = (b - \bar{c} + e_i) / 3$. 

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and the marginal cost falls, the marginal benefit of a quantity increase goes up for two reasons. First, because a larger price-cost margin is made on each unit. And second, an increase in output also puts downward pressure on the rivals output, which in turn puts upward pressure on the market price, increasing the price-cost margin, and hence the marginal benefit from an output, further. This second effect reflects strategic substitutability of output decisions. This strategic effect disappears under monopoly, and accordingly the marginal impact of an effort increase on output is smaller post-merger.

For the same reason, the impact of effort on price-cost margins will fall with the merger: \( \frac{\partial (E P_m - c_m)}{\partial e_m} = 1/2 < 2/3 = \frac{\partial (E P_i - c_i)}{\partial e_i} \). An increase in effort reduces the marginal cost of production in the same way pre- and post merger. It also leads to an increase in the firm’s output, which in turn lowers the market price, in both cases. But in the pre-merger case - because of strategic substitutability - this increase in output reduces the equilibrium output of the rival firm, and this has a mitigating effect on the downward pressure on the market price. This mitigating effect is absent post-merger, leading to the stated result.

Thus, on the one hand the merger, by eliminating the externality associated with duopoly, tends to have a positive impact on the marginal product of effort. On the other hand, by eliminating the strategic substitutability, the merger has a negative impact on the marginal product of effort. In this model, the externality elimination effect dominates, leading to \( \frac{\partial \pi_m(e^*_{m})}{\partial e_m} > \frac{\partial \pi_i(e^*_i, e^*_j)}{\partial e_i} \) and hence \( e^*_m = (b - \bar{c}) > \frac{4}{5} (b - \bar{c}) = e^* \). Finally, as a result of higher managerial effort, the marginal cost of production falls following the merger: \( e^*_m = 2\bar{c} - b < \bar{c} - \frac{4}{5} (b - \bar{c}) = e^* \).

4 Second-Best Case: Non-Verifiable Costs

4.1 Equilibrium Pre-Merger

Having examined the benchmark case where costs are verifiable and hence contractible, we now turn to the second-best case where costs, though observable, cannot be verified in court and hence cannot be contracted upon.

Contractual Form

Since costs are not contractible, agent i’s payoff must be made contingent on profits. We
focus on linear contracts of the form \( W = \alpha \Pi + \beta \). One can easily verify that it is optimal for principal \( i \) to set \( \alpha_i \geq 0 \) and \( \beta_i = 0 \), for the following reasons: Solving IC constraint (1), it becomes clear that setting \( \alpha_i \geq 0 \) is necessary to elicit non-negative effort from the agent.

As for the optimality of setting \( \beta_i = 0 \), it can be explained as follows: First, note that since \( \beta_i \) has no impact on agent incentives, and is a fixed cost for the principal, she wants to set it as low as possible. Second, the W constraint (3) requires \( \beta_i \) to be non-negative, otherwise in the bad state of the world when realized profits are zero, the agent would get a strictly negative payoff, an outcome ruled out by limited liability. Third, given \( \alpha_i \geq 0 \), the IR constraint (2) and the W constraint (3) can be shown not to bind for any non-negative \( \beta_i \), and can thus be ignored.\(^{25}\) Accordingly, \( \beta_i = 0 \) is the smallest value of \( \beta_i \) that satisfies (1), (2) and (3): It is the optimal choice for principal \( i \).

The wealth constraint is the source of agency costs in the model (together with the non-verifiability of costs of production): if the agent were not protected by limited liability, or if his initial wealth were sufficiently large, the principal could set \( \alpha_i = 1 \), thus making the agent residual on the firm’s profits, and extracting the expected value of such profits with a sufficiently negative \( \beta_i \). But because \( \beta_i \) must be set to zero, fraction \( \alpha_i \) of profits relinquished to the agent is lost for good, and it optimal to set \( \alpha_i < 1 \), thus providing less than first-best incentives and eliciting a second-best level of effort.

Before determining the equilibrium, let us discuss briefly the importance of our assumptions about uncertainty in this model. First, consider the price uncertainty generated by the noise term \( \varepsilon \). One can readily show that without it the first-best effort could be reached with a “forcing contract” (similar to one described in section 3) specifying a minimum level of profit for a bonus to be paid out, and zero otherwise. Having this price uncertainty, however, prevents this outcome,\(^{26}\) and helps motivate our choice of linear contract as a second-best alternative.

Our assumption about market uncertainty, together with regularity condition \( m \leq (6/11) (b - \overline{\tau}) \) ensures the concavity of the principal’s program in our model. To see this, suppose that there

\(^{25}\)With \( \alpha_i \geq 0 \) and \( \beta_i \geq 0 \), the agent’s equilibrium expected payoff is non-negative since it can be no smaller than what he would get with zero effort (otherwise he would choose zero effort), which itself is non-negative: \((1/2) \left[ \alpha_i \pi_i (c_i (\hat{e}_i)), c_j (\hat{e}_j) + \beta_i \right] - K_i (\hat{e}_i) \geq (1/2) \left[ \alpha_i \pi_i (c_j (0)) , c_j (\hat{e}_j) + \beta_i \right] > 0 \). The W constraint is \( \alpha_i \Pi_i (c_i (\hat{e}_i), c_j (\hat{e}_j), -m) + \beta_i \geq 0 \), which always holds when \( m \leq (6/11) (b - \overline{\tau}) \), \( \alpha_i \geq 0 \) and \( \beta_i \geq 0 \).

\(^{26}\)Because the agent anticipates that under some realizations of demand he will receive nothing, and his incentives to exert effort are mitigated below the first-best level.
were no bad state of the world, i.e. that we had $\gamma = 1$. In that case the lowest possible compensation for the agent would depend only on the price uncertainty associated with $\varepsilon$, and the W constraint (3) would simplify to $\beta_i = \alpha_i \Pi_i (c_i (\bar{e}_i), c_j (\bar{e}_j), -m)$. But in our model, substituting $\beta_i$ back into the principal’s objective function “linearizes” that program, yielding trivial corner solutions. Now adding market uncertainty, the W constraint (3) becomes $\beta_i = \max (0, \alpha_i \Pi_i (c_i (\bar{e}_i), c_j (\bar{e}_j), -m))$, or $\beta_i = 0$ if and only if $m \leq (6/11) (b - \bar{c})$, in which case the principal’s program remains strictly concave.

**Backward Induction**

At date 3, in the good state, principal $i$ and agent $i$ receive $(1 - \alpha_i) \pi_i (q_i, c_i (e_i), q_j, c_j (e_j))$ and $\alpha_i \pi_i (q_i, c_i (e_i), q_j, c_j (e_j))$, respectively. In the bad state, neither the principal nor the agent receive any payment.

At date 2, if the good state is realized, after observing marginal costs $c_i (e_i)$ and $c_j (e_j)$, agent $i$ chooses output $q_i$. Since agents $i$ and $j$ maximize profits (or a fraction thereof) as in the first-best, the equilibrium output choices and the resulting expressions of profits as functions of effort levels $e_i$ and $e_j$ can be expressed as in (5) and (6), respectively. If the bad state of the world is realized, there is no demand for the product and agent $i$ optimally chooses not to produce any output.

At date 1, agent $i$ selects the effort level that maximizes his expected payoff, $\frac{1}{2} \alpha_i \pi_i (e_i, e_j) - K_i (e_i)$. He chooses effort $\bar{e}_i$ as a function of $\alpha_i$ and $e_j$, such that:

$$\frac{1}{2} \alpha_i \frac{\partial \pi_i (\bar{e}_i, e_j)}{\partial e_i} = \frac{\partial K_i (\bar{e}_i)}{\partial e_i}$$

(13)

At date 0, principal $i$ chooses $\bar{e}_i$ and $\alpha_i$ to maximize $\frac{1}{2} (1 - \alpha_i) \pi_i (\bar{e}_i, e_j)$ subject to constraints (2), (3) and (13). To solve this program, we first use (13) to express $\alpha_i$ as a function of $\bar{e}_i$:

$$\alpha_i^{**} (\bar{e}_i, e_j) = \frac{9 \bar{e}_i}{4 (b - \bar{c} + 2 \bar{e}_i - e_j)}$$

(14)

We then choose $e_i^{**}$ to maximize $\left[ \frac{1}{2} \pi_i (\bar{e}_i, e_j) - \frac{1}{2} \alpha_i^{**} (\bar{e}_i, e_j) \pi_i (\bar{e}_i, e_j) \right]$. Taking the first-order
condition (FOC), we obtain:

\[
\frac{1}{2} \frac{\partial \pi_i(e_i^{**}, e_j)}{\partial e_i} = \frac{1}{2} \frac{\partial \alpha_i^{**}(e_i^{**}, e_j)}{\partial e_i} \pi_i(e_i^{**}, e_j).
\]

(15)

Solving the system of two FOCs for principals \(i\) and \(j\), we get \(e_i^{**} = e_j^{**} = e^{**} = \frac{7}{11} (b - c)\), yielding marginal costs \(c_i^{**} = c_j^{**} = c^{**} = c = \frac{7}{11} (b - c)\) and (in the good state) outputs \(q_i^{**} = q_j^{**} = q^{**} = \frac{6}{11} (b - c)\). This outcome is obtained by offering fraction of profits \(\alpha_i^{**} = \alpha_j^{**} = \alpha^{**} = \frac{7}{8}\) to agents \(i\) and \(j\). One can readily verify that the IR and W constraints (2) and (3) hold here (see footnote 25). We summarize these results in the following lemma:

**Lemma 3** With non-verifiable marginal costs, the pre-merger equilibrium is as follows: At date 0, both principals offer a fraction \(\alpha^{**} = \frac{7}{8}\) of profits to their agent. At date 1, agents exert pre-merger second-best effort \(e^{**} = \frac{7}{11} (b - c)\), yielding marginal cost \(c^{**} = c = \frac{7}{11} (b - c)\). At date 2, firms produce \(q^{**} = \frac{6}{11} (b - c)\) in the good state of the world; and zero otherwise. Finally, at date 3 the initial contracts between principals and their agent are honored.

**Proof.** See appendix. ■

**Second-Best Versus First-Best Scenario**

A direct implication of lemmas 1 and 3 is that pre-merger merger effort is lower in the second-best than in the first-best. To understand the intuition behind this result, let us return to equilibrium condition (15), which we can express as follows:

\[
\frac{1}{2} \frac{\partial \pi_i(e_i^{**}, e_j)}{\partial e_i} = \frac{\partial K_i(e_i^{**})}{\partial e_i} + \frac{1}{2} \frac{\partial \alpha_i^{**}(e_i^{**}, e_j)}{\partial e_i} \pi_i(e_i^{**}, e_j).
\]

(16)

The marginal benefit from eliciting an increase in effort in the second-best, on the left-hand side of (16), is the same as in the first-best. The difference with the first-best comes from the marginal cost of inducing effort - represented on the right-hand side of (16) - which can be expressed as the sum of the first-best marginal cost of effort \(\partial K_i(e_i^{**}) / \partial e_i\) and the agency marginal cost of effort \((1/2) (\partial \alpha_i^{**}(e_i^{**}, e_j) / \partial e_i) \pi_i(e_i^{**}, e_j)\).27 The agency marginal cost

---

27To see this, note that \(\frac{1}{2} \frac{\partial \alpha_i^{**}(e_i^{**}, e_j)}{\partial e_i} \pi_i(e_i^{**}, e_j) \frac{\partial e_i^{**}}{\partial e_i} = \frac{1}{2} \alpha_i^{**} \frac{\partial e_i^{**}}{\partial e_i} + \frac{1}{2} \frac{\partial \alpha_i^{**}}{\partial e_i} \pi_i.\) Substituting (13) in the first factor yields the required result. A similar intuition in the context of a Hotelling duopoly can be found in Baggs and Bettignies (2007).
measures the increase in the fraction of profits which must be paid out to the agent, to induce him to increase effort. Using (6) and (14), we can easily express the marginal agency cost as

\[(1/2) \left( \frac{\partial \alpha_i^* (e_i^*, e_j)}{\partial e_i} \pi_i (e_i^*, e_j) \right) = \frac{b - \overline{e} - e_j}{4}.\]

Clearly, the agency marginal cost is strictly positive at the first-best level of effort \(e_i^* = e_j^* = e^*\) implying that, at that effort level, the marginal cost of eliciting effort must be higher than the marginal benefit at \(e_i^*\). Principal \(i\) thus optimally responds by reducing the elicited effort. Principal \(j\) reasons the same way and the second-best equilibrium is reached at \(e_i^{**} = e_j^{**} = e^{**} < e^*\). Note that output levels are also lower in the second-best than in the first-best. In the second-best, lower equilibrium effort leads to higher marginal cost of production, which in turn reduces the marginal benefit from an output increase and implies a reduction in equilibrium output.

### 4.2 Equilibrium Post-Merger

The contractual form is the same post-merger as it was pre-merger: The principal specifies the fraction \(\alpha_m\) of realized profits to be relinquished to the agent at date 3. As before, we determine the equilibrium by backward induction as follows:

**At date 3**, in the good state, the principal and agent receive \((1 - \alpha_m) \pi_m (q_m, c_m (e_m))\) and \(\alpha_m \pi_m (q_m, c_m (e_m))\), respectively. In the bad state, both players receive zero.

**At date 2**, if the good state is realized, after observing marginal cost \(c_m (e_m)\) and \(c_j (e_j)\), the agent chooses output \(q_m\) to maximize profits (or a fraction thereof) as in the first-best. Thus the equilibrium output choice and the resulting expression of profit as functions of effort \(e_m\), can be obtained from (9) and (10), respectively. In the bad state of the world, chooses zero output.

**At date 1**, the agent exerts effort \(\hat{e}_m\) to maximizes his expected payoff, \(1/2 \alpha_m \pi_m (e_m) - K_m (e_m)\); such that:

\[
1/2 \alpha_m \frac{\partial \pi_m (\hat{e}_m)}{\partial e_m} = \frac{\partial K_m (\hat{e}_m)}{\partial e_m}.
\]

**At date 0**, the principal chooses \(\hat{e}_m\) and \(\alpha_m\) to maximize \(1/2 (1 - \alpha_m) \pi_m (\hat{e}_m)\) subject to constraints (2), (3) and (17). To solve this program, we start by using (17) to express \(\alpha_m\) as a function of \(\hat{e}_m\):

\[
\alpha_m^{**} (\hat{e}_m) = \frac{2\hat{e}_m}{(b - \overline{e} + \hat{e}_m)}.
\]
We then choose \( e_m^{**} \) to maximize \( \left[ \frac{1}{2} \pi_m (\hat{e}_m) - \frac{1}{2} \alpha_m^{**} (\hat{e}_m) \pi_i (\hat{e}_m) \right] \). Taking the FOC yields:

\[
\frac{1}{2} \frac{\partial \pi_m (e_m^{**})}{\partial \hat{e}_m} = \frac{1}{2} \frac{\partial \alpha_m^{**} (e_m^{**}) \pi_m (e_m^{**})}{\partial \hat{e}_m} = \frac{\partial K_m (e_m^{**})}{\partial \hat{e}_m} + \frac{1}{2} \frac{\partial \alpha_m^{**} (e_m^{**})}{\partial \hat{e}_m} \pi_m (e_m^{**}).
\]  

(19)

Solving for \( e_m^{**} \), we get \( e_m^{**} = 0 \), yielding marginal cost \( c_m^{**} = \bar{c} \) and (in the good state) output \( q_m^{**} = (b - \bar{c}) / 2 \). Here a fraction \( \alpha_m^{**} = 0 \) of profits is offered to the agent. We summarize these results in the following lemma:

**Lemma 4** With non-verifiable marginal costs, the post-merger equilibrium is as follows: At date 0, the principal offers a fraction \( \alpha_m^{**} = 0 \) of profits to the agent. At date 1, the agent exerts effort \( e_m^{**} = 0 \), yielding marginal cost \( c_m^{**} = \bar{c} \). At date 2, the firm produces \( q_m^{**} = (b - \bar{c}) / 2 \), in the good state of the world; and zero otherwise. Finally, at date 3 the initial contracts between principals and their agent are honored.

**Proof.** See appendix. ■

Comparing lemmas 2 and 4, we note that - as in the pre-merger case - effort is lower in the second-best than in the first-best. The intuition is identical to that in the pre-merger case: The marginal cost of eliciting effort in the second-best includes not only the agent’s marginal cost of effort \( \partial K_m (\hat{e}_m) / \partial \hat{e}_m \) (as in the first-best), but also an agency marginal cost \( (1/2) (\partial \alpha_m^{**} (e_m^{**}) / \partial \hat{e}_m) \pi_m (e_m^{**}) \). Using (10) and (18), the agency marginal cost can be expressed as \( (1/2) (\partial \alpha_m^{**} (e_m^{**}) / \partial \hat{e}_m) \pi_m (e_m^{**}) = (b - \bar{c}) / 2 > 0 \). It is this “additional” and positive agency marginal cost in the second-best that leads to lower equilibrium effort post merger. Again, here as in the pre-merger case, output is lower in the second-best than in the first-best, for the same reason: A lower effort leads to higher marginal cost of production, which in turn reduces the marginal benefit from an output increase, and leads to a drop in output.

### 4.3 Mergers and Managerial Effort in Second-Best

Interestingly, lemmas 3 and 4 suggest that in sharp contrast to the first-best:

\[\text{The result that } e_m^{**} = 0 \text{ in equilibrium comes from our assumption about cost of effort: } K (e) = e^2 / 4. \]  
\[\text{If we used a more general functional form } K' (e) = ke^2 / 4 \text{ with } k > 8/9 \text{ (a sufficient condition for second-order conditions to hold), we would have the same results, but with } e_m^{**} > 0 \text{ for all } k \in (8/9, 1). \]  
\[\text{Having } k \in (8/9, 1) \text{ rather than } k = 1 \text{ adds little in terms of insight, and complicates the analysis; hence our decision to assume } k = 1. \]
Proposition 2: In the second-best, the merger leads to weaker managerial incentives, to a decrease in managerial effort, and to an increase in the marginal cost of production.

Proof. Follows directly from above.

To understand this result, note first that the Schumpeterian effect of the merger on the marginal product of effort - which we discussed in subsection 3.3 - is still present here: The positive effect on the marginal product of effort of eliminating the externality associated with duopoly still dominates the negative effect of eliminating strategic substituability; and this puts upward pressure on equilibrium effort after the merger.

What is different here is the presence of the agency marginal cost of eliciting effort, which also changes with the merger. Indeed, comparing the agency marginal cost pre- and post-merger, it is clear that it increases significantly after the merger, from \((b - \bar{e} - e_j) / 4\) to \((b - \bar{e}) / 2\). The negative impact of this increase in marginal agency cost on effort is substantial enough to dominate the positive Schumpeterian effect identified above, leading to a decrease in effort post-merger.

The question remaining, then, is why does the agency marginal cost go up after the merger? This result is mainly driven by two factors: 1) Individual firm profits \(\pi\) go up after the merger; and 2) Concomitant with the elimination - after the merger - of the strategic behavior observed pre-merger, the agent’s responsiveness to incentives decreases. As a result, then, the increase in the power of incentives needed to generate a given increase in effort, \(\partial \alpha^*/\partial \bar{e}_m\), increases after the merger. Together, these two factors imply an increase in the marginal agency cost \((1/2)(\partial \alpha^*/\partial \bar{e}_m) \pi_m\) post-merger. It thus becomes more expensive to elicit an increase in effort after the merger, and in equilibrium effort levels fall.

5 Welfare Analysis

As shall become clear below, a key advantage of our common but simple linear Cournot demand specification is that makes the welfare analysis very intuitive and analytically easy to perform.
5.1 In the First-Best

The social surplus in this framework can be expressed simply as the consumer value generated at the industry output, minus the cost of producing this output, which includes the total cost of production and the cost(s) of managerial effort. The pre- and post-merger social surpluses can thus be expressed as follows, respectively:

\[ S^* = \frac{1}{2} \left[ \int_{0}^{2q^*} (b - q) \, dq - 2q^* c^* \right] - 2 \left( \frac{1}{4} e^{*2} \right) \]

\[ S^m = \frac{1}{2} \left[ \int_{0}^{q^m} (b - q) \, dq - q^* c^* \right] - \frac{1}{4} e^{*2} \]  

(20)

Computing the values of these social surpluses at equilibrium, it is easy to show that:

**Proposition 3** In the first-best, the merger has an unambiguously positive impact on social welfare: \( \Delta S^* = S^m - S^* = (b - \tau)^2 / 10 > 0 \).

**Proof.** See appendix. ■

Intuitively, the merger has three effects on social surplus: First, as can be gleaned from lemmas 1 and 2, the merger leads to a well-known decrease in industry output, \( q^* m < 2q^* \), which has a negative effect on social surplus. Second, as discussed in subsection 3.3, the merger leads to a Schumpeterian increase in managerial effort and to a decrease in the marginal cost of production: \( c^* m < c^* \). Naturally this has a positive effect on social surplus. Finally, the merger avoids the duplication of managerial costs of effort, and this also has a positive on social surplus. As \( \Delta S^* > 0 \) suggests, in our model the two cost-related effects of the merger dominate the output effect, and overall the merger has an unambiguously positive effect on social surplus.

5.2 In the Second-Best

In the second-best, the pre- and post-merger social surpluses can be expressed similarly as follows, respectively:

[29]See, for example, Tirole (1988, pp.218-221). In a recent paper, Marino and Zábojnik (2006) show - in a dynamic model - that the ease and speed of entry in the industry may mitigate these anti-competitive effects of mergers.
\[ S^{**} = \frac{1}{2} \left[ \int_0^{2q^{**}} (b - 2q) dq - q^{**}c^{**} \right] - 2 \left( \frac{1}{4}c^{**2} \right) \]
\[ S^{**}_{m} = \frac{1}{2} \left[ \int_0^{q^{**}_m} (b - q_m) dq_m - q^{**}_mc^{**}_m \right] - \frac{1}{4}c^{**2} \]

(21)

Computing the equilibrium values of these social surpluses, we can show that:

**Proposition 4** In the second-best, the merger has an unambiguously negative impact on social welfare: \( \Delta S^{**} = S^{**}_{m} - S^{**} = -(593/1936)(b - \bar{c})^2 < 0. \)

**Proof.** See appendix. ■

Again, here the merger has three effects on social surplus: First, we deduce from lemmas 3 and 4 that, as in the first-best and for the same reasons, the merger still leads to a decrease in industry output: \( q^{**}_m < 2q^{**} \). This has a negative effect on the social surplus. Second, and as in the first-best, the merger avoids the duplication of managerial costs of effort, and this also has a positive on social surplus. The key difference with the first-best here is that, as discussed in subsection 4.3, in the second-best the positive Schumpeterian effect of the merger on managerial effort is dominated by an increased agency marginal cost effect, resulting in a decrease in managerial effort and an increase in the marginal cost of production, post-merger: \( c^{**}_m > c^{**} \). Naturally this has a negative effect on social surplus, sufficiently negative in fact to make the overall impact of the merger on social surplus negative.

### 5.3 Policy Implications

**Ignoring Agency Costs.** The foregoing discussion yields important policy implications. It suggests that, since \( \Delta S^{*} > \Delta S^{**} \), ignoring agency costs when assessing the benefits and costs of a merger may lead to systematic overestimation of the net benefits of the merger, and to authorizations of mergers which should in fact be prevented. Assuming a first-best world, one would predict an increase in effort and a decrease in marginal cost following the merger, due to the Schumpeterian effect discussed above. But this would ignore the increased agency cost of the merger, which pushes managerial effort down and marginal costs up. Indeed, in our model the latter effect of the merger dominates the former, thus leading to a significant overestimation of the net benefit of the merger:
Proposition 5 Ignoring agency costs in this model would lead to an overestimate of the net benefit of the merger, by an amount equal to \((\Delta S^{**} - \Delta S^*) / \Delta S^{**} = 133\% \) of the true net benefit.

Proof. Follows directly from above. ■

Ignoring Managerial Effort. In practice, however, managerial effort may be prohibitively costly to estimate, and policy makers may be able to estimate neither first- nor second-best worlds. They may actually ignore managerial effort altogether, and assume instead that marginal costs of production remain the same after the merger as the ones observed pre-merger. In that case the “no effort” pre-merger marginal cost for firm \(i\) can be obtained from Lemma 3: 
\[ \hat{c}_{i \text{ne}} = \hat{c}^{**} - \frac{b}{11} (b - \hat{c}) \]
leading to firm output \(q_{i \text{ne}}^{\text{ne}} = \hat{q}^{**} = \frac{6}{11} (b - \hat{c}) \). The no effort pre-merger social surplus can then be expressed as 
\[ S_{\text{ne}}^{\text{ne}} = \frac{1}{2} \left[ \int_0^{\hat{q}^{\text{ne}}} (b - q) dq - 2q_{\text{ne}}^{\text{ne}} \hat{c}_{\text{ne}}^{\text{ne}} \right] \]
After the merger, if one assumes the same marginal cost as before the merger, \( \hat{c}_{m \text{ne}}^{\text{ne}} = \hat{c}_{i \text{ne}}^{\text{ne}} \), the chosen output is 
\[ q_{m \text{ne}}^{\text{ne}} = \frac{9}{11} (b - \hat{c}) \]
and post-merger social surplus is 
\[ S_{m \text{ne}}^{\text{ne}} = \frac{1}{2} \left[ \int_0^{q_{m \text{ne}}^{\text{ne}}} (b - q) dq - q_{m \text{ne}}^{\text{ne}} \hat{c}_{m \text{ne}}^{\text{ne}} \right] \]. The impact of the merger can then easily be derived as 
\[ \Delta S_{\text{ne}}^{\text{ne}} = S_{m \text{ne}}^{\text{ne}} - S_{\text{ne}}^{\text{ne}} = - (171/484) (b - \hat{c})^2 < 0 \]
Again here \( \Delta S_{\text{ne}}^{\text{ne}} > \Delta S^{**} \), suggesting that ignoring managerial effort altogether, just like ignoring agency costs, leads to a systematic overestimation of the net benefits of the merger:

Proposition 6 Ignoring managerial effort altogether in this model would lead to an overestimate of the net benefit of the merger, by an amount equal to \((\Delta S^{**} - \Delta S^{\text{ne}}) / \Delta S^{**} = 15\% \) of the true net benefit.

Proof. See appendix. ■

Interestingly, the bias resulting from ignoring effort altogether is much smaller than the bias generated when effort is taken into account but agency costs are ignored. Thus this appears to be a case where “ignorance is bliss”... When effort is ignored altogether, the increased agency cost effect of the merger, which works against mergers, is no longer the only omitted effect: The Shumpeterian effect of the merger, as well as the avoided duplication of managerial costs of effort, are also ignored. These latter two effects work in favor of mergers, and hence ignoring them creates a negative bias that partially offsets the positive bias created by the ignored increased agency effect.
6 Applying the Theory

The question naturally arises regarding how the results of this paper could be used to improve merger policy and enforcement: making predictions on the potential for inefficiencies as a result of agency costs will be fraught with challenges. Of course, all merger reviews involve making predictions of likely future effects – whether on prices or new efficiencies. However, for these predictions we have developed techniques to more or less ground the work. We are not really at that stage with respect to increased agency costs or other X-inefficiencies. Therefore we must proceed with caution.

At a high, conceptual, level we should recognize that the potential for X-inefficiencies is very real, with the result that many mergers will not achieve the benefits expected by their proponents. This should affect how we balance the risks of harm to competition against the potential for product efficiencies in a Williamson-style tradeoff analysis.

In policy terms, a first step would be to incorporate considerations of agency costs and X-inefficiency in the cases when they are most likely to arise: mergers to monopoly or near monopoly. Where mergers leave a number of significant competitors, it is still difficult to predict the effect on competition – weakening our ability to provide predictions about both price and X-inefficiency post-merger. The effects on competition of mergers to near monopoly are not so difficult to predict. In merger-to-monopoly cases we are, at the same time, most confident of our predictions of price increases with the associated allocative inefficiencies and of our predictions for X-inefficiencies. In cases in which the negative effects from reduced competition post merger roughly balance the efficiencies expected by merger proponents, it certainly makes sense to consider X-inefficiencies that might tip the balance against approving the merger.

A Canadian case provides an illustrative example. The Superior Propane case\(^\text{30}\) involved the merger, in 1998, of Superior Propane and ICG Propane, the two largest marketers of propane in Canada at the time. While the actual size and number of players in the market was somewhat contentious, the Canadian Competition Tribunal reviewing the case determined that the merged entity would have had approximately 70% of the market nationally (and monopolies in many local markets) facing much smaller competitors and protected by barriers to entry. There was

\(^{30}\) The Commissioner of Competition vs. Superior Propane Inc., 2000 Comp. Trib. 15.
also recognition that the merger had the potential to produce significant efficiencies.

A classic Williamsonian tradeoff analysis was performed on the merger, under the Canadian law that allows a merger that lessens competition to nevertheless proceed if the efficiencies are “greater than, and will offset” the anticompetitive effects. After hearing the evidence, the Tribunal found that the anticompetitive harm, as measured by new deadweight loss attributable to higher prices, would amount to approximately $3 million per year. The Tribunal accepted efficiencies amounting to $29 million per year. As a result the Tribunal permitted the merger to proceed. However, as explained by Mathewson and Winter (2000), there was an important error made in the calculation of deadweight loss – the analysis that had produced the $3 million estimate had not taken into account the fact that premerger prices were already well in excess of marginal costs. A proper evaluation of the new deadweight loss attributable to higher post-merger prices would have estimated the size of a deadweight loss trapezoid rather than the much smaller triangle presented into evidence. Using public data produced through the hearings, Mathewson and Winter (2000, p. 91) estimate that the true new deadweight loss would be 8.5 times higher than previously thought – or approximately $25.5 million per year. The two deadweight loss calculations are illustrated in Figure 2.

[Insert figure 2 here.]

In this figure (P0, Q0) and (P1, Q1) represent the pre- and post-merger price-quantity pairs, respectively and c* is the marginal cost of production. In the Tribunal decision, the only recognized deadweight loss from the higher prices was the triangle ABD which would have been correct had the original price been equal to marginal cost. Because it was not, however, there were additional deadweight losses as each unit lost from consumption represented a larger amount of social surplus lost. The actual deadweight loss, as pointed out by Mathewson and Winter is equal to the area of trapezoid ABEF.

As a result, we have a case in which the efficiencies are close in value to the deadweight loss attributable to rising prices. Significant X-inefficiencies could certainly tip the balance in such a case. To get a sense of what the magnitude of these inefficiencies could be in this case we

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31 *Competition Act (Canada)*, R.S.C. 1985, c. C-34 as am., Section 96(1).

32 See also Sanderson (2002, p. 4.). Mathewson and Winter further suggest that the efficiencies may have been overstated as they did not take into account the expected reduction in volumes resulting from higher post-merger prices.
supplement the Mathewson-Winter analysis by including costs associated with agency effects (or other X-inefficiencies).

As indicated, agency costs that inflate marginal costs will have two harmful effects on total surplus in uncompetitive markets. First, the higher costs represent a loss in productive efficiency as more resources than necessary are used to produce any quantity of the good. Second, the higher marginal costs will lead the seller to raise its price still higher, leading to a further restriction in output and additional allocative inefficiencies. The shaded area in Figure 3 illustrates both of these losses for a post-merger increase in marginal cost from $c^*$ to $c^{**}$ and a resulting increase in price from $P_1$ to $P_2$.

[Insert figure 3 here.]

To estimate the potential magnitude of these losses in the Superior Propane case we make the following simplifying assumptions: (i) the merged firm can be treated as a monopolist;\(^{33}\) (ii) the merged firm is facing a (locally) linear demand curve of the (inverse) form: $P = b - mQ$; and (iii) and marginal costs ($c$) are constant with respect to quantity.

With this demand curve it is straightforward to show that the total surplus (producers’ plus consumers’ surplus), $S$, earned in the market, assuming profit-maximizing output-setting by the dominant firm can be expressed as:

$$ S = 3(b - c)2/8m $$

Then we can see that the effect of an increasing marginal cost, given the adjustments in price it would produce, will lead to the following effect on total surplus:

$$ \frac{dS}{dc} = -3(b - c)/4m = -3Q/2 $$

Notice that this is 150% of the harm that would have been suffered with only a production inefficiency effect (in that case $dS/dc = -Q$).

In the *Superior Propane* case, the merged entity would have been expected to sell, post

\(^{33}\)We could also treat the merged firm as dominant firm facing a competitive fringe of independent suppliers. This would only slightly complicate the analysis without fundamentally altering the results.
merger, approximately 2 billion litres of propane per year. Annual reports indicate that the two firms were earning a gross margin (revenue less the cost of purchased propane) of approximately 11 cents per litre. Therefore a one cent increase in marginal costs, cutting into those pre-merger margin by 9%, would have at this rate produced a additional harm of $30 million per year – a magnitude greater than either the estimated efficiencies or corrected deadweight loss estimates (before X-inefficiencies). Obviously, even much smaller degrees of X-inefficiency would still tip the balance in favor of blocking the merger.

7 Conclusion

Our hope has been to encourage a discussion within the antitrust community about including considerations of X-inefficiency, whether caused by agency costs or otherwise, in merger review. With further work on models of X-inefficiency and more empirical research to give us a better sense of the likely magnitudes of these effects, we will be in a better situation to add these effects into our trade-off analyses.

Our work here has shown that a consideration of agency issues can add to the costs associated with the lost of competitive tension in a market post-merger. In contrast to the effect of the

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34 Superior Propane Income Fund, Annual Report 1999. Combined sales in 1998 were approximately 2.15 million litres and in 1999 were approximately 2.08 million litres.
36 While our purpose here is not to establish the amount of X-inefficiency experienced by Superior Propane post-merger, some data from its Annual Reports is interesting and suggestive. Using, as a measure of costs, the cash operating and administrative costs (this excludes the cost of acquiring the propane) per litre of propane sold, the combined Superior Propane – ICG would have had, in 1997 (the last full year before the merger), a combined cost per litre of about 9.8¢ and it enjoyed a weighted average sales margin (on propane sales) of 12.0¢. By 2001, costs were up to 11.4¢ per litre and margins had risen to 15¢ per litre. By 2005 costs were up to 12.7¢ per litre and margins were even higher than in 2001 – up to 15.8¢ per litre. (These costs have continued to climb since 2005, with a particularly sharp spike in 2008 bringing them up to 15.1¢ per litre.) These costs, in the years immediately prior to the merger, where not obviously on an upward trajectory – Superior’s costs in 1995 were almost exactly the same as its costs in 1997 (9.9¢ per litre), though there was some increase in the smaller ICG’s costs (from 8.2¢ to 9.7¢) over this period. Margins were in fact declining between 1994 and 1996 for both firms (from 11.5¢ to 10.5¢ for a weighted average of the two firms) before beginning to climb the year of the merger. While hardly proof that the merger created both market power and X-inefficiency, these numbers are consistent with both of these effects. Notice also that the increase in costs per litre from 1997 to 2005 is almost three cents per litre, triple the increase considered in the text above. Of course, general inflation could play a role in this increase, but both the consumer price index and GDP deflator index suggested general inflation in the range of 18% from 1997 to 2005, compared to an increase in these costs of just under 30%. Recall as well, that claimed efficiencies should have pushed cost numbers down, other things equal – $29 million of annual efficiencies would amount to about a 1.2¢ per litre reduction in costs (based upon combined 1997 quantities). Additional data in this footnote came from the Superior Propane Income Fund, Annual Reports 1997, 1998 and 2005 and the Superior Propane Income Fund Annual Information Form 1996.
merger in a first-best (i.e. no agency problems) environment – where the reduced competition leads to greater cost reduction effort, more than making up for the loss of output – in the second-best environment, less competition leads to greater control problems and higher agency costs. The result is an unambiguous loss of welfare.

Clearly, our model is highly stylized. Further work could test the robustness of our findings by pushing the theory in a number of different directions. For example, here we have a significant economy of scale built in to the model, in the sense that one effort can work for both firms post merger – and this drives part of the efficiency results. If we wanted to acknowledge that the effort to operate the larger firm would need to be greater, we could contemplate a cost of effort function of the sort $K = (\phi e^2)/4$, where $1 \leq \phi \leq 2$. In this case, $\phi$ is effectively 1 (as a normalization) when the firms are separate but then the cost of effort associated with running the larger company is somewhat higher post-merger (because it is operating on a larger firm). This would allow us to explore the sensitivity of the results to less powerful economies of scale. It should be clear, however, that our current assumption of strong economies of scale in effort would, if anything, bias our results in favour of approving mergers. In this sense, our results can be seen as conservative with respect to their arguments for intervention.

Other extensions that would seem useful would include the consideration of: (i) different models of competition, for example differentiated products Bertrand-style competition; (ii) mergers that concentrate markets short of creating monopoly; and (iii) a more general approach to market uncertainty – for example one in which the “bad” state is not so bad as to lead to the complete disappearance of the market. We look forward to exploring these possibilities in future research.

A Appendix

Proof of Lemma 1:

From expression (6) we know that $\pi_i (e_i, e_j) = \left[\frac{b - \bar{e} + 2e_i - e_j}{3}\right]^2$. Since by assumption $K_i (e_i) = e_i^2/4$, first-order condition (FOC) (7) can then be expressed as:

$$\frac{1}{2} \frac{\partial \pi_i (e_i^*, e_j)}{\partial e_i} = \frac{1}{2} \frac{4}{3} \left(\frac{b - \bar{e} + 2e_i^* - e_j}{3}\right) = \frac{1}{2} e_i^* = \frac{\partial K_i (e_i^*)}{\partial e_i},$$

(24)
yielding $e_i^* = 4(b - \bar{c} - e_j)$ for $i = 1, 2$. Solving for $e_1^*$ and $e_2^*$ yields $e_1^* = e_2^* = (4/5) (b - \bar{c})$. This in turn implies marginal costs $c_1^* = c_2^* = \bar{c} - (4/5) (b - \bar{c})$.

Thus at date 0, principal $i$ offers to pay agent $i$ a wage $w_i^* = K_i(e_i^*) = K ((4/5) (b - \bar{c}))$, or $w_i^* = (8/25) (b - \bar{c})^2$ if marginal cost $c_i^*$ is verified, and 0 otherwise. At date 1 agent exerts effort $e_i^*$ yielding marginal cost $c_i^*$. Substituting equilibrium efforts in (5), we get can determine the outputs $q_1^* = q_2^* = 3/5 (b - \bar{c})$ produced at date 2 in the good state of the world. At date 3 the initial contracts are honored. □

**Proof of Lemma 2:**

From expression (10) we know that $\pi_m(e_m) = [b - \frac{\tau + e_m}{2}]^2$. Since $K_m(e_m) = e_m^2/4$, FOC (11) can then be expressed as:

$$\frac{1}{2} \frac{\partial \pi_m(e_m)}{\partial e_m} = \frac{1}{2} \left( \frac{b - \bar{c} + e_m^*}{2} \right) = \frac{1}{2} e_m^* = \frac{\partial K_m(e_m)}{\partial e_m},$$

yielding $e_m^* = (b - \bar{c})$, and marginal cost $c_m^* = \bar{c} - (b - \bar{c})$.

Thus at date 0, the principal offers to pay the agent a wage $w_m^* = K_m(e_m^*) = K ((b - \bar{c}))$, or $w_i^* = (b - \bar{c})^2 /2$ if marginal cost $c_m^*$ is verified, and 0 otherwise. At date 1 agent exerts effort $e_m^*$ yielding marginal cost $c_m^*$. Substituting equilibrium efforts in (9), we get can determine the output $q_m^* = (b - \bar{c})$ produced at date 2 in the good state of the world. At date 3 the initial contract is honored. □

**Proof of Lemma 3:**

Using the profit function described in (6), and the cost of effort $K_i(e_i) = e_i^2/4$, we can express agent $i$ incentive compatibility constraint (13) as follows:

$$\frac{1}{2} \alpha_i \frac{\partial \pi_i(\hat{e}_i, e_j)}{\partial \hat{e}_i} = \frac{1}{2} \alpha_i \frac{4}{3} \left( \frac{b - \bar{c} + 2\hat{e}_i - e_j}{3} \right) = \frac{1}{2} \hat{e}_i = \frac{\partial K_i(\hat{e}_i)}{\partial \hat{e}_i},$$

which we can express in terms of $\alpha_i$ as follows:

$$\alpha_i^{**}(\hat{e}_i, e_j) = \frac{9\hat{e}_i}{4(b - \bar{c} + 2\hat{e}_i - e_j)}.$$  

Substituting $\alpha_i^{**}(\hat{e}_i, e_j)$ into the principal’s objective function, one then chooses $e_i^{**}$ to max-
imize the following expression:

\[
\frac{1}{2} [1 - \alpha_i^* (\widehat{e}_i, e_j)] \pi_i (\widehat{e}_i, e_j) = \frac{1}{2} \left[ 1 - \frac{9 \widehat{e}_i}{4 (b - \overline{c} + 2 \widehat{e}_i - e_j)} \right] \left[ \frac{b - \overline{c} + 2 \widehat{e}_i - e_j}{3} \right]^2. \tag{28}
\]

Differentiating with respect to \( \widehat{e}_i \), we obtain:

\[
\frac{\partial}{\partial \widehat{e}_i} \left[ \frac{1}{2} [1 - \alpha_i^* (\widehat{e}_i, e_j)] \pi_i (\widehat{e}_i, e_j) \right] = \frac{17 (b - \overline{c} - e_j) - 4 \widehat{e}_i}{36} = 0. \tag{29}
\]

Solving the FOC for \( \widehat{e}_i \) yields \( e_i^{**} = \frac{7}{4} (b - \overline{c} - e_j) \) for \( i = 1, 2 \). Solving for \( e_1^{**} \) and \( e_2^{**} \) yields \( e_1^{**} = e_2^{**} = (7/11) (b - \overline{c}) \). This in turn implies shares of profits \( \alpha_1^{**} = \alpha_2^{**} = 7/8 \), and marginal costs \( c_1^* = c_2^* = \overline{c} - (7/11) (b - \overline{c}) \). One can easily verify that in equilibrium, in the good state but when price is at its lowest realized value (\( \varepsilon = -m \)), the agent’s W

constraint \( \alpha_i^{**} \Pi_i (c_i (e_i^{**}), c_j (e_j^{**}), -m) + \beta_i^{**} = (6/11) (b - \overline{c}) - m \geq 0 \) holds if and only if \( m \leq (6/11) (b - \overline{c}) \).

Thus at date 0, principal \( i \) offers to pay agent \( i \) a share \( \alpha_i^{**} = 7/8 \) of realized profits at the end of the game. At date 1 the agent exerts effort \( e_i^{**} \) implying marginal cost \( c_i^* = \overline{c} - (7/11) (b - \overline{c}) \). Substituting equilibrium efforts in (5), we get can determine the outputs \( q_1^{**} = q_2^{**} = (6/11) (b - \overline{c}) \) produced at date 2 in the good state of the world. At date 3 the initial contracts are honored. \( \square \)

**Proof of Lemma 4:**

Using the profit function described in (6), and the cost of effort \( K_m (e_m) = e_m^2 / 4 \), we can express agent \( i \) incentive compatibility constraint (17) as follows:

\[
\frac{1}{2} \alpha_m \frac{\partial \pi_m (\widehat{e}_m)}{\partial e_m} = \frac{1}{2} \alpha_m \left( \frac{b - \overline{c} + \widehat{e}_m}{2} \right) = \frac{1}{2} \frac{\partial e_m}{\partial e_m} = \frac{\partial K_m (\widehat{e}_m)}{\partial e_m}, \tag{30}
\]

which we can express in terms of \( \alpha_m \) as follows:

\[
\alpha_m^{**} (\widehat{e}_m) = \frac{2 \widehat{e}_m}{(b - \overline{c} + \widehat{e}_m)}. \tag{31}
\]

Substituting \( \alpha_m^{**} (\widehat{e}_m) \) into the principal’s objective function, one then chooses \( e_m^{**} \) to maxi-
mize the following expression:

\[
\frac{1}{2} \left[ 1 - \alpha_m^* (\hat{e}_m) \right] \pi_m (\hat{e}_m) = \frac{1}{2} \left[ 1 - \frac{2\hat{e}_m}{(b - \bar{c} + \hat{e}_m)} \right] \left[ \frac{b - \bar{c} + \hat{e}_m}{2} \right]^2.
\]  

(32)

Differentiating with respect to \( \hat{e}_m \), we obtain:

\[
\frac{\partial}{\partial \hat{e}_m} \left[ \frac{1}{2} \left[ 1 - \alpha_m^* (\hat{e}_m) \right] \pi_m (\hat{e}_m) \right] = \frac{1}{2} \left[ -\frac{b - \bar{c}}{2} + \frac{b - \bar{c} - \hat{e}_m}{2} \right] = 0.
\]  

(33)

Solving the FOC for \( \hat{e}_m \) yields \( e_m^* = 0 \). This in turn implies shares of profits \( \alpha_m^* = 0 \), and marginal costs \( c_m^* = \bar{c} \). One can easily verify that in equilibrium, the agent’s W constraint is trivially satisfied regardless of the state of world (good or bad) and of the realization of price.

Thus at date 0, principal \( m \) offers to pay the agent a share \( \alpha_m^* = 0 \) of realized profits at the end of the game. At date 1 the agent exerts effort \( e_m^* = 0 \) implying marginal cost \( c_m^* = \bar{c} \). Substituting equilibrium efforts in (5), we get can determine the outputs \( q_m^* = (b - \bar{c})/2 \) produced at date 2 in the good state of the world. At date 3 the initial contract is honored. \( \square \)

**Proof of Proposition 3:**

Using (20), we can express the social surplus differential \( \Delta S^* = S_m^* - S^* \) as:

\[
S_m^* - S^* = \frac{1}{2} \left[ \int_0^{q_m^*} (b - q) dq - q_m^* c_m^* \right] - \frac{1}{4} e_m^* - \frac{1}{2} \left[ \int_0^{2q^*} (b - q) dq - 2q^* c^* \right] + 2 \left( \frac{1}{4} e^* \right).
\]  

(34)

This simplifies to:

\[
S_m^* - S^* = \frac{1}{2} \left[ bq - \frac{1}{2} q^2 \right]_{q_m^*}^{2q^*} - \frac{1}{2} \left[ q_m^* c_m^* - 2q^* c^* \right] - \frac{1}{4} \left[ e_m^* - 2e^* \right].
\]  

(35)

or:

\[
S_m^* - S^* = \frac{1}{2} \left[ b(q_m^* - 2q^*) - \frac{1}{2} (q_m^* - 4q^*) \right] - \frac{1}{2} \left[ q_m^* c_m^* - 2q^* c^* \right] - \frac{1}{4} \left[ e_m^* - 2e^* \right].
\]  

(36)
Using \( c_m^* = \bar{c} - c_m^* \) and \( c^* = \bar{c} - c^* \), we can write:

\[
S_m^* - S^* = \frac{1}{2} \left[ q_m^* (b - \bar{c} + e_m^*) - 2q^* (b - \bar{c} + e^*) - \frac{1}{2} \left( q_m^{*2} - 4q^*2 \right) \right] - \frac{1}{4} \left[ e_m^{*2} - 2e^*2 \right]. \tag{37}
\]

Substituting \( e_m^* = \frac{4}{5} (b - \bar{c}) \), \( e^* = (b - \bar{c}) \), \( q_m^* = (b - \bar{c}) \), and \( q^* = \frac{2}{3} (b - \bar{c}) \), and rearranging, we obtain the required result: \( S_m^* - S^* = (b - \bar{c})^2 / 10 > 0. \quad \square \)

**Proof of Proposition 4:**

Using (21), we can express the social surplus differential \( \Delta S^{**} = S_m^{**} - S^{**} \) as:

\[
S_m^{**} - S^{**} = \frac{1}{2} \left[ \int_0^{q_m^{**}} (b - q) \, dq - q_m^{**} c_m^{**} \right] - \frac{1}{4} e_m^{**2} - \frac{1}{2} \int_0^{2q^{**}} (b - q) \, dq - 2q^{**} c^{**} - 2 \left( \frac{1}{4} e^{**2} \right). \tag{38}
\]

This simplifies to:

\[
S_m^{**} - S^{**} = \frac{1}{2} \left[ bq - \frac{1}{2} q^2 \right]^{q_m^{**}}_{2q^{**}} - \frac{1}{2} \left[ q_m^{**} c_m^{**} - 2q^{**} c^{**} \right] - \frac{1}{4} \left[ e_m^{**2} - 2e^{**2} \right], \tag{39}
\]

or:

\[
S_m^{**} - S^{**} = \frac{1}{2} \left[ b (q_m^{**} - 2q^{**}) - \frac{1}{2} \left( q_m^{**2} - 4q^{**2} \right) \right] - \frac{1}{2} \left[ q_m^{**} c_m^{**} - 2q^{**} c^{**} \right] - \frac{1}{4} \left[ e_m^{**2} - 2e^{**2} \right]. \tag{40}
\]

Using \( c_m^{**} = \bar{c} - c_m^{**} \) and \( c^{**} = \bar{c} - c^{**} \), we can write:

\[
S_m^{**} - S^{**} = \frac{1}{2} \left[ q_m^{**} (b - \bar{c} + e_m^{**}) - 2q^{**} (b - \bar{c} + e^{**}) - \frac{1}{2} \left( q_m^{**2} - 4q^{**2} \right) \right] - \frac{1}{4} \left[ e_m^{**2} - 2e^{**2} \right].
\]

Substituting \( e_m^{**} = 0 \), \( e^{**} = \frac{7}{11} (b - \bar{c}) \), \( q_m^{**} = \frac{1}{2} (b - \bar{c}) \), and \( q^{**} = \frac{6}{11} (b - \bar{c}) \), and rearranging, we obtain the required result: \( S_m^{**} - S^{**} = -\frac{593}{1936} (b - \bar{c})^2 < 0. \quad \square \)

**Proof of Proposition 6:**

Let \( \Delta S^{ne} = S_m^{ne} - S^{ne} \), with \( S_m^{ne} = \frac{1}{2} \left[ \int_0^{q_m^{ne}} (b - q) \, dq - q_m^{ne} c_m^{ne} \right], S^{ne} = \frac{1}{2} \left[ \int_0^{2q^{ne}} (b - q) \, dq - 2q^{ne} c^{ne} \right] \).
and $c^{ne} = c^e_m = \overline{c} - \frac{7}{12} (b - \overline{c})$. Then:

$$S^{ne}_m - S^{ne} = \frac{1}{2} \left[ \int_0^{q^{ne}_m} (b - q) \, dq - q^{ne}_m c^e_m \right] - \frac{1}{2} \left[ \int_0^{2q^{ne}} (b - q) \, dq - 2q^{ne} c^{ne} \right]. \quad (41)$$

This simplifies to:

$$S^{ne}_m - S^{ne} = \frac{1}{2} \left[ bq - \frac{1}{2} q^2 \right]^{q^{ne}_m} - \frac{1}{2} \left[ (q^{ne}_m - 2q^{ne}) c^{ne} \right], \quad (42)$$

or:

$$S^{ne}_m - S^{ne} = \frac{1}{2} \left[ b (q^{ne}_m - 2q^{ne}) - \frac{1}{2} (q^{ne}_{m2} - 4q^{ne2}) - (q^{ne} c^{ne}_m - 2q^{ne} c^{ne}) \right] \quad (43)$$

or:

$$S^{ne}_m - S^{ne} = \frac{1}{2} \left[ (q^{ne}_m - 2q^{ne}) (b - c^{ne}) - \frac{1}{2} (q^{ne}_{m2} - 4q^{ne2}) \right]. \quad (44)$$

Substituting $c^{ne} = \overline{c} - \frac{7}{12} (b - \overline{c})$, and outputs $q^{ne}_m = \frac{9}{12} (b - \overline{c})$ and $q^{ne}_n = q^{**} = \frac{6}{12} (b - \overline{c})$, we get $S^{ne}_m - S^{ne} = -\frac{171}{484} (b - \overline{c})^2$.

Substituting the values of $\Delta S^{ne}$ and $\Delta S^{**}$ into $(\Delta S^{**} - \Delta S^{ne}) / \Delta S^{**}$ yields $(\Delta S^{**} - \Delta S^{ne}) / \Delta S^{**} = 15\%$. □

References


American Economic Review, 58, 18-36.


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Figure 1: Timing of the Game

<table>
<thead>
<tr>
<th>date 0</th>
<th>date 1</th>
<th>date 2</th>
<th>date 3</th>
<th>date 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal i offers contract to agent i.</td>
<td>Agent i exerts effort $e_i$.</td>
<td>State of the world (good or bad) is realized.</td>
<td>Agent i chooses output $q_i$.</td>
<td>Profits are realized and contracts are honoured.</td>
</tr>
</tbody>
</table>
Figure 2: Estimation of Deadweight Loss in Superior Propane Case, Ignoring X-inefficiency
Figure 3: Estimation of Deadweight Loss in Superior Propane Case, Including X-inefficiency