# Interest Rates and the Exchange Rate: A Non-Monotonic Tale


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**Abstract**

What is the relationship between interest rates and the exchange rate? The empirical literature in this area has been inconclusive. We use an optimizing model of a small open economy to rationalize the mixed empirical findings. The model has three key margins. First, higher domestic interest rates raise the demand for deposits, and, hence, the money base. Second, firms need bank loans to finance the wage bill, which reduces output when domestic interest rates increase. Lastly, higher interest rates raise the government's fiscal burden, and, therefore, can lead to higher expected inflation. While the first effect tends to appreciate the currency, the remaining two effects tend to depreciate it. We then conduct policy experiments using a calibrated version of the model and show the central result of the paper: the relationship between interest rates and the exchange rate is *non-monotonic*. In particular, the exchange rate response depends on the size of the interest rate increase and on the initial level of the interest rate. Moreover, we show that the model can replicate the heterogeneous responses of the exchange rate to interest rate innovations in several developing economies.

**JEL Classification:** F3, F4

**Keywords:** Interest rate policy, flexible exchange rates, currency depreciation

# 1 Introduction

In this paper we show, both theoretically and quantitatively, that the relationship between nominal interest rates and the nominal exchange rate is inherently non-monotonic. We formalize a small open economy model where higher interest rates have three effects. They raise the fiscal burden on the government, reduce output due to higher working capital costs, and raise the demand for domestic currency assets. The first two effects tend to depreciate the currency while the last effect tends to appreciate the currency. The net exchange rate response to interest innovations depends on the relative strengths of these opposing forces. We calibrate the model to developing economies. We use the quantitative model to show that a permanent increase in the policy-controlled interest rate has a non-monotonic effect on the steady state exchange rate: for small increases in the interest rate the exchange rate appreciates but for larger increases it depreciates. We also compute model-generated impulse responses of the exchange rate to temporary interest rate innovations and demonstrate that these are similarly non-monotonic. As a final test, we demonstrate that the model can replicate the disparate responses of the exchange rate to interest rate innovations across three different countries: Thailand, Korea, and Brazil. We interpret our results as providing a rationalization for the failure of standard empirical methods to detect any systematic relationship between interest rates and exchange rates in the data.

The relationship between interest rates and exchange rates has long been a key focus of international economics. Most standard theoretical models of exchange rates predict that exchange rates are determined by economic fundamentals, one of which is the interest rate differential between home and abroad. However, a consistent result in the empirical literature is that a random-walk exchange rate forecasting model usually outperforms fundamental-based forecasting models. In other words, most models do not explain exchange rates movements (see Meese (1990)). This fact is highlighted in Obstfeld and Rogoff (2000) who call it the "exchange rate disconnect puzzle". Moreover, studies that have directly examined the relationship between interest rates and exchange rates have typically found mixed and/or conflicting results. Thus, Eichenbaum and Evans (1995) find that for the G7 countries interest rate innovations tend to appreciate the currency. On the other hand, Calvo and Reinhart (2002) find that for developing countries there is no systematic relationship between the two variables. On a related theme, Drazen and Hubrich (2006) show that during the European Exchange Rate Mechanism (ERM) crisis of 1992, currency forecasts reacted non-monotonically to interest rate increases with near term appreciations being accompanied by long term depreciations of forecasted currency values.

The absence of a clear empirical relationship between interest rates and the exchange rate is even more problematic from the perspective of applied practitioners. A short-term interest rate is the typical policy instrument used by policymakers to affect currency values (and monetary conditions more generally).[1] If there is no clear relationship in the data, then why do policymakers persist in using the interest rate instrument to affect exchange rates? Or, for example, why do observers today argue that the rapid reduction in the Federal Funds rate over the last six months is partly responsible for the continuous slide of the dollar in international currency markets? The goal of our work is thus two-fold. First, we want to explore in greater conceptual detail the relationship between interest rates (both policy-controlled and market determined) and the nominal exchange rate and in the process clarify the tradeoffs that are typically faced by policymakers. Second, we want to use insights from the first exercise to facilitate a rationalization of the mixed empirical results regarding this relationship.

We should note at the outset that our paper is *not* concerned with the relationship between the nominal market interest rate and the rate of currency depreciation. There is a voluminous literature which attempts to document and/or explain this relationship. This literature is concerned with the failure of the uncovered interest parity (UIP) condition (the "forward premium anomaly"). In our model interest parity holds for internationally traded bonds. Hence, we do not shed any new light on the observed deviations from UIP. Instead, our main focus is on the effects of policy-induced changes in nominal interest rates on the *level* of the exchange rate.

In the context of the model described above, we find that the relationship between interest rates and the nominal exchange rate is non-monotonic under fairly general conditions. In particular, as long as the interest elasticity of money demand is increasing in the relevant opportunity costs, an increase in the policy-controlled interest rate will raise the elasticity of cash demand since the nominal interest rate rises. Simultaneously, the elasticity of demand for deposits falls since the opportunity cost of deposits (the nominal interest rate less the deposit rate) declines. Hence, the negative effect on money demand coming from cash rises with the domestic interest rate while

[1] We should note that, in the aftermath of the Asian crisis in the late 1990s, there was a contentious debate regarding the soundness of the International Monetary Fund's (IMF) advice to affected countries of raising interest rates to stabilize the rapidly depreciating domestic currencies. IMF critics (like Joseph Stiglitz and Jeffrey Sachs) were of the view that this policy may not even worked and, even if it did, its costs would be just too high. In fact, a large empirical literature on the topic has failed to unearth a systematic effect of higher interest on the currency values during the crisis period in the affected Asian economies (see Kraay (2003)).

the positive effect coming from deposits becomes gradually smaller. This implies that as long as deposit demand is more elastic than cash demand for low domestic interest rates, initially money demand will rise with the interest rate since the demand deposit effect dominates. However, beyond a certain point, the negative cash effect overwhelms the positive demand deposit effect and money demand begins to fall. This non-monotonicity of real money demand maps into a non-monotonicity of the nominal exchange rate: for small increases in the domestic interest rate the exchange rate appreciates but once money demand begins to fall the exchange rate depreciates. The associated negative effect on output of changes in the domestic interest rate adds an additional negative effect to money demand but does not change this basic intuition.

While the theoretical linkages are instructive, our main interest lies in determining whether these non-monotonicities can arise in a realistically parameterized model that embodies the channels specified above. Toward that end, we calibrate our model to Argentinean data so that the model can reproduce the key unconditional moments of real variables in the Argentinean economy between 1983-2002. We then use the calibrated model to conduct interest rate experiments. We show three main results. First, the steady state response of the nominal exchange rate to increases in the steady state domestic policy-controlled interest rate is non-monotonic. Second, the impulse response of the exchange rate to a temporary one standard deviation increase in the interest rate is also non-monotonic: for economies with low steady state interest rates, a temporary increase in the interest rate appreciates the exchange rate while for high interest rate economies the same sized increase in interest rates depreciates the currency. Third, the response of the exchange rate to transitory interest rate innovations depends on the size of the shock. For small increases in the interest rate, the domestic currency appreciates; for large increases in the interest rate, it depreciates. We also provide a quantitative assessment for the relevant ranges of the interest rate.

As a final step, we recalibrate our model to three developing countries: Thailand, Korea, and Brazil. Despite their similarities along many dimensions, these countries are distinct in that they exhibit different exchange rate responses to interest rate innovations. We show that the model-generated impulse responses of exchange rates can reproduce the data-generated impulse responses. These results suggest that our model has the capacity to explain different exchange rate responses based on country-specific measurable parameters.

The rest of the paper is organized as follows. The next section presents some empirical evidence from a number of developing and developed countries detailing the mixed results on the relationship between interest rates and the exchange rate. Section 3 presents the model while Section 4 builds some analytical insights by studying some special cases of the model. Section 5 discusses how the model is calibrated and solved, while Section 6 presents our quantitative results using the calibrated model. Section 7 compares the country specific impulse responses from the model with the data. The last section concludes.

## 2 Empirical motivation

We start off by empirically documenting our motivating issue (the lack of a systematic relationship between interest rates and the exchange rate) through a look at the data. We study the relationship by estimating unrestricted vector autoregressions (VARs) on a country-by-country basis for a sample of ten countries. Our sample includes six developing countries – Brazil, Korea, Mexico, Thailand, Peru and Philippines – and four developed countries – Canada, Germany, Italy, and the United States.

For our sample we chose a representative mix of developed and developing countries. We focus on periods during which the exchange rate in these countries was floating, so the starting date varies from country to country. Here we report the empirical results from the subsample ending in 2001.[2] We also investigated the evidence for the entire period ending in 2007. The results remained qualitatively unchanged for all countries except for Brazil and Thailand. We explore these two countries and their changing responses in greater detail below in Section 7. We chose to examine the periods 1997 – 2001 and 1997 – 2007 separately as there was a perceptible difference in monetary conditions between the two. During the period 1997-2001 the interest rate was used very actively by a number of central banks to defend against speculative attacks on their currencies. In contrast, the period 2001-2007 was one of relative tranquility in international currency markets. As a result interest rates were much less volatile during the latter phase relative to the first sub-sample.

We estimate country-specific four variable VARs using monthly data on nominal exchange rates (domestic currency units per U.S. dollar, period average), short term interest rate differentials between home and abroad (domestic minus U.S. Federal Funds rate), industrial production (index, 2000 base year) and government fiscal balance.[3] We express the fiscal balance data in terms of

[2] The sample periods are as follows: Brazil 1998:12 – 2001:03; Mexico 1995:01 – 2001:03; Peru 1992:01 – 2001:03; Korea 1997:07 – 2001:03; Thailand 1997:07 – 2001:03; and Phillipines 1983:01 – 2001:03. We chose four major developed countries as a control group. The sample period for developed countries is 1974:01 – 2001:03.

[3] Whenever available, we use the interest payments data instead of the fiscal balance data in order to have more accurate correspondence with the model-generated series.

U.S. currency and then normalize the resulting numbers across countries by expressing them in per capita terms.[4] For the United States, the exchange rate is expressed in terms of dollar per yen while the interest rate differential is the U.S. minus the Japanese short term interest rate. Since monthly fiscal data for all countries in our sample is highly seasonal and volatile, we use the 11-month centered moving average instead. Our data is from the International Financial Statistics (IFS).[5]

We use the estimated VARs to calculate the impulse response of the exchange rate, industrial production, and the fiscal balance to an orthogonalized one standard deviation innovation in the interest rate differential between home and abroad for each country. Following Eichenbaum and Evans (1995) we compute the impulse responses using the following ordering: industrial production, interest rate differential, exchange rate, and fiscal balance.[6]

Figure 1 depicts the impulse response of the nominal exchange rate (with a one standard deviation significance band) to a one standard deviation orthogonalized innovation in the interest rate differential. The picture reveals mixed results. Within the set of developed countries, in Canada, Germany and Italy there is a significant appreciation of the currency in response to an increase in the interest rate differential. This is the well-known result of Eichenbaum and Evans (1995). For the developing group the effect is mostly the opposite. Except for Thailand, in all countries a positive innovation in the interest rate differential between home and the United States induces a significant depreciation of the currency. Thailand, on the other hand, shows a significant appreciation of the currency in response to an interest rate innovation.

However, even for the developed countries the relationship between interest rates and exchange rates is not stable over time. Thus, for the U.S. we split the sample into two sub-periods – 1974:01 – 1990:05 and 1990:06 – 2001:03. Note that the first sub-period corresponds to the period analyzed in Eichenbaum and Evans (1995). As can be seen from the last row of Figure 1, the exchange rate effect of an interest rate innovation is different in the two sub-periods. For 1974:01–1990:05 we see the standard result - a positive innovation in the interest rate differential between the US and Japan causes a significant exchange rate appreciation. However, this relationship is reversed for the latter period in which the dollar depreciates relative to the yen in response to an innovation in

[4]We express the fiscal balance in this form in order to keep the measured fiscal balance in the data consistent with the way it is measured in the model that we present later.

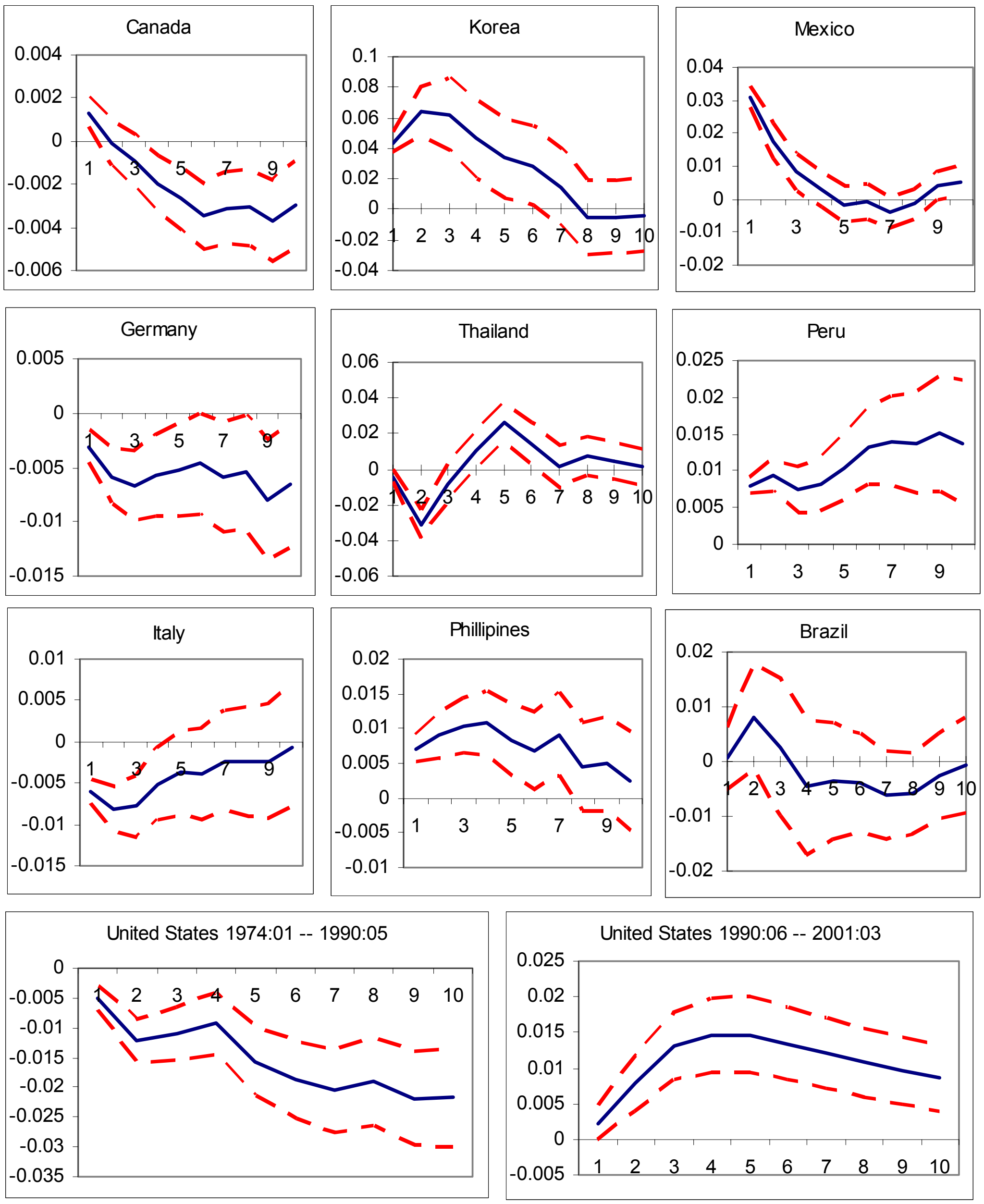
[5]For the interest rate data, we use short term money market rates. For the U.S., the series used is the Federal Funds rate.

[6]The Akaike criterion was used to choose the lag length.

the same interest rate differential.[7]

Our evidence is thus consistent with the lack of a systematic relationship between interest rates and the exchange rate in the data. As can be seen in Figure 1, this puzzle exists both on a cross-country basis as well as on a time series basis.

Figure 1. VAR: Nominal exchange rate response to interest rate innovations




[7]We also estimated the VAR system for these countries for the entire sample ending in 2007:10. The results remained qualitatively unchanged for all countries except for Brazil and Thailand. We explore these two countries and their changing responses in greater detail below in Section 7.

# 3 The model

Consider a representative household model of a small open economy that is perfectly integrated with the rest of the world in both goods and capital markets. The infinitely-lived household receives utility from consuming a (non-storable) good and disutility from supplying labor. The world price of the good in terms of foreign currency is fixed and normalized to unity. Free goods mobility across borders implies that the law of one price applies. The consumer can also trade freely in perfectly competitive world capital markets by buying and selling real bonds which are denominated in terms of the good and pay $r$ units of the good as interest at every point in time.

## 3.1 Households

Household's lifetime welfare is given by

$$V = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, x_t\right), \tag{1}$$

where $c$ denotes consumption, $x$ denotes labor supply, and $\beta(> 0)$ is the exogenous and constant rate of time preference. We assume that the period utility function of the representative household is given by

$$U(c, x) = \frac{1}{1-\sigma}\left(c - \zeta x^{\nu}\right)^{1-\sigma}, \qquad \zeta > 0, \quad \nu > 1.$$

Here $\sigma$ is the intertemporal elasticity of substitution, $\nu - 1$ is the inverse of the elasticity of labor supply with respect to the real wage. These preferences are well-known from the work of Greenwood, Hercowitz and Huffman (1988), which we will refer to as GHH.[8]

Households use cash, $H$, and nominal demand deposits, $D$, for reducing transactions costs. Specifically, the transactions costs technology is given by

$$s_t = v\left(\frac{H_t}{P_t}\right) + \psi\left(\frac{D_t}{P_t}\right), \tag{2}$$

where $P$ is the nominal price of goods in the economy, and $s$ denotes the non-negative transactions costs incurred by the consumer. Let $h$ $(= H/P)$ denote cash and let $d$ $(= D/P)$ denote interest-bearing demand deposits in real terms. We assume that the transactions technology is strictly

[8] These preferences have been widely used in the real business cycle literature as they provide a better description of consumption and the trade balance for small open economies than alternative specifications (see, for instance, Correia, Neves, and Rebelo (1995)). As will become clear below, the key analytical simplification introduced by GHH preferences is that there is no wealth effect on labor supply.

convex. In particular, the functions $v(h)$ and $\psi(d)$, defined for $h \in [0, \bar{h}]$, $\bar{h} > 0$, and $d \in [0, \bar{d}]$, $\bar{d} > 0$, respectively, satisfy the following properties:

$$\begin{aligned} v &\geq 0, \quad v' \leq 0, \quad v'' > 0, \quad v'(\bar{h}) = v(\bar{h}) = 0, \\ \psi &\geq 0, \quad \psi' \leq 0, \quad \psi'' > 0, \quad \psi'(\bar{d}) = \psi(\bar{d}) = 0. \end{aligned}$$

Thus, additional cash and demand deposits lower transactions costs but at a decreasing rate. The assumption that $v'(\bar{h}) = \psi'(\bar{d}) = 0$ ensures that the consumer can be satiated with real money balances.

In addition to the two liquid assets, households also hold a real internationally-traded bond, $b$, and physical capital, $k$, which they can rent out to firms. The households flow budget constraint in nominal terms is

$$\begin{aligned} & P_t b_{t+1} + D_t + H_t + P_t \left(c_t + I_t + s_t + \kappa_t\right) \\ = \; & P_t \left(R b_t + w_t x_t + \rho_t k_{t-1} + \tau_t + \Omega_t^f + \Omega_t^b\right) + \left(1 + i_t^d\right) D_{t-1} + H_{t-1}. \end{aligned}$$

Foreign bonds are denominated in terms of the good and pay the gross interest factor $R (= 1 + r)$, which is constant over time. $i_t^d$ denotes the deposit rate contracted in period $t - 1$ and paid in period $t$. $w$ and $\rho$ denote the wage and rental rates. $\tau$ denotes lump-sum transfers received from the government. $\Omega^f$ and $\Omega^b$ represent dividends from firms and banks respectively. $\kappa$ denotes capital adjustment costs

$$\kappa_t = \kappa\left(I_t, k_{t-1}\right), \quad \kappa_I > 0, \kappa_{II} > 0, \tag{3}$$

i.e., adjustment costs are convex in investment. Lastly,

$$I_t = k_t - (1 - \delta)\, k_{t-1}. \tag{4}$$

In real terms the flow budget constraint facing the representative household is thus given by

$$\begin{aligned} & b_{t+1} + h_t + d_t + c_t + I_t + s_t + \kappa_t \\ = \; & R b_t + w_t x_t + \rho_t k_{t-1} + \frac{h_{t-1}}{1 + \pi_t} + \left(\frac{1 + i_t^d}{1 + \pi_t}\right) d_{t-1} + \Omega_t^f + \Omega_t^b, \end{aligned} \tag{5}$$

where $\Omega^f$ and $\Omega^b$ denote dividends received by households from firms and banks, respectively. $1 + \pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross rate of inflation between periods $t - 1$ and $t$. It is useful to note that the uncovered interest parity condition dictates that expected returns from investing in domestic nominal bonds and international real bonds must be equalized. Hence, recalling that $P_{t+1}/P_t = 1 + \pi_{t+1}$,

$$1 + i_{t+1} = R\mathbb{E}_t \left(1 + \pi_{t+1}\right).$$

Households maximize their lifetime welfare equation (1) subject to equations (2), (3), (4) and (5).

### 3.2 Firms

The representative firm in this economy produces the perishable good using a constant returns to scale technology over capital and labor

$$y_t = F\left(k_{t-1}, A_t l_t\right) = A_t k_{t-1}^{\alpha} l_t^{1-\alpha}, \tag{6}$$

with $\alpha > 0$, and $A_t$ denoting the current state of productivity which is stochastic. $l$ is labor demand. At the beginning of the period, firms observe shocks for the period and then make production plans. They rent capital and labor. However, a fraction $\phi$ of the total wage bill needs to be paid upfront to workers. Since output is only realized at the end of the period, firms finance this payment through loans from banks. The loan amount along with the interest is paid back to banks next period.[9] Formally, this constraint is given by

$$N_t = \phi P_t w_t l_t, \quad \phi > 0, \tag{7}$$

where $N$ denotes the nominal value of bank loans. The assumption that firms must use bank credit to pay the wage bill is needed to generate a demand for bank loans.

The firm's flow constraint in nominal terms is given by

$$P_t b_{t+1}^f - N_t = P_t\left(Rb_t^f + y_t - w_t l_t - \rho_t k_{t-1} - \Omega_t^f\right) - \left(1 + i_t^l\right) N_{t-1},$$

where $i^l$ is the lending rate charged by bank for their loans and $\Omega^f$ denotes dividends paid out by the firms to their shareholders. $b^f$ denotes foreign bonds held by firms which pay the going world interest factor $R$. In real terms the flow constraint reduces to

$$b_{t+1}^f - n_t = Rb_t^f - \left(\frac{1 + i_t^l}{1 + \pi_t}\right) n_{t-1} + y_t - w_t l_t - \rho_t k_{t-1} - \Omega_t^f.$$

[9] Alternatively, we could assume that bank credit is an input in the production function, in which case the derived demand for credit would be interest rate elastic. This would considerably complicate the model without adding any additional insights.

Define

$$a_{t+1}^f \equiv b_{t+1}^f - \frac{\left(1+i_{t+1}^l\right)}{R\left(1+\pi_{t+1}\right)} n_t.$$

Substituting this expression together with the credit-in-advance constraint into the firm's flow constraint in real terms gives

$$a_{t+1}^f + \Omega_t^f = Ra_t^f + y_t - \rho_t k_{t-1} - w_t l_t \left[1 + \phi \left\{ \frac{1+i_{t+1}^l - R\left(1+\pi_{t+1}\right)}{R\left(1+\pi_{t+1}\right)} \right\}\right]. \tag{8}$$

Note that $\phi \left\{ \frac{1+i_{t+1}^l - R(1+\pi_{t+1})}{R(1+\pi_{t+1})} \right\} w_t l_t = \left\{ \frac{1+i_{t+1}^l - R(1+\pi_{t+1})}{R(1+\pi_{t+1})} \right\} n_t$ is the additional resource cost that is incurred by firms due to the credit-in-advance constraint.[10]

The firm chooses a path of $l$ and $k$ to maximize the present discounted value of dividends subject to equations (6), (7) and (8). Given that households own the firms, this formulation is equivalent to the firm using the household's stochastic discount factor to optimize. The first order conditions for this problem are given by two usual conditions and an Euler equation which is identical to the household's Euler equation. The two usual conditions are standard – the firm equates the marginal product of the factor to its marginal cost. In the case of labor the cost includes the cost of credit. This is proportional to the difference between the nominal lending rate and the nominal interest rate.

### 3.3 Banks

The banking sector is assumed to be perfectly competitive. The representative bank holds foreign real debt, $d^b$, accepts deposits from consumers and lends to both firms, $N$, and the government in the form of domestic government bonds, $Z$.[11] It also holds required cash reserves, $\theta D$, where $\theta > 0$ is the reserve-requirement ratio imposed on the representative bank by the central bank. Banks face a cost $q$ (in real terms) of managing their portfolio of foreign assets. Moreover, we assume that banks also face a constant proportional cost $\phi^n$ per unit of loans to firms. This is intended

[10] We should note that the credit-in-advance constraint given by equation (7) holds as an equality only along paths where the lending spread $1+i^l - R(1+\pi)$ is strictly positive. We will assume that if the lending spread is zero, this constraint also holds with equality.

[11] Commercial bank lending to governments is particularly common in developing countries. Government debt is held not only as compulsory (and remunerated) reserve requirements but also voluntarily due to the lack of profitable investment opportunities in crisis-prone countries. This phenomenon was so pervasive in some Latin American countries during the 1980's that Rodriguez (1991) aptly refers to such governments as "borrowers of first resort". For evidence, see Rodriguez (1991) and Druck and Garibaldi (2000).

to capture the fact that domestic loans to private firms are potentially special as banks need to spend additional resources in monitoring loans to private firms.[12] The nominal flow constraint for the bank is

$$
\begin{aligned}
N_t + Z_t - (1-\theta) D_t + P_t q_t - P_t d^b_{t+1} \quad = \quad & \left(1 + i^l_t - \phi^n\right) N_{t-1} + (1 + i^g_t) Z_{t-1} \\
& - \left(1 + i^d_t\right) D_{t-1} + \theta D_{t-1} - P_t R d^b_t - P_t \Omega^b_t, \qquad (9)
\end{aligned}
$$

where $i^g$ is the interest rate on government bonds. We assume that banking costs are a convex function of the foreign debt held by the bank:

$$
q_t = q\left(d^b_{t+1}\right) \ , \qquad q' > 0, \ q'' > 0,
$$

where $q'$ denotes the derivative of the function $q$ with respect to its argument, while $q''$ denotes the second derivative. The costly banking assumption is needed to break the interest parity condition between domestic and foreign bonds. Throughout the paper we assume that the banking cost technology is given by the quadratic function:

$$
q_t = \frac{\gamma}{2}\left(d^b_{t+1}\right)^2, \qquad (10)
$$

where $\gamma > 0$ is a constant parameter.[13]

Deflating the nominal flow constraint by the price level gives the bank's flow constraint in real terms:

$$
\Omega^b_t = \left[\frac{R(1+\pi_t) - 1}{1+\pi_t}\right]\left[(1-\theta) d_{t-1} - n_{t-1} - z_{t-1}\right] + \frac{i^l_t - \phi^n}{1+\pi_t} n_{t-1} + \frac{i^g_t}{1+\pi_t} z_{t-1} - \frac{i^d_t}{1+\pi_t} d_{t-1} - q_t, \qquad (11)
$$

where we have used the bank's balance sheet identity: $P_t d^b_{t+1} = N_t + Z_t - (1-\theta) D_t$. Note that this is equivalent to setting the bank's net worth to zero at all times. Also, the quadratic specification for banking costs along with the zero net worth assumption implies that these banking costs can also be reinterpreted as a cost of managing the portfolio of net domestic assets since $d^b_{t+1} = \frac{N_t + Z_t - (1-\theta) D_t}{P_t}$.

The representative bank chooses sequences of $N$, $Z$, and $D$ to maximize the present discounted value of profits subject to equations (9) taking as given the paths for interest rates $i^l$, $i^d$, $i^g$, $i$, and

[12] We should note that this cost $\phi^n$ is needed solely for numerical reasons since, as will become clear below, it gives us a bigger range of policy-controlled interest rates to experiment with. Qualitatively, all our results would go through with $\phi^n = 0$.

[13] Similar treatment of banking costs of managing assets and liabilities can be found in Diaz-Gimenez et al (1992) and Edwards and Végh (1997). This approach to breaking the interest parity condition is similar in spirit to Calvo and Végh (1995).

the value of $\theta$ and $\phi^n$. We assume that the bank uses the household's stochastic discount factor to value its profits. Note that $i^g_{t+1}, i^l_{t+1}$ and $i^d_{t+1}$ are all part of the information set of the household at time $t$.

The bank optimality conditions imply that we must have

$$\begin{aligned} i^l_{t+1} &= i^g_{t+1} + \phi^n, \qquad (12) \\ i^d_{t+1} &= (1-\theta)\, i^g_{t+1}. \qquad (13) \end{aligned}$$

These conditions are intuitive. Loans to firms and loans to the government are perfect substitutes from the perspective of commercial banks up to the constant extra marginal cost $\phi^n$ of monitoring loans to private firms. Hence, equation (12) says that the interest rate charged by banks on private loans should equal the rate on loans to the government plus $\phi^n$. For every unit of deposits held the representative bank has to pay $i^d$ as interest. The bank can earn $i^g$ by lending out the deposit. However, it has to retain a fraction $\theta$ of deposits as required reserves. Hence, equation (13) shows that at an optimum the deposit rate must equal the interest on government bonds net of the resource cost of holding required reserves. We should note that the parameter $\phi^n$ plays no role in the theoretical results that we derive below. Hence, in our main propositions we set $\phi^n = 0$. This parameter is useful in the quantitative sections later where it allows us to calibrate some key interest rate spreads.

It is instructive to note that as the marginal banking costs becomes larger the bank will choose to lower its holdings of foreign assets. This can be checked from the bank first order conditions; all of them imply that $\lim_{\gamma\to\infty} d^b_{t+1} = 0$. Hence, in the limit as banking costs becomes prohibitively large, the bank will choose to economize by shifting to a closed banking sector with no external assets or liabilities.

### 3.4 Government

The government issues high powered money, $M$, and domestic bonds, $Z$, makes lump-sum transfers, $\tau$, to the public, and sets the reserve requirement ratio, $\theta$, on deposits. Domestic bonds are interest bearing and pay $i^g$ per unit. Since we are focusing on flexible exchange rates, we assume with no loss of generality that the central bank's holdings of international reserves are zero. We assume that the government's transfers to the private sector are fixed exogenously at $\bar{\tau}$ for all $t$. Hence, the consolidated government's nominal flow constraint is

$$P_t\bar{\tau} + (1 + i^g_t)\, Z_{t-1} = M_t - M_{t-1} + Z_t.$$

As indicated by the left-hand-side of this expression, total expenditures consist of lump-sum transfers, debt redemption and debt service. These expenditures may be financed by issuing either high powered money or bonds. In real terms the government's flow constraint reduces to

$$\bar{\tau} + \frac{1+i_t^g}{1+\pi_t} z_{t-1} = m_t + z_t - \frac{1}{1+\pi_t} m_{t-1}. \tag{14}$$

Lastly, the rate of growth of the nominal money supply is given by:

$$\frac{M_t}{M_{t-1}} = 1 + \mu_t, \qquad M_0 \text{ given.} \tag{15}$$

It is worth noting that from the central bank's balance sheet the money base in the economy is given by

$$M_t = H_t + \theta D_t.$$

Hence, $M$ can also be interpreted as the level of nominal domestic credit in the economy.

The consolidated government (both the fiscal and monetary authorities) has three policy instruments: (a) monetary policy which entails setting the rate of growth of nominal money supply; (b) interest rate policy which involves setting $i^g$ (or alternatively, setting the composition of $m$ and $z$ and letting $i^g$ be market determined); and (c) the level of lump sum transfers to the private sector $\tau$. Given that lump-sum transfers are exogenously-given, only one of the other two instruments can be chosen freely while the second gets determined through the government's flow constraint (equation (14)). Since the focus of this paper is on the effects of interest rate policy, we shall assume throughout that $i^g$ is an actively chosen policy instrument. This implies that the rate of money growth $\mu$ adjusts endogenously so that equation (14) is satisfied.

### 3.5 Resource constraint

By combining the flow constraints for the consumer, the firm, the bank, and the government (equations (5), (8), (11) and (14)) and using equations (6) and (7), we get the economy's flow resource constraint:

$$a_{t+1} = Ra_t + y_t - c_t - I_t - \kappa_t - s_t - q_t, \tag{16}$$

where $a = b + b^f - d^b$. Note that the right hand side of equation (16) is simply the current account.

### 3.6 Equilibrium relations

We start by defining an equilibrium for this model economy. The three exogenous variables in the economy are the productivity process $A$ and the two policy variables $\bar{\tau}$ and $i^g$. We denote the entire state history of the economy till date $t$ by $s^t = (s_0, s_1, s_2, ..., s_t)$. An equilibrium for this economy is defined as:

*Given a sequence of realizations* $A(s^t)$, $i^g(s^t)$, $r$ *and* $\bar{\tau}$, *an equilibrium is a sequence of state contingent allocations* $\{c(s^t), x(s^t), l(s^t), h(s^t), d(s^t), k(s^t), b(s^t), b^f(s^t), d^b(s^t), n(s^t), z(s^t)\}$ *and prices* $\{P(s^t), \pi(s^t), i(s^t), i^d(s^t), i^l(s^t), w(s^t), \rho(s^t)\}$ *such that (a) at the prices the allocations solve the problems faced by households, firms and banks; (b) factor markets clear; and (c) the government budget constraint (equation (14)) is satisfied.*

Combining the government flow constraint with the central and commercial bank balance sheets yields the combined government flow constraint:

$$\bar{\tau} = h_t - \left(\frac{1}{1+\pi_t}\right) h_{t-1} + \theta\left(d_t - \frac{d_{t-1}}{1+\pi_t}\right) + z_t - \left(\frac{1+i_t^g}{1+\pi_t}\right) z_{t-1}. \tag{17}$$

Lastly, for future reference, the nominal interest rate in this economy is given by the standard no arbitrage condition between a one-period nominal bond bought at time $t$ which pays $i_{t+1}$ as interest in domestic currency at $t+1$ and an international real bond which pays $r$ as interest in terms of the good:

$$1 + i_{t+1} = R\mathbb{E}_t\left(1 + \pi_{t+1}\right). \tag{18}$$

## 4 Exchange rates and interest rate policy: Analytical results

In order to build intuition about the workings of this model, in this section we specialize the model to derive some analytical results. In particular, we shall focus on stationary environments in which both the policy-controlled interest rate, $i^g$, and government transfers, $\bar{\tau}$, are constant for all $t$. We eliminate capital by setting $\alpha = 0$ and $A_t = 1$ for all $t$. This also reduces the model to perfect foresight since there is now no source of uncertainty. Lastly, we set the loan administering cost to zero so that $\phi^n = 0$. This implies that $i^g = i^l$ at all times.

### 4.1 Perfect foresight stationary equilibrium

We first derive the perfect foresight equilibrium path. Under perfect foresight, the first order conditions for optimal cash and demand deposits imply that $h_t = \tilde{h}\left(\frac{i_{t+1}}{1+i_{t+1}}\right)$ and $d_t = \tilde{d}\left(\frac{i_{t+1}-i_{t+1}^d}{1+i_{t+1}}\right)$.

Moreover, we know that $i_t^d = (1-\theta)\, i_t^g$ and $i_t^l = i_t^g$. Moreover, the first order condition for bank loans to the government implies an equilibrium relationship between deposits and bank loans to the government:

$$z_t = (1-\theta)\, d_t - n_t + \left( \frac{i_{t+1} - r}{\gamma\,(1+i_{t+1})} \right). \tag{19}$$

where we have used equation (18) to substitute out for $R\,(1+\pi_{t+1})$.

Using the relationships above it is easy to see the government flow constraint (see equation (17)) becomes a first-order difference equation in $i$. In the following we shall use $\eta_h \equiv -\frac{\tilde{h}'}{h}\frac{i}{1+i}$ to denote the absolute value of the interest elasticity of cash, $\eta_d \equiv -\frac{\tilde{d}'}{d}\frac{i-i^d}{1+i}$ for (the absolute value of) the opportunity-cost elasticity of demand deposits, and $\eta_n \equiv -\frac{\tilde{n}'}{n}\frac{i^l-i}{1+i}$ to denote the corresponding elasticity of loans to firms. In order to economize on notation we shall also use $I^d = i - i^d$ and $I^g = i^g - i$ to denote the interest spreads on deposits and loans respectively.

It is easy to check that, in a local neighborhood of the steady state, $\left.\frac{di_{t+1}}{di_t}\right|_{i_t=i_{t+1}} > 1$, i.e., the government flow constraint is an unstable difference equation in $i$, if and only if

$$\begin{aligned} & Rh\left[1 - \left(\frac{i-r}{i}\right)\frac{\eta_h}{R}\right] + \left(1+i^d\right) Rd\left[1 - \left\{1 - \left(\frac{r\,(1+i^d)}{I^d}\right)\right\}\frac{\eta_d}{R}\right] \\ > \quad & (1+i^g)\, Rn\left[1 - \left\{1 + \left(\frac{r\,(1+i^g)}{I^g}\right)\right\}\frac{\eta_n}{R}\right] + \frac{(r-i^g)\, I^g}{\gamma\,(1+i)}\left(1 + \frac{(1+2r)\,(1+i^g)}{I^g}\right). \end{aligned} \tag{20}$$

To ensure a unique convergent perfect foresight equilibrium path we shall restrict attention to parameter ranges for which the stability condition holds.[14] As long as this condition is satisfied, along any perfect foresight equilibrium path with constant $\bar{\tau}$ and $i^g$, $i$ will also be constant over time.

A constant $i$ and $i^g$ imply that $\pi$, $i - i^d$ and $i^g - i$ must all be constant over time. In conjunction with the first order conditions for households, firms and banks, these results imply that consumption, $c$, output, $x$, cash demand, $h$, deposit demand, $d$, and demand for bonds, $z$, must all remain constant as well. The constancy of both $h$ and $d$ implies that money demand is constant over time. Lastly, the stationary level of government transfers is given by

$$\bar{\tau} = \left(\frac{i-r}{1+i}\right)(h + \theta d) - \left(\frac{I^g + r\,(1+i^g)}{1+i}\right) z. \tag{21}$$

Before proceeding further it is useful to note that the left hand side of equation (20) reflects the well-known possibility of a Laffer curve relationship between revenues from money printing

[14] Note that if instead $\left.\frac{di_{t+1}}{di_t}\right|_{i_t=i_{t+1}} < 1$ then the economy would exhibit equilibrium indeterminacy. There would be an infinite number of equilibrium paths all converging to the unique steady state.

and the opportunity cost of holding money. As is standard, and to ensure that the economy is always operating on the "correct" side of the Laffer curve, we will assume throughout that $h\left[1-\left(\frac{i-r}{i}\right)\frac{\eta_h}{R}\right]+\left(1+i^d\right)d\left[1-\left\{1-\left(\frac{r\left(1+i^d\right)}{I^d}\right)\right\}\frac{\eta_d}{R}\right]>0$.

In the rest of this section we shall make one additional assumption which will simplify the analytics greatly. In particular, we shall assume that $\gamma=\infty$. This corresponds to assuming that the cost of managing a non-zero net foreign asset position is prohibitively high for domestic banks. The implication of this assumption is that commercial banks will set $d_t^b=0$ for all $t$. Hence, the commercial bank balance sheet will reduce to $z_t+n_t=(1-\theta)\,d_t$ for all $t$.

## 4.2 Two special cases

We now turn to the central issue of the paper – the effects of interest rate policy on the nominal exchange rate. We want to ask the following questions: how does the nominal exchange rate change when the policy-controlled interest rate, $i^g$, changes? What is the relationship between the market interest rate, $i$, and the exchange rate? Our goal is to demonstrate that in the model just described, there is an inherent tendency for the relationship between interest rates (both $i^g$ and $i$) and the exchange rate to be non-monotonic. Since uncovered interest parity dictates that $1+i_t=R\,(1+\pi_t)$, for the *rate* of currency depreciation to be non-monotonic in $i^g$ we need $i$ to be a non-monotonic function of $i^g$. For the *level* of the nominal exchange rate to have a similar non-monotonicity, $m\,(=h+\theta d)$ must be a non-monotonic function of $i^g$. With nominal money supply at time 0 given, a rise in $m$ will imply that the price level must decrease, i.e., the nominal exchange rate appreciates. Similarly, a fall in $m$ will be associated with a nominal depreciation.

We also want to determine the minimal elements that are needed to generate such non-monotonicities. To this effect, we will show that (i) in the presence of only one money (interest-bearing deposits), both the fiscal and the output effects are needed to generate a non-monotonic relationship between interest rates and the nominal exchange rate, and (ii) in the presence of two monies (cash and deposits), the fiscal effect is all that is needed to generate a non-monotonic relationship. As will be discussed below, these two cases illustrate the general principle that two sources of fiscal revenues are needed to generate a non-monotonic relationship between interest rates and exchange rates. In (i), the two sources of revenues are demand deposits and, indirectly, bank lending to firms (through its effect on banking lending to the government), whereas in (ii) the two sources of revenues are the two monies.

### 4.2.1 Case 1: The one-money case

In this one-money case, higher interest rates have both an output effect and a fiscal effect and both effects are key in generating the non-monotonic relationship between interest rates and exchange rates. Formally, we set $v(h) \equiv 0$ for all $h$. Hence, the demand for cash is zero at all times (i.e., $\tilde{h}\left(\frac{i}{1+i}\right) \equiv 0$). This implies that the entire money base is held by the banking sector. In particular, $m = \theta d$. Hence, after setting $h = 0$ and $\gamma = \infty$, the stability condition (20) remains valid.[15]

The relationship between the policy-controlled interest rate, $i^g$, and the nominal exchange rate, $E$, has three potential non-monotonicities which we describe through the following proposition:

**Proposition 1** *A permanent, unanticipated change in the policy-controlled interest rate, $i^g$, has a non-monotonic effect on the equilibrium nominal interest rate along three dimensions: (i) the initial level of the nominal exchange rate is falling or rising with $i^g$ as $i^g \lesseqgtr \hat{\imath}^g$; (ii) the steady-state depreciation rate falls or rises with $i^g$ as $i^g \lesseqgtr \bar{\imath}^g$ where $\bar{\imath}^g < \hat{\imath}^g$; and (iii) in the range $i^g \in (\bar{\imath}^g, \hat{\imath}^g)$, a rise in $i^g$ appreciates the currency on impact but depreciates it at some point in the future.*

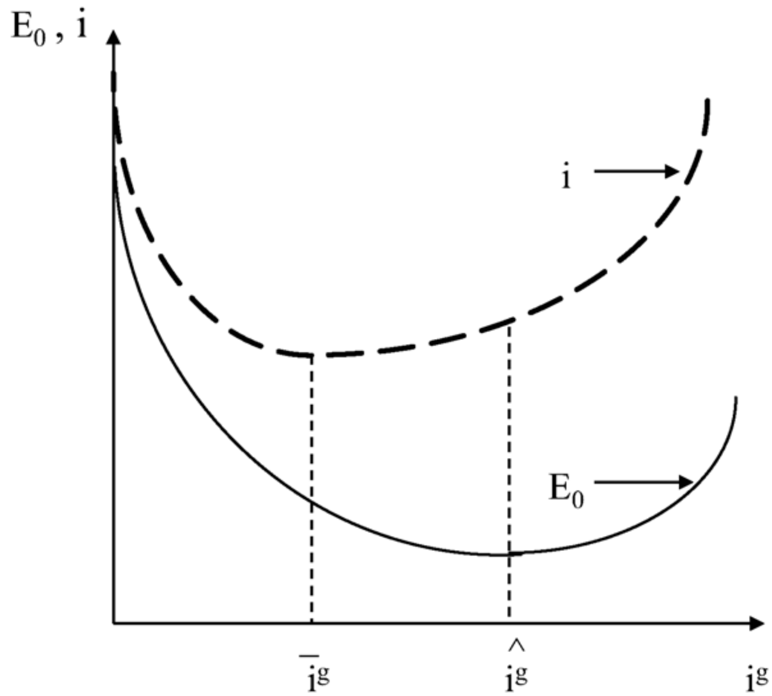
**Proof.** See appendix. ■

Figure 2 illustrates part (i) of this proposition. Specifically, the initial level of the exchange rate, $E_0$, is a U-shaped function of the policy-controlled interest rate, $i^g$, with the minimum being reached at $i^g = \hat{\imath}^g$. The intuition is as follows. Recall that the opportunity cost of demand deposits is $\frac{I^d}{1+i} \equiv \frac{i-i^d}{1+i}$. A rise in $i^g$, in and of itself, increases the deposit rate, $i^d$ – recall (13) – and therefore tends to reduce $\frac{I^d}{1+i}$ and appreciate the currency. A rising $i^g$, however, also raises $I^g$ which induces a fall in bank credit to firms, $n$. This effect tends to reduce fiscal revenues because the counterpart of a falling $n$ is an increase in $z$ (i.e., an increase in liabilities of the central bank held by commercial banks), which increases the government's debt service. In order to finance this fall in revenues, the inflation rate (i.e., the rate of depreciation) must increase. This effect tends to increase $i$ and hence $I^d$. For all $i^g > \hat{\imath}^g$, the higher debt service overwhelms the increase in the deposit rate, and further increases in $i^g$ actually raise $I^d$.

Figure 2 also illustrates part (ii) of this proposition, by showing that the market interest rate, $i$, is also a U-shaped function of $i^g$. For $i^g < \bar{\imath}^g$, the direct effect on revenues of an increase in $i^g$ (due to a higher demand for real demand deposits) is so large that it facilitates a cut in the inflation tax. However, for $i^g > \bar{\imath}^g$ the indirect effect of a fall in $n$ becomes large enough to require

[15] Also notice that, in this particular case, the condition ensuring that the economy is operating on the correct side of the Laffer curve reduces to $1 - \left\{1 - \left(\frac{r\left(1+i^d\right)}{I^d}\right)\right\} \frac{\eta_d}{R} > 0$.

Figure 2. Initial exchange rate and the nominal interest rate




an increase in $\pi$ (or equivalently, the rate of money growth $\mu$) in order to finance fiscal spending.

Aside from the non-monotonicity of both the initial level of the exchange rate and the steady-state depreciation rate, Proposition 1 also shows that an increase in the policy-controlled interest rate often induces an intertemporal trade-off in the path of the nominal exchange rate. In particular, the instantaneous appreciation of the currency that is generated by a higher $i^g$ comes at the cost of a more depreciated level of the nominal exchange rate at some time in the future (relative to the path with a lower stationary $i^g$). This occurs for values of $i^g \in (\bar{\imath}^g, \hat{\imath}^g)$ because in that range, as Figure 2 illustrates, a rise in $i^g$ reduces $E_0$ but increases the rate of depreciation.

Finally, the results of Proposition 1 also imply that the relationship between the nominal exchange rate and the market interest rate is highly non-monotonic. This can be visualized using Figure 2. For "low" values of $i^g$ (i.e., $i^g < \bar{\imath}^g$), both the initial exchange rate ($E_0$) and the market interest rate ($i$) fall (positive comovement). For "high" values of $i^g$ (i.e., $i^g > \hat{\imath}^g$), both increase (also positive comovement). In contrast, for "intermediate" values of $i^g$, (i.e., $\bar{\imath}^g < i^g < \hat{\imath}^g$), $E_0$ falls while $i$ increases, indicating a negative comovement.

#### 4.2.2 Case 2: The two-money case

We now turn to our second special case of the general model. Here, we reintroduce a transactions role for cash so that $s_t = v(h_t) + \psi(d_t)$. Thus, this economy has two liquid assets – cash and demand deposits. However, we now assume that $\phi = 0$ so that loan demand by firms is zero (i.e., $\tilde{n}(I^g) \equiv 0$). Hence, there is no output effect of higher interest rates.

After setting $\tilde{n}(I^g) = 0$, the equilibrium transfer equation (21) continues to be valid. We restrict parameter ranges so that the stability condition continues to hold in this case. Hence, the rate of devaluation remains constant along a convergent perfect foresight equilibrium path.[16]

The interest elasticity of cash $\eta_h$ is a function of the nominal interest rate $i$, which is an implicit function of the policy controlled interest rate $i^g$ through the equilibrium transfer equation (21). Define $\bar{\imath}^g$ by the relation $\eta_h\left(\frac{\tilde{\imath}(\bar{\imath}^g,\bar{\tau})}{1+\tilde{\imath}(\bar{\imath}^g,\bar{\tau})}\right) = \frac{R\tilde{\imath}(\bar{\imath}^g,\bar{\tau})}{\tilde{\imath}(\bar{\imath}^g,\bar{\tau})-r}$. We will now assume that the demands for cash and deposits satisfy the following conditions:

**Condition 1:** $\eta_h\left(\frac{\tilde{\imath}(0,\bar{\tau})}{1+\tilde{\imath}(0,\bar{\tau})}\right) < \theta\eta_d\left(\frac{\tilde{\imath}(0,\bar{\tau})}{1+\tilde{\imath}(0,\bar{\tau})}\right)$.

**Condition 2:** $\eta_h$ and $\eta_d$ are both increasing in their respective arguments.

Condition 1 requires that for "low" inflation rates (i.e., inflation rates corresponding to a non-activist interest rate policy), the interest elasticity of cash be lower than that of deposits (adjusted by the reserve requirement ratio). The idea is that cash is kept mainly for transactions and is therefore relatively interest inelastic for low inflation rates. This intuition is consistent with the evidence for the United States provided in Moore, Porter, and Small (1990).[17]

We can now state the main proposition of this section:

**Proposition 2** *Under Conditions 1 and 2, the initial nominal exchange rate is a non-monotonic (U-shaped) function of the policy-controlled interest rate, $i^g$. In particular there exists an $\hat{\imath}^g \in (0, \bar{\imath}^g)$ such that $\frac{\partial E_0}{\partial i^g} \lesseqgtr 0$ as $i^g \lesseqgtr \hat{\imath}^g$.*

**Proof.** See appendix. ■

This proposition shows that, as in the previous case, the initial level of the exchange rate is a U-shaped function of the policy-controlled interest rate. Intuitively, for low values of $i^g$, the positive money demand effect dominates the fiscal effect (i.e., the inflationary consequences of a higher $i^g$). Beyond a certain point, however, further increases in $i^g$ have such a large impact on the rate of inflation that money demand begins to fall and hence the currency depreciates. The role

[16] To be consistent with the one-money case, we will also maintain the assumption that $\left(1 - \frac{I^d - r}{I^d}\eta_d\right) d > 0$.

[17] Condition 2 is satisfied by, among others, Cagan money demands, which provide the best fit for developing countries (see Easterly, Mauro, Schmidt-Hebbel (1995)).

of *Condition* 1 is to ensure that, around $i^g = 0$, the demand for cash falls by less than the amount by which demand for bank deposits rises, so that overall real money demand increases.[18] Lastly, Proposition 2 also implies that, for $i^g < \bar{\imath}^g$, a rise in $i^g$ appreciates the currency on impact but increases the depreciation rate. Hence, there is again an intertemporal trade-off in the sense that a higher $i^g$ buys a more appreciated currency in the short-run at the expense of a more depreciated currency in the future.

# 5 Calibration

Policy experiments performed in the next section intend to demonstrate the central result of the paper: the relationship between interest rates and exchange rates may be non-monotonic. Towards this end we calibrate a discrete time version of the model developed above and assess its quantitative relevance for understanding the relationship between interest rates and the exchange rate. In this section we calibrate the parameters of the model as well as the processes for productivity, interest rate, and fiscal policy shocks. For our benchmark calibration we use the data for Argentina during 1983-2002. The model calibration is such that one period in the model corresponds to one quarter.

## 5.1 Functional forms and parameters

We assume that the capital adjustment cost technology is given by

$$\kappa(I_t, k_{t-1}) = \frac{\xi}{2} k_{t-1} \left( \frac{I_t - \delta k_{t-1}}{k_{t-1}} \right)^2, \qquad \xi > 0,$$

with $\xi$ being the level parameter.

Following Rebelo and Vegh (1995), we assume that the transactions costs functions $v(.)$ and $\psi(.)$ have quadratic forms given by

$$s_{\varkappa} \left( \varkappa^2 - \varkappa + \frac{1}{4} \right),$$

where $\varkappa$ represents cash or demand deposits, $\varkappa = \{h, d\}$ and $s_{\varkappa}$ are the level parameters. This formulation implies that the demand for money components are finite and that transaction costs are zero when the nominal interest rate is zero.[19]

The transaction technology for the banks is given by a quadratic function

$$q_t = \frac{\gamma}{2} \left( d^b_{t+1} \right)^2,$$

[18] If condition 1 is not satisfied, then an increase in $i^g$ would always lead to a *depreciation* of the currency.

[19] Notice as well that condition 2 is met.

where $d^b_{t+1} = \frac{N_t + Z_t - (1-\theta) D_t}{P_t}$.

Most of our parameter values are borrowed from Neumeyer and Perri (2005). In particular, we set the coefficient of relative risk aversion, $\sigma$, to 5, while the curvature of the labor, $\upsilon$, is set to 1.6, which is within the range of values used in the literature.[20] This implies the elasticity of labor demand with respect to real wage, $\frac{1}{\upsilon - 1}$, equal to 1.67, consistent with the estimates for the U.S. Labor weight parameter $\zeta$ in the utility function is chosen to match the average working time of 1/5 of total time and is set to 2.48. Subjective discount factor, $\beta$, is set to 0.97, as in Uribe and Yue (2006).

Capital income share, $\alpha$, is chosen to be equal to 0.38, while a depreciation rate for capital, $\delta$, of 4.4% per quarter. We calibrate the share of wage bill paid in advance, $\phi$, to be equal to 0.26, which is chosen to match the ratio of domestic private business sector credit to GDP in Argentina over our sample period. The number for private credit to GDP in Argentina is borrowed from the Financial Structure Dataset assembled by Beck, Demirgüç-Kunt and Levine (2000) and is equal to 0.21. The share of business sector credit in the total private sector credit is calculated using activity based financing reports from the Central Bank of Argentina, and is equal to 78% over our sample period. Capital adjustment costs parameter $\xi$ is calibrated to replicate the volatility of investment relative to the volatility of output in Argentina. Parameter $\theta$ determines the reserve requirement ratio in the model and is set to match its value of 0.4 in Argentina.

The level coefficients in the transactions costs technology, $s_h$ and $s_d$, are set to 10 and 5, respectively, in order to match seignorage revenues in Argentina equal to 7% of GDP. This number is taken from Kiguel (1989) and was used by Rebelo and Vegh (1995) to calibrate the transaction costs technology. We follow their strategy in our calibration exercise. Parameters $s_h$ and $s_d$ also allow us to pin down the initial values of money demand elasticities, $\eta_h$ and $\eta_d$, such that *Condition* 1 in Section 4.2.2 is satisfied. The proportional cost parameter $\phi^n$ in the banking sector' problem is chosen to match the average spread of nominal lending rate over money market rate equal 10% in Argentina over our sample period. Lastly, we pick $\gamma$ to match the economy-wide steady state bond holdings. Table 1 summarizes parameter values under our benchmark parametrization.[21]

[20] For example, Mendoza (1991) uses $\upsilon$ equal to 1.455 for Canada, while Correia, et.al (1995) set $\upsilon$ to 1.7 for Portugal.

[21] For all the experiments reported below we checked to ensure that the implied inflation tax revenues are on the upward sloping portion of the Laffer-curve.

| Table 1. Benchmark parameter values | | |
|---|---|---|
| PREFERENCES | | |
| discount factor | $\beta$ | 0.971 |
| risk-aversion | $\sigma$ | 5 |
| labor curvature | $\upsilon$ | 1.6 |
| labor weight | $\zeta$ | 2.48 |
| TECHNOLOGY | | |
| capital income share | $\alpha$ | 0.38 |
| depreciation rate | $\delta$ | 0.044 |
| share of wage-in-advance | $\phi$ | 0.26 |
| capital adjustment costs | $\xi$ | 4.5 |
| MONEY | | |
| reserve requirement | $\theta$ | 0.4 |
| transaction cost technology | $\sum_{\varkappa=h,d} s_{\varkappa}(\varkappa^2 - \varkappa + 1/4)$ | $s_h = 10$, $s_d = 5$ |
| banks cost technology | $\gamma$ | 1 |
| per unit loans costs | $\phi^n$ | 0.1 |

## 5.2 Calibration of the shock processes

There are two sources of uncertainty in our benchmark model: exogenous productivity realizations, $A$, and the policy-controlled interest rate realizations, $i^g$. We now describe how we calibrate the total factor productivity (TFP) and the process for interest rates. We will use a "hat" over a variable to denote the deviation of that variable from its balanced growth path.

Following Neumeyer and Perri (2005) we assume that productivity, $\hat{A}_t$, in Argentina is an independent AR(1) process with autoregressive coefficient, $\rho_{\text{A}}$, equal to 0.95. The innovations, $\varepsilon^{\text{A}}$, to this process are assumed to be independent and identically normally distributed with the standard deviation, $\sigma(\varepsilon^{\text{A}})$, equal to 0.0195.[22]

To calibrate the process for the policy-controlled interest rate $i^g$, we use data on the money market rate in Argentina during 1992:2-2002:3. During this period the average level of $i^g$ was 13%. We obtain $\hat{i}^g$ as a differential of Argentinean money market rate from the U.S. Federal Funds rate,

[22]This process is commonly used to describe total factor productivity in the U.S. In the absence of quarterly data on Argentinean employment, we rely on it to calibrate the dynamics of $\hat{A}_t$, as in Neumeyer and Perri (2005).

and then estimate the first-order autoregressive process for $\hat{\imath}^g$ as

$$\hat{\imath}_t^g = \rho_g \hat{\imath}_{t-1}^g + \varepsilon_t^g,$$

where $\varepsilon_t^g$ are i.i.d. normal innovations. The ordinary least squares (OLS) estimation of this equation gives $\rho_g = 0.97$, and $\sigma(\varepsilon^g) = 0.0353$.[23]

To simplify the analytics of the model, up to now we have assumed that the lump-sum transfers paid by the government to the private sector, $\tau$, are fixed at $\bar{\tau}$. We measure $\bar{\tau}$ as the average (seasonally-adjusted) ratio of government consumption to GDP over our sample period, which gives $\bar{\tau} = 13\%$.[24]

Once the shock processes and other parameter values are set, we solve the model using the perturbation method (Judd (1998), Schmitt-Grohe and Uribe (2004)). In particular, we take the second-order approximations of the model equilibrium conditions around the non-stochastic steady state, and then solve the resulting system of equations following the procedure described in Schmitt-Grohe and Uribe (2004). [25]

# 6 Results

We analyze the equilibrium properties of the model by conducting a series of three experiments. First, we perform the steady state comparisons of our model for different levels of policy-controlled interest rate. Second, we discuss how the variables in the model respond to temporary interest rate shocks of different magnitudes; and third, we compare the model responses to same sized interest rate shocks, but for different steady state levels of the interest rate.

[23]Measuring $\hat{\imath}^g$ as a deviation of the money market rate from world interest rate proxied by U.S. Federal Funds rate is consistent with the VAR specification reported in Section 2. Moreover, it also provides a detrend of the interest rate data.

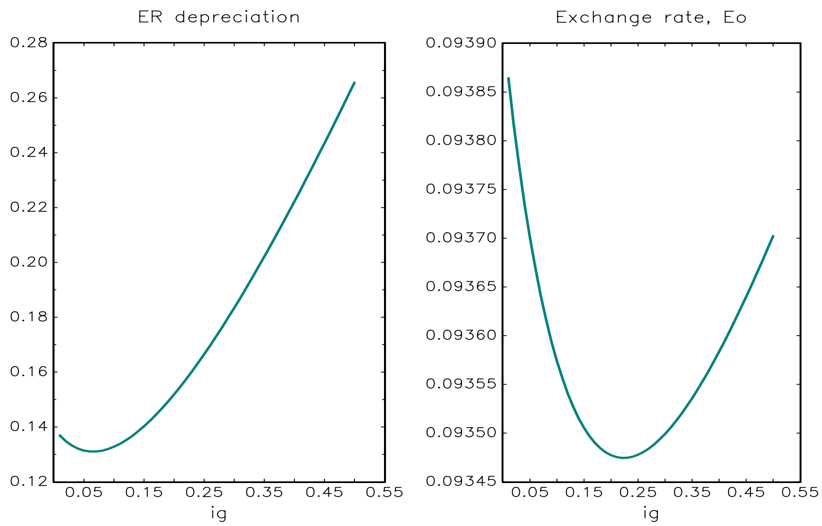
[24]As part of a sensitivity analysis we relaxed the assumption of a fixed $\tau$ and evaluated the model's implications when the process for fiscal spending is stochastic. We found that the exchange rate response to fiscal shocks are monotonic both in levels and rates.

[25]In our economy, international bonds may follow a unit root process. To account for this potential non-stationarity, we impose a small quadratic bond holding cost, $\Phi(a_t) = \frac{\vartheta}{2} y_t \left(\frac{a_t}{y_t} - \bar{a}\right)^2$, where $\bar{a}$ denotes the steady state ratio of bond holdings to GDP, and $\vartheta$ is a level parameter. This does not alter the model dynamics substantially, and therefore when discussing the results, we focus on the case with no bond holding costs.

### 6.1 Steady state comparative statics

Figures 3 and 4 present the results of our first experiment. Figure 3 shows the responses of the steady state level of exchange rate and the rate of currency depreciation to changes in the steady state level of the policy-controlled interest rate, $i^g$. Both variables exhibit a non-monotonic relationship with $i^g$. Small increases in $i^g$ appreciate the currency and reduce the rate of currency depreciation; more aggressive increases in $i^g$ depreciate the currency and increase the rate of currency depreciation.

Figure 3. Steady state comparisons: Exchange rate level and rate of change

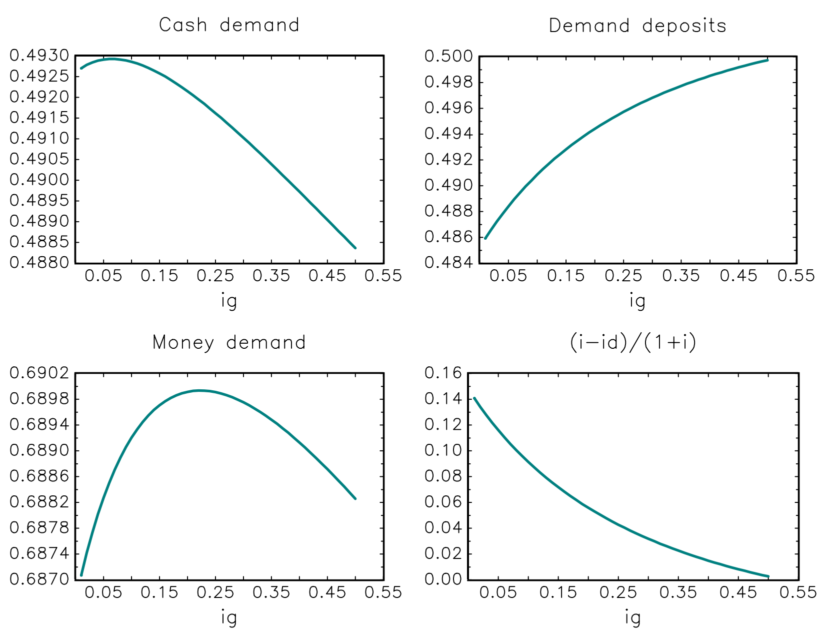



The intuition for this result can be gained by using the analytical insights derived in section 4. In particular, consider Figure 4. It summarizes the steady state responses of the money demand, its components, and opportunity costs to changes in the steady state level of policy-controlled interest rate, $i^g$. An increase in $i^g$ has three effects in our model. First, higher $i^g$ creates an inflationary pressure on the government by increasing the interest burden on the outstanding government debt. This "fiscal" effect raises the required seignorage revenues and tends to increase the rate of currency depreciation and thus the nominal interest rate, $i$.

Second is the "output" effect, which arises due to the working capital constraint in this economy. When $i^g$ rises, the cost of borrowing faced by the firm goes up, leading to lower employment and output. With lower outstanding loans, banks increase their holdings of government bonds to balance their balance sheets. This increases the government's fiscal burden and raises the required seignorage revenue to finance the budget. Thus, both the "fiscal" and "output" effects will push the market interest rate, $i$, up.

Our third effect is the "money demand" effect. It captures the response of the money base, $m(= h + \theta d)$, to changes in domestic interest rates. With higher $i$, the opportunity cost of holding cash rises, thus lowering the demand for cash by households. At the same time, an increase in $i^g$ is accompanied by an increase in the interest paid on demand deposits, $i^d$. Lower opportunity costs of holding deposits lead to a higher demand for deposits, which, as can be seen from Figure 4 is monotonically rising in $i^g$. The response of money demand, and thus the direction of the "money demand" effect, depends on which component dominates. For low steady state levels of $i^g$ the increase in demand deposits is large enough to swamp the fall in cash demand, and leads to an appreciation in the level of the currency. It is also strong enough to counterweight the "fiscal" and "output" effects, thus reducing the rate of depreciation of the exchange rate. For high levels of $i^g$, deposits do not respond sufficiently to overtake the negative effect on cash demand coming through the fiscal and output effects. As a result, both the level and the rate of change of the exchange rate go up. For intermediate levels of $i^g$ (i.e., $0.08 \leq i^g \leq 0.22$ in our benchmark calibration), there exists an intertemporal trade-off in the path of the nominal exchange rate. In particular, the level of exchange rate continues to appreciate, but comes with a higher rate of currency depreciation.

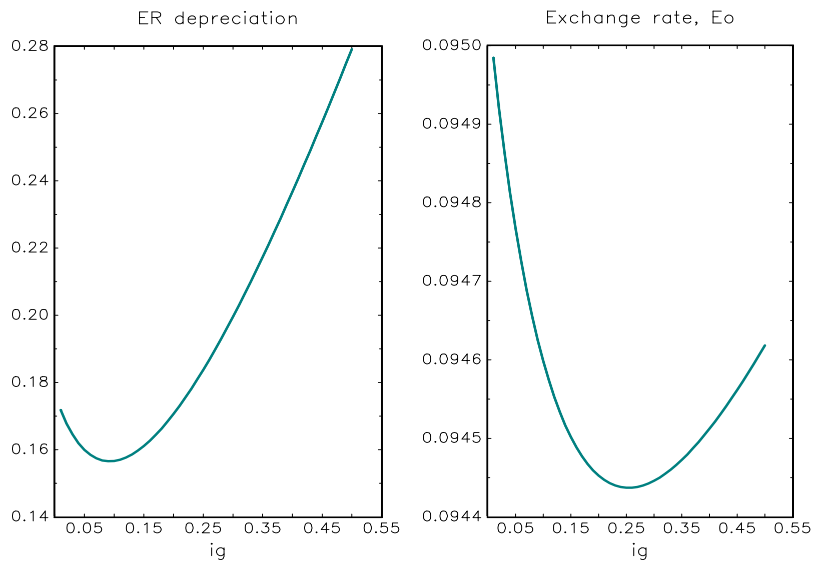
Figure 4. Steady state comparisons: Money demand and opportunity costs




Overall, we get a hump-shaped response of the money base in the economy and, therefore, a U-shaped response in the level of exchange rate. The response of the rate of currency depreciation, and thus the nominal interest rate, is also U-shaped, but reaches its minimum for a lower steady state $i^g$.[26] As was highlighted in section 4, the key property of the model needed to generate such responses is that the elasticity of money demand is rising in the opportunity cost of holding money. Our calibration also ensures that the deposit elasticity for low $i^g$ is greater than the elasticity of cash demand, which is also required for non-monotonicity.

It is informative to evaluate the effect of the level of $\bar{\tau}$ on the exchange rate non-monotonicities. Figure 5 presents the steady state comparisons of the exchange rate and its rate of change for $\bar{\tau} = 15\%$. The latter change increases the fiscal burden. As a result, the U-shape in the response of the rate of exchange rate depreciation becomes more pronounced; the minimum steady state level in the responses of both the exchange rate level and the rate shifts to the right.

Figure 5. Steady state comparisons: Higher level of $\tau = 15\%$



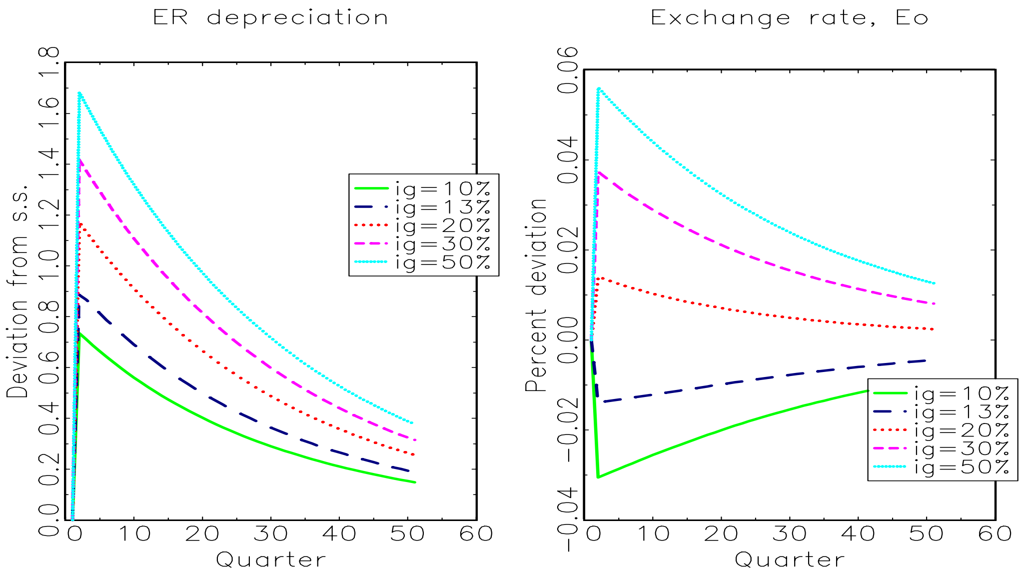

## 6.2 Temporary shocks around different steady state $i^g$'s

Our remaining two experiments study the dynamics of the model around its non-stochastic steady state. First, we look at the impulse responses of the exchange rate and the rate of currency depreciation to a positive one standard deviation innovation to the policy-controlled interest rate $i^g$. We conduct this experiment for different steady state levels of $i^g$. The results of this exercise are presented in Figure 6.

For the benchmark parameterization with low steady state interest rates, a temporary increase

[26] Since demand for cash is a monotonically decreasing function of $i$, and under our benchmark parametrization, $i$ is a U-shaped function of $i^g$, cash demand is a hump-shaped function of $i^g$ as Figure 4 shows.

Figure 6. Transitions: Different steady state $i^g$



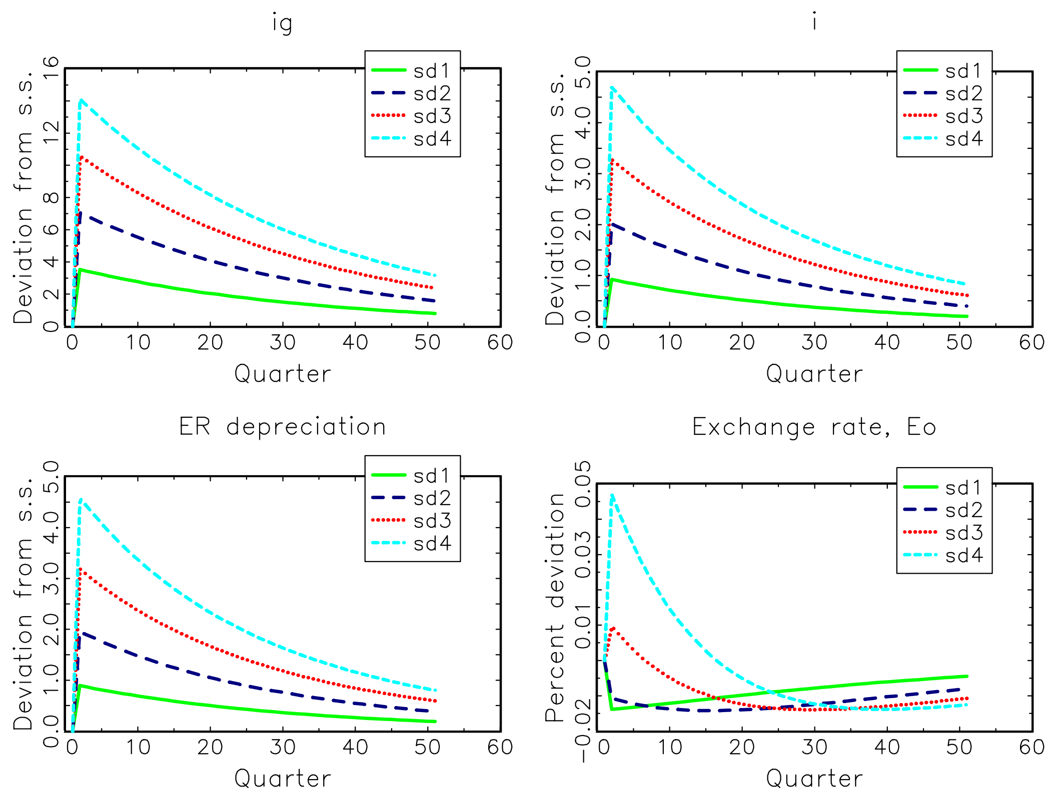

in $i^g$ tends to appreciate the currency. Under high steady state interest rates, on the other hand, a temporary increase in $i^g$ has the reverse effect: both the level and the rate of currency depreciation increase. This comovement between the level and the rate of the exchange rate breaks down for intermediate levels of the steady state interest rate. Here a positive one standard deviation shock to $i^g$ will generate an appreciation in the nominal exchange rate, accompanied by an increase in the rate of depreciation. The market interest rate also rises in this range. These results show that interest rates (both policy-controlled and market determined) and exchange rates can be positively or negatively related. Hence, if one were to conduct a similar exercise for a cross-section of countries with different initial levels of interest rates no systematic relationship between the two variables need necessarily arise.

## 6.3 Different sized shocks to steady state $i^g$

Finally, we turn to our last experiment in which we study the impulse responses of interest rates and exchange rates to innovations in $i^g$ of different sizes. We analyze these responses around an invariant steady state $i^g$. We calibrate the steady state $i^g$ to the average money market rate in Argentina over 1992:2-2002:3, equal to 13%. The results are presented in Figure 7.

It is easy to see that increases in interest rates up to 3 standard deviations cause the domestic currency to appreciate. However, more aggressive rises in $i^g$ would lead to currency depreciation. The rate of currency depreciation and the nominal interest rate always increase in response to

Figure 7. Transitions: Different size of shocks to $i^g$, $i^g_{ss} = 13\%$.




interest rate shocks, independent of the magnitude of the shock. The reason is that for relatively high levels of steady state $i^g$ (13% in our case), the interest burden of government debt is so high that it requires large seigniorage revenues to maintain the government budget. Therefore, the steady state inflation rate and the rate of currency depreciation are also high. Any further increase in $i^g$ raises the interest rate burden, thereby increasing the inflation rate some more. The money demand, on the other hand, continues to exhibit non-monotonicity in response to changes in $i^g$. In particular, at $i^g = 13\%$, the elasticity of demand for deposits is sufficiently high, while the cash elasticity is sufficiently low to generate a net increase in money demand for small innovations to $i^g$. This positive "money demand" effect will appreciate the currency. However, when innovations to $i^g$ become large, the "money demand" effect turns negative. As a result, the domestic currency depreciates. These findings highlight that the non-monotonicity in the relation between interest rates and exchange rates exists on the time-series basis as well.

In summary, we have shown that the relationship between interest rates and exchange rates is non-monotonic along three dimensions: First, the relationship is non-monotonic in the steady state; second, controlling for the steady state, the relationship is non-monotonic in the size of the interest rate changes. Lastly, controlling for the size of the innovations, the relationship is non-monotonic along the cross-sectional dimension.

# 7 Matching country impulse responses

This section describes the final test of our model. Consider again the empirical impulse responses presented at the beginning of the paper. We used those impulse responses to illustrate that it is hard to find any systematic relationship between interest rates and exchange rates in the data. For instance, we showed that for Thailand, a positive innovations to the interest rate differential induced a significant exchange rate appreciation. Korea, on the other hand, exhibited a depreciated exchange rate in response to an interest rate innovation. Finally, Brazil's exchange rate has shown no significant reaction to a change in interest rate differential. The objective of this section is to rationalize these opposite dynamics in the context of our model. For this purpose, we re-calibrate our model for Thailand, Korea and Brazil, and derive their corresponding theoretical impulse responses to interest rate innovations. We conduct this exercise for the two periods over which our chosen economies had flexible exchange rate regime: 1997-2001 which corresponds to a period of active interest rate defence in these economies; and the complete sample period of 1997-2007. We show that our model can qualitatively match the data for both periods.

The choice of the periods were dictated by two motivations. The first sub-period (1997-2001) was characterized by a number of currency crisis episodes in emerging countries. During this period the interest rate instrument was used very actively by these countries as a weapon to fight attacks on domestic currencies. The period from 2001 onward, on the other hand, has been one of relative tranquility. As a result the use of the interest rate instrument has been more active. Thus, during the period 1997-2001 the mean interest rate is about 340 basis points higher and the standard deviation of the interest rates is 40 percent higher relative to the full sample (1997-2007). Second, after 2001 there was a big and prolonged change in the US monetary policy with the Federal Funds rate dropping sharply in response to a developing recession. As a result, during the period 1997-2001 the mean level of the Federal Funds rate was about 170 basis points higher, while the standard deviation was 70 percent lower relative to the entire period of 1997-2007. Given that a key exogenous driver in the model is the spread between the policy controlled domestic interest rate and the exogenous world interest rate, both facts outlined above suggest that the properties of this spread may have changed significantly after 2001. Moreover, our counterfactual policy experiments above showed that the properties of the interest rate process are key components of potential non-monotonicities in the exchange rate response to interest rate innovations. Hence, studying the two periods separately appears to be a natural test of the model.

## 7.1 Active interest rate period

We keep all of the preference parameters and most of the technology and money parameters unchanged. We only recalibrate two key parameters: the share of wage-in-advance, $\phi$, to match the ratio of domestic business credit to GDP; and the reserve requirement ratio, $\theta$, in our three economies.[27] In calibrating all parameters and shock processes we use the time periods considered in the construction of the empirical impulse responses: 1997:q3 - 2001:q1 for Thailand and Korea; and 1998:q4 - 2001:q1 for Brazil. [28]

As before, we measure $i_t^g$ as the money market rate in a given country. During the sample periods the average level of $i^g$ was 7% for Thailand, 9% for Korea, and 23% for Brazil. To calibrate the process for the policy-controlled interest rate, we estimate a first-order autoregressive process on $\hat{\imath}_t^g$, an interest rate differential between the money market rate in a given country and a U.S. Federal Funds rate for a panel of three countries. We restrict this process to be the same across the three countries we consider. This approach is intended to capture the dynamics of $\hat{\imath}_t^g$ in an average emerging market economy. It also allows us to isolate cross-country differences in exchange rate dynamics driven by a limited set of structural country characteristics. We find $\rho^g = 0.92$ and $\sigma(\varepsilon_t^g) = 0.0413$.[29] Finally, we also calculate country-specific values for the lump-sum transfers from the government to the private sector, $\bar{\tau}$. As before, we measure $\bar{\tau}$ as the average ratio of government consumption to GDP over the corresponding sample period. The new parameter

[27] The numbers for private credit to GDP in Thailand, Korea and Brazil are borrowed from the Financial Structure Dataset assembled by Beck, Demirgüç-Kunt and Levine (2000) as in the case of Argentina. The share of business credit in total private credit is calculated using the financial institutions reports from the Bank of Thailand, Korea and the Central Bank of Brazil. These numbers are 66% for Korea, 67% for Thailand, and 78% for Brazil. The reserve requirement ratios for these countries are calculated following Brock (1989). See web appendix for detailed data sources (available at http://www.econ.ubc.ca/vhnatkovska/research.htm). We also raise the value of $\phi^n$ for Thailand to 0.13 from 0.10 in order to ensure that the relevant interest rate spreads remain positive.

[28] We should note that due to the small number of observations in the 1997-2001 subsample we had to use monthly data in our empirical VARs. We continue using monthly data in our empirical estimation on the full 1997-2007 sample in order to retain comparability of results.

[29] We also estimated country-specific processes for $\hat{\imath}^g$, and found them to be along the lines of the aggregate estimates.

values are summarized in Table 2.

Table 2. Parameter values: Thailand, Korea, Brazil for 1997-2001

| | Thailand | Korea | Brazil |
|---|---|---|---|
| share of wage-in-advance, $\phi$ | 1.11 | 0.72 | 0.36 |
| reserve requirement, $\theta$ | 0.03 | 0.03 | 0.27 |
| Average $i^g$ | 7% | 9% | 23% |
| Average $\tau$ | 11% | 12% | 20% |

We solve the model under these new parameterizations and generate the responses of the exchange rate to the interest rate innovations. The resulting impulse responses along with the corresponding data impulse responses are presented in Figure 8.

Figure 8. Exchange rate impulse responses for Thailand, Korea and Brazil 1997-2001

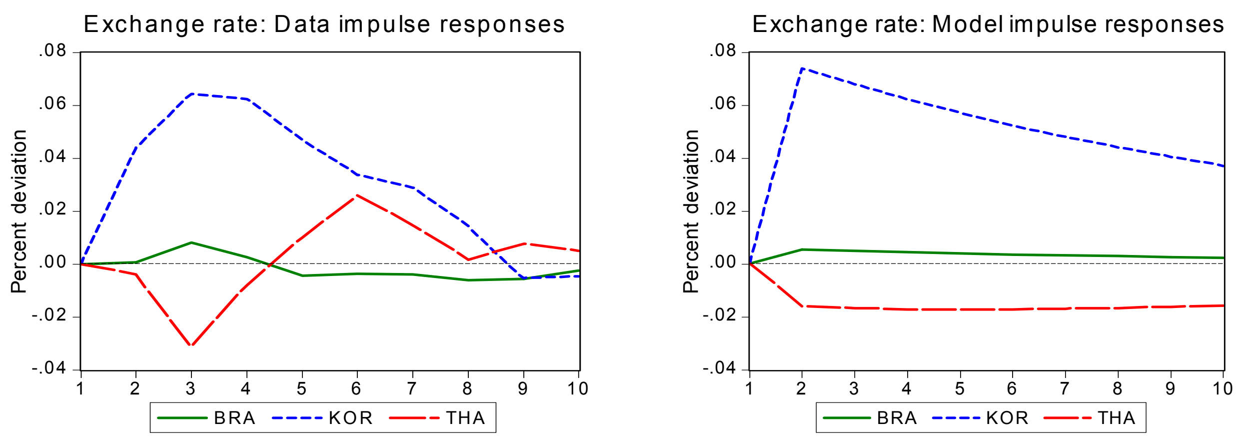



Figure 8 shows that our model delivers impulse responses that qualitatively match those in the data: The exchange rate appreciates in Thailand, depreciates in Korea, and exhibits no significant change in Brazil following a one standard deviation innovation to interest rate. The opposing responses of exchange rates in Korea and Thailand are especially striking given that the two countries are very similar along a number of dimensions: the steady state levels of $i^g$ and $\tau$; the process for $i^g$; as well as most of the structural parameters. The key difference between the two countries in the context of our model is in the level of business loans to GDP, captured by the parameter $\phi$. This parameter determines the strength of the output effect through working capital cost; and affects the intensity of the money demand effect through the steady state levels of interest rates in the model. In Thailand, where the share of private credit is larger, the money demand effect of $\phi$ dominates and the exchange rate appreciates. The opposite is true in Korea leading to depreciated exchange rate.

The success of our model, however, is not limited to capturing the behavior of exchange rate. We want to ensure that matching dynamics of exchange rates in our model does not come at the expense of unrealistic dynamics of other variables. This is not the case. We again return to the empirical VARs estimated at the beginning of the paper. Besides the exchange rate the estimated VAR systems also included output and fiscal balance to capture, respectively, the output and fiscal effect of interest rate changes. Here we present the empirical impulse responses of output and fiscal balance and compare them with their theoretical counterparts. Figure 9 shows the impulse response of output to interest rate shocks, while Figure 10 shows the corresponding impulse responses for the fiscal balance in the three countries. The left-hand-side panel of Figure 9 shows that both Korea and Brazil experienced a contraction in industrial production following a positive interest rate innovation. The output response in Thailand is more mixed as it fluctuates around zero. The right-hand-side panel of Figure 9 shows that in the model Thai, Korean and Brazilian output decline after a positive interest rate shock. The figures suggest that the model does well in reproducing the output response in Korea and Brazil to interest rate innovations while it fails to do so for Thailand.

Figure 9. Output impulse responses for Thailand, Korea and Brazil 1997-2001

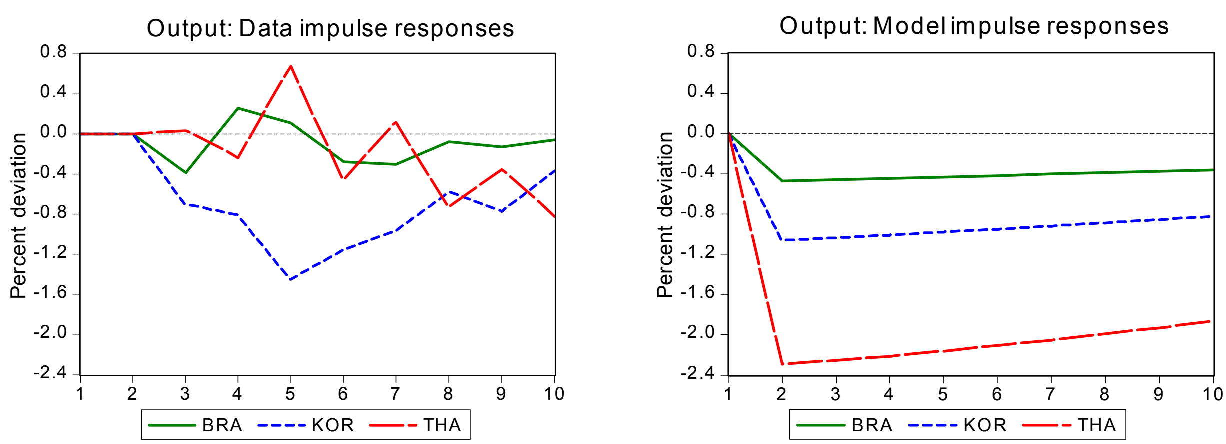
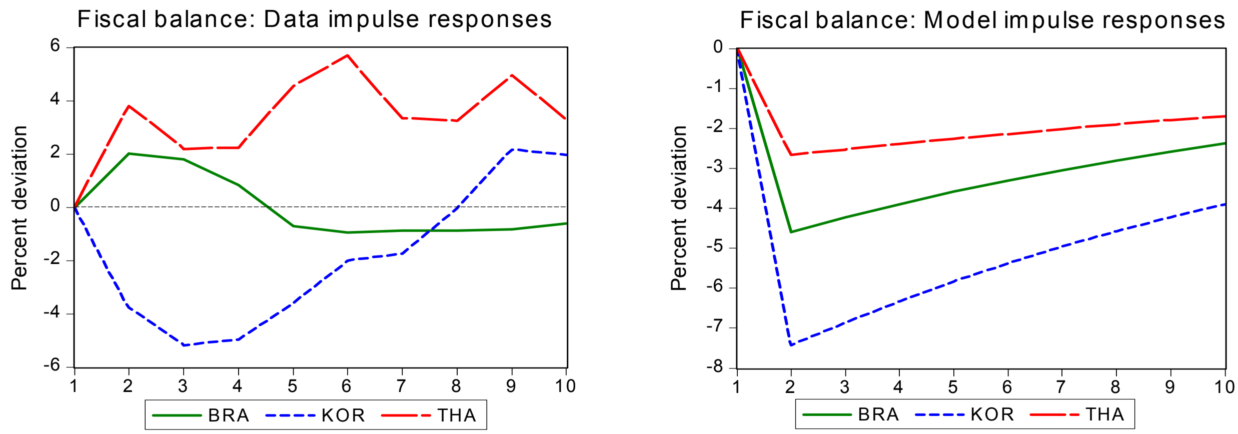



Figure 10 shows the response of the fiscal balance to a positive innovation in the interest rates in our three countries. The model produces a worsening fiscal balance in all three countries while in the data the evidence is mixed. The fiscal balance worsens in Korea, improves in Thailand and fluctuates around zero in Brazil. While the model doesn't reproduce the fiscal responses in Brazil and Thailand, it does reproduce the ordering of the fiscal responses.

Figure 10. Fiscal balance impulse responses for Thailand, Korea and Brazil 1997-2001




### 7.2 Stable period

In this subsection we aim to verify whether our model is successful in capturing the interest rate-exchange rate interplay when interest rates instruments are not actively used by a country's policymakers. For this purpose we repeat the exercise in the previous subsection for the full sample of 1997-2007. Table 3 summarizes the updated model parameters.

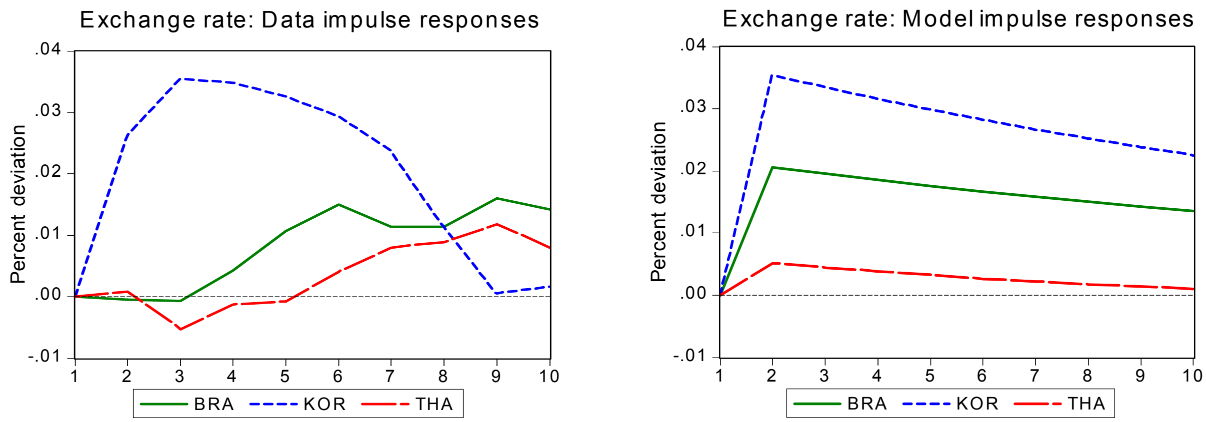
Table 3. Parameter values: Thailand, Korea, Brazil for 1997-2007

| | Thailand | Korea | Brazil |
|---|---|---|---|
| share of wage-in-advance, $\phi$ | 0.9 | 0.73 | 0.31 |
| reserve requirement, $\theta$ | 0.03 | 0.03 | 0.176 |
| Average $i^g$ | 4.1% | 6% | 19% |
| Average $\tau$ | 11% | 13% | 20% |

The key difference from the parameters in the short sample is lower steady state level for $i^g$, as well as more stable dynamics for $i^g$ in all three countries. Furthermore, Thailand and Brazil experienced a fall in the share of business sector credit in total credit, which lead to lower parameter $\phi$. That number for Korea became slightly higher. We also re-calibrate the process for $i^g$ and find $\rho^g = 0.95$ and $\sigma(\varepsilon_t^g) = 0.0276$. As before, we restrict this process to be the same across our three economies. Lastly, we set $\phi^n = 0.14$ in all three countries.

Figure 11 contrasts the empirical and model-generated impulse responses of the exchange rate to interest rate shocks. In the data the exchange rate depreciates in response to an increase in the interest rate for all three countries. The model impulse responses replicate precisely this pattern as well as the size-ordering of the individual country responses.

Figure 11. Exchange rate impulse responses for Thailand, Korea and Brazil 1997-2007




We also compare the model and data impulse responses of output and the fiscal balance to interest rate innovations in these countries. The left panel of Figure 12 shows the data impulse response of output to a one standard deviation interest rate innovation while the right panel shows the corresponding impulse responses generated by the model. Both in the model and the data output falls on impact in all three countries. As in the shorter sample, the model does well in reproducing the data in terms of the response of output to interest rate innovations in Brazil and Korea but fails to pick up the relative lack of a response in Thailand.

Figure 12. Output impulse responses for Thailand, Korea and Brazil 1997-2007

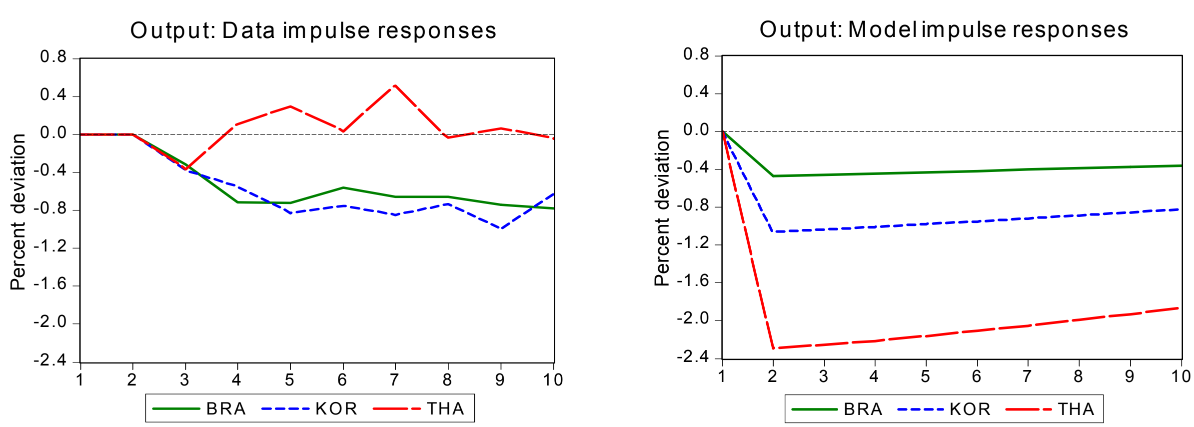
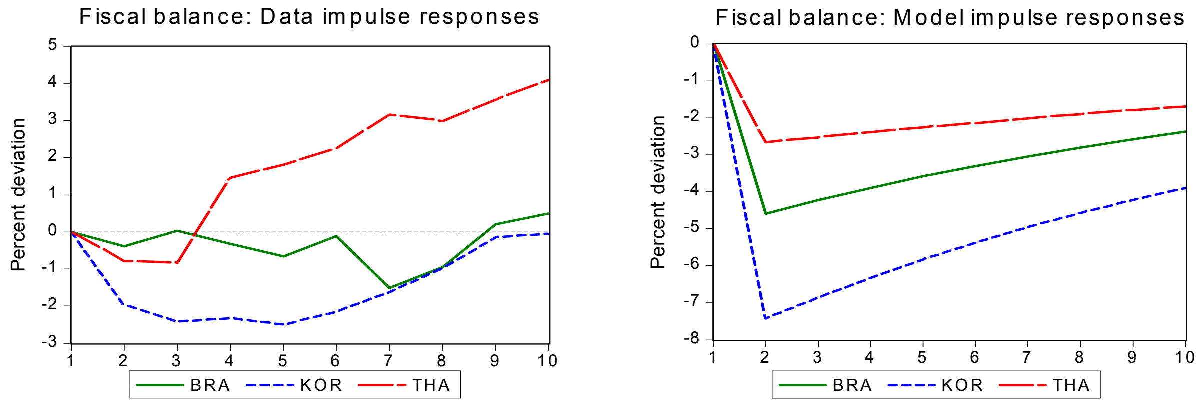


Figure 13 depicts the data and model generated impulse response for the fiscal balance. For all three countries the fiscal balance clearly worsens on impact both in the data and the model.

Figure 13. Fiscal balance impulse responses for Thailand, Korea and Brazil 1997-2007




Overall, we interpret the evidence in Figures 8-13 as being supportive of the margins isolated in the model. Given that the model was not calibrated to match impulse responses and the fact that the parameterization was changed along just three dimensions for these three countries, we find the results to be very encouraging for the ability of the model to explain exchange rate dynamics in different countries.

# 8 Conclusions

The relationship between interest rates and the exchange rate has been the focus of a spirited academic and policy debate for a long time. This paper has developed a simple model to rationalize the mixed and often conflicting results that have been obtained by a large body of empirical work in this area. We have shown – both analytically and numerically – that the relationship between changes in interest rates (both the interest rate controlled by policymakers as well as the market-determined interest rate) and the level of the exchange rate is inherently non-monotonic. Hence, there is no reason to expect to find a monotonic relationship between the two variables in the data.

To make our points as sharply as possible, in our analytical analysis we have focused on two particular cases of a general property of this class of monetary models. Our two cases illustrate the fact that if the government has at least two sources of revenues, then a non-monotonic relationship is to be expected. In our numerical analysis we studied the full version of the model that encompasses all three effects. We have shown that the non-monotonicity in the relation between interest rates and exchange rates exists along both the cross-sectional (in response to the interest rate changes around different steady state levels) and the time-series (in response to the different magnitude shocks around the same steady state) dimensions. In sum, we believe that these non-monotonic results are quite general and not specific to the particular formulation that we may have chosen.

The analysis in our model has treated the policy-controlled interest rate as an exogenous driving process. This simplified treatment of the interest rate allowed us to isolate the channels through which it can affect the nominal exchange rate. While being convenient for our purposes, this simplification is clearly unrealistic. Understanding equilibrium exchange rate dynamics in an environment with endogenous interest rate policy is an area that we intend to pursue in future work.

**Appendices**

# A Proof of Proposition 1

(i) Since the nominal money supply is given at time 0, any change in $m_0$ due to a change in $I_0^d$ has to be accommodated by a change in the exchange rate $E_0$ in the opposite direction. Hence, to uncover the effect of $i^g$ on $E_0$ we need to determine the effect of $i^g$ on $I_0^d$. Since $I^d = i - i^d$ we need to first determine the sign of $\frac{\partial i}{\partial i^g}$. After setting $\tilde{h}(i) = 0$, we can totally differentiate (21) to implicitly solve for $i$ as a function of $i^g$ and $\bar{\tau}$ with

$$\frac{\partial i}{\partial i^g} = \frac{(1-\theta)\, d \left[1 - \left\{1 - \left(\frac{r\left(1+i^d\right)}{I^d}\right)\right\} \frac{\eta_d}{R}\right] - \left[1 - \left\{1 + \left(\frac{r\left(1+i^g\right)}{I^g}\right)\right\} \frac{\eta_n}{R}\right] \theta n}{\chi_1/\left(1+i\right)} \tag{22}$$

where $\chi_1 \equiv \left(1+i^d\right) d \left[1 - \left\{1 - \left(\frac{r\left(1+i^d\right)}{I^d}\right)\right\} \frac{\eta_d}{R}\right] - \left(1+i^g\right) n \left[1 - \left\{1 + \left(\frac{r\left(1+i^g\right)}{I^g}\right)\right\} \frac{\eta_n}{R}\right]$. Define this implicit solution for $i$ as $\tilde{\imath}(i^g; \bar{\tau})$.

We can substitute $\tilde{\imath}(i^g; \bar{\tau})$ into $I^g$ to implicitly solve for $\frac{I^g}{1+i} = \tilde{I}^g(i^g; \bar{\tau})$ where

$$\frac{\partial \tilde{I}^g}{\partial i^g} = \frac{\left[1 - \left\{1 - \left(\frac{r\left(1+i^d\right)}{I^d}\right)\right\} \frac{\eta_d}{R}\right] \theta d}{\chi_1\left(1+i\right)} > 0. \tag{23}$$

The sign of this expression follows directly from our assumption $1 > \left\{1 - \left(\frac{r\left(1+i^d\right)}{I^d}\right)\right\} \frac{\eta_d}{R}$ and the stability condition. Define $\hat{\imath}^g$ such that $\hat{I}^g = \tilde{I}^g(\hat{\imath}^g; \bar{\tau})$.

Using equation (22) along with the definition of $I^d$ and the equilibrium relation $i^d = (1-\theta)\, i^g$ we can derive the implicit function $\frac{I^d}{1+i} = \tilde{I}^d(i^g; \bar{\tau})$, where

$$\frac{\partial \tilde{I}^d}{\partial i^g} = -\frac{\left[1 - \left\{1 + \left(\frac{r\left(1+i^g\right)}{I^g}\right)\right\} \frac{\eta_n}{R}\right] \theta n}{\chi_1\left(1+i\right)}. \tag{24}$$

Under our assumed production function for firms (equation (6) the steady state loan demand is given by $n = \phi\left(1-\alpha\right)^{\frac{\nu}{\nu-1+\alpha}} \left(\nu\zeta\right)^{\frac{\alpha-1}{\nu-1+\alpha}} \left[\frac{1}{1+\phi\left\{\frac{i^g-i}{1+i}\right\}}\right]$. Using this to solve for $\eta_n$ $(= -\frac{\tilde{n}'}{n}\frac{i^g-i}{1+i})$ in equation (24) above, it is easy to check that $1 \gtreqless \left[1 + \left(\frac{r\left(1+i^g\right)}{I^g}\right)\right] \frac{\eta_n}{R}$ as $1 + \phi\left(\frac{I^g}{1+i}\right) \lesseqgtr \nu\left(1 - \frac{\phi r}{R}\right)$. Hence, if $1 < \nu\left(1 - \frac{\phi r}{R}\right)$, then $\frac{\partial \tilde{I}^d}{\partial I^g} < 0$ for low values of $\frac{I^g}{1+i}$ but $\frac{\partial \tilde{I}^d}{\partial i^g} > 0$ for all $\frac{I^g}{1+i} > \left[\nu\left(1 - \frac{\phi r}{R}\right) - 1\right]/\phi \equiv \hat{I}^g$. The proof then follows from the fact that $\partial \tilde{I}^g/\partial i^g > 0$.

(ii) From (i) above we know that $\frac{\partial i}{\partial i^g}\Big|_{i^g=\hat{\imath}^g} = (1-\theta)\left(\frac{1+\tilde{\imath}(\hat{\imath}^g,\bar{\tau})}{1+(1-\theta)\hat{\imath}^g}\right) > 0$. Moreover, $1 < \left\{1 + \left(\frac{r\left(1+i^g\right)}{I^g}\right)\right\} \frac{\eta_n}{R}$ for all $i^g > \hat{\imath}^g$. Since $1 > \left\{1 - \left(\frac{r\left(1+i^d\right)}{I^d}\right)\right\} \frac{\eta_d}{R}$ by assumption and $\chi_1 > 0$ from the stability condition, it directly follows from equation (22) that $\frac{\partial i}{\partial i^g} > 0$ for all $i^g \geq \hat{\imath}^g$.

Is it possible for $\frac{\partial i}{\partial i^g} < 0$ when $i^g < \hat{\imath}^g$? From part (i) of this Proposition we know that $\left\{1 + \left(\frac{r(1+i^g)}{I^g}\right)\right\} \frac{\eta_n}{R}$ is monotonically increasing in $i^g$. Since $n$ is monotonically decreasing in $\tilde{I}^g$ and $\tilde{I}^g$ is increasing in $i^g$, we know that $n$ is monotonically decreasing in $i^g$. Hence $\left[1 - \left\{1 + \left(\frac{r(1+i^g)}{I^g}\right)\right\} \frac{\eta_n}{R}\right] \theta n$ is monotonically falling in $i^g$ for all $i^g$. We also know from part (i) of this proposition that $d$ is increasing with $i^g$ for $i^g < \hat{\imath}^g$ since $\frac{\partial \tilde{I}^d}{\partial i^g} > 0$ in this range. Define

$$A \equiv \left\{1 - \left(\frac{r\left(1+i^d\right)}{I^d}\right)\right\} \frac{\eta_d}{R}$$

Note thas this can be rewritten as

$$A \equiv \left(1 - \frac{r}{R\tilde{I}^d}\right) \eta_d$$

It is easy to check that $\frac{\partial \eta_d}{\partial \tilde{I}^d} > 0$ (a maintained assumption) is a sufficient condition for $A$ to be decreasing in $i^g$ for $i^g < \hat{\imath}^g$. This implies that the first term in the numerator is $(1-\theta)\, d\, [1-A]$ is mononically increasing in $i^g$ for all $i^g < \hat{\imath}^g$ (since $d$ is also rising with $i^g$ in this range). Combining this with the fact that $\left[1 - \left\{1 + \left(\frac{r(1+i^g)}{I^g}\right)\right\} \frac{\eta_n}{R}\right] \theta n$ is monotonically decreasing in $i^g$ for all $i^g$ implies that the numerator of equation (22) is monotonically increasing in $i^g$ for all $i^g < \hat{\imath}^g$. Hence, as we lower $i^g$ below $\hat{\imath}^g$, the numerator of equation (22) falls monotonically. By arguments of continuity, there exists an $\bar{\imath}^g < \hat{\imath}^g$ such that $\frac{\partial i}{\partial i^g} \gtreqless 0$ for all $i^g \gtreqless \bar{\imath}^g$.[30]

(iii) From (i) and (ii) we know that for $i^g \in (\bar{\imath}^g, \hat{\imath}^g)$ an increase in $i^g$ appreciates the currency on impact but also increases the steady-state depreciation rate.

# B Proof of Proposition 2

Totally differentiating the government transfer equation (21), evaluating it around the steady state and rearranging gives

$$\frac{\partial i}{\partial i^g} = \frac{(1-\theta)\, d \left[1 - \left(1 - \frac{Ri^d}{i-r}\right)\left(\frac{i-r}{i-i^d}\right)\frac{\eta_d}{R}\right]}{\chi_2/(1+i)} > 0 \tag{25}$$

where

$$\chi_2 = h\left[1 - \left(\frac{i-r}{Ri}\right)\eta_h\right] + \left(1+i^d\right) Rd \left[1 - \left\{1 - \left(\frac{r\left(1+i^d\right)}{i-i^d}\right)\right\}\frac{\eta_d}{R}\right]. \tag{26}$$

[30] It is fairly easy to restrict parameters such that $\bar{I}^g > 0$ where $\bar{I}^g = \tilde{I}^g(\bar{\imath}^g, \bar{\tau})$. This restriction is necessary to guarantee a non-monotonicity of $i$ within the permissible range of $I^g$.

The inequality follows from our restriction that $1 > \left(1-\frac{Ri^d}{i-r}\right)\left(\frac{i-r}{i-i^d}\right)\frac{\eta_d}{R}$ (which we imposed to maintain comparability with the one money case). It is easy to check that

$$\frac{\partial \tilde{I}^d}{\partial i^g} = \frac{-(1-\theta)\,h}{(1+i)\,\chi_2}\left[1-\left(\frac{i-r}{Ri}\right)\eta_h\right]. \tag{27}$$

where $\tilde{I}^d = \frac{i-i^d}{1+i}$ and where $\chi_2$ is given by equation (26).

To determine the relationship between the level of the nominal exchange rate and $i^g$ we need to determine the effect of $i^g$ on total real money demand $m(=h+\theta d)$. Using equation (25) it follows that

$$\frac{\partial m}{\partial i^g} = \frac{(1-\theta)hd\eta_d}{i\chi_2}\left[\frac{\eta_d-\eta_h}{\eta_d} - \frac{r(1-\theta)i^g}{i\tilde{I}^d}\frac{\eta_h}{R} + \frac{(1-\theta)I^g}{I^d}\left(1-\frac{i-r}{i}\frac{\eta_h}{R}\right)\right]. \tag{28}$$

Since $i$ is always rising in $i^g$, demand for cash $(h)$ is always falling in $i^g$. Hence, $m$ must necessarily fall with a higher $i^g$ if demand deposits are non-increasing in $i^g$. Noting that the interest elasticity of cash $\eta_h$ is a function of the nominal interest rate $i$, define $\bar{\imath}^g$ by the relation $\eta_h\left(\frac{\tilde{\imath}(\bar{\imath}^g,\bar{\tau})}{1+\tilde{\imath}(\bar{\imath}^g,\bar{\tau})}\right) = \frac{R\tilde{\imath}(\bar{\imath}^g,\bar{\tau})}{\tilde{\imath}(\bar{\imath}^g,\bar{\tau})-r}$. Hence, from equation (27) we have $\left.\frac{\partial \tilde{I}^d}{\partial i^g}\right|_{i^g=\bar{\imath}^g} = 0$. But this implies that $\left.\frac{\partial d}{\partial i^g}\right|_{i^g=\bar{\imath}^g} = 0$ and $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=\bar{\imath}^g} < 0$.

Since $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=\bar{\imath}^g} < 0$, the proof of the non-monotonicity of $m$ in $i^g$ hinges on showing that $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=0} > 0$. First, note that since $i$ is increasing in $i^g$ from equation (25) and $\frac{\partial \eta_h}{\partial i} > 0$ (from Condition 2) we must have $\eta_h\left(\frac{\tilde{\imath}(0,\bar{\tau})}{1+\tilde{\imath}(0,\bar{\tau})}\right) < \frac{R\tilde{\imath}(0,\bar{\tau})}{\tilde{\imath}(0,\bar{\tau})-r}$. Hence, $\left.\frac{\partial \tilde{I}^d}{\partial i^g}\right|_{i^g=0} < 0$. But this implies that $\left.\frac{\partial d}{\partial i^g}\right|_{i^g=0} > 0$. Noting that $I^d = i = -I^g$ around $i^g = 0$, it is easy to check that equation (28) gives

$$\left.\frac{\partial m}{\partial i^g}\right|_{i^g=0} = \frac{(1-\theta)\,hd\eta_d}{i\chi_2}\left[\frac{\theta\eta_d\left(\frac{\tilde{\imath}(0,\bar{\tau})}{1+\tilde{\imath}(0,\bar{\tau})}\right)-\eta_h\left(\frac{\tilde{\imath}(0,\bar{\tau})}{1+\tilde{\imath}(0,\bar{\tau})}\right)}{\eta_d\left(\frac{\tilde{\imath}(0,\bar{\tau})}{1+\tilde{\imath}(0,\bar{\tau})}\right)} + \left(\frac{1-\theta}{R}\right)\left(\frac{\tilde{\imath}(0,\bar{\tau})-r}{\tilde{\imath}(0,\bar{\tau})}\right)\eta_h\left(\frac{\tilde{\imath}(0,\bar{\tau})}{1+\tilde{\imath}(0,\bar{\tau})}\right)\right].$$

Hence, Condition 1 is sufficient for $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=0} > 0$. Moreover, $\frac{\partial i}{\partial i^g} > 0$ and $\frac{\partial \eta_h}{\partial i} > 0$ jointly imply that $\left(\frac{\tilde{\imath}(i^g,\bar{\tau})-r}{\tilde{\imath}(i^g,\bar{\tau})}\right)\eta_h\left(\frac{\tilde{\imath}(i^g,\bar{\tau})}{1+\tilde{\imath}(i^g,\bar{\tau})}\right)$ is rising in $i^g$. Moreover, for all $i^g < \bar{\imath}^g$ we know from above that $\frac{\partial \tilde{I}^d}{\partial i^g} < 0$. From Condition 2 $\frac{\partial \eta_d}{\partial \tilde{I}^d} > 0$. Hence, $\eta_d$ must be falling with $i^g$ for all $i^g < \bar{\imath}^g$. By arguments of continuity, there must therefore exist an $\hat{\imath}^g$ such that $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=\hat{\imath}^g} = 0$. The proof is completed by noting that $E_0 = M_0/m$ and that nominal money supply at time 0 is given (see equation (15)). Hence, $E_0$ moves inversely with $m$.

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