Consumption of traded goods and real exchange rate dynamics with Epstein-Zin preferences.

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Abstract

The paper analyzes the consumption-real exchange rate anomaly in a multi-country model with recursive preferences, complete markets, long-run risks and stochastic volatility. Recent research indicates that such a model is successful in replicating low correlation between consumption and real exchange rates as in the data, thus resolving the anomaly. The paper focuses on another implication of the model that bilateral real exchange rates are linked to consumption in both countries and to the ratio of domestic consumption of home endowment relative to exports of domestic goods to the foreign country. This implication finds little empirical support with data for 9 OECD economies. (*JEL* F31, F41)

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I. INTRODUCTION

One of the puzzles of international finance is that real exchange rates are disconnected from consumption differentials across countries. This is the so-called consumption-real exchange rate anomaly or the Backus-Smith puzzle (Backus and Smith, 1993; Kollmann, 1995). Recently it has been proposed in the literature to resolve the puzzle with the help of recursive preferences and long-run risks (Colacito and Croce, 2013).

The long-run risk literature is initiated by Bansal and Yaron (2004) who combine Epstein-Zin (1989) preferences with a small long-run predictable component in consumption to explain a variety of asset market phenomena. The model is successful in explaining the equity puzzle and low risk-free rate, among other puzzles. The long-run risk literature has made its way into the field of international finance. Colacito and Croce (2011) show that a model with highly correlated long-run risks and recursive preferences can explain high correlation of international stock market returns without high correlation of fundamentals. Colacito and Croce (2013) study the implication of a two-good two-country economy for the Backus-Smith puzzle and the forward premium anomaly. Kollmann (2016) and Backus et al. (2016) analyze volatility shocks. Colacito et al. (2016) build up a multicountry economy model to analyze the cross-section of currency returns.

The Backus-Smith puzzle arises in a standard open economy model with additive preferences and complete markets. Backus and Smith (1993) analyze the puzzle in a multi-country economy with N countries, N nontraded goods and one traded good. A positive endowment shock to the nontraded good in one country depreciates the real exchange rate and increases domestic consumption, resulting in the perfect correlation between the growth of the real exchange rate, $\Delta ln e_{ijt}$, and the consumption differential, $\Delta ln(C_{it}/C_{jt})$:

$$\Delta ln\ e_{ijt}\ \sim\ \Delta ln(C_{it}/C_{jt})$$

In data, the correlation between these variables is low or even negative.[1]

The long-run risk literature changes the additive preferences to recursive Epstein-Zin preferences

[1] The correlation is also low or negative when the variables are analyzed in levels rather than growth rates. See Chari et al. (2002), Corsetti et al. (2008), among others.

(Epstein and Zin, 1989) and introduces a slowly moving predictable growth component as in Bansal and Yaron (2004). With these modifications the link between the real exchange rate growth and the growth of the consumption differential breaks down:

$$\Delta ln\ e_{ijt} \sim \Delta ln\lambda_t^* + \Delta ln(C_{it}/C_{jt}).$$

The variable that breaks the relationship is the modified Pareto weight, λ_t^*, in the social planner problem that maximizes utility of country i subject to utility promises to country j. In the language of Backus et al. (2016), the modified Pareto weight creates a wedge between the real exchange rate and the consumption differential.[2] This separation makes it possible that the real exchange rate is not highly correlated with the consumption differential. This is indeed what is found in the literature (Colacito and Croce, 2013, Kollmann, 2016, Backus et al., 2016) by means of simulations.

This paper analyzes the consumption-real exchange rate anomaly in a standard setting of the long-run risk literature with recursive preferences and traded goods. Like Colacito et al. (2016) I consider a multi-country economy. The focus is different, though. Colacito et al. (2016) use the model to analyze the cross-section of currency returns. I consider the implication of the model for the consumption-real exchange rate anomaly. The model of this paper allows for long-run risk shocks, as in Colacito and Croce (2013), and for volatility shocks as in Backus et al. (2016) and Kollmann (2016). I derive a different implication of the model for the consumption-real exchange rate relationship:

$$\Delta ln\ e_{ijt} \sim \Delta ln(C_{it}/C_{jt}) - \Delta ln(x_{it}^i/x_{jt}^i),$$

where x_{jt}^i is the consumption of good i in country j. The success of the long-run risk literature can be seen in breaking the relationship between the real exchange rate and the consumption ratio. The modified Pareto weight creates a wedge between the former and the latter. In practice, Pareto weights are not observable. The relationship proposed in this paper is observable and testable. The social planner in order to achieve the socially desirable reallocation of wealth, would change the consumption of traded goods among the countries. If in practice, this reallocation is done optimally from the social point of view, we should observe the link between the real exchange rate growth and

[2]The Pareto problem is formally introduced in Section III.

the growth of the consumption differential and the ratio of the consumption growth of domestic endowment relative to the growth of exports of the domestic good to the foreign country.

Such implication of the multi-country model with recursive preferences is tested with data for 9 OECD economies. To foresee the results of the paper, I find little empirical support for the proposed relationship. The contribution of the paper is that unobservable Pareto weights can be linked to observable macroeconomic variables. The conclusion we can draw from the weak empirical evidence is that consumption of traded goods is not reallocated optimally.

II. MULTI-COUNTRY ECONOMY

The economy consists of N countries and N traded goods, denoted $X_{it}, \quad i = 1, ..., N, \ t = 0, ..., \infty$. Each country is represented by an infinitely-lived agent. The consumption of i^{th} agent is described by the CES function over all traded goods:

$$C_{it} = (\alpha_i^{1-\eta}(x_{it}^i)^\eta + \sum_{j \neq i}^{N} \alpha_{ij}^{1-\eta}(x_{it}^j)^\eta)^{1/\eta}, \quad \text{with} \quad \sum_{j \neq i}^{N} \alpha_{ij} = 1 - \alpha_i, \tag{1}$$

where x_{it}^j is consumption of good j in country i at time t. The parameters $0 \leq \alpha_i < 1$ for any i, and $\eta \leq 1$. The elasticity of substitution is $1/(1-\eta)$. Each country is endowed at every point in time with the single good most preferred by its agent, i.e. $\alpha_i > 1/2$. Therefore, the consumption function exhibits home bias towards domestic traded good. This assumption is crucial to generate real exchange movements in line with the terms of trade.

Iceberg-type trade costs are assumed for shipments of goods from one country to another; for each good shipped, $1/(1+\tau)$ goods arrive at destination. The resource constraint for each traded good thus is:

$$x_{it}^i + \sum_{j \neq i}(1 + \tau_{ij})x_{jt}^i = X_{it}. \tag{2}$$

The endowment of each country evolves according to the error-correction mechanism:

$$\Delta ln(X_{it}) = g + z_{it-1} - k \ (lnX_{it-1} - W_{t-1}) + exp(\sigma_{t-1})\varepsilon_{xt}^i, \quad \varepsilon_x^i \sim N(0, \sigma_x^2) \tag{3}$$

$$W_t = \frac{1}{N} \sum_{j=1}^{N} lnX_{jt} \tag{4}$$

$$z_{it} = \psi_i z_{it-1} + \varepsilon^i_{zt}, \quad 0 < \psi_i < 1, \quad \varepsilon^i_{zt} \sim N(0, \sigma^2_z) \tag{5}$$

$$\sigma_t = v\sigma_{t-1} + \nu_t, \quad 0 < v < 1, \quad \nu_t \sim N(0, \sigma^2_\nu), \tag{6}$$

where W_t is the world endowment of traded goods. The (log) endowment for each country has a unit root. The (log) endowments for all countries are cointegrated. The adjustment coefficient k is small in practice; the convergence to the cointegrating relationship is slow. The growth of the endowment has persistent growth component z_i. This assumption is important. First, long-run shocks ε_z do not have an impact at time t on the endowment process. Second, long-run shocks are small but have long-lasting impact on the endowment. The volatility process follows a persistent dynamics as well. Both long-run and volatility shocks can generate consumption-real exchange rate disconnect. Short-run shocks ε^i_{xt} are correlated among the countries with the correlation coefficient $corr(\varepsilon^i_{xt}, \varepsilon^j_{xt}) = \rho_x$, $j \neq i$. Parameter ρ_x is a small number in practice. Long-run shocks are highly correlated among the countries with $corr(\varepsilon^i_{zt}, \varepsilon^j_{zt}) = \rho_z$. Shocks $(\varepsilon^i_{xt}, \varepsilon^j_{zt}, \nu_t)$ are not correlated among themselves and there is no autocorrelation.

Agents have Epstein-Zin recursive preferences (Epstein and Zin, 1989):

$$U_{it} = [(1-\beta)C^\theta_{it} + \beta[E_t(U^\gamma_{it+1})]^{\theta/\gamma}]^{1/\theta}, \tag{7}$$

where U_{it} is utility of agent i at time t. The parameters $0 < \beta < 1$, $\theta < 1$, and $\gamma < 1$. The additive preferences can be obtained as a special case when $\theta = \gamma$. The degree of risk-aversion is $1 - \gamma$. The intratemporal elasticity of substitution is $1/(1-\theta)$. The interaction of Epstein-Zin preferences with long-run risk shocks or with volatility shocks is what drives an interesting dynamics in the relationship between the real exchange rate and consumption in the model. As shown by Colacito and Croce (2013), assuming log-normality for the ordinary equivalent transformation $J_t = U^\theta/\theta$:

$$J_t = (1-\beta)C^\theta_t + \beta[E_t(U^\gamma_{it+1})]^{\theta/\gamma} \approx (1-\beta)C^\theta_t + \beta E_t(J_{t+1}) - \frac{\beta}{2}\frac{\theta-\gamma}{\theta}\frac{Var_t[J_{t+1}]}{E_t[J_{t+1}]}.$$

The approximation shows that agents are willing to trade the level of expected utility for the expected volatility when $\theta > \gamma$. Long-run shocks reduce uncertainty about the future due to their persistence, allowing agents to reduce consumption today given reduced expected volatility in the future. Volatility shocks can play a similar role in this economy. If volatility is reduced, consumption can fall today to achieve the same level of utility (Kollmann, 2016; Backus et al., 2016).

The real exchange rate between countries i and j is defined as:

$$e_{ijt} = P_{jt}/P_{it}, \tag{8}$$

where P_{jt} and P_{it} are price indices corresponding to the consumption aggregator in (1). The rise in the real exchange rate represents a depreciation of currency i with respect to currency j.

$$P_{it} = [\alpha_i p_i^{\frac{\eta}{\eta-1}} + \sum_{j \neq i} \alpha_{ij} p_j^{\frac{\eta}{\eta-1}}]^{\frac{\eta-1}{\eta}}. \tag{9}$$

Without loss of generality, prices are normalized such that $\sum_i p_{it} = N$. The demand for traded goods is given as:

$$x_{it}^i = \alpha_i \left(\frac{p_i}{P_i}\right)^{\frac{1}{\eta-1}} C_{it} \tag{10}$$

$$x_{it}^j = \alpha_{ij} \left(\frac{(1+\tau_{ji})p_j}{P_i}\right)^{\frac{1}{\eta-1}} C_{it}. \tag{11}$$

The intertemporal marginal rate of substitution (IMRS) between aggregate consumption at periods $t-1$ and t is as follows:

$$M_{it} = \beta \left(\frac{C_{it}}{C_{it-1}}\right)^{\theta-1} \left(\frac{U_{it}}{[E_{t-1}(U_{it}^{\gamma})]^{1/\gamma}}\right)^{\gamma-\theta}. \tag{12}$$

Financial markets are assumed to be complete so that the real exchange rate and IMRS between any two countries are linked:

$$e_{ijt+1}/e_{ijt} = M_{jt+1}/M_{it+1}. \tag{13}$$

Equations (12) and (13) emphasize the importance of the Epstein-Zin preferences. If $\gamma = \theta$

the preferences collapse to the regular additive case and IMRS depends only on the consumption growth. In this case, the real exchange rate is proportional to the ratios of domestic to foreign consumption growth, and we are back to the Backus-Smith puzzle. The last term of equation (12) allows IMRS to be more volatile under long-run or volatility shocks, implying higher volatility of the real exchange rate (13) and potentially resolving the exchange rate volatility puzzle. In fact, IMRS and real exchange rates may be too volatile unless IMRS are highly correlated (Brandt et al., 2006). Colacito and Crose (2011) show that high correlation of IMRS is achieved by assuming that long-run growth components are highly correlated.

III. PARETO PROBLEM

It is common in the literature to find a competitive equilibrium as a solution to the social planner problem. This approach was used in Colacito and Crose (2013), Colacito et al. (2016), Kollmann (2015, 2016), Tretvoll (2011), Backus et al. (2016). In particular, I extend Backus et al. (2016), Tretvoll (2011) in the formulation of the Pareto model to the multi-country setting.

The planner maximizes the utility of agent i with respect to consumption and utility promises made to the other agents:

$$U_{it}(U_{jt}) = \max_{C_{it}, U_{jt+1}} V[C_{it}, U_{it+1}(U_{jt+1})],$$

$$\begin{aligned} s.t. \quad & V(C_{jt}, U_{jt+1}) \geq U_{jt}, \quad \text{for } j \neq i, \\ s.t. \quad & \text{resource constraint (2)}, \\ s.t \quad & \text{consumption aggregator (1)}, \end{aligned}$$

where $V[C_{it}, U_{it+1}] = [(1-\beta)C_{it}^{\theta} + \beta[E_t(U_{it+1}^{\gamma})]^{\theta/\gamma}]^{1/\theta}$ is the value function. For simplicity, I drop the notation with respect to the states. The Lagrange multiplier on the utility promises is denoted as λ_{jt}, on the resource constraint as q_{it}, and on the consumption aggregator as P_{it}.

The first-order conditions are the following:

$$U_{jt+1}: \quad U_{it}^{1-\theta}[E_t(U_{it+1}^{\gamma})]^{\frac{\theta-\gamma}{\gamma}} U_{it+1}^{\gamma-1} \frac{\partial U_{it+1}}{\partial U_{jt+1}} = -\lambda_{jt} U_{jt}^{1-\theta}[E_t(U_{jt+1}^{\gamma})]^{\frac{\theta-\gamma}{\gamma}} U_{jt+1}^{\gamma-1} \tag{14}$$

$$x_{it}^{i}: \quad P_{it} C_{it}^{1-\eta} \alpha_i^{1-\eta} (x_{it}^{i})^{\eta-1} = q_{it} \tag{15}$$

$$x_{jt}^{i}: \quad \frac{1}{1+\tau_{ij}} P_{jt} C_{jt}^{1-\eta} \alpha_{ji}^{1-\eta} (x_{jt}^{i})^{\eta-1} = q_{it} \tag{16}$$

$$C_{it}: \quad U_{it}^{1-\theta}(1-\beta) C_{it}^{\theta-1} = P_{it} \tag{17}$$

$$C_{jt}: \quad \lambda_{jt} U_{jt}^{1-\theta}(1-\beta) C_{jt}^{\theta-1} = P_{jt} \tag{18}$$

By the envelope condition, $\frac{\partial U_{it+1}}{\partial U_{jt+1}} = -\lambda_{jt+1}$. Following Backus et al. (2016), I transform Pareto weights in the following way: $\lambda_{jt}^{*} = \lambda_{jt} U_{jt}^{1-\theta} / U_{it}^{1-\theta}$. With these changes equation (14) becomes:

$$\left(\frac{U_{it+1}^{\gamma}}{E_t(U_{it+1}^{\gamma})} \right)^{\frac{\gamma-\theta}{\gamma}} \lambda_{jt+1}^{*} = \lambda_{jt}^{*} \left(\frac{U_{jt+1}^{\gamma}}{E_t(U_{jt+1}^{\gamma})} \right)^{\frac{\gamma-\theta}{\gamma}}. \tag{19}$$

The implication of this equation is that when preferences are additive, i.e. $\gamma = \theta$, the transformed Pareto weights are constant, $\lambda_{jt+1}^{*} = \lambda_{jt}^{*}$. Another interesting result follows from dividing equation (18) by equation (17):

$$e_{ijt} = \lambda_{jt}^{*} (C_{jt}/C_{it})^{\theta-1}, \tag{20}$$

where I used the definition of the real exchange rate (8). In the language of Backus et al. (2016), the transformed Pareto weight creates a wedge between the real exchange rate and the consumption ratio. This makes it possible that the correlation between the real exchange rate and the consumption ratio is low if the correlation between the former and the Pareto weight is high. It is shown by Colacito and Croce (2013) by means of simulations that in a recursive model with long-run risks the correlation between the real exchange rate and the consumption ratio is indeed low, thus potentially resolving the Backus-Smith puzzle.

It however remains a question whether the Pareto weights are highly correlated with the real exchange rate in data. To get an insight into this question I analyze conditions (15) and (16)[3]:

$$e_{ijt} = (1+\tau_{ij})(\frac{\alpha_i}{\alpha_{ji}})^{1-\eta}(C_{jt}/C_{it})^{\eta-1}/(x^i_{jt}/x^i_{it})^{\eta-1} \tag{21}$$

Taking the log-difference of the last two equations implies:

$$\Delta ln(e_{ijt}) = \Delta ln(\lambda^*_{jt}) + (\theta-1)\Delta(ln(C_{jt}) - ln(C_{it})), \tag{22}$$

and

$$\Delta ln(e_{ijt}) = (\eta-1)\Delta(ln(C_{jt}) - ln(C_{it}) - ln(x^i_{jt}) + ln(x^i_{it})). \tag{23}$$

Equation (22) can not be analyzed directly because the optimal Pareto weights are not observable and they are also time-varying. If preferences are additive then the Pareto weights are constant and equation (22) collapses to the regular Backus-Smith puzzle, in which the real exchange rate and the consumption differential are perfectly correlated. Equation (23) is on the other hand testable. It relates exchange rate movements to movements in the consumption differential and to the optimal variation in domestic consumption of the domestic good (x^i_{it}) and exports of good i to country j, x^i_{jt}. The value of parameter η does not matter for the correlation of the changes in the (log) real exchange rate and the right-hand side of equation (23), unless $\eta = 1$. If goods are perfect substitutes, $\eta = 1$, prices of all goods will be the same and the real exchange rate does not move. To my knowledge, the implication of recursive preferences and long-run risks and/or volatility as implied by equation (23) has not been tested empirically.

Equation (23) is derived under the minimal set of assumptions. It requires the consumption aggregator (1) and the resource constraint (2). It would hold independently of the risk aversion parameter or the intertemporal rate of substitution. The rest of the model would determine the optimal consumption and relative prices. Once those are determined, equation (23) links the real exchange rate, consumption ratios and demand for individual goods.

By combining equations (22) and (23), one can see that the modified Pareto weights also create a wedge between the consumption differential and the consumption of domestic goods relative to

[3]This result can also be obtained from (10) and (11). Note that $e_{ijt} = 1/e_{jit}$.

exports:

$$\Delta ln(C_{jt}/C_{it}) = \frac{1}{\eta-\theta}\Delta ln\lambda_t^* + \frac{1-\eta}{\eta-\theta}\Delta ln(x_{it}^i/x_{jt}^i).$$

Under additive preferences, λ_t^* =constant, and the consumption differential is proportional to $\Delta ln(x_{it}^i/x_{jt}^i)$. In this case, equation (23) collapses to the Backus-Smith puzzle. If preferences are recursive, there is a wedge between the real exchange rate and the consumption differential, and between the consumption differential and the consumption of domestic goods relative to exports of domestic goods. This paper focuses on the second implication.

IV. MODEL IMPLICATIONS

Since endowments have a unit root, country i consumption and utility are divided by its endowment X_i to stationarize the model. The stationarized model is then solved using the perturbation method around the deterministic steady states with the help of DYNARE 4.4.3.

Calibration Table 1 presents the benchmark calibration of the model.

TABLE 1: CALIBRATION

Parameter	Value	Comments
γ	- 9	Degree of risk aversion $= 1-\gamma = 10$
θ	- 1	Intertemporal elasticity of substitution $= 1/(1-\theta) = 1/2$
β	0.98	
α_i	0.9	For any i. The degree of home bias
α_{ij}	0.025	For any $i \neq j$. Assuming the number of countries $N = 5$
η	0.33	The elasticity of substitution among traded goods is $1/(1-\eta) = 1.5$
g	0.02	The growth of endowments, 2%
k	0.0005	The adjustment parameter for the error-correction term
ψ_i	0.985	Persistence in the long-run growth component of the endowment
σ_x	1.87%	Standard deviation of the short-run shocks. Colacito and Croce (2013)
σ_z	0.26%	Standard deviation of the long-run shocks. Colacito and Croce (2013)
υ	0.99	Persistence in volatility shocks. Kollmann (2016)
σ_ν	5.8%	Standard deviation of volatility shocks. Kollmann (2016)
τ_{ij}	0	

The degree of risk aversion, 10, is in line with the long-run risk literature, e.g. Bansal and Yaron (2004). Colacito et al. (2015) assume the degree of risk aversion as 6.5, and Colacito and Croce (2013) as 8. Bansal et al. (2013) estimate the degree to be around 7.4. The degree of intertemporal elasticity of substitution is 1/2 as in Backus et al. (2016). The long-run risk literature usually assumes the intertemporal elasticity of substitution greater than 1 since this is what is required to explain the equity puzzle, e.g. Bansal and Yaron (2004). Since this paper does not consider the equity premium, the elasticity does not have to be greater than 1. Colacito and Croce (2013) consider the intertemporal elasticity to be 1.5 in the benchmark specification. Bansal et al. (2013) estimate it around 2.1. Since the degree of risk aversion is bigger than the intertemporal elasticity, the agents have a preference for an early resolution of uncertainty. The qualitative results of this paper hold for various degrees of risk aversion and intertemporal elasticity as assumed in the long-run risk literature.

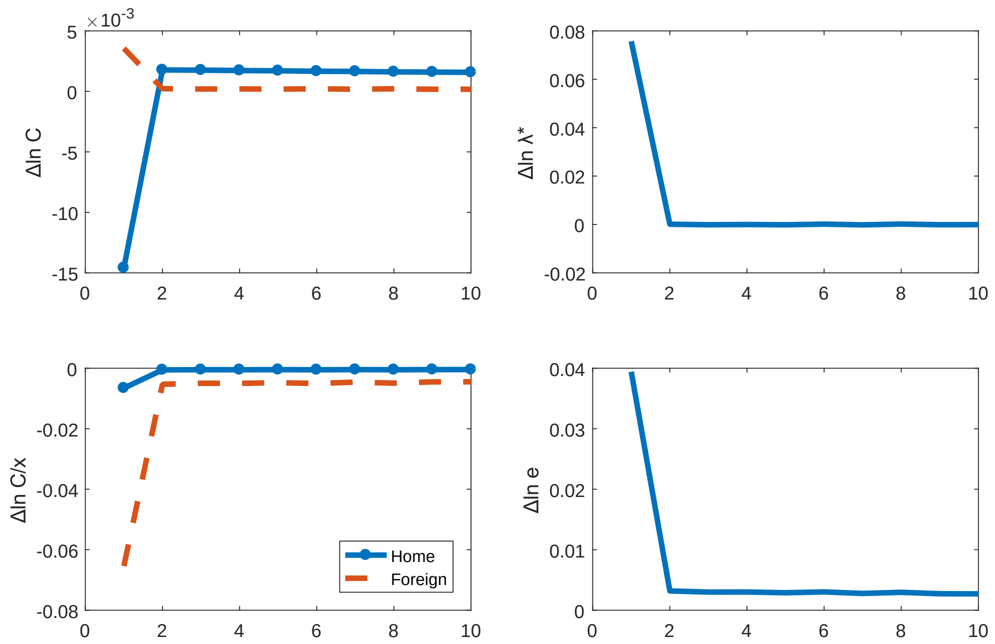
The elasticity of substitution among traded goods is assumed to be 1.5. Usually, the consumption aggregator is assumed to be the Cobb-Douglas which implies that the elasticity of substitution is 1 (Backus et al., 2016, Colacito and Croce, 2013). Empirical studies point toward a higher degree of substitution in the range of 5 to 6 (see an overview in Obstfeld and Rogoff, 2000).

Impulse Response Functions

Figure 1 presents the impulse response functions with respect to one standard deviation long-run shock. The long-run growth component of country 1 is shocked and responses are denoted as 'Home' for simplicity. The responses of the other four countries are identical due to symmetry and denoted 'Foreign'. The left-top corner of the figure shows the responses of consumption growth in the countries. Home consumption falls upon receiving good news for the domestic endowment in the future. Note that the endowment per se is not affected in period 1 when the shock hits. It is only the long-run component that is affected, and the endowment starts to grow at a declining rate in period 2. Home consumption falls in period 1 to provide risk-sharing with foreign countries. Consequently, foreign consumption rises on the impact. In order to encourage foreign agents to buy domestic goods, the terms of trade for the home country need to deteriorate and the bilateral real exchange rates between the home and foreign countries depreciate as shown in the right-bottom of the figure. Since the shock is favorable for the home country, the social planner assigns more weight to utility promises given to the other countries in the right-top corner of the figure.

The home country reduces consumption of domestic endowment and imported goods, but due to falling terms of trade, consumption of domestic endowment falls less. We see from the left-bottom diagram, that the domestic consumption growth relative to the growth of consumption of domestic endowment is negative in period 1. Since the real exchange rate depreciates, exports of domestic goods to foreign countries increase strongly. Foreign consumption relative to exports from the home country falls strongly as well.

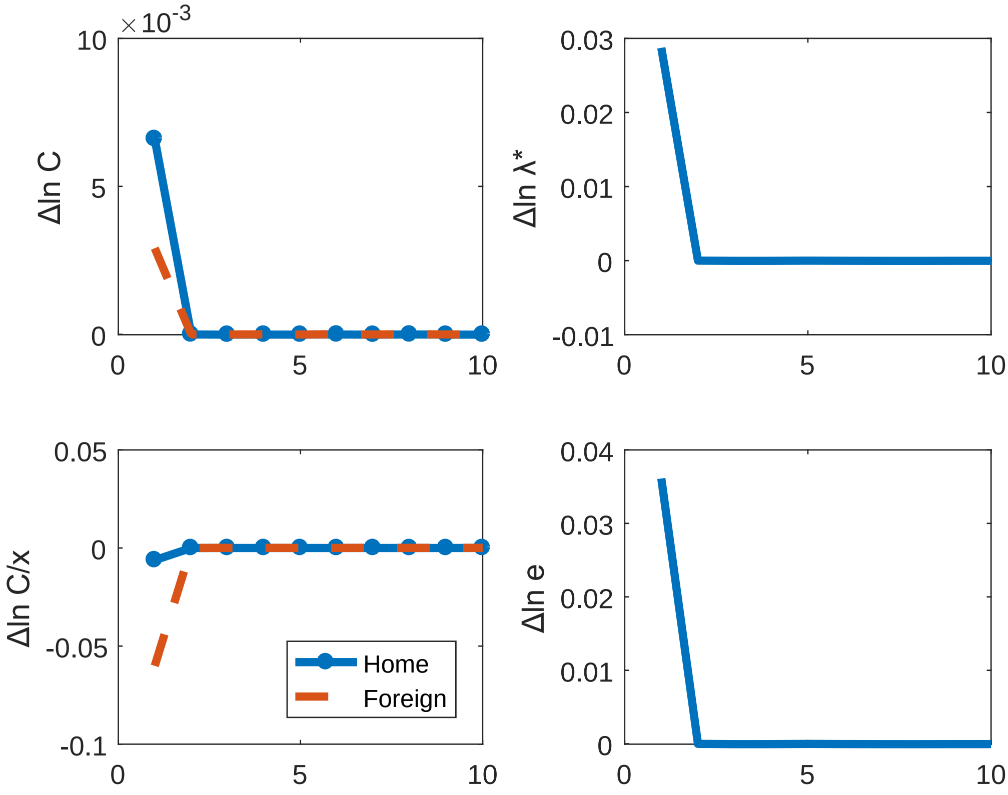
Figure 1: Long-Run Shock



The case of the short-run shock to the endowment of country 1 is shown in Figure 2. This shock affects endowment in period 1. This is a standard endowment shock. As endowment is increased, terms of trade deteriorate and the real exchange rate depreciates. Both domestic and foreign consumption rise with the former rising more strongly that the latter. The real exchange rate and the consumption differential (home minus foreign) are perfectly positively correlated as in the Backus-Smith puzzle. The behavior of the domestic consumption relative to the consumption of home endowment and the foreign consumption relative to exports from the home country is similar to the long-run shock. This is because these measures capture the substitution effect between home and foreign goods. Since the domestic terms of trade deteriorate in both cases, agents substitute foreign goods for domestic goods.

A positive shock to volatility of domestic endowment is now negative news for the home country. In line with perfect risk-sharing, foreign countries insure the domestic agent. Domestic consumption rises and foreign consumption falls upon arrival of the negative shock. The terms of trade improve and the real exchange rates appreciate to facilitate the adjustment. Due to the appreciation, the substitution effect favors foreign goods. Domestic consumption rises relative the consumption of the home endowment, and foreign consumption rises with respect to exports from the home country. As can be seen, the real exchange rate and consumption differential move in the opposite directions. Thus, long-run shocks and volatility shocks

Figure 2: Short-Run Shock



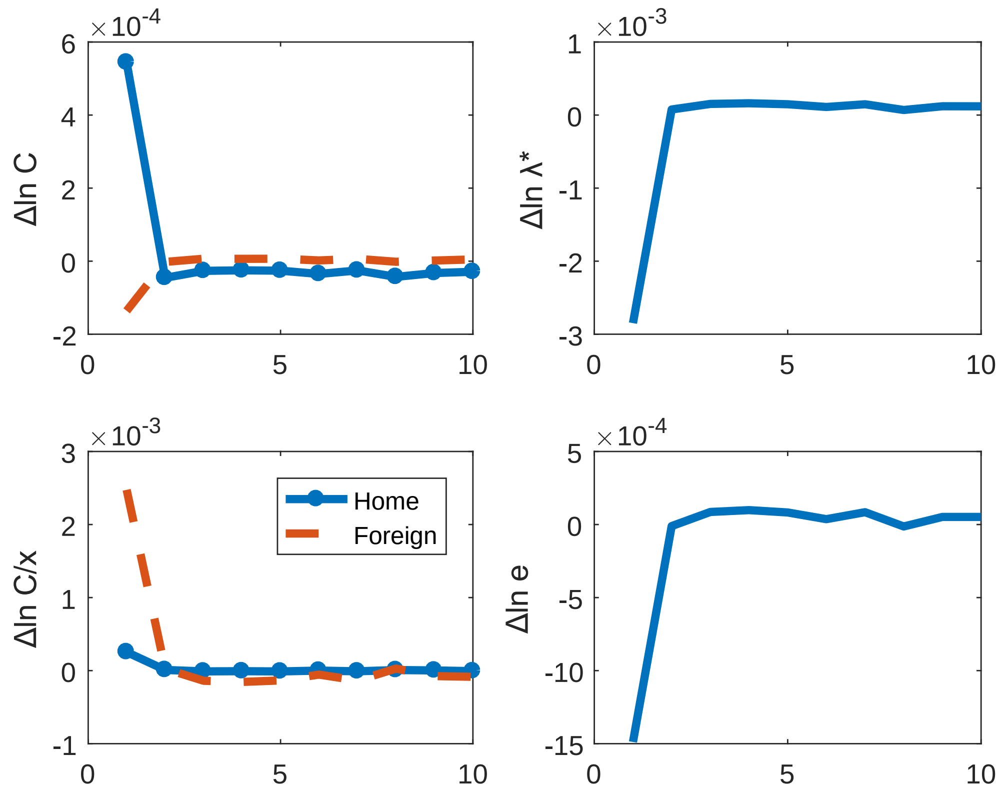
can potentially resolve the Backus-Smith puzzle.

Simulated correlations

To get a better insight into the correlation between various variables of interest, I simulate the model and report the simulated correlation in Table 2. I consider different combinations of shocks. The first row of the table shows correlations when the five countries are hit by the short-run shocks only. In this case, the real exchange rate, consumption differential, the optimal Pareto weights, and the relative consumption differential $(dln(C_i/x_i^i) - dln(C_j/x_j^i))$, are all perfectly positively correlated. In row 2, I turn on only long-run shocks. As can be seen from the table, the correlation between the real exchange rate and the consumption differential is negative now. Mixing the long-run and short-run shocks as in Colacito and Croce (2013) in row 3 helps to resolve the Backus-Smith puzzle. The correlation is low at 0.12. Going back to equation (22), we can see that this possible since the real exchange rate is highly correlated with the optimal Pareto weights.

The last row shows the correlations when the model is hit with short-run and volatility shocks only.

Figure 3: Volatility Shock



Despite the ability of volatility shocks to lower the correlation, the correlation is still high for the benchmark parametrization. Kollmann (2016) shows that the degree of risk aversion needs to be around 40 (much higher that 10, what is assumed for the calibration in Table 2) to produce a reasonably low correlation with volatility shocks.

Table 2: Simulated correlations

	$dln(C_i/C_j)$	$dln(C_i/x_i^i) - dln(C_j/x_j^i)$	$dln\lambda^*$	Shocks
$dlne_{ij}$	1	1	1	Short-run only
$dlne_{ij}$	-0.97	1	0.99	Long-run only
$dlne_{ij}$	0.12	1	0.87	Short-run and long-run
$dlne_{ij}$	0.98	1	1	Short-run and volatility

Independently of type of shocks, the correlation between the real exchange rate and the relative consumption differential $(dln(C_i/x_i^i) - dln(C_j/x_j^i))$ is always perfectly positive. This is because the measure captures the substitution effect with respect to real exchange rate movements. The next section turns to the empirical analysis of this prediction.

IV. EMPIRICAL ANALYSIS

Data

I consider 9 OECD countries for the empirical analysis: New Zealand, Australia, United States, Canada, Germany, United Kingdom, Japan, Sweden, and Norway. Since trade is important for risk-sharing and the optimal outcome is achieved by substitution among traded goods, I focus on main trading partners (among OECD countries) for the nine countries considered. Appendix B presents the list of country pairs and their abbreviations. The data are annual over the period 1973-2015. Nominal exchange rates are from the IMF IFS database; national currency per US dollar, end of the period. Real exchange rates are constructed using bilateral nominal exchange rates adjusted using CPI from the IFS. Consumption is final consumption expenditure of households in constant prices (OECD base year) from the OECD. Overall imports of goods and services in constant prices are from the OECD as well. Bilateral exports of goods are from the IMF Direction of Traded Statistics. Volume of exports is constructed using the value of exports, free on board, deflated by the corresponding unit value of exports index.

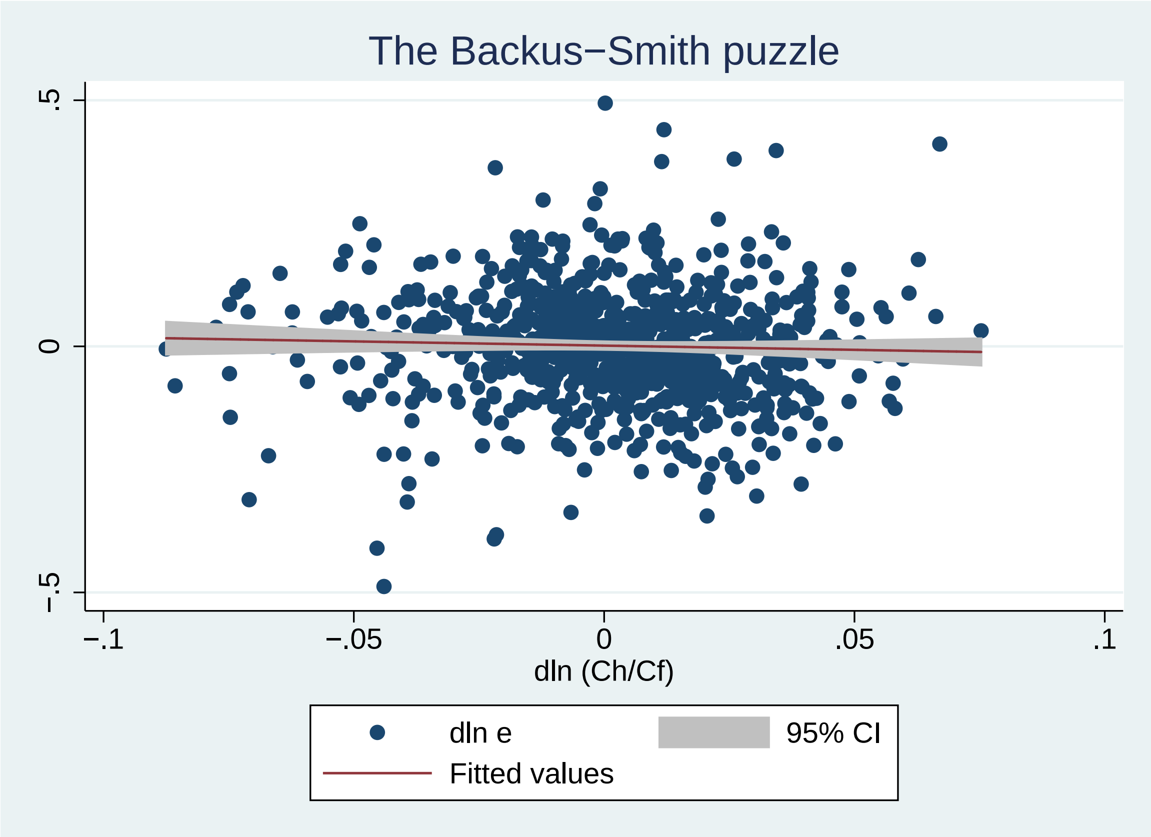
Analysis

Figure 4 presents the growth of the real exchange rates and the consumption differentials pooled over time and across country pairs. As can be seen there is insignificant negative correlation in the data. This is the manifestation of the Backus-Smith puzzle. Instead of perfect positive correlation we find low and negative correlation with our data.

Figure 5 focuses on testing the other implication of the model that the growth of the real exchange rates should be linked to the consumption differential relative to the growth of home consumption of the domestic endowment versus the growth of exports of domestic goods to foreign countries. This time the correlation is positive and significant but still remains low. The model predicts that it should be perfectly positive. Thus, the prediction of the model is rejected. Interestingly there is a lot of variation in the relative consumption measure; the volatility is higher than the volatility in the real exchange rate growth. Usually macroeconomic variables are found to be less volatile than real exchange rates. This is not the case here due to the inclusion of imports and exports which are very volatile in the data. In the left and right sides of the figure, there are some observations that can be influential in affecting the regression results.

To have a closer look at the relationship, I restrict the growth of the relative consumption measure by -0.5 and 0.5 in line with the real exchange rate deviations. Figure 6 demonstrates that there is still low

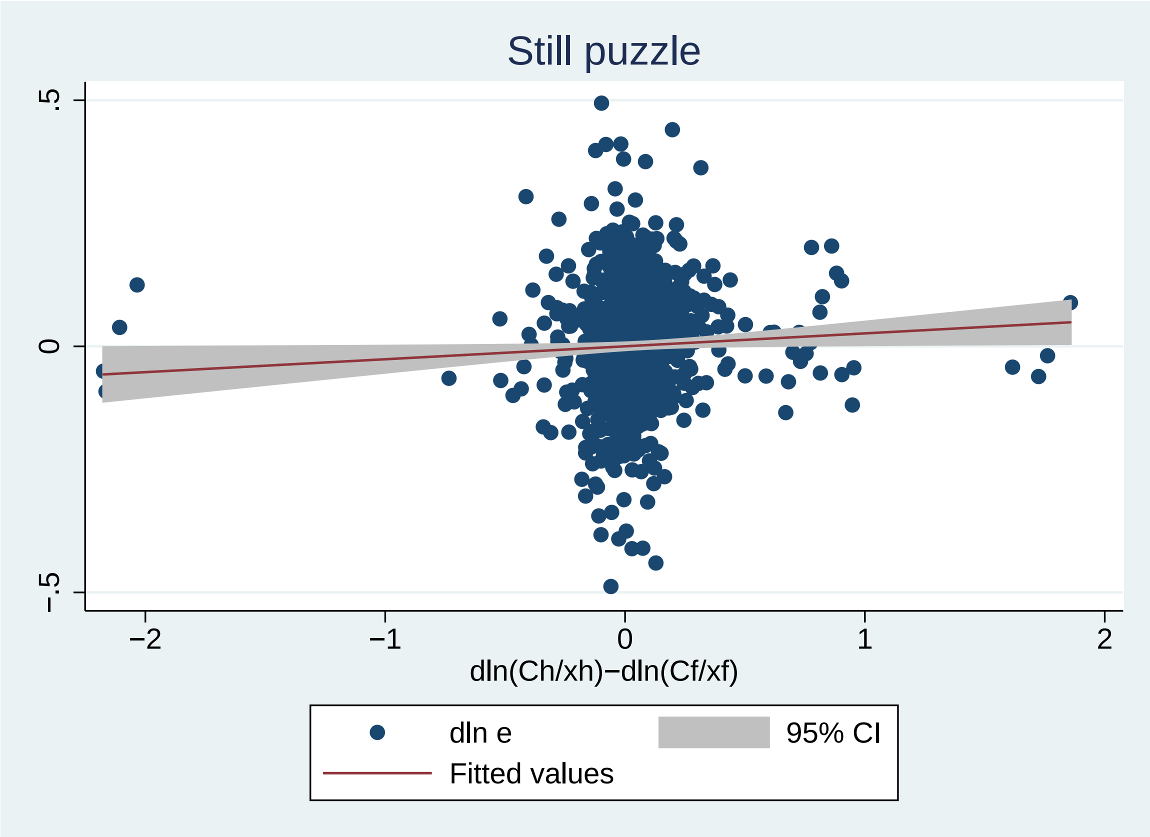
Figure 4: Consumption-real exchange rate anomaly



correlation even when influential observations are removed.

As will be discussed in Table 3, most of the explanatory power comes from cross-sectional variation. To observe the fit of the model across observations, Figure 7 presents observations for each pair averaged across time. Not surprisingly, we observe positive and negative deviations for the real exchange rates. This would be consistent with some countries receiving positive news in the sample, and some countries receiving negative news. The relative consumption measure is on the other hand is always positive when averaged over time. This would be consistent with all pairs receiving goods news, which is not possible. In the sample, consumption (ΔlnC_i and ΔlnC_j) grew at 2.3% rate over the period on average. Consumption of domestic endowment (Δlnx_i^i) felt at 0.3%, and exports of domestic goods (Δlnx_j^i) grew at 3.5%. Mostly due to the growth of exports, all of the relative consumption measures ($\Delta lnC_i/x_i^i - \Delta lnC_j/x_j^i$) are positive in the sample on average. In the right part of the figure, we see pairs for Sweden. In the sample, consumption for Sweden grew at 1.8%, while for its major trading partners in OECD, consumption grew at 1.7%. Consumption of domestic endowment felt at 9.3% and exports of Swedish goods grew at 3.4%. This would be consistent with Sweden receiving good news for endowment growth (either short-run, long-run or reduced volatility). According to the model, the Swedish real exchange rate is expected to depreciate by 10-13% in the sample.

Figure 5: Modified consumption-real exchange rate anomaly



We observe only modest depreciation by 1% and even an appreciation with respect to Germany.

The black line on the figure represents the prediction of the model assuming the elasticity of substitution is 1.5 among traded goods. As can be seen, with the exception of two pairs, most observations do not fall on the line. The fit can be improved somewhat for higher elasticities of substitution (the line would become flatter) but the model would still fail to explain observations when the real exchange rates appreciate. The cross-sectional fit of the model is slightly better than over time, but still remains low.

Figure 6: Closer look at the modified consumption-real exchange rate anomaly

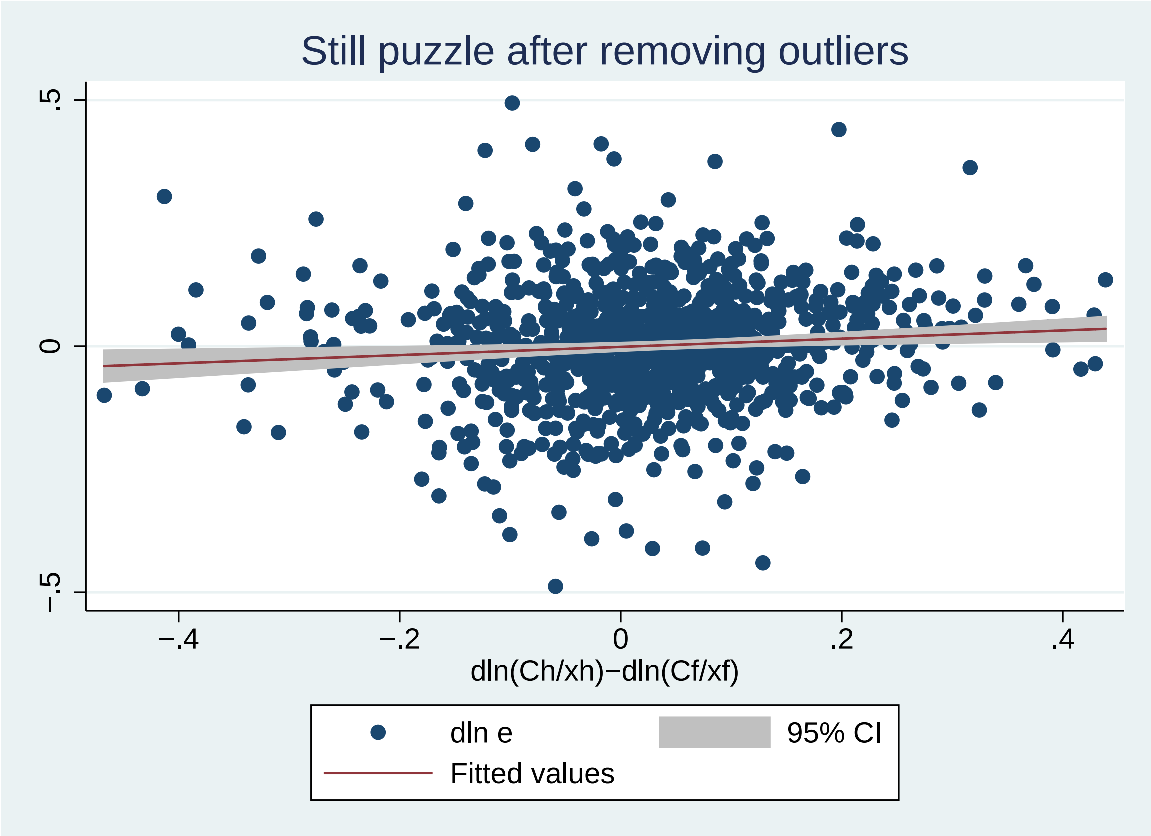
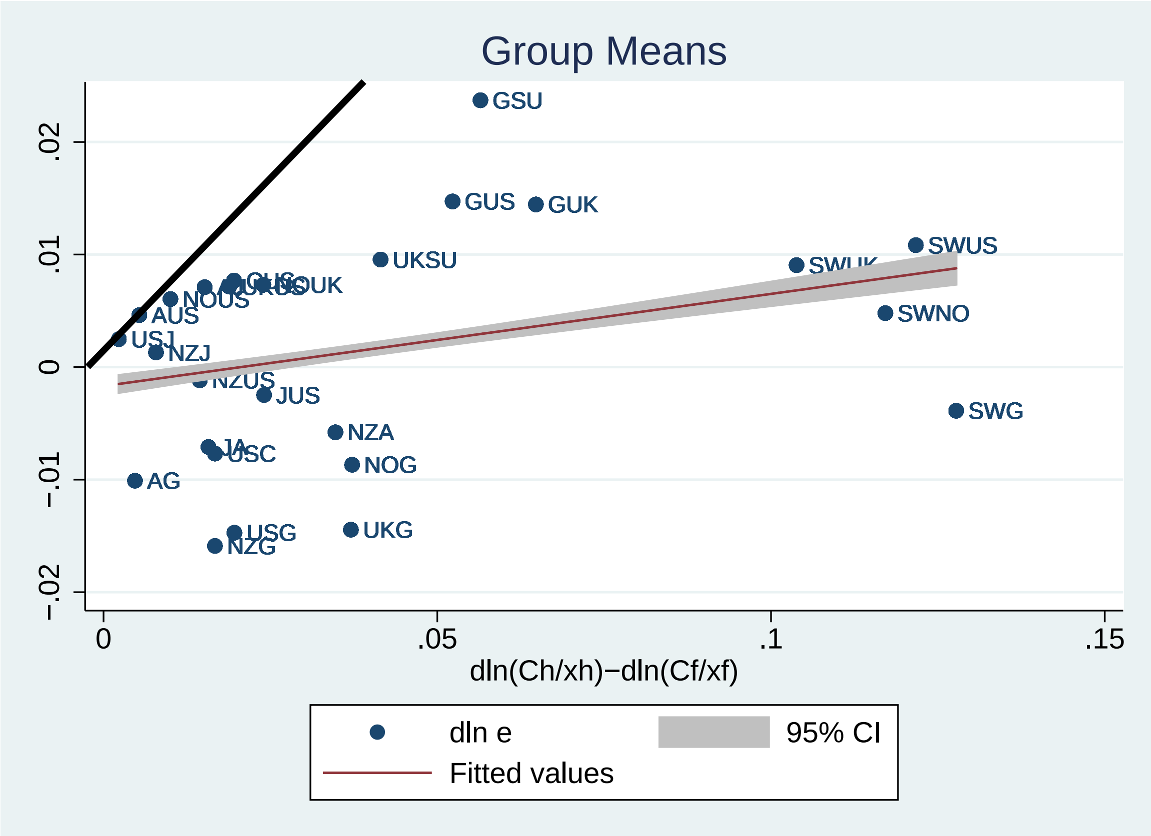


Figure 7: Cross-sectional variation over the period 1973-2015



Panel Data Analysis

Panel data analysis is useful in the presence of unobserved heterogeneity. Equation (21) shows that the real exchange rate, when analyzed in levels, depends on trade costs and consumption shares which are heterogeneous among the countries. I analyze the relationship in growth rates so this unobserved heterogeneity disappears provided it is constant over time. An argument can be made that trade costs decrease over time. In this case, fixed effects show up even when analyzing the relationship in growth rates assuming that trade costs decline at a constant rate over time. Table 3 presents the panel data estimation.

Table 3: Panel data estimation

	Pooled	Between-effects	Fixed effects	Fixed effects
$dln(C_i/x_i^i) - dln(C_j/x_j^i)$	0.026	0.079	0.025	-0.01
$p - value$	0.077	0.086	0.103	0.560
Time dummies			No	Yes
$R^2_{overall}$	0.003			0.090
$R^2_{between}$		0.118		0.126
R^2_{within}			0.003	0.091
p-value F-test				
all cross fixed effects=0			0.26	0.30

Consistent with Figure 5, the first column shows that there is very weak relationship between the real exchange rate growth and modified consumption measure. The relationship is positive and statistically significant at 10%, but the coefficient is small at 0.026 and R^2 is very low at 0.003. Averaging data for each pair over time, between-effects, shows that the model can explain 11.8% of the cross-sectional variation. The coefficient for the modified consumption measure is at 0.086 and is statistically significant at 10%. This is in line with Figure 7. The next two columns look at how the relationship changes by introducing fixed effects. Column 3 considers only cross-sectional fixed effects to account for trade costs assuming they are not time-varying when considering growth rates. The results are similar to the pooled estimation. This not surprising since the joint hypothesis that all coefficients corresponding to fixed effect dummies are not statistically significant is not rejected with p-value at 0.26. Allowing for time fixed effects (in addition to cross-sectional fixed effects) makes the coefficient for the modified consumption slightly negative at -0.01. It becomes statistically insignificant. Introduction of fixed effects makes the relationship weaker.

Predictive Regressions

Several variables are found to be predictive of the long-run growth component (Bansal et al., 2013, Colacito and Croce, 2013, among others). In particular I follow Colacito and Croce (2013) in using the following set of variables for the United Kigdom:

pd	price-dividend ratio
pd, rf	as above plus the risk free rate
pd, rf, ΔlnC, Δcy	as above plus consumption growth, and (log) consumption-output ratio

All variables are lagged one period in the regressions. The data on predictable variables and consumption come from Colacito and Croce (2011). To this set of variables, I add the consumption of domestic endowment $\Delta ln(x_{UK}^{UK})$ and UK's exports $\Delta ln(x_{US}^{UK})$. The sample size is 1948-2006.[4]

The first column in Table 4 shows the p-value associated with the hypothesis that predictive variables are not significant. There is evidence that the relative consumption growth in UK and US is predictable. This is consistent with the long-run risk literature (Colacito and Croce, 2011). the associated R^2 are presented in the second column. They demonsrate that a substantial amount of variation is explained by variables associated with the long-run component.

In the third and forth columns, I analyze if the same variables affect my measure of the consumption differential, in which consumption is adjusted by consumption of domestic endowment or exports. We can see again that the long-run shock variables are driving the modified consumption differential in line with the theory. These variables are statistically and economically significant. Finally, in the last two columns I consider if the same variables affect the growth of the real UK-US exchange rate. Here, we find no impact. The result is not surprising in the light of the exchange-rate disconnect puzzle (Obstfeld and Rogoff, 2000).

Table 4: Predictive regressions

	$\Delta ln(C_{UK}/C_{US})$		$\Delta(ln(C_{UK}/x_{UK}^{UK}) - ln(C_{US}/x_{US}^{UK}))$		$\Delta ln(Q_{UKUS})$	
	F-stat p-value	R^2	F-stat p-value	R^2	F-stat p-value	R^2
pd, risk free	0.002	0.21	0.002	0.21	0.842	0.00
all	0.004	0.25	0.010	0.22	0.638	0.05
pd only	0.035	0.08	0.000	0.21	0.721	0.00

[4] The sample used in Colacito and Croce (2011) is 1929-2006. However, UK's exports and imports are not available over 1939-1946.

This finding explains why we do not find much support for the model. If the long-run shocks drive the consumption differentials and other macroeconomic variables, but are not affecting the real exchange rates much, the Backus-Smith puzzle remains unresolved.

V. CONCLUSIONS

It has been proposed in the literature to resolve the consumption-real exchange rate anomaly by introducing Epstein-Zin preferences and by allowing a slowly-moving predictable component or by allowing time-varying volatility in in the consumption growth. The success of this literature is based on the ability to break the link between the real exchange rate and the consumption differential. In response to long-run shocks or volatility shocks the consumption differential and the real exchange rate move in opposite directions, thus weakening the link between the two.

This paper argues that these movements in the opposite directions are associated with the reallocation of traded goods among the countries. This implies that the real exchange rate is linked to a broader measure of consumption in which the consumption differential is considered relative to the consumption of domestic endowments and exports. This broader measure of consumption fails to be linked to the real exchange rates empirically as shown in this paper. The consumption-real exchange rate anomaly remains unresolved when this broader measure of consumption is considered.

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APPENDIX B

The list of country pairs used in the empirical analysis.

Table B1: Country pairs

Source	Destination	Abbrevation
New Zealand	Australia, Germany, Japan, United States	NZA, NZG, NZJ, NZUS
Australia	Germany, Japan, United States	AG, AJ, AUS
United States	Germany, Japan, Canada	USG, USJ, USC
Canada	United States	CUS
Germany	Switzerland, United Kingdom, United States	GSU, GUK, GUS
United Kingdom	Germany, Switzerland, United States	UKG, UKSU, UKUS
Japan	Australia, United States	JA, JUS
Sweden	Germany, Norway, United Kingdom, United States	SWG, SWNO, SWUK, SWUS
Norway	Germany, United Kingdom, Unites States	NOG, NOUK, NOUS