

# Spatiotemporal Model for the Optimal Harvesting Schedule of Biological Stocks at Risk of Disease Transmission

By

Trevor A. Knechtel

A Thesis Submitted in Partial Fulfillment of the Requirements for the  
Degree of Bachelor of Sciences, Honours  
in the Department of Economics  
University of Victoria  
April 2018

Supervised by Dr. Daniel Rondeau

For

Dr. Elisabeth Gugl, Honours co-advisor

Dr. Martin Farnham, Honours co-advisor

# OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

# OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

## Abstract

This paper defines the optimal adaptive harvesting schedule of a biological stock which is under threat of disease transmission. Consider a firm that manages a growing stock which is sold at a constant competitive market price. The stock is broken up into discrete subpopulations and a disease is present at one subpopulation. With some probability the disease will transmit from the infected subpopulation to the healthy ones. I describe an adaptive harvesting schedule which maximizes the net present value (NPV) of each healthy subpopulation. Once a disease appears in one subpopulation the optimal management schedule of any other subpopulation is immediately different from the disease-free steady state. All subpopulations are harvested more aggressively than at the disease-free steady state. The level of this increased harvesting is defined by the probability of disease transmission, the biological effects of the disease, and the economic factors that define the value of the stock.

*Keywords:* Epidemiology, Aquaculture, Resource Management

# OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

## Table of Contents

1. Introduction.....	4
2. Literature Review.....	6
3. The Model.....	9
Steady States.....	11
Disease Dynamics.....	12
Computing Numerical Solutions.....	13
4. Results.....	15
Numerical Solution.....	15
Static Analysis.....	19
5. Discussion.....	19
Additions to the Literature.....	19
Policy Implications.....	20
Further Research.....	20
6. Conclusion.....	21
7. Works Cited.....	22
8. Appendices and Figures.....	25
Appendix A.....	25
Appendix B.....	26

# OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

## Spatiotemporal Model for the Optimal Harvesting Schedule of Biological Stocks at Risk of Disease Transmission

### **Introduction**

In this paper I use a spatiotemporal model to define the optimal harvesting strategy of biological stocks at risk of becoming infected by an infectious disease. The appearance of disease outbreaks are challenging if not impossible to predict. Requiring management organizations and private firms to prepare to adjust their actions immediately once an outbreak begins. My research is directly applies to the management of aquaculture, fisheries, forestry, and farming.

There are countless examples of disease outbreaks which have devastated biological stocks. Causing large decreases in stock populations and devastating economic. In British Columbia the impacts of the Mountain Pine Beetle are well documented. It is estimated that the beetle destroyed 50% of all Lodgepole Pine in the province since 1990. In Europe the return of Foot and Mouth Disease on pig farms in the early 2000's caused the loss of millions of dollars and is still a hot topic in farming and management communities. In Norway, aquaculture, a rapidly growing industry, has suffered from multiple infectious disease outbreaks. In all cases disease result in the loss of economic value, decreases in biological stocks, and losses in consumer confidence.

Disease outbreaks are commonly managed in one of three ways. The first common tactic is to continue harvesting at the disease-free steady state level. This is not optimal because the future expected value of the stock is lower due to the possibility of becoming infected. The trade-off between current and future benefits tips in favour of harvesting more today. As a result

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

harvesting should increase as soon as a disease appears increasing further as the risk of infection increases.

The second common practice is a complete moratorium on harvesting. Stopping harvesting completely is detrimental to the value of the stock because it increases the population under threat of infection. I show increasing the level of harvesting, in order to reduce the susceptible stock, is always a better strategy. By harvesting more today the cost of the disease decreases in the future.

The third common strategy is to invest in methods of risk reduction. In some cases this is a necessary process to protect stocks from diseases which cause large economic and biological losses. If the costs associated with reducing the risk of a disease are lower than the cost that the disease is likely to have on the stock, then this is the correct action. The cost of risk reduction whether through antibiotics, advancements in sanitizing equipment, or the engineering of new holding pens, must be measured relative to the cost of the disease when the stock is managed optimally. In most cases investment in risk reduction occurs because it is less expensive than the losses associated with the disease if the stock is managed as if it were in steady state throughout the time leading up to becoming infected. Using the value associated with steady state harvesting levels as the opportunity cost of investing in risk reduction methods overstates the benefits of reducing the risk of disease transmission.

My research identifies the flaws in the first two common practices. The third, investing in risk reduction needs to be more carefully measured against the value of an optimally managed stock in order to decide if it is the correct net present value (NPV) maximizing action. This research serves as a litmus test for whether or not investment in risk reduction is the optimal choice.

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

In order to identify the optimal decision between harvesting and investing in prevention measures I model the NPV maximizing harvesting schedule. This model applies to any growing stock, broken into discrete subpopulations, with a competitive market price. The results describe an adaptive harvesting schedule which adjusts the level of harvesting each period. Once a disease appears in one subpopulation the optimal management schedule of each other subpopulation is immediately different from the disease-free steady state. All subpopulations are drawn down to a lower stock level which is dependent upon the exact probability of infection and the biological effects of the disease.

This research has its foundations in the work of Conrad and Rondeau (2015). Working closely with biologists and Australian fisheries management to model the optimal harvesting schedule of subpopulations of abalone along the coast of the State of Victoria, Australia. Abalone Viral Ganglioneuritis (AVG), a deadly virus which caused up to a 90% loss of harvestable stock on infected reefs, transmitted along the coast through the water column from reef to reef. Conrad and Rondeau make one restrictive assumption; the disease only moves from one reef to its immediate neighbor. The model presented in this paper applies more generally to any disease and any arrangement of subpopulations.

Generalizing their model, I find similar results to Conrad and Rondeau. The correct harvesting procedure when a disease appears at another subpopulation is to immediately increase the level of harvesting. Increasing harvesting further anytime the probability of becoming infected increases. The implications of my model apply to the optimal management of biological stocks at risk of infectious diseases and for planning the locations and distribution of future, high-risk, farming and aquaculture operations.

## Literature Review

Research at the intersection of economics and epidemiology focuses on a micro analysis of the motion of diseases through individuals in a population. The most common approach is to divide a population into groups of individuals who are susceptible to the disease, individuals already infected, and in some cases individuals who are recovered. Models structured this way are called susceptible-infected (SI) or susceptible-infected-recovered (SIR) models. Fenichel *et al.* (2010) use a basic SI model to demonstrate the effects that different ecological and economic parameters have on the optimal harvesting level of a diseased stock. The disease transmits through direct contact between infected and susceptible individuals. Horan and Wolf (2005) and Horan *et al.* (2010) employ an SIR model by introducing recovery from disease to further study the dynamics of diseased wildlife stocks. They describe the effects that human involvement and policy changes have on infected stocks of wild white-tailed deer in Michigan.

The factors that drive disease transmission between discrete subpopulations are climate, geography, and many biological and chemical factors specific to the species and disease of interest. Earn and Tien (2009) include waterborne transmission in a classic SIR model where infected individuals can pass a disease to susceptible individuals through direct contact or by way of infecting a shared water supply. In the waterborne pathogen model Earn and Tien find the biology of the disease and the water environment are key factors of disease transmission. Conrad and Smith (2012) describe how a 'patchy' population model can show the efficiency and inefficiency of marine protected areas on fish populations. Horan *et al.* (2005) use a patchy model to study the dynamics of a disease transmitting between two subpopulations which interact through migratory behavior. I apply a basic patchy SI model in this paper.

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

The major difference between the spatiotemporal model I propose and patchy SI and SIR models most commonly used in epidemiology is the scope at which I focus. SI and SIR take what an economist might call a micro look at the effects on individuals and their interactions that relate to disease spread while the spatiotemporal model in this paper takes a broader more macro approach. The effects of the disease are assumed to be immediate and growth and harvesting occur in discrete time. It is discrete and aggregate level information that managers require to make informed harvesting decisions.

To motivate my modelling of disease transmission further, I provide examples of industries which are commonly victims of infectious diseases.

First, aquaculture, a rapidly expanding industry, now supplies half of all fish for human. Despite rapid growth aquaculture faces criticism for the abundance of diseases which affect stocks of penned fish. Rabe and Salama (2013) investigate the environmental factors which affect disease transmission in Scottish Atlantic Salmon raised in open water pens. Currents, winds, temperature, light exposure, and distance between host sources all play important roles in the probability of disease transmission between aquaculture sites. Garver *et al.* (2013) find that Infectious Hematopoietic Necrosis Virus (IHNV), a salmonid virus that has devastated salmon farms off the coast of British Columbia, transmits between open cage salmon pens. The rate of transmission of IHNV depends on sunlight, currents, distance, and concentration of infected material. These studies demonstrate the challenges management faces when dealing with diseases in the aquaculture industry.

Second, my model applies to waterborne diseases in shellfish, mollusks, and other filter feeders. Bidegain *et al.* (2016) describe a few special cases that exist for disease transfer in filter feeders in marine environments. Filter feeders in large numbers may be able to reduce the

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

likelihood of a disease outbreak. Their ability to filter particulate from can allow large groups of filter feeders to filter infected material from the water avoiding possible outbreaks. In smaller populations some filter feeders may be more susceptible to disease. Filter feeders are also likely to pass on infected material in their environment after they die similar to terrestrial animals.

Unlike pelagic fish species who fall to the ocean floor away from other susceptible fish. These two cases offer some interesting caveats for future research focused on specific application of a population dependent spatiotemporal model to these species.

Finally, my model applies to the infection of plants, such as parasitic infestations of tree stands, a major topic in Canada due to the mountain pine beetle infestation in Canadian forests. Environmental factors responsible for the spread of pine beetles in Canadian forests are becoming better understood. With improving knowledge comes more accurate predictions of the probability that specific tree stands will become infected. This understanding combined with my model could result in more efficient management of forestry operations, decreasing losses attributable to the pine beetle.

These three examples are only a small subset of the number of instances in which modelling disease transmission is of value. They also demonstrate the reliance management has on other fields of research to determine the environmental and biological factors of disease transmissions in biological stocks. Collaboration between management and scientists will be a requirement to have optimally managed stocks.

### **The Model**

I create a spatial model where a population is broken into discrete subpopulations. It is not necessary, but for simplicity I will assume that all subpopulations are homogeneous.

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

Let the stock of the  $i^{\text{th}}$  subpopulation in time period  $t$  be denoted by the variable  $X_{i,t}$  and the amount harvested from the  $i^{\text{th}}$  subpopulation in period  $t$  be denoted by  $y_{i,t}$ .  $X_{i,t}$  can take on any value from zero to  $K$  and  $y_{i,t}$  can take on any value from zero to  $X_{i,t}$ .  $K$  defines the carrying capacity of each subpopulation. Each period the stock level of a subpopulation,  $X_{i,t}$ , grows at some rate  $F(X_{i,t})$ :

$$(1) \quad X_{i,t+1} = X_{i,t} + F(X_{i,t}) - y_{i,t}$$

In this paper I will use a logarithmic growth function of the form:

$$(2) \quad F(X_{i,t}) = r * X_{i,t} * \left(1 - \frac{X_{i,t}}{K}\right)$$

$r \in [0,1]$  is the intrinsic growth rate. This form of growth function is common in fisheries, shellfish, and forestry. Equation (2) describes the law of motion and it serves as the connection between one period and the next. It will be crucially important in understanding the trade-off between present and future costs and benefits. This definition also shows the timing of events that will occur in the model. Growth of the stock occurs before harvesting in each time period.

I denote the market price of each unit harvested as  $p$ . Revenue each period for a given subpopulation is price multiplied by the amount harvested. I define fixed costs as  $FC$ . I define variable costs by the equation:

$$(3) \quad VC = c * y_{i,t} * \ln \left[ \frac{X_{i,t} + F(X_{i,t})}{X_{i,t} + F(X_{i,t}) - y_{i,t}} \right]$$

Equation (3) yields increasing marginal costs. Increasing marginal costs capture the idea that it is more challenging to harvest from a stock that has a lower population than from a stock that has a higher population. (3) Also implies that it is infinitely costly to reduce a stock to zero. Such high costs may or may not be justified. It is unlikely that (3) is a perfectly accurate representation of

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

true costs due to the infinite limit as  $y_{i,t}$  approaches  $X_{i,t}$ . However, I find that the optimal solution remains well above this limit so it does not have negative impacts on the result.

Combining fixed costs, variable costs, and revenue, profits from the  $i^{\text{th}}$  subpopulation in time  $t$  are defined as:

$$(5) \quad \pi_{i,t} = p * y_{i,t} - FC - c * y_{i,t} * \ln \left[ \frac{X_{i,t} + F(X_{i,t})}{X_{i,t} + F(X_{i,t}) - y_{i,t}} \right]$$

Using these equations we can define the value of harvesting from each subpopulation at any time period.

### Steady States

In most cases of harvesting from a growing biological stock there exists a steady state stock and harvesting level. In my model the steady state stock level of each subpopulation will serve as the initial condition when a disease first appears. All subpopulations are initially at their steady state levels and after the disease arrives, they return to a new steady state level. Solving for a steady state stock and harvest level in discrete time requires a simple application of optimal control using a Lagrange function. Equations (1) and (5) give us the constraint and the value function of the Lagrangean:

$$(6) \quad L = \sum_{t=0}^{\infty} \sum_{i=1}^N \beta^t * \pi_{i,t} + \beta^{t+1} * \lambda_{t+1} * [X_{i,t} + F(X_{i,t}) - y_{i,t} - X_{t+1}]$$

$\beta$ , the discount factor, is defined as  $\frac{1}{1+\delta}$  where  $\delta$  is the social discount rate.

In steady state there is no change in harvesting or stock levels between periods.

In steady state:

$$X_{i,t} = X_{i,t+1} = \bar{X}$$

$$y_{i,t} = y_{i,t+1} = \bar{y}$$

$$\lambda_{i,t} = \lambda_{i,t+1} = \bar{\lambda}$$

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

Resulting in the three first order conditions:

$$(7) L_X = 0 = -c\bar{y}\ln\left[\frac{1+r-\frac{2r\bar{X}}{K}}{\bar{X}+r\bar{X}-\frac{r\bar{X}^2}{K}}\right] + \beta\bar{\lambda}\left[r - \frac{2r\bar{X}}{K}\right]$$

$$(8) L_y = 0 = -c\ln\left[\frac{\bar{X}+F(\bar{X})}{\bar{X}+F(\bar{X})-\bar{y}}\right] - c\frac{\bar{y}}{\bar{X}+F(\bar{X})-\bar{y}} - \beta\bar{\lambda}$$

$$(9) L_\lambda = 0 = \bar{X} + F(\bar{X}) - \bar{y} - \bar{X}$$

This system of three equations and three unknowns can be solved for the steady state values of  $\bar{X}$ ,  $\bar{y}$ , and  $\bar{\lambda}$ . The steady state level is the initial condition of all subpopulations for the analysis of the optimal harvesting schedule.  $\lambda$  is not a focus in this analysis but it is still important to understand how to interpret it.  $\lambda_{i,t}$  is the value of adding one unit of stock to subpopulation  $i$  at time  $t$  given the harvesting schedule is adjusted to optimize over all periods after the additional unit has been added.

### Disease Dynamics

The appearance of a disease removes subpopulations from steady state. In this model a disease has two effects on the stock. A mortality rate, and a prolonged decrease in the intrinsic growth rate. The immediate mortality rate, denoted by  $M$ , describes the proportion of the subpopulation that dies during the period infection occurs. A subpopulation that becomes infected has a stock at the end of the period of infection equal to:

$$(10) X_{i,t+1} = (1 - M) * [X_{i,t} + F(X_{i,t}) - y_{i,t}]$$

The decrease in population growth rate is defined by a decrease in  $r$  post disease. The lower intrinsic growth rate post disease is denoted by  $r_I$ . A decrease in intrinsic growth rate changes the growth function. I denote the growth function post infection as  $F_I$ . Any infected subpopulation exhibits growth such that:

$$(11) X_{i,t+1} = X_{i,t} + F_I(X_{i,t}) - y_{i,t}$$

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

The probability of disease transmission between any two subpopulation  $i$  and  $j$  is defined as  $P_{i,j}$ . I study the case where the probability of transmission is exogenously determined by environmental factors.

$$(12) P_{i,j} = \alpha_{i,j}$$

$\alpha_{i,j}$  is a constant determined by the environmental factors of subpopulations  $i$  and  $j$ . This implies that when two subpopulations are infected with a disease the cumulative probability that a third subpopulation becomes infected is equal to:

$$\alpha_{1,3} + \alpha_{2,3} - (\alpha_{1,3} * \alpha_{2,3})$$

The literature suggests that  $\alpha$  depends on the disease and species in question as well as climate and geographical factors. Temperature, distance, sunlight, altitude, air/water currents, and countless other exogenous factors define  $\alpha$ . Initially, I treat distance as a proxy for these factors. In reality  $\alpha$  is determined by chemical and biological research on the stock and disease of interest.

### **Computing Numerical Solutions**

Computing numerical solutions requires large computational power. In order to have exact results available for this paper I have solved for scenarios which include four subpopulations. Any number of subpopulations can be analyzed but for simplicity a smaller number is easier to present here.

To compute numerical solutions outside of steady state I use backwards induction. Specifying a time period,  $T$ , as the terminal period the solution no longer requires an infinite time horizon. In determining the value of an infected stock which becomes infected with some stock  $X_{i,t}$  by the time the terminal period the system has reached the post disease steady state given the post disease growth function. The value of remaining in this steady state from period  $T$  to infinity

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

can be computed and is used as the salvage value of the system at time  $T$ . It is now possible to compute the solution to the function:

$$\begin{aligned} & MAX_y L_{inf}(X_{i,0}) \\ & = \sum_{t=0}^T \sum_{i=1}^N \beta^t * \pi_{i,t} + \beta^{t+1} * \lambda_{t+1} * [X_{i,t} + F_1(X_{i,t}) - y_{i,t} - X_{t+1}] + \beta^{T+1} S(X_{i,T+1}) \end{aligned}$$

For any initial  $X_{i,0}$ . The solution to this equation would be a sequence of harvesting amounts which maximize the NPV of an infected subpopulation.

Once the value of an infected subpopulation with any possible initial stock level is computed the process moves one step back to the state of the world where all subpopulations except for one is infected. In this state of the world the Lagrange function has the form:

$$\begin{aligned} & MAX_y L_{1,2,3}(X_{i,0}) \\ & = \sum_{t=0}^T \sum_{i=1}^N \beta^t * \pi_{i,t} + \beta^t * P_i * L_{inf}(X_{i,t+1}) * \lambda_{t+1} * [X_{i,t} + F(X_{i,t}) - y_{i,t} - X_{t+1}] + \beta^{T+1} L_{inf} \end{aligned}$$

$P_i =$  The cumulative probability of all  $P_{j,i}$

Maximizing  $L_{1,2,3}(X_{i,0})$  with respect to  $y$  results in the optimal harvesting schedule of subpopulation 4 over  $T$  periods when subpopulations 1, 2, and 3 are all infected for any initial stock level  $X_{i,0}$ . Continuing the backwards induction process until we arrive in the state of the world where only the first subpopulation is infected results in the optimal management strategy of subpopulation 4 in every possible state of the world.

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

$$MAX_y L_1(X_{i,0} = \bar{X})$$

$$= \sum_{t=0}^T \sum_{i=1}^N \beta^t * \pi_{i,t} + \beta^{t+1}$$

$$* [P_{1,2} * (1 - P_{1,3}) * (1 - P_{1,4}) * L_{1,2}(X_{i,t+1}) + P_{1,3} * (1 - P_{1,2}) * (1 - P_{1,4}) * L_{1,3}(X_{i,t+1})$$

$$+ P_{1,2} * P_{1,3} * (1 - P_{1,4}) * L_{1,2,3}(X_{i,t+1}) + P_{1,4} * L_{inf}(X_{i,t+1})] + \beta^{t+1} * \lambda_{t+1}$$

$$* [X_{i,t} + F(X_{i,t}) - y_{i,t} - X_{t+1}] + \beta^{T+1} S(X_{i,T+1})$$

To numerically solve this system I use Wolfram Mathematica software.

### Results

My results agree with Conrad and Rondeau (2015)'s model that is now a special case of my more general result. I find if a disease is present in a group of subpopulations the harvesting schedule should be adjusted immediately, increasing harvesting at first and maintaining a lower stock level. The size of the increase in harvesting level depends on all of the parameters but is especially sensitive to the disease mortality rate and the probability of disease transmission.

### Numerical Results

In this section I present a detailed solution examining one set of possible parameters. The solution that follows uses the parameters in Table 1.

Table 1

Carrying Capacity of Each Subpopulation	<i>K</i>	100
Intrinsic Growth Rate Pre-Disease	<i>r</i>	0.30
Intrinsic Growth Rate Post-Disease	<i>r<sub>l</sub></i>	0.18
Fixed Costs	<i>FC</i>	300
Market Price	<i>p</i>	3000
Discount Rate	<i>δ</i>	0.01
Mortality Rate	<i>M</i>	0.40
Marginal Cost Constant	<i>c</i>	23431

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

Steady State Stock Pre-Disease	$X_{0SS}$	39.2882
Steady State Stock Post-Disease	$X_{1SS}$	22.2293
	$P_{i,j}$	$\alpha_{i,j}$
Probability of Transmission from Subpopulation i to j	$P_{1,2}$	0.30
	$P_{1,3}$	0.01
	$P_{1,4}$	0.001
	$P_{2,3}$	0.30
	$P_{2,4}$	0.01
	$P_{3,2}$	0.30
	$P_{3,4}$	0.30

I focus on the management of subpopulation 4 assuming that a disease first arrives at subpopulation 1. I find it useful to refer to Figure 1 to get a sense of the possible disease transmission pathways. The longer an arrow between two subpopulations is, the lower the probability of transmission between them.

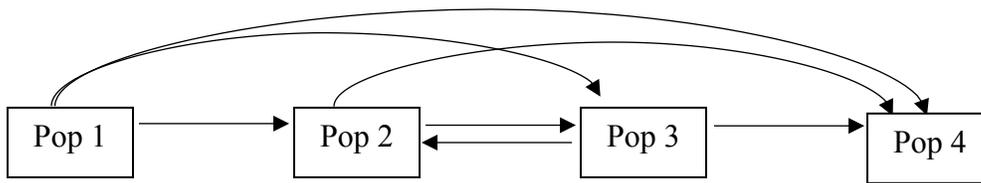
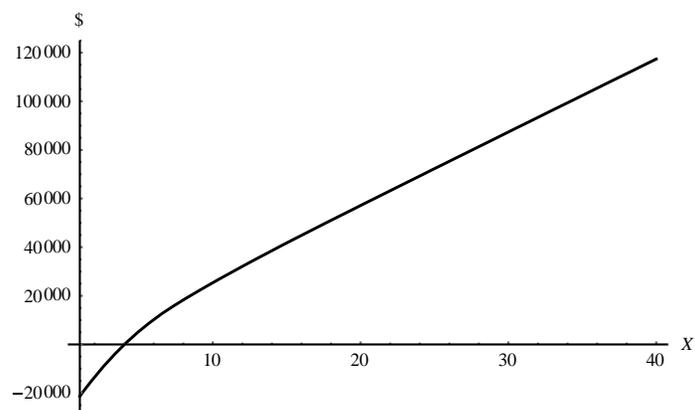


Figure 1. Visualization of disease transmission pathways.

It is possible to construct any number of arrangements of subpopulations and transmission connections in order to fit any industry, disease, or species. In this case, there are six possible states of the world from the perspective of subpopulation 4. First, trivially, the disease has not appeared yet and no populations are infected. Second, the disease has arrived at subpopulation 1 and remains there. Third, subpopulation 1 and 2 are both infected. Fourth, subpopulations 1 and 3 are infected. Fifth, subpopulations 1, 2, and 3 are infected. And finally, subpopulation 4 is infected. With the focus being on subpopulation 4, once it becomes infected the state of all other subpopulations is no longer relevant to its management. Using backwards induction we start by solving the optimal harvesting schedule of subpopulation 4 once it has

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS



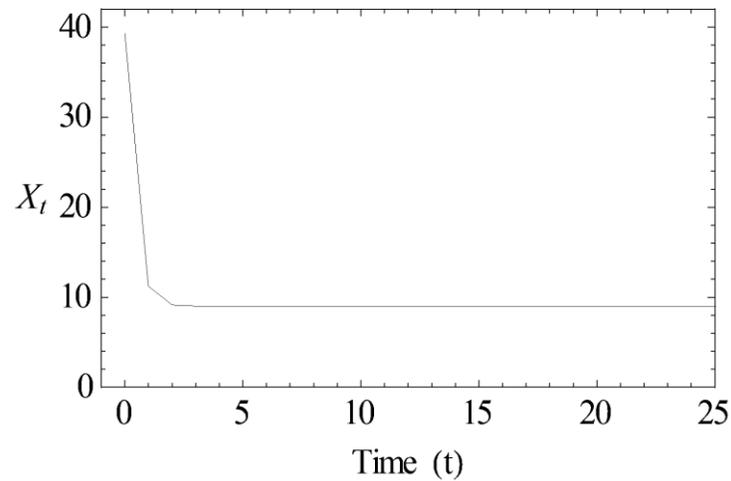
become infected. Figure 2 plots the NPV of an optimally managed subpopulation post-disease for any stock level at the time the disease arrived.

*Figure 2.* NPV of any stock level X at time of infection.

Having a stock level equal to 1 unit at the time of infection results in a net present value of about -\$20,000 compared to \$120,000 when the stock is at its disease-free steady state level at the time of infection.

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

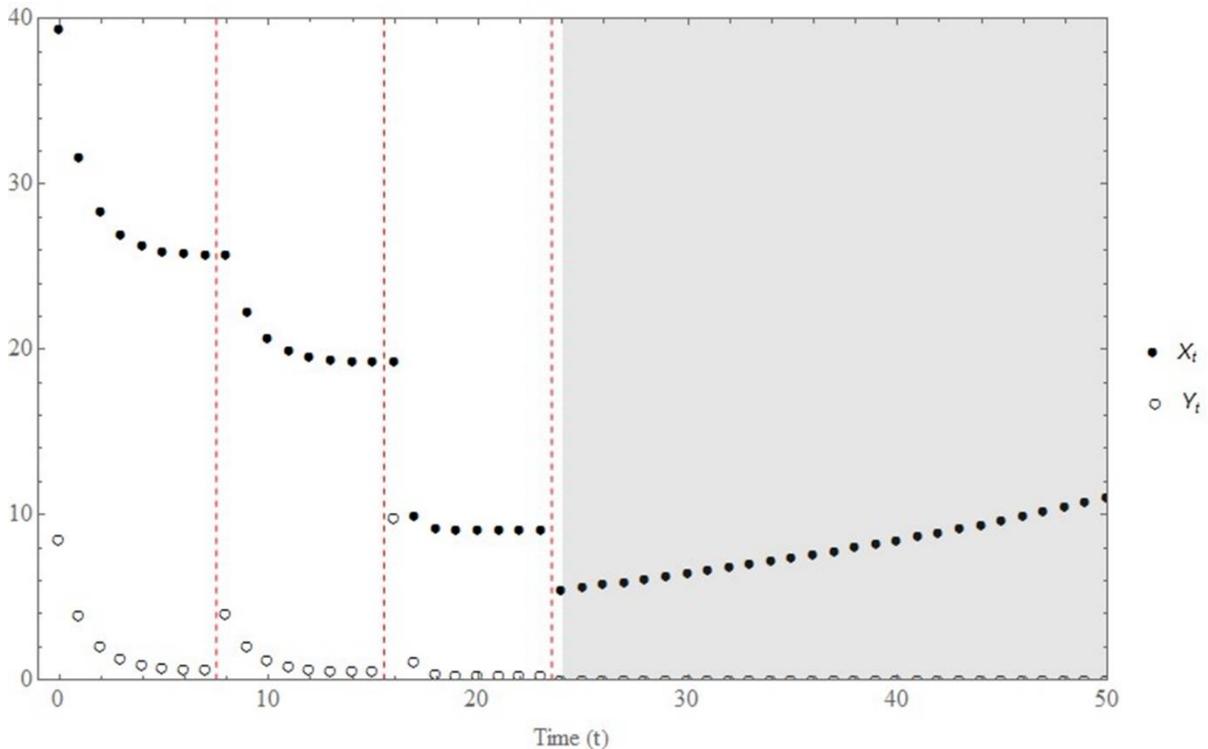
Continuing the backwards induction process solving for the NPV maximizing harvesting strategy of subpopulation 4 when all other subpopulations are infected. Figure 3 shows the stock level associated with the optimal harvesting strategy of subpopulation 4 when all other subpopulations are infected over 25 periods. Starting from an initial stock level equal to the disease-free steady state.



*Figure 3.* Optimal stock level of subpopulation 4 when all other subpopulations are infected starting from the disease free steady-state.

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

Figure 3 demonstrates for a given set of parameters there exists a disease-prone steady state. The NPV maximizing strategy for managing a subpopulation in this state of world is to harvest from its stock until it reaches this new disease-prone steady state level. In this case the disease-prone steady state stock level is just below 9 units. Figure 1A outlines this process for each other state of the world under the same parameter values.



*Figure 4.* Optimal stock and harvesting strategy of subpopulation 4 when disease moves forward once every 8 periods.

Figure 4 is the final result of my model. The stock level (black dots) and periodic harvesting level (white dots) adjust over time. Red dashed lines correspond to the time the state of the world changes and the disease transmits to another subpopulation. The grey background shows all period where the level of harvesting is equal to zero.

When the disease first arrives at subpopulation 1 the probability that subpopulation 4 will become infected the following period is only 0.1%. However, the disease may transmit through other subpopulations more rapidly. The optimal strategy is to significantly reduce the stock of

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

subpopulation 4 immediately. When the state of the world changes and the disease transmits to another subpopulation the probability of infection increases so the stock is harvested from more aggressively until it reaches another disease-prone steady state. This process occurs each time the state of the world changes. The speed at which the stock level approaches each disease-prone steady state depends on the probability of immediate infection.

### **Static Analysis**

The two parameters which have the largest impact on harvesting and stock levels are increasing marginal costs and the disease mortality rate. Increasing marginal costs set an upper bound on the amount harvested in each period. The functional form I present here does not permit the stock to be completely eliminated in order to decrease or even stop future disease transmission. If increasing marginal costs of this form are not present in an industry it is possible that completely eradicating a single subpopulation is the optimal strategy.

A high mortality rate greatly reduces the future value of a stock. As the expected future value decreases the level of harvesting before the disease arrives increases. The mortality rate has the larger effect than a decrease in intrinsic growth rate because the losses due to mortality are imposed immediately when the disease arrives. The reduction in intrinsic growth rate has a smaller impact over a long time period which, after being discounted, has a lower impact on the present value of the stock.

### **Discussion**

#### **Additions to the Literature**

My results highlight the importance of effective management of biological stocks at risk of disease. Most epidemiological studies focus on the continuous motion of diseases as they

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

transmit between individuals. From a resource management perspective, knowledge of the discrete macro level processes allows for more effective regulations and policies.

Allowing for multiple disease pathways makes this, more general solution, an effective tool for managers to have on hand in the event of a disease outbreak.

### **Policy Implications**

The policy implications of this research are broader than is immediately apparent. The obvious use of this research is to aid management institutions or private owners in managing a biological stock optimally when faced with a spreading disease. Less obvious are the policy applications of entry to industries in which diseases are known to cause damage. Aquaculture as an industry is criticized for the high concentration of diseases that persist in farmed marine animals. When two open water aquaculture sites are nearby diseases can transmit between them. If there is little regulation in this thriving industry more sites will likely appear; increasing the probability of disease transmission between them. In some cases an additional pathway for disease transmission decreases the expected value of all other stocks enough that it is not worth creating a new site. Policy makers, with help from researchers, can apply my model to determine the true cost of introducing an additional aquaculture site in a given region.

### **Further Research**

Moving forward, I believe my model can be expanded to include an endogenous process to determine the transmission rate of the disease. I would first expand by incorporating an SI model at each subpopulation and allowing the number of infected individuals within a population to determine the probability of disease transmission at a given time. The probability of disease transmission could then be defined as:

$$P_{i,j} = \alpha_{i,j} * g(X_{i,t}, X_{j,t})$$

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

This function would result in more detailed results which would take into account the value of harvesting a stock in order to reduce the probability of transmission between it and other subpopulations in the future. With disease transmission of this form competing firms will harvest differently than a social planner who is maximizing the total social NPV. With exogenous disease transmission private and social maximizers follow the same harvesting schedule.

### **Conclusion**

I find, similar to past research, managing a biological stock which is at risk of becoming infected by disease requires immediate and aggressive action. The level of harvesting is increased as soon as a disease appears in order to maximize the net present value of the stock. The size of this increase is dependent on the biological parameters governing the effects of the disease and the stock. The non-biological factors, increasing marginal costs of harvesting and the social discount rate are important in determining the amount of stock which is left to become infected in order to sustain future harvesting post disease.

The results obtained are directly applicable to fisheries, forestry, aquaculture, and farming and with some modifications any other industries which are commonly victim to contagious diseases. Results can be used aid owners and managers of biological stocks in the valuation of their stocks when a disease is a serious threat to its future value. These results can also be implemented to determine the true opportunity cost of investing in risk reduction methods, or in choosing to deny entry to new firms wanting to operate near active, high-risk, farm or aquaculture sites.

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

### References

- Bidegain, G., Powell, E. N., Klinck, J. M., Ben-Horin, T., & Hofmann, E. E. (2016). Marine infectious disease dynamics and outbreak thresholds: contact transmission, pandemic infection, and the potential role of filter feeders. *Ecosphere*, *7*(4). doi:10.1002/ecs2.1286
- Bondad-Reantaso, M. G., Subasinghe, R. P., Arthur, J. R., Ogawa, K., Chinabut, S., Adlard, R., . . . Shariff, M. (2005). Disease and health management in Asian aquaculture. *Veterinary Parasitology*, *132*(3-4), 249-272. doi:10.1016/j.vetpar.2005.07.005
- Brooks-Pollock, E., Jong, M. D., Keeling, M., Klinkenberg, D., & Wood, J. (2015). Eight challenges in modelling infectious livestock diseases. *Epidemics*, *10*, 1-5. doi:10.1016/j.epidem.2014.08.005
- Cabello, F. C. (2007). Salmon Aquaculture and Transmission of the Fish Tapeworm. *Emerging Infectious Diseases*, *13*(1), 169-171. doi:10.3201/eid1301.060875
- Conrad, J. M., & Smith, M. D. (2011). Nonspatial And Spatial Models In Bioeconomics. *Natural Resource Modeling*, *25*(1), 52-92. doi:10.1111/j.1939-7445.2011.00102.x
- Cooke, B. J., & Carroll, A. L. (2017). Predicting the risk of mountain pine beetle spread to eastern pine forests: Considering uncertainty in uncertain times. *Forest Ecology and Management*, *396*, 11-25. doi:10.1016/j.foreco.2017.04.008
- Fenichel, E. P., Horan, R. D., & Hickling, G. J. (2010). Management of infectious wildlife diseases: bridging conventional and bioeconomic approaches. *Ecological Applications*, *20*(4), 903-914. doi:10.1890/09-0446.1
- Garver, K. A., Mahony, A. A., Stucchi, D., Richard, J., Woensel, C. V., & Foreman, M. (2013). Estimation of Parameters Influencing Waterborne Transmission of Infectious

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

- Hematopoietic Necrosis Virus (IHNV) in Atlantic Salmon (*Salmo salar*). *PLoS ONE*, 8(12). doi:10.1371/journal.pone.0082296
- Horan, R. D., & Wolf, C. A. (2005). The Economics of Managing Infectious Wildlife Disease. *American Journal of Agricultural Economics*, 87(3), 537-551. doi:10.1111/j.1467-8276.2005.00746.x
- Horan, R., Wolf, C. A., Fenichel, E. P., & Mathews, K. H. (2005). Spatial Management of Wildlife Disease\*. *Review of Agricultural Economics*, 27(3), 483-490. doi:10.1111/j.1467-9353.2005.00248.x
- Horan, R. D., Fenichel, E. P., Wolf, C. A., & Gramig, B. M. (2010). Managing Infectious Animal Disease Systems. *Annual Review of Resource Economics*, 2(1), 101-124. doi:10.1146/annurev.resource.012809.103859
- Huang, B., & Perrings, C. (2017). Managing the Risks of Sea Lice Transmission Between Salmon Aquaculture and Wild Pink Salmon Fishery. *Ecological Economics*, 142, 228-237. doi:10.1016/j.ecolecon.2017.03.012
- Mccallum, H., Harvell, D., & Dobson, A. (2003). Rates of spread of marine pathogens. *Ecology Letters*, 6(12), 1062-1067. doi:10.1046/j.1461-0248.2003.00545.x
- Moberg, E. A., Kellner, J. B., & Neubert, M. G. (2015). Bioeconomics and biodiversity in harvested metacommunities: a patch-occupancy approach. *Ecosphere*, 6(11). doi:10.1890/es14-00503.1
- Robinson, M., Stilianakis, N. I., & Drossinos, Y. (2012). Spatial dynamics of airborne infectious diseases. *Journal of Theoretical Biology*, 297, 116-126. doi:10.1016/j.jtbi.2011.12.015

## OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

Salama, N., & Rabe, B. (2013). Developing models for investigating the environmental transmission of disease-causing agents within open-cage salmon aquaculture. *Aquaculture Environment Interactions*, 4(2), 91-115. doi:10.3354/aei00077

Strohm, S., Reid, M., & Tyson, R. (2016). Impacts of management on Mountain Pine Beetle spread and damage: A process-rich model. *Ecological Modelling*, 337, 241-252. doi:10.1016/j.ecolmodel.2016.07.010

The State of World Fisheries and Aquaculture 2016. (n.d.). Retrieved March 12, 2018, from [http://www.bing.com/cr?IG=60529098B73446E29F636BF3C40B25BB&CID=27AEBA33BE7C66341559B183BFD3671F&rd=1&h=ZHDBFXLj4T46R3HKs8qNtE\\_Hkz\\_HldlXaTTFxI381QU&v=1&r=http%3a%2f%2fwww.fao.org%2f3%2fa-i5555e.pdf&p=DevEx,5067.1](http://www.bing.com/cr?IG=60529098B73446E29F636BF3C40B25BB&CID=27AEBA33BE7C66341559B183BFD3671F&rd=1&h=ZHDBFXLj4T46R3HKs8qNtE_Hkz_HldlXaTTFxI381QU&v=1&r=http%3a%2f%2fwww.fao.org%2f3%2fa-i5555e.pdf&p=DevEx,5067.1)

Tien, J. H., & Earn, D. J. (2010). Multiple Transmission Pathways and Disease Dynamics in a Waterborne Pathogen Model. *Bulletin of Mathematical Biology*, 72(6), 1506-1533. doi:10.1007/s11538-010-9507-6

Appendix A

Stock Levels in Every State of the World

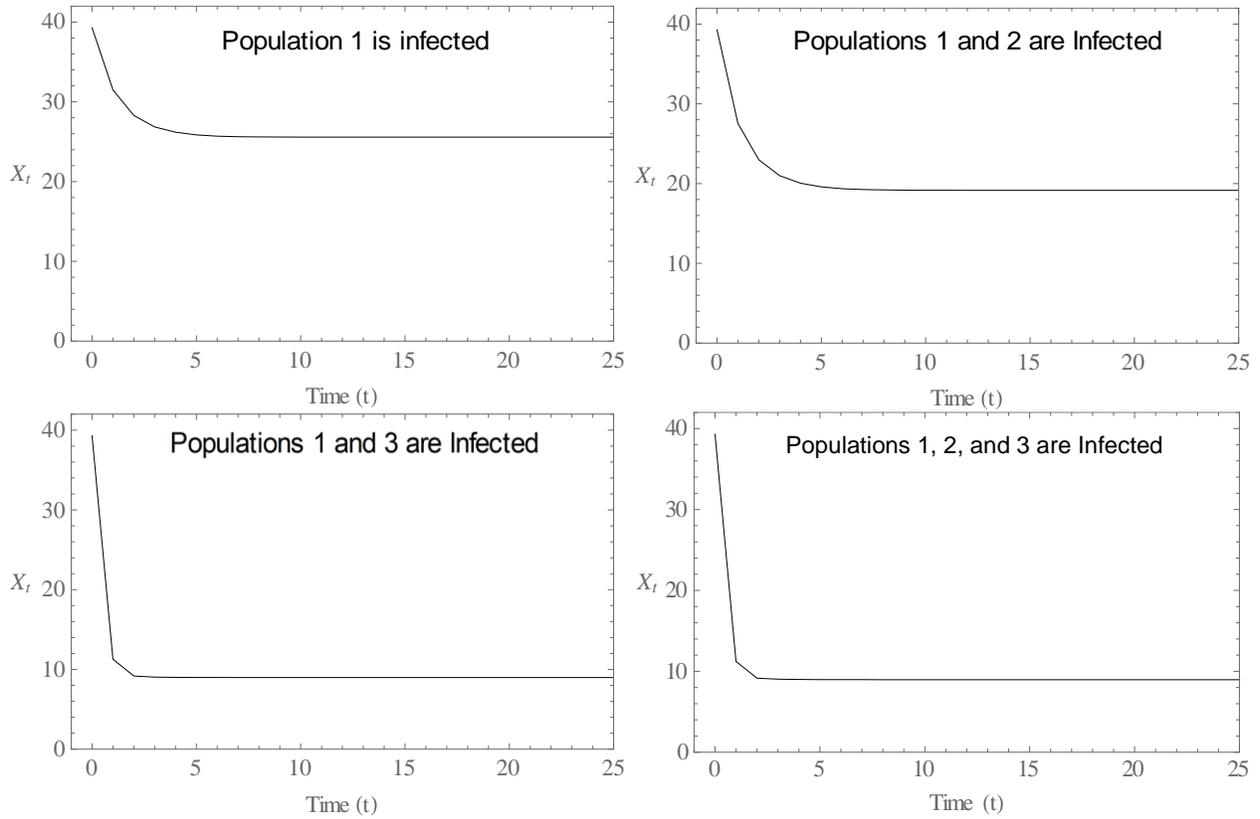


Figure 1A. Optimal stock level in each state of the world under original parameter values.

Appendix B

Results with Varying Parameter Values

Each of the following figures corresponds to a change in one parameter compared to the results presented in the text. Changing the probability of transmission, the post disease intrinsic growth rate, and the disease mortality rate all change the optimal harvesting schedule. These results demonstrate the importance of having accurate estimates of each of these parameters.

A change in the probabilities of disease transmission.

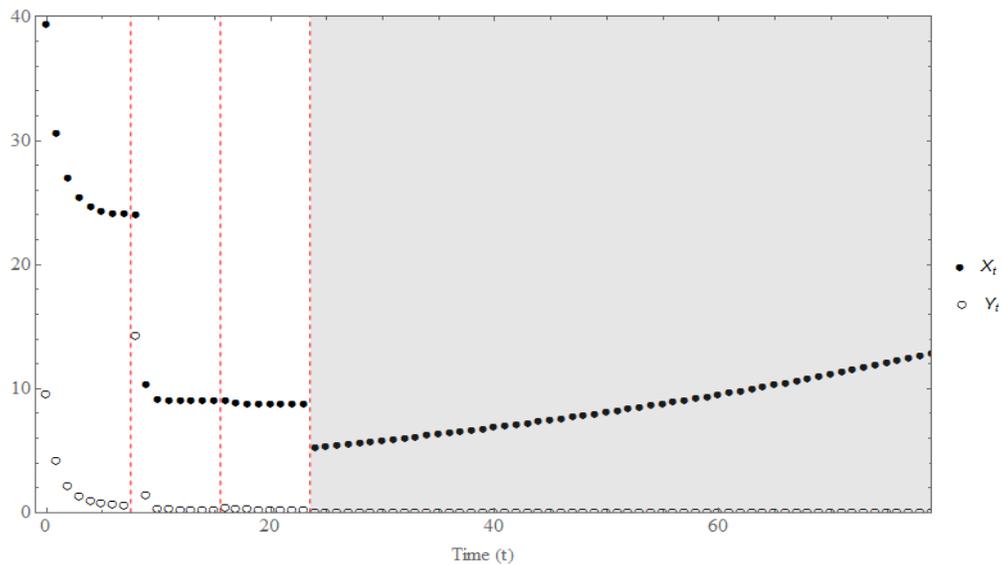


Figure 1B.  $P_{12} = 0.3$ ,  $P_{13} = 0$ ,  $P_{14} = 0$ ,  $P_{23} = 0.3$ ,  $P_{24} = 0.3$ ,  $P_{34} = 0.3$ .

# OPTIMAL HARVESTING SCHEDULE OF AT RISK STOCKS

A change in the post-disease intrinsic growth rate  $r_1$ .

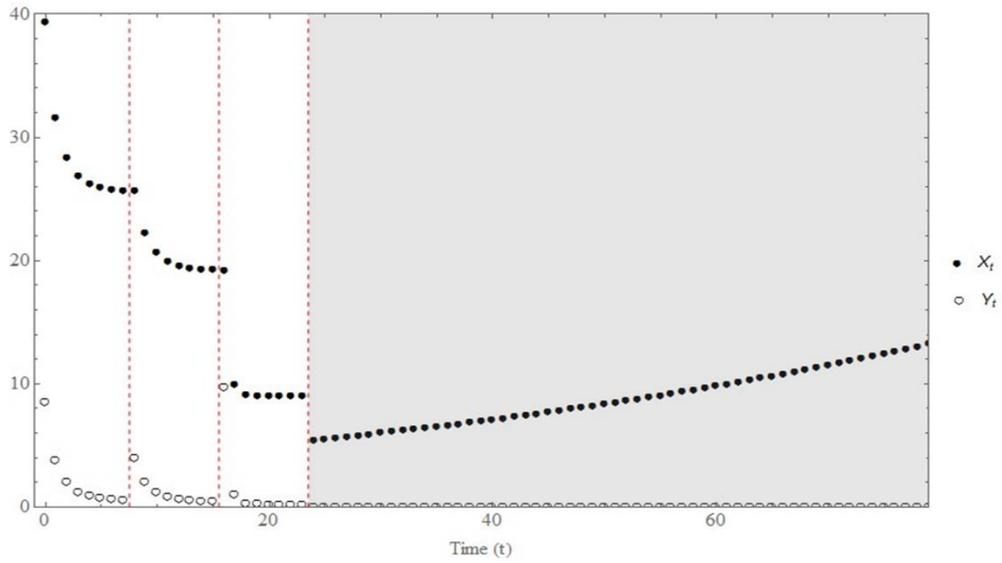


Figure 2B. Intrinsic growth rate post disease  $r_1 = 0.25$ .

A change in the mortality rate  $M$ .

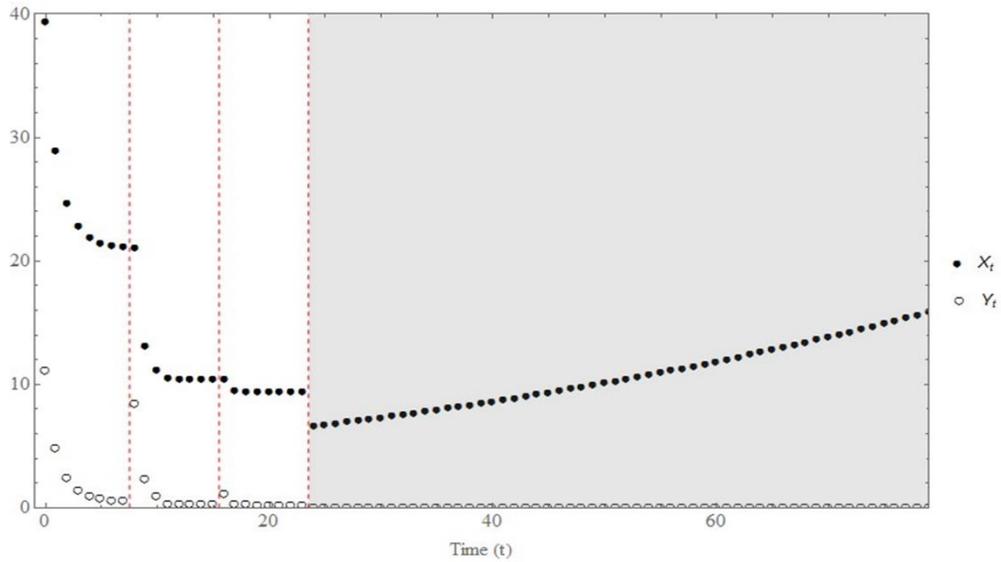


Figure 3B. Mortality rate  $M = 0.3$ .