Artificial Intelligence and Automation: Effects of Worker-Replacing Technology

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Abstract

This paper investigates the effects of human-replacing machine labour on growth, the number of machines, output, and consumption. I use a production function with human labour and machine labour as perfect substitutes. There exists a threshold price of machines, below which the economy can end up in economic singularity. The growth rate of the economy becomes exponential, and humans are completely replaced in the labour force. Above the threshold price, there can be a steady-state with no machines, or a positive steady-state number of machines. Wages are unchanged in this model because of the linear production function. I add depreciation to the model and find that it lowers the threshold price, making economic singularity less likely to occur. Finally, I allow for the price of machines to be decreasing. A decreasing price has the opposite effect from depreciation, and it makes the economic singularity occur much more quickly. Simulations corroborate that a zero-machine steady-state, and positive machine steady-state, and economic singularity are all possible depending on the price of machines, and the depreciation of machines.

Keywords: economic singularity, human-replacing machine labour

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1. Introduction

As machines become more capable of intelligence and performing human tasks, they threaten to replace human labour entirely. In this paper, I consider a situation where machines are capable of completely replacing humans in the labour market. I investigate the effects of human-replacing machines on the share of human labour, the number of machines, output, and consumption.

The bleakest possible outcome of artificial intelligence (AI) is known as the "technological singularity" or just the "singularity." First noted by Hungarian mathematician John von Neumann in a conversation with Stanislaw Ulam, the singularity is the point in time where technology has advanced beyond the need for humans (Ulam, 1958). Important to the singularity is the fact that AI has surpassed human levels of general intelligence and can reproduce and improve itself without human intervention. In the technological singularity, machines can do everything a human can do, but better, and cheaper. Another important concept is the economic singularity.

By contrast, economic singularity occurs when machines become cheaper and more efficient workers than humans. As a result, there will be no more humans in the labour force and the economy will experience exponential growth (Korinek and Stiglitz, 2017a). Exponential economic growth results because once machines can rapidly reproduce themselves, the size of the human population is no longer a limiting factor of growth.

To investigate the effects of machine displacement of human labour share, wages, and economic growth, I extend a model proposed by Korinek and Stiglitz' (2017a). Korinek and Stiglitz use a simple general equilibrium model with Constant Relative Risk Aversion (CRRA) utility function, and a production function in which human labour and machine labour are perfect substitutes. Both forms of labour are equally productive, so the firm always demands the lower-cost form of labour. If the wage of machines falls below that of humans, then the outcome is the economic singularity. Only machines are employed, human labour share falls to zero, and—because machines can reproduce themselves rapidly—there is a sudden increase in economic growth (Korinek & Stiglitz, 2017a). Korinek and Stiglitz find only two possible equilibria: either an economy with no machines, or economic singularity.

I extend the Korinek and Stiglitz model by introducing depreciation. This introduces a third possible equilibrium: with a high depreciation rate and/or a high price of machines there could be a positive steady-state level of machines. I further extend the Korinek and Stiglitz model by considering a situation where the price of machines declines over time. Finally, I perform simulations on a calibrated model using DYNARE**.**

Present-day technological advancements are not the beginning of human-replacing technology. Aghion *et al.* (2017) argue that AI is simply the next step in a process of automation that started with the industrial revolution. Acemoglu and Restropo (2016) take a similar view. Nordhaus (2015) finds that based on six tests that measure current economic trends, an economic singularity is not anywhere in the near future, predicting at least a century before humanity should be concerned about an economic singularity.

As a second result, Korinek and Stiglitz find that growth may be limited by factors that are complementary to machines, primarily energy and land. On the path to a singularity, the number of machines increases rapidly. The massive increase in the labour supply means that the amount of the limiting factor per unit of labour is decreasing. For example, if the number of machines in the labour force is increasing rapidly, then that amount of land per unit of labour will be less, as there is only a limited amount of land available to all workers. This causes a decrease in the real wage and leads to greater imbalance between human workers and human factor owners (Korinek & Stiglitz, 2017a).

While Korinek and Stiglitz use a simple general equilibrium model, others use a task-based framework. A task-based model uses a production function describing the production of a final good, Y. The final good is produced by a finite set of tasks that can be completed by people or capital. Acemoglu and Restropo (2016) model automation as an increase in the tasks that are completed by capital that were previously completed by people. Their model allows for new tasks to be created and for humans to have the comparative advantage in performing newly created tasks. As a result, though automation of tasks causes employment of humans to decrease, the creation of new tasks for which humans have the comparative advantage causes employment to increase. They conclude with two possible outcomes. If capital is cheap relative to labour, then there will be a singularity with rapidly advancing technologies and redundant labour in the long-run. If capital rental rates are not sufficiently cheap, there will be a long-run balanced growth path (Acemoglu and Restropo, 2016).

Aghion et al. (2017) consider several task-based models. They model both the production of goods and the production of ideas. They identify conditions that lead to a balanced growth path despite near complete automation. However, they also find that a singularity could occur in the production of ideas even without full automation because ideas are non-rival.

Hanson (2001) models machines as both complements and substitutes to human labour. He considers a continuum of jobs that machines or human can perform. Every job on the continuum is a complement to every other job. The increased productivity of jobs that are automated complements the jobs done by humans. The behaviour of the key variables depends on the fraction of jobs done by machines. As with other models I have discussed, Hanson finds that when machines are expensive and therefore do a small share of jobs, there is little effect of automation on wages or growth. However, when machines are cheap and therefore doing a large share of jobs, human wages will decrease unless owners of capital place an extremely high value on the few jobs that human do. As in Korinek and Stiglitz (2017a), growth increases as it is no longer limited by the slow growth of the human labour force.

Income distribution is another important aspect of the economy that could be affected by the rise of AI. Korinek and Stiglitz (2017b) address this. In their model, automation leads to an increasing income gap between workers and the owners of capital. In an ideal world, where redistribution is costless and markets are perfect, technological progress can benefit everyone (Korinek and Stiglitz, 2017b). However, if redistribution is costly then it may be difficult to compensate the jobless human workers.

Acemoglu and Restropo (2016) also address the possible issue of increased income inequality. In their model, new tasks are created by increased technology, and high-skill workers will have a comparative advantage in the new tasks, which makes them better off. Low-skill workers are the first to be replaced by machines and are also unable to benefit from the creation of new tasks. As a result, the income gap between low-skill and high-skill workers increases. Acemoglu and Restropo (2018) extend their task-based model to include both low-skill and high-skill automation. They find that low-skill automation increases inequality. Low-skill automation complements high-skill labour, making them more productive, while low-skill labour can no longer find jobs. High-skill automation decreases inequality. With high-skill automation, high-skill workers also cannot find work. The overall effect of automation on inequality is ambiguous because the two types of automation have opposite effects on the income gap.

2. The Mechanical Model with Exogenous Saving

I first take a mechanical approach to the model, focusing on the equation of motion for machines and considering what outcomes are possible. Similarly to Korinek and Stiglitz (2017a), I look for the threshold price that will change the path the economy will take. Equation (1) is the equation of motion for machines.

$$m_{t+1} - m_t = \frac{sA}{\gamma}(h + m_t) - \delta m_t \qquad (1)$$

Production Function: $Y = AF(h + m_t) = A(h + m_t)$

$m_t \equiv$ machine labour

$m_t \geq 0$

$h \equiv$ human labour held constant

$\gamma \equiv$ Price of Machines in 'corn' per machine

$s \equiv$ saving rate

$\delta \equiv$ depreciation rate

$A \equiv$ scale on production

Equation (1) shows the change in the number of machines from one period to the next. This change consists of the amount of production used from the previous period to create more machines less the depreciation of old machines. The production function is constant returns to scale, and machine labour and human labour are perfect substitutes. For simplicity, human labour is held constant. Equation (1) can be rearranged to obtain a first-order linear difference equation.

$$m_{t+1} = \frac{sA}{\gamma}h + \left(\frac{sA}{\gamma} + 1 - \delta\right)m_t \qquad (2)$$

Figure 1 shows the two possible solutions to equation (2): a steady-state where equation (2) crosses the 45-degree line, and a runaway growth case where equation (2) and the 45-degree line never cross.

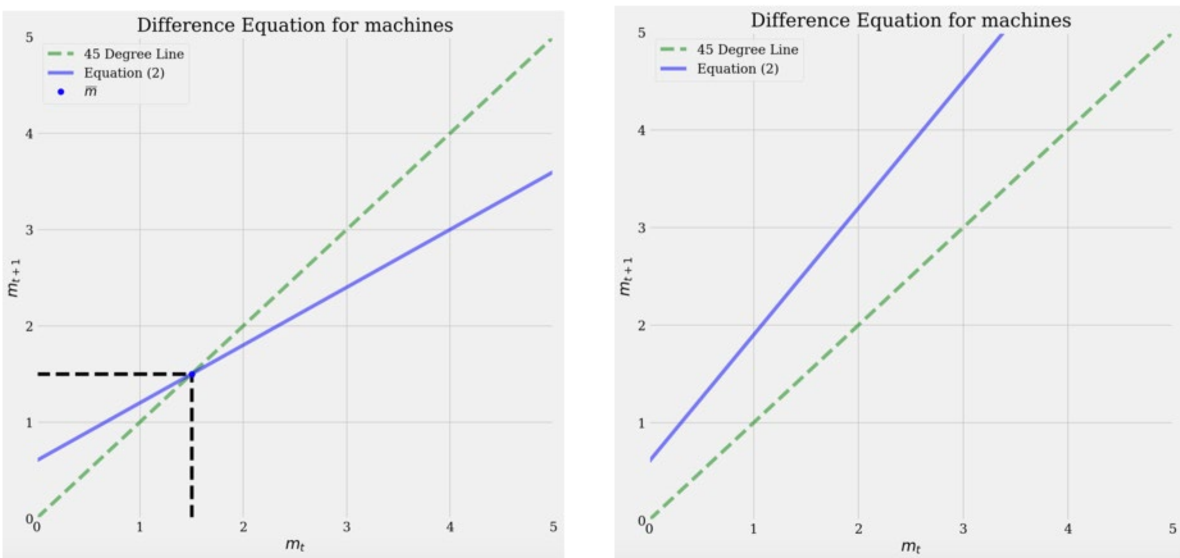

Figure 1: The Difference equation for machines with a steady-state (left) and without a steady-state (right)

Equation (3) is the solution to the first order, linear, difference equation (2). This solution allows for the number of machines to be found at any time period, as long as the initial value, m_0, is known.

$$m_t = \left(\frac{sA}{\gamma} + 1 - \delta\right)^t \left(m_0 - \frac{\frac{sA}{\gamma}h}{1 - \left(\frac{sA}{\gamma} + 1 - \delta\right)}\right) + \frac{\frac{sA}{\gamma}h}{1 - \left(\frac{sA}{\gamma} + 1 - \delta\right)} \qquad (3)$$

From equation (3) one can see that convergence or divergence of machines depends on: $sA/\gamma + 1 - \delta$. There will be a critical threshold price of machines, $\gamma^* \equiv sA/\delta$ such that two outcomes are possible: convergence to a steady-state level of machines or divergence to infinite machines in the long-run. Whether the economy has the number of machines converging to a steady-state or diverging to infinity depends on the price of machines. The stationary point, or steady-state is the point where the number of machines is staying the same from period to period. That is, the number of machines is not increasing or decreasing. If $m_0 = \bar{m}$ then the number of machines stays constant.

$$\text{Steady-state: } m_{t+1} = m_t = \bar{m} = \frac{\frac{sA}{\gamma}\bar{h}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)} = \frac{\frac{sA}{\gamma}\bar{h}}{\delta-\frac{sA}{\gamma}}$$

To find the growth rate of machines I take the following limit:

$$\lim_{t\to\infty}\frac{m_{t+1}}{m_t} = \frac{\left(\frac{sA}{\gamma}+1-\delta\right)^{t+1}\left(m_0 - \frac{\frac{sA}{\gamma}\bar{h}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)}\right) + \frac{\frac{sA}{\gamma}\bar{h}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)}}{\left(\frac{sA}{\gamma}+1-\delta\right)^{t}\left(m_0 - \frac{\frac{sA}{\gamma}\bar{h}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)}\right) + \frac{\frac{sA}{\gamma}\bar{h}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)}}$$

2.1. Lemma 1

(i) If $\gamma > \gamma^*$ then the price of machines is sufficiently expensive and the number of machines will converge to the steady-state-level of $\bar{m}$.

(ii) If $\gamma < \gamma^*$ then machines are now sufficiently cheap so that in the long-run the number of machines, and therefore output, will go to infinity. In this scenario, the growth rate of machines converges to $sA/\gamma + 1 - \delta > 1$ in the limit. The human labour share goes to zero and the machine labour share goes to one in the long-run. This is an economic singularity.

Proof: Convergence or divergence only depends on $sA/\gamma + 1 - \delta$ because it is the only part of equation (3) that changes with time. If $0 < sA/\gamma + 1 - \delta < 1$ then there is convergence. If $sA/\gamma + 1 - \delta > 1$ then there is divergence in the number of machines. These conditions give the threshold value $\gamma^* \equiv sA/\delta$. This is the threshold price of machines that would cause the behavior of the economy to change. *This concludes the proof.* ■

Now I consider the implications of m_0 on the convergent and divergent outcomes (outcomes (i) and (ii)). For the runaway growth outcome, outcome (ii), machines will go to positive infinity, regardless of the initial value of machines (assuming a non-negative initial value). Figure 2 shows machines diverging to infinity with zero machines initially.

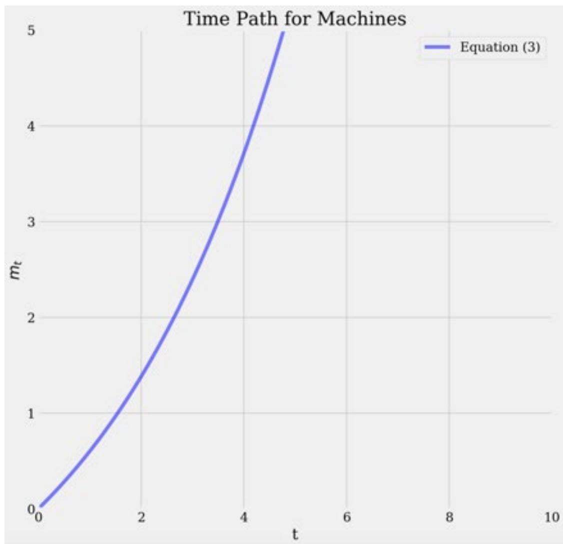

Figure 2: Time path for machines diverging with $m_0 = 0$

The steady-state outcome, outcome (i) is a bit more complicated. In case (i), whether $m_0 - ((sA/\gamma)\ h/(\delta - (sA/\gamma)))$ is positive or negative will depend on the initial number of machines. However, it does not actually matter whether or not $m_0 - ((sA/\gamma)\ h/(\delta - (sA/\gamma)))$ is positive or negative because it will always be multiplied by a fraction between zero and one. This means that even if it turns out to be negative, it will still be smaller in absolute value than the positive steady-state, thus making the number of machines positive and approaching the steady-state from below. If it ends up positive, then the number of machines will approach the steady-state from above. Figure 3 shows the number of machines converging from below and above. The only case where the initial number of machines matters is when the initial number of machines is already at the steady-state, in which case by definition the number of machines will remain constant.

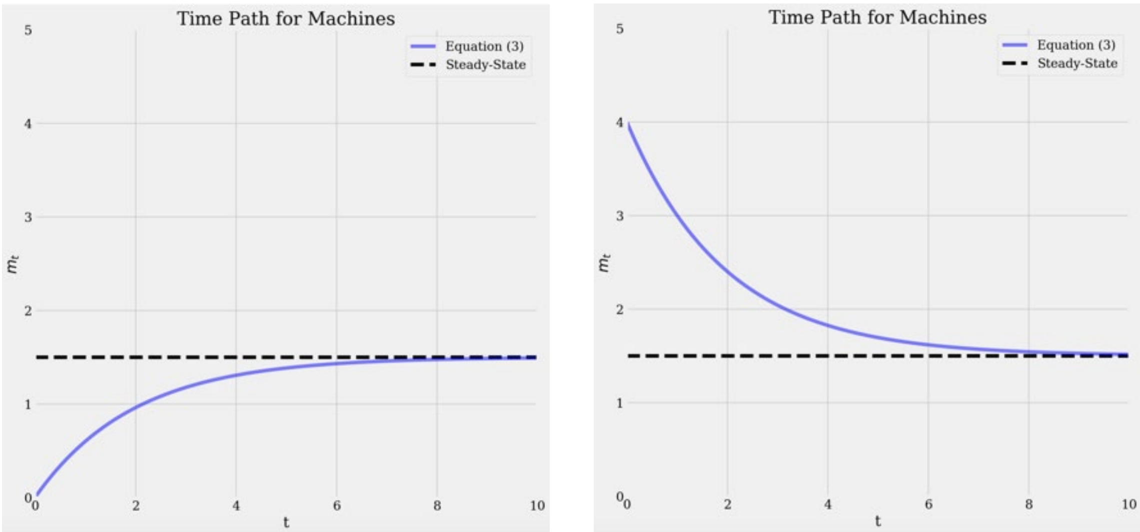

Figure 3: Time path for machines with a low m_0 (left) and high m_0 (right)

Depreciation is the variable that sets this analysis apart from that of Korinek and Stiglitz (2017a) as their model has zero depreciation. The threshold price of machines, that determines which state the economy is in, is negatively related to depreciation. That is, when depreciation is larger, the threshold price of machines is lower. This means that the price of machines would have to be very small for the divergence case to take place. Likewise, when there is less depreciation of machines, the threshold price is larger and it is much more likely that the price of machines will be low enough to trigger the divergence case.

2.2. The Mechanical Model with Endogenous Saving

The second scenario I consider assumes a planner who sets the saving rate. If the planner chooses a positive saving rate then machines could either reach a positive steady-state or go to infinity depending on the price, as in Lemma 1. However, if the planner chooses a saving rate of zero, perhaps because the price of machines is too high to warrant investing in more machines, a third outcome is possible: a steady-state with no machines. Since $\gamma^* \equiv sA/\delta$ depends on the saving rate, if the saving rate is zero, then the price of machines would have to be negative in order for the runaway case to take place. Since negative prices are not possible, if 1) a planner chooses a saving rate of zero; 2) there is full depreciation of machines; and 3) the economy starts with no machines, then the economy will remain with no machines. In fact, even if the economy starts with a positive number of machines, if the saving rate is zero and there is full depreciation, then there will be no machines.

The question remains whether there is a price of machines that would cause a planner to save nothing. The planner will choose the saving rate that maximizes consumption, so if the cost of machines is such that there is nothing left to consume, then saving to invest in machines will no longer make sense. Equation (4) is the equation for consumption per capita.

$$\frac{C}{\bar{h}} = (1-s)\frac{Y}{\bar{h}}$$

C = Consumption

$$\frac{C}{\bar{h}} = (1-s)A\frac{(\bar{h}+m_t)}{\bar{h}} \qquad (4)$$

Consumption per capita consists of whatever remains of output after saving. In this model, a world without machines means that everything produced is consumed.

$$\frac{C}{\bar{h}} = \frac{Y}{\bar{h}} = A$$

In order for the planner to choose to save and end up in a world with machines, the consumption per capita must be greater than or equal to A. That is, there must be higher consumption with machines than without.

2.3. Lemma 2

(i) If $(1-s)\frac{Y}{\bar{h}} < A$, then it is optimal to not invest in machines and the steady-state number of machines is zero. Whether or not the initial number of machines is zero, there are no machines in this economy in the long-run.

(ii) If $(1-s)\frac{Y}{\bar{h}} > A$, then it is optimal to invest in machines.

There is a new threshold price of machines, $\tilde{\gamma} \equiv A/\delta$, above which a planner chooses not to save towards creating more machines. The number of possible outcomes depends on if the new threshold, $\tilde{\gamma}$, is greater than or less than the threshold from Lemma 1, $\gamma^* \equiv sA/\delta$. If γ^* is greater than or equal to $\tilde{\gamma}$, then there are two possible outcomes: convergence to a steady-state of zero or divergence to infinity. If γ^* is less than $\tilde{\gamma}$, then there are three possible outcomes: convergence to a steady-state number of machines of zero, convergence to the positive steady-state number of machines, $\bar{m}$, or divergence to infinite machines. In order for a positive steady-state to be possible in this case, γ must be less than $\tilde{\gamma}$. Since the saving rate is between 0 and 1, γ^* is always less than $\tilde{\gamma}$ so all three outcomes—no machines, a positive steady-state of machines, or infinite machines—are all possible.

2.4. Proposition 1

(i) If $\gamma > \tilde{\gamma} > \gamma^*$ then the optimal savings rate of zero, and the steady-state number of machines is zero.

(ii) If $\tilde{\gamma} > \gamma > \gamma^*$ then the price is low enough that it is optimal to save, but still too high for the runaway case. The number of machines will converge to a positive steady-state, $\bar{m}$.

(iii) If $\tilde{\gamma} > \gamma^* > \gamma$ then the price of machines is now low enough for the planner to choose to save and for the runway case to occur. The number of machines will diverge and go to infinity.

Proof: To find the new threshold price, $\tilde{\gamma}$, I find the per capita consumption at the steady-state for machines. Next, I determine the price which makes the per capita consumption with the steady-state number of machines higher than the per capita consumption in the economy with no machines.

$$\frac{Y}{\bar{h}} = A\frac{(\bar{h} + m_t)}{\bar{h}} = A + A\frac{m_t}{\bar{h}}$$

Positive steady-state for machines: $\bar{m} = \frac{\frac{sA}{\gamma}\bar{h}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)}$

$$\frac{Y}{\bar{h}} = A + A\frac{\bar{m}}{\bar{h}}$$

$$\frac{Y}{\bar{h}} = A + A\frac{\frac{\frac{sA}{\gamma}\bar{h}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)}}{\bar{h}}$$

$$\frac{Y}{\bar{h}} = A + \frac{\frac{sA^2}{\gamma}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)}$$

Need $\frac{C}{\bar{h}} = (1-s)\frac{Y}{\bar{h}} > A$ for a positive steady of machines

$$(1-s)\left(A + \frac{\frac{sA^2}{\gamma}}{1-\left(\frac{sA}{\gamma}+1-\delta\right)}\right) > A$$

$$\gamma < \frac{A}{\delta} \rightarrow \tilde{\gamma} = \frac{A}{\delta}$$

This concludes the proof. ■

Case (ii) of Proposition 1 requires further consideration. In case (ii), the optimal saving rate will approach 1, which will make γ^* and $\tilde{\gamma}$ very close to each other. This will make it unlikely for the actual price of machines to be between the two thresholds and thus unlikely that the positive steady-state case will occur.

3. Model Simulations

So far, I have examined the theoretical results of worker-replacing technology in a mechanical model. Now, in order to illustrate some possible paths the economy could take when machines are capable of replacing humans in the labour force, I perform simulations using DYNARE and MATLAB. Model calibrations can be found in the appendix.

3.1. The Korinek and Stiglitz model

I begin simulations with the Korinek and Stiglitz (2017a) model as a baseline. The Korinek and Stiglitz (2017a) model starts with a labour only economy with a representative agent with CRRA preferences.

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{where} \quad u(c_t) = (c_t)^{1-\theta}/(1-\theta)$$

They use a constant returns to scale production function with human labour and machine labour as perfect substitutes.

$$\text{Production Function: } Y = AF(h + m_t) = A(h + m_t)$$

The representative agent maximizes utility subject to the budget constraint.

$$\max_{\{c_t, m_t \geq 0\}} U \quad \text{s.t.} \quad c_t + \gamma m_{t+1} = A(h + m_t)$$

Using the first order conditions at generic time t, Korinek and Stiglitz are able to determine the threshold price below which the economy experiences exponential long-run growth, and the human labour share falls to zero. As a benchmark, I simulate this model in DYNARE in order to see the paths of output, consumption, and the number of machines over time when the price of machines is above and below the threshold. Figures 4.1, 4.2, and 4.3 show the time paths for the number of machines, output, and consumption respectively when the price of machines is above the threshold. That is, when machines are too expensive. In this case, there are no machines in the economy, and consumption and output are fixed. There are only humans in the labour force in this scenario.

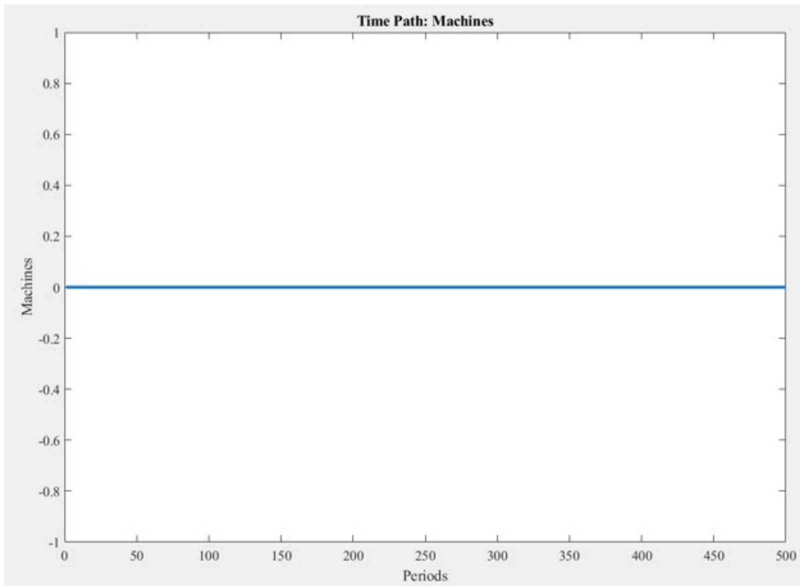

Figure 4.1: Time path for machines when the price of machines is above the threshold

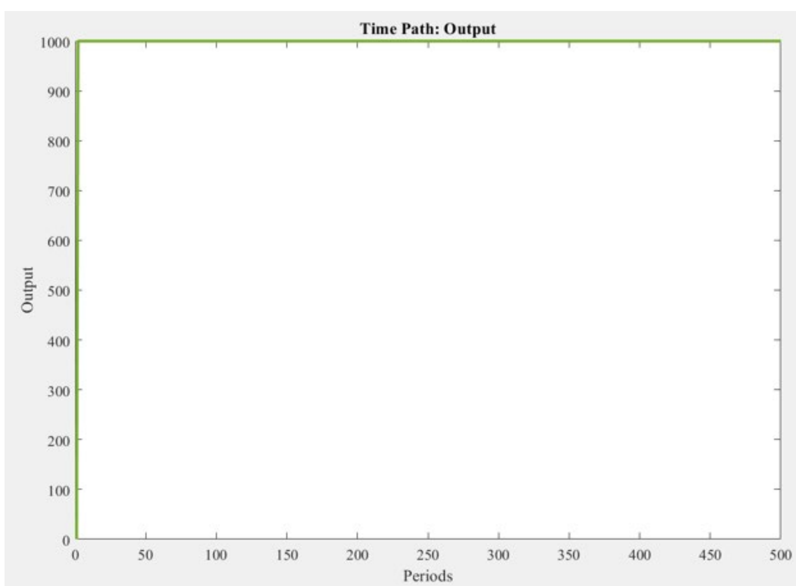

Figure 4.2: Time path for output when the price of machines is above the threshold

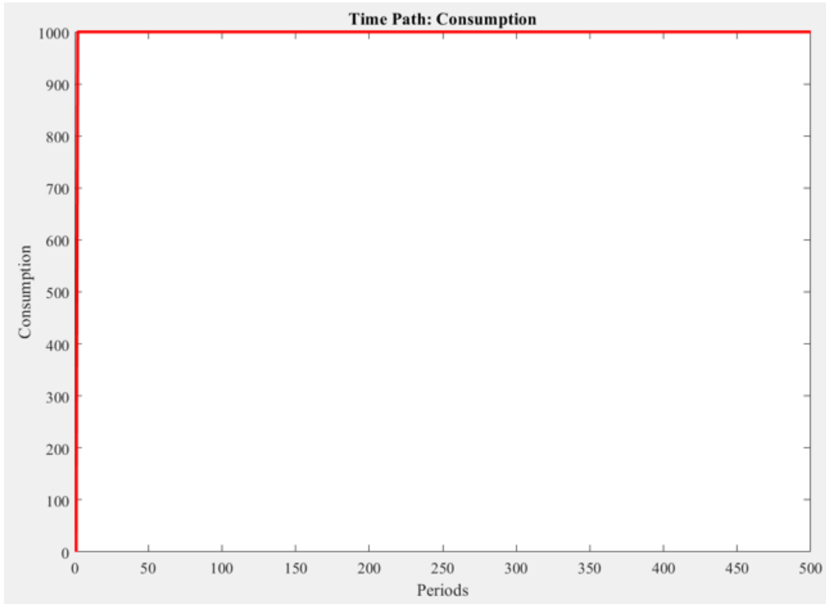

Figure 4.3: Time path for consumption when the price of machines is above the threshold

Since the assumption in this model is that the number of humans is constant, output and consumption are also constant. Figures 5.1, 5.2, and 5.3 show the time paths for the number of machines, output, and consumption respectively when the price of machines is below the threshold. In this case, machines are now cheap enough that the singularity outcome occurs.

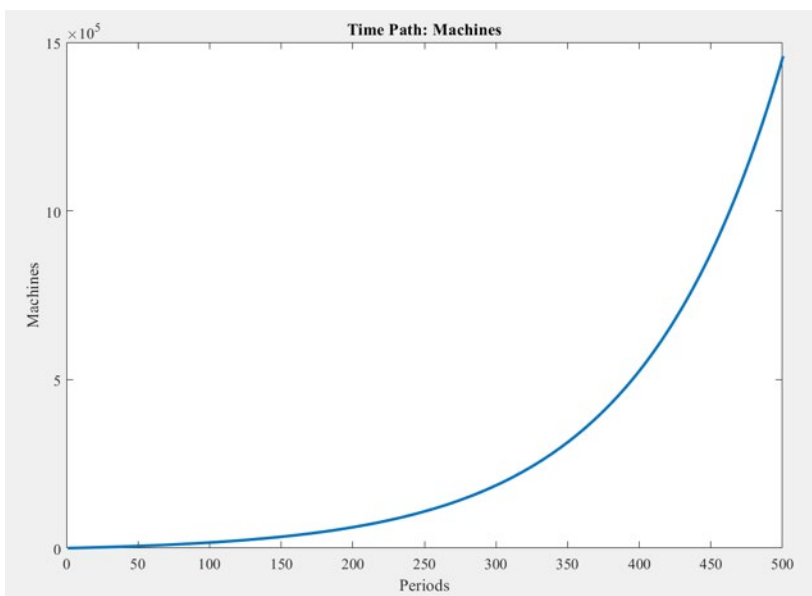

Figure 5.1: Time path for machines when the price of machines is below the threshold

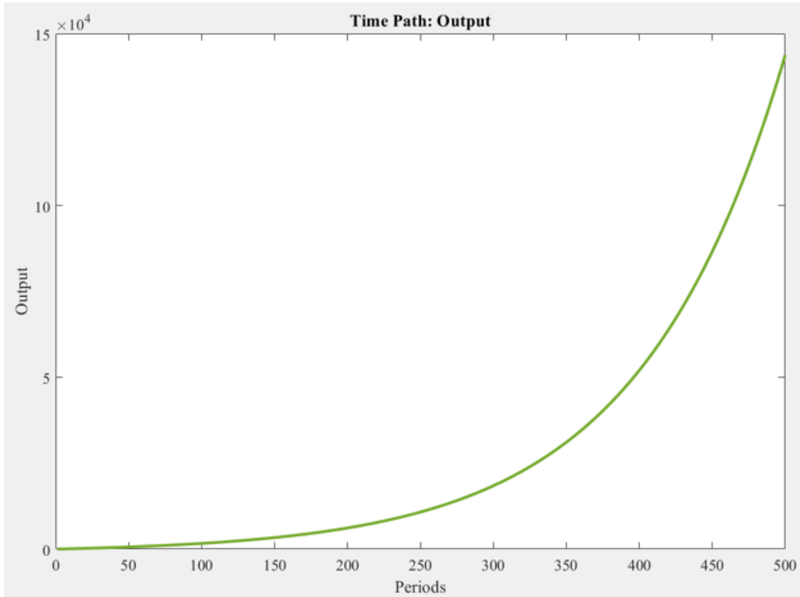

Figure 5.2: Time path for output when the price of machines is below the threshold

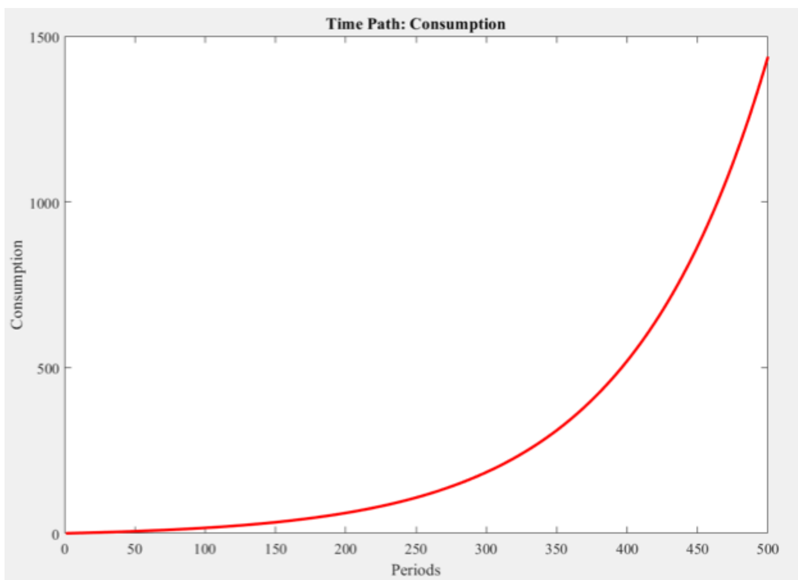

Figure 5.3: Time path for consumption when the price of machines is below the threshold

Machines, output, and consumption all go to infinity in the long-run. The results of these simulations are consistent with the theoretical outcomes predicted in Korinek and Stiglitz' (2017a) model.

3.2. The Model with Depreciation

Using the same preferences and production, I add depreciation to the budget constraint. I once again simulate the model in DYNARE with both high and low depreciation, to see the effects on the time paths of output, consumption, and the number of machines. In this case, I only simulate the model when the price is below the threshold because when the price of machines is too high, the economy will always remain in the no machine steady-state. Figures 6.1, 6.2, and 6.3 show the time paths for machines, output, and consumption respectively with a very low depreciation rate ($\delta = 0.01$).

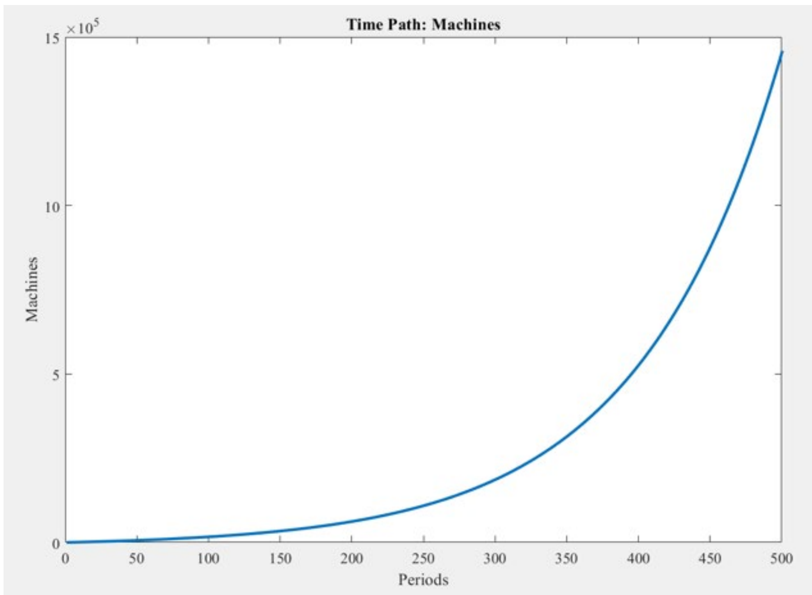

Figure 6.1: Time path for machines when the price of machines is below the threshold and depreciation is very low

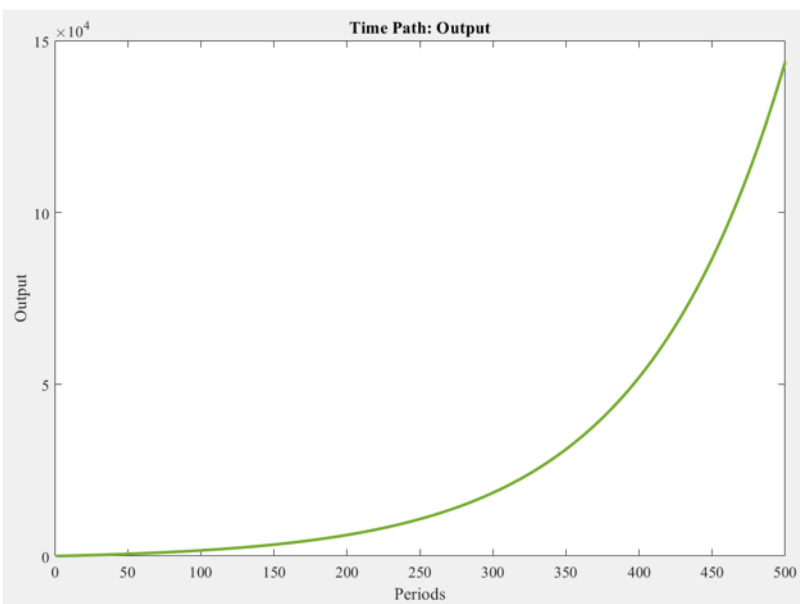

Figure 6.2: Time path for output when the price of machines is below the threshold and depreciation is very low

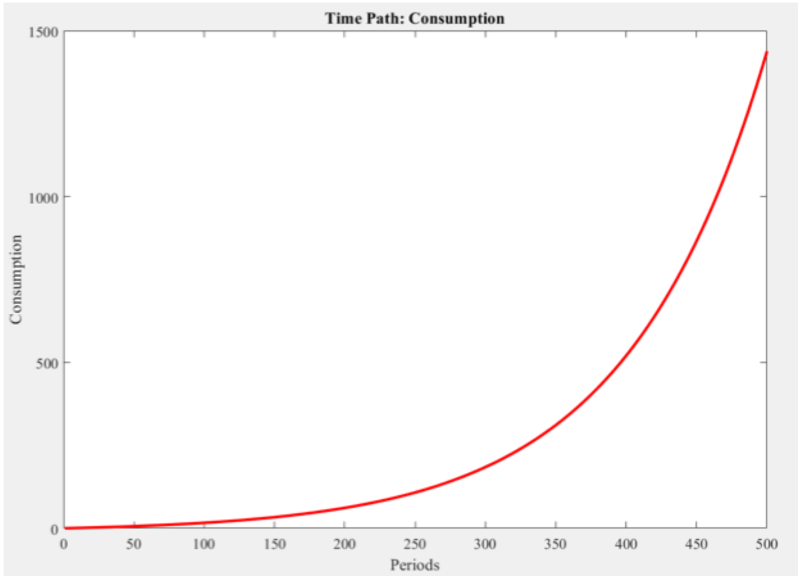

Figure 6.3: Time path for consumption when the price of machines is below the threshold and depreciation is very low

A small amount of depreciation is not sufficient to prevent an economic singularity, and as such the resulting time paths look much like those from the simulation of the Korinek and Stiglitz (2017a) model when the price of machines is below the threshold. Figures 7.1, 7.2, 7.3 show the time paths for machines, output, and consumption with a very high depreciation rate ($\delta = 0.99$).

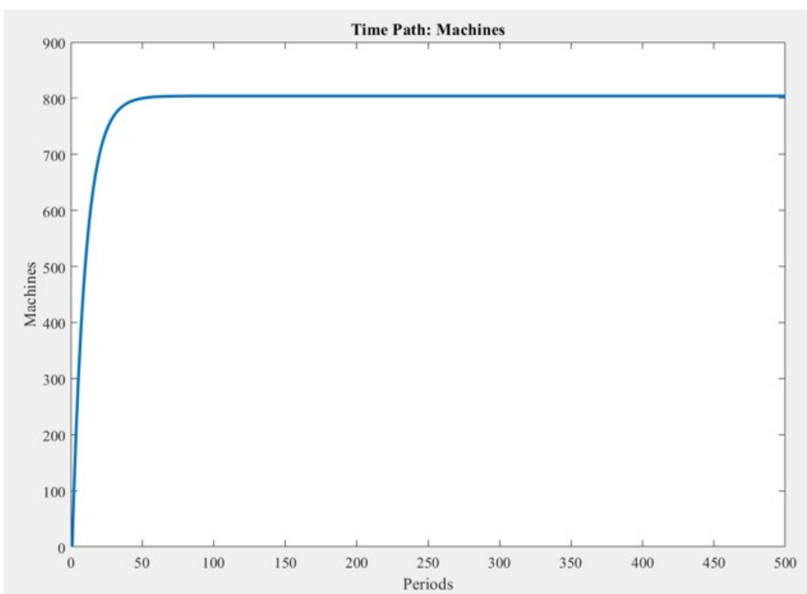

Figure 7.1: Time path for machines when the price is below the threshold, but depreciation is very high

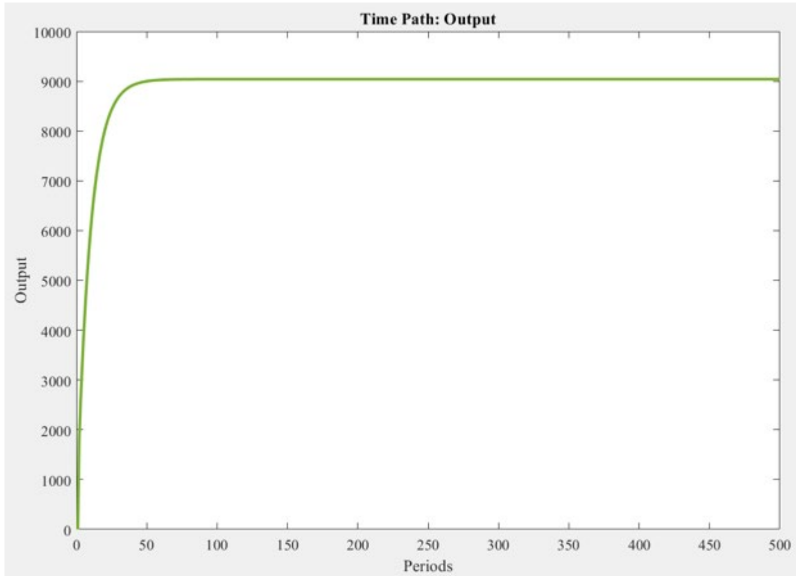

Figure 7.2: Time path for output when the price is below the threshold, but depreciation is very high

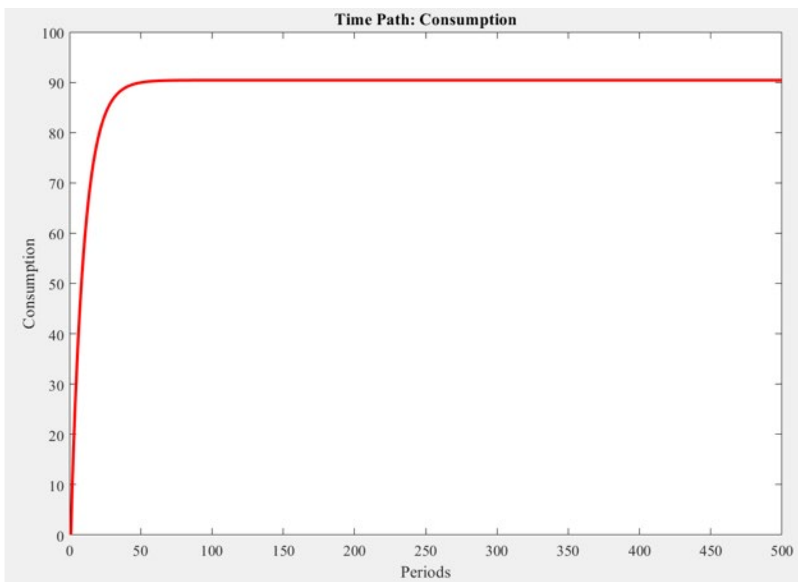

Figure 7.3: Time path for consumption when the price is below the threshold, but depreciation is very high

In this case, the depreciation each period is enough to offset the low price of machines, and in the long-run the number of machines goes to a positive steady-state. Because the number of humans is held constant in this model, and the number of machines is eventually constant, output and consumption also end up constant in the long-run at positive steady-state levels.

3.3. The Model with Depreciation and Decreasing Price

It is likely that as machines become more advanced, they also decrease in price. To capture this, the price of machines will now be a decreasing function rather than a static parameter. The following simulations are done with the high depreciation rate ($\delta = 0.99$) in order to see if a new result can be obtained. Figures 8.1, 8.2, 8.3 show the time paths for machines, output, and consumption respectively when the price of machines decreases 1 percent each period.

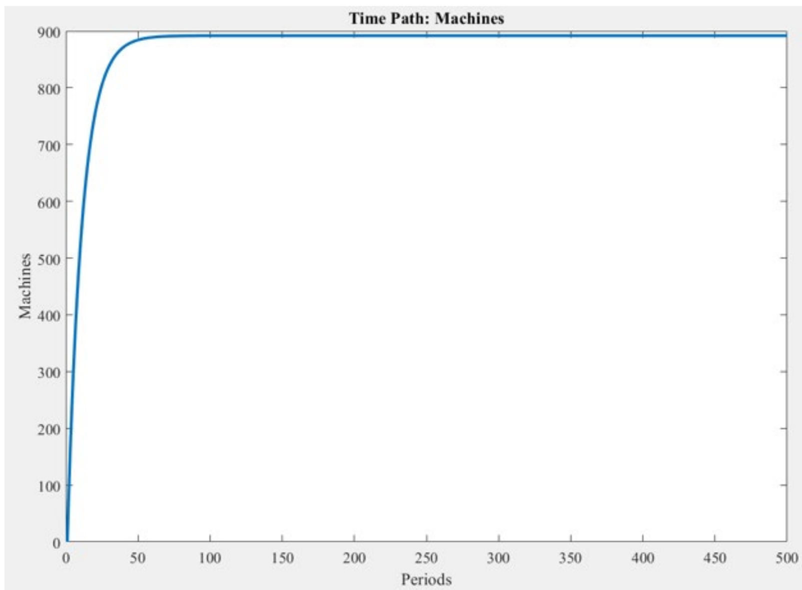

Figure 8.1: Time path for machines when depreciation is high and the price decreases slowly

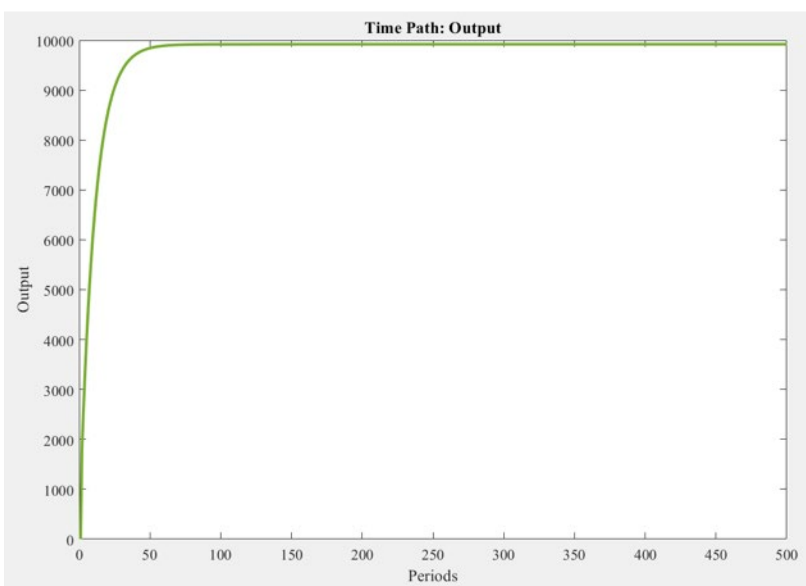

Figure 8.2: Time path for output when depreciation is high and the price decreases slowly

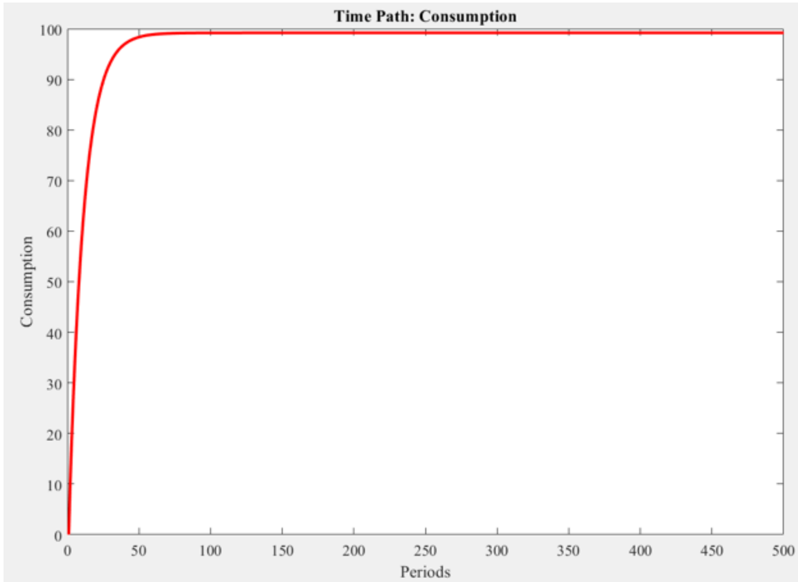

Figure 8.3: Time path for consumption when depreciation is high and the price increases slowly

In this case, the decreasing price of machines is not enough to offset the high depreciation rate and the number of machines, output, and consumption all converge to positive steady-state values that are higher than the steady-state values when the price is not decreasing. Figures 9.1, 9.2, 9.3 show the time paths for machines, output, and consumption respectively when the price of machines decreases 99 percent each period.

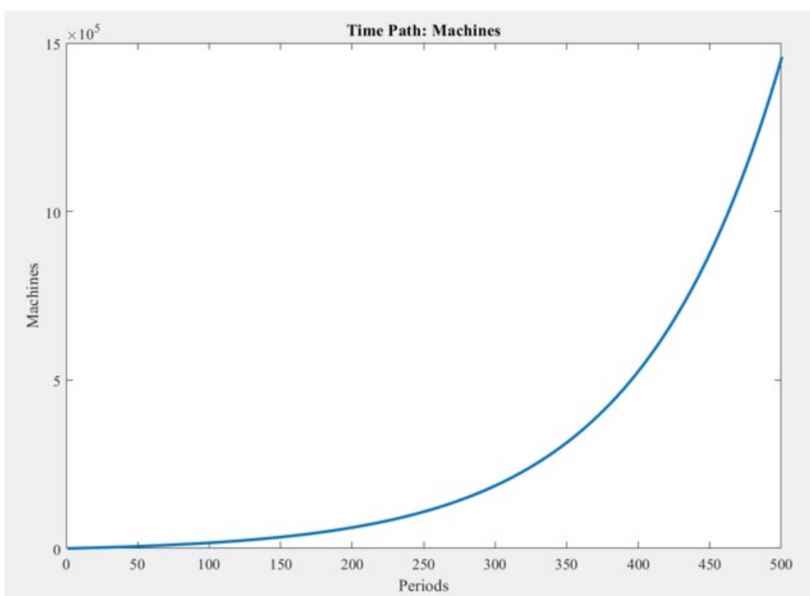

Figure 9.1: Time path for machines when depreciation is high, but the price of machines is decreasing quickly

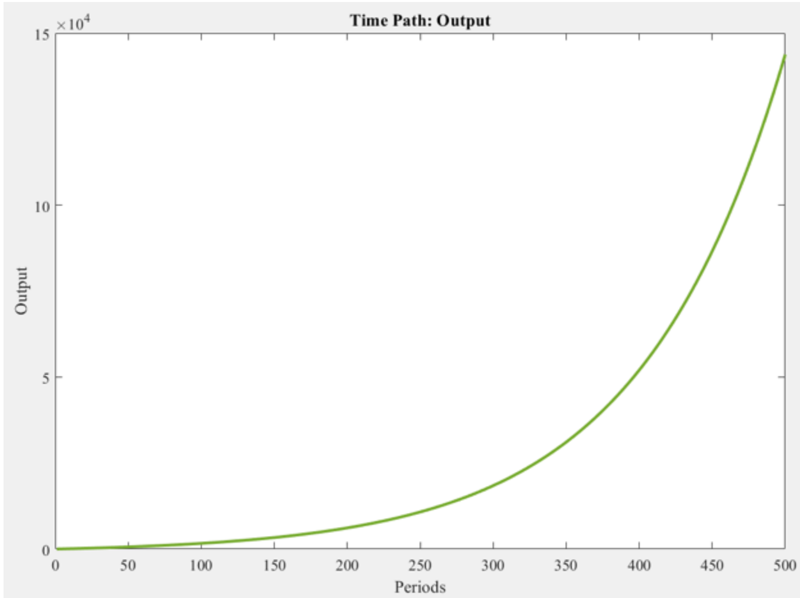

Figure 9.2: Time path for output when depreciation is high, but the price of machines is decreasing quickly

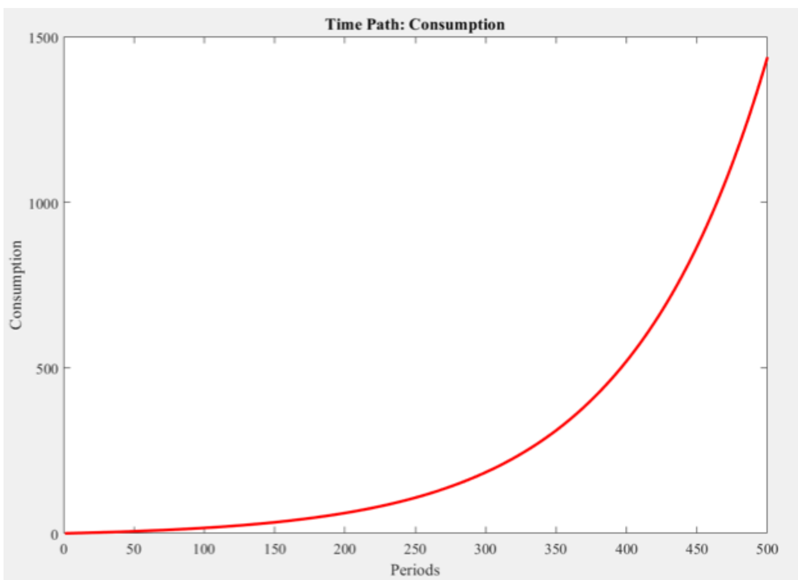

Figure 9.3: Time path for consumption when depreciation is high, but the price of machines is decreasing quickly

In this case, the price of machines is decreasing so quickly that it overwhelms the effect of depreciation and the number of machines, output, and consumption all diverge to infinity in the long-run.

4. Conclusion

In this paper, I have considered a few of the possible outcomes of automation on the economy. From theoretical analysis, I find there are three main outcomes that are possible depending on the price of machines. When machines are expensive, there can either be no machines or a positive steady-state of machines, depending on how expensive machines are. Once machines have a low enough price, they will replace humans in the labour force and output will diverge to infinity in the long-run. The growth rate of output converges to an exponential rate. Because of the linear production function in my model, human wages remain unchanged.

Simulations of the Korinek and Stiglitz (2017a) model with the price of machines above and below the critical threshold corroborate their theoretical predictions. When the price of machines is above the threshold, an economy that starts with no machines stays with no machines, and because human labour is constant in the model, output and consumption are constant. When the price of machines is below the critical threshold, there is an economic singularity. The number of machines, output, and consumption all go to infinity in the long-run. The growth rates of machines, output, and consumption all approach a steady-state.

When a small amount of machine depreciation is added to the simulation of the model, the outcome does not change. If the price of machines is below the critical threshold, the result is economic singularity. However, when depreciation is very high the simulation shows the number of machines, output, and consumption all converging to positive steady-state levels in the long-run. This is consistent with one of the theoretical results from my extended model with endogenous saving.

The final set of simulations show possible outcomes when the price of machines is decreasing and depreciation is high. If the price of machines is decreasing at a very small rate, the effect of the depreciation of machines is larger than the effect of the decreasing price of machines. In this case, the number of machines, output, and consumption all go to positive steady-state levels. However, when the price of machines is decreasing at a rapid rate, the effect of depreciation is not enough to offset the effect of the decreasing price of machines. In this case, the number of machines, output, and consumption all diverge to infinity, and there is once again an economic singularity.

Overall, the three main outcomes from the theoretical model (zero steady-state, positive steady-state, and economic singularity) are shown to be possible when performing simulations. However, this model has limitations, particularly with its linear production function. Further simulations should be done with production functions that are non-linear in order to be able to examine the effect of worker-replacing technology on wages.

I have not considered the effects of scarce factors such as land or energy as Korinek and Stiglitz (2017a) do. Limiting factors, such as land or energy, will not necessarily prevent a singularity. Since their introduction, computers have gone from taking up entire rooms to the palm of a hand. It is conceivable that future AI will take up very little space. It is also conceivable that the limited land on earth will no longer be a constraint as humans expand further into the universe. Current levels of technology certainly are constrained by the amount of energy required to run efficiently. However, once again future inventions could mean that technology runs on very little energy with renewable energy sources. Because of the linear form of production in this model, so far wages have been unaffected by the influx of machines. Because the wage is equal to the marginal product of labour, and the production function is linear, the wage rate remains constant. However, if a non-reproducible factor, for example land, is added to production, it has the potential to cause human wages to fall. Furthermore, the non-reproducible factor will put a limit on growth. However, growth will not be stopped all together, it will just be very small. Further research should consider this in more depth.

Another important line of research I have not considered are the implications of a machine labour world on inequality, or policies to mitigate possible negative effects. Korinek and Stiglitz (2017b) find that the economic singularity outcome is harmless if workers care only about absolute level of labour earnings. This is because of the linear production function of their model and the fact that wages remain constant as a result. Since the economy is experiencing such rapid growth in economic singularity, it is possible to redistribute the increased wealth to compensate the humans who have lost their jobs, making them at least as well off as before, or perhaps better off. The issue becomes more complex when non-reproducible factors are considered. The non-reproducible factors put a limit on growth, and human wages fall while non-reproducible factor owners get all the rents (Korinek & Stiglitz, 2017a). A solution Korinek and Stiglitz (2017b) suggest is to tax the non-reproducible factor and redistribute the tax revenues to the displaced human workers. However, this result could be examined further in future research.

5. References

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6. Appendix

Model Calibrations:

Note: all model simulations were performed with 1000 periods. However, only 500 periods are shown because the behavior of the economy does not change over the last 500 periods.

Korinek and Stiglitz (2017a):

Variable	Calibration
β	0.99
θ	0.9
γ	10
A	10
h	100
m_0	0
Number of periods simulated	1000

Table 1: Calibration of Korinek and Stiglitz with the price of machines above the threshold γ. The threshold price is 9.9, found by multiplying β and A. Corresponding to figures: 4.1, 4.2, and 4.3

Variable	Calibration
β	0.99
θ	0.9
γ	9.8
A	10
h	100
m_0	0
Number of periods simulated	1000

Table 2: Calibration of Korinek and Stiglitz with the price of machines below the threshold γ. The threshold price is 9.9, found by multiplying β and A. Corresponding to figures: 5.1, 5.2, and 5.3

The Model with Depreciation:

Variable	Calibration
β	0.99
θ	0.9
γ	9.8
δ	0.01
A	10
h	100
m_0	0
Number of periods simulated	1000

Table 3: Model calibration with the price of machines below the threshold γ and with low depreciation. Corresponding to figures: 6.1, 6.2, and 6.3

Variable	Calibration
β	0.99
θ	0.9
γ	9.8
δ	0.99
A	10
h	100
m_0	0
Number of periods simulated	1000

Table 4: Model calibration with the price of machines below the threshold γ and with high depreciation. Corresponding to figures: 7.1, 7.2, and 7.3

The Model with Depreciation and Decreasing Price:

To have the price decreasing, γ now enters the model as a variable with time rather than a parameter. For the simulations with price decreasing slowly, each period γ is updated as 99 percent of the previous γ (the functional form: $\gamma_t = 0.99\gamma_{t-1}$). For the simulations with price decreasing quickly, each period γ is updated as 1 percent of the previous γ (the functional form: $\gamma_t = 0.01\gamma_{t-1}$). The price of machines will never be less than or equal to zero.

Variable	**Calibration**
β	0.99
θ	0.9
δ	0.99
A	10
h	100
m_0	0
γ_0	20
Number of periods simulated	1000

Table 5: Model calibration high depreciation and the price decreasing slowly. Corresponding to figures: 8.1, 8.2, and 8.3

Variable	**Calibration**
β	0.99
θ	0.9
δ	0.99
A	10
h	100
m_0	0
γ_0	20
Number of periods simulated	1000

Table 6: Model calibration high depreciation and the price decreasing quickly. Corresponding to figures: 9.1, 9.2, and 9.3