An Udder One Bites The Dust

Optimal Grazing Management and Land Degradation

by

Hector Demarco

A Thesis Submitted in Partial Fulfillment of the Requirements for the

Degree of Bachelor of Science, Honours

in the Department of Economics

University of Victoria

April, 2019

Supervised by Dr. Daniel Rondeau

for

Dr. Elizabeth Gugl, Honours co-advisor

Dr. Martin Farnham, Honours co-advisor

Abstract

Land degradation affects at least 3.2 billion people through biodiversity loss, extinction of species, increasing food and water insecurity, and exacerbating climate change. The management of grazing lands is one of the most important direct drivers of land degradation. For this reason, understanding the extent to which farmers' economic incentives contribute to land degradation is possibly an important step towards reducing the rate of land degradation and devising policies to improve the welfare of individuals across the globe. This study develops a model of optimal grazing by an individual farmer who is trying to maximize the discounted lifetime profits they obtain from their private land. The model is simulated as a dynamic system of three differential equations linking soil, grass biomass, and the number of animals the farmer puts on the land. The results show that the optimal grazing strategy is contingent on the initial levels of soil. If soil is abundant, the farmer's optimal grazing strategy is to choose a grazing rate that never degrades the land. If the soil levels are low, the farmer would intentionally deplete the land. If the soil levels are below a certain threshold, the land degradation would be imminent, regardless of the grazing rate chosen by the farmer. When the soil levels are low, governments can stop land degradation by introducing subsidies for farmers who keep livestock levels below a certain target.

Keywords: dynamics, grazing, optimal farm management, land degradation, desertification

Contents

1 Introduction

Land degradation is currently affecting at least 3.2 billion people through the biodiversity loss, extinction of species, increasing food and water insecurity, and exacerbating climate change by reducing plant cover, which then reduces greenhouse gas mitigation (Scholes et al., 2018). “[T]he management of croplands and grazing lands is currently the most extensive direct driver of land degradation”. These agricultural activities affect the condition of the land directly in both the short-run and the long-run, and currently represent almost 80% of all agricultural land. In fact, even though grazing takes place in 25% of the global land surface, making it the single most extensive form of land use on the planet (Asner et al., 2004), the demand of meat production is projected to double by 2050 (FAO, n.d.).

Over the years, many studies have used various different definitions for land degradation and desertification, generating some confusion in the literature (D'Odorico et al., 2013). In my research “land degradation” refers to “reduction, or loss, of the biological or economic productivity and complexity (...) resulting from (...) human activities and habitation patterns...” (UN, 1997). “Desertification” refers to “land degradation in arid, semi-arid, and dry-sub-humid areas resulting from various factors, including climatic variations (drought) and human activities (overexploitation of drylands)”(UN, 1997).

Understanding the extent to which the farmers' economic incentives contribute to land degradation is possibly an important step towards reducing the rate of land degradation and devising policies to improve the welfare of individuals across the globe. This study develops a model of optimal grazing by an individual farmer who is trying to maximize the discounted lifetime profits they obtain from their private livestock breeding farm. The model is simulated as a dynamic system of three differential equations linking soil, grass biomass, and the number of animals the farmer puts on the land. The results show that the grazing strategy that maximizes discounted profits is contingent on the initial levels of soil. If soil is abundant, the optimal strategy is choosing a grazing rate that never degrades the land. If the soil level is low, the optimal grazing strategy is case dependent (it can only be solved numerically), and the farmer intentionally degrades the land. Finally, if the soil level is below a certain threshold, land degradation is imminent, regardless of the farmer's decision. All of these results hold, even for reasonably large changes in market prices for feed and livestock. This suggests that if land degradation occurs in lands with abundant soil, then lack of farmer education (land dynamics/land quality are unknown), market imperfections than do not allow an easy transition of land ownership (asymmetric information of land quality), and/or political instability that does not allow for long term farmer financial planning can be the causes of land degradation. If the land degradation occurs on lands with low levels of soil, the farmer intentionally causes the land degradation, and governments can address this behaviour by implementing subsidies contingent on the livestock levels on their farm. If the land degradation occurs in lands with insufficient soil, the grazing rate decision by the farmer cannot stop this from happening.

I focus on livestock breeding farms as often the lower quality lands (such as dry-lands) are not suitable for crop production (Pennsylvania State University, 2018), and land degradation in these low-quality lands tend to have the most negative consequences on its population. D'Odorico et al. (2013) report that dryland degradation costs developing countries 4{8% of their National Gross Domestic Product, and that about 135 million people in 1995 were at risk of mass starvation due to land degradation. In addition, I focus on privately owned farms because much of the grazing land in the world is starting to move into private holdings. This is a common historical process in Australia, and North America (in 2001, 88% of the grazing lands in the U.S were privately owned (Follett & Reed, 2010)), but now Africa, Central Asia, and China are following the same trend (Galvin et al., 2008).

Many economists are concerned with land degradation. They try to understand what are the main economic drivers of grazing choices and predict a farmer's future course of action. With this information, the economists can conclude under what conditions it is in the farmer's best interest to deplete their land, and then work with government officials to assess when intervention is recommended and what are the most effective polices that can be implemented to stop the land degradation.

The literature on livestock breeding farm management is quite extensive, but these papers fail to fully address the problem of predicting long-term farmer behaviour and their effects on land degradation, due to the complexity of the underlying biology. This motivates my research to further investigate these long-term dynamics. For example, Bowen & Chudleigh (2018), and MacLeod et al. (2004) run simulations in search of the grazing rate that would maximize a farmer's discounted profits over time, based on some initial land conditions. The issue with these papers is that the authors use normative models with transitions, which may or may not appropriately reflect the dynamics of pasture growth. The conclusions that they derive rely on outputs associated with the initial conditions of the land in the simulations, which does not provide any insight on the farmer's decision making process nor the biological dynamics of the system.

Typically, dynamic analysis allows for more comprehensive understanding of all the evolving elements of the system at hand. This methodology allows the researcher to modify all these moving parts (parameters, variables, and equations) and interpret the long-term consequences, by observing the interaction of the elements of the system through time. Dynamic modelling is extremely relevant to the 7 principles for the conservation of wild living resources established by Mangel et al. (1996) (most particularly, principles III, IV, V, and VI). The only downside if these models is that closed form solutions are not always available, due to the mathematical implication of solving complex systems of differential equations. As White (2000) points out, "The curse of dimensionality means that optimisation models are not applicable to multi-species ecosystems; (...) Most models are for no more than two species with only the local stability of the steady state assessed, [and] the behavioural assumptions of many models are unrealistic...". There is a trade-off between making the model as realistic as possible, and being able to find a true optimal solution.

In spite of the benefits associated with this methodology, the literature on dynamic economic analysis of livestock breeding farms and land degradation is not very extensive. There is only one paper that does a dynamic economic analysis of a livestock-grass-soil system: Ibañez et al. (2007)/Valderrama (2005). Other studies, such as Huffaker & Wilen (1989) and Noy-Meir (1975), do an economic analysis only on the dynamics of livestock and grass. By omitting the dynamics of the soil, their conclusions are not practical for assessing the long-term consequences of a farmer's actions, since soil is the main measure of land productivity and land degradation (Doran, 1996). Meanwhile, papers by Synodinos et al. (2018) and Williams & Albertson (2006), have more complex biological models with stochastic components (such as rain variability), but they ignore the economic dynamics. These last studies answer the question of what happens if a farmer were to choose a given grazing rate, but not whether a real farmer would choose said rate.

The problem with Valderrama's paper, is that they model a myopic farmer. The farmer they simulate is not only not aware of the long-run consequences of their actions, but they would also never sacrifice profits today to substantially increase the net present value of their lifetime profits. I find this assumption unrealistic. It is well documented that farmers use agricultural techniques such as crop rotation and land resting, to insure that they can increase their lifetime profits by forgoing some of their present profits (Hunt et al., 2014). Also, if a myopic farmer happens to be running a farm, a well educated investor would take advantage of the opportunity for arbitrage. The investor would buy the farm from this myopic farmer, paying them the same amount that this farmer would make from their inefficient myopic farming management, and then sell the farm to a farmer that cares about maximizing the net preset value of their lifetime profits. Since these forward looking farmers would be willing to pay the maximum NPV that can be achieved from an optimal farm management strategy, myopic farmers would be driven out of the market.

Therefore, my contribution is to restructure the economic aspect of Valderrama's model, so that the farmer's actions simulate those of a farmer that is maximizing their discounted lifetime profits, rather than those of a farmer that maximizes present profits.

2 The Original Model

I base my model on Valderrama's. In this section I explain their model, and in the next section I describe my own. Their model incorporates a commercial livestock breeding farm on privately owned lands. The farms consist of livestock that can be either be fed by the grass on the land, or supplementary feed provided by the farmer in case of grass shortage. The purpose of the farm is to maximize the birth rate of livestock, and sell these newborns right after their breastfeeding period (the newborns do not consume grass). Then, using data gathered from available sources, the authors build three different sets of parameters that correspond to a cattle farm and a sheep farm in the Spanish region of Extremadura, and a goat farm from the south-east of Iberia.

This model consists of a system of three differential equations for the livestock "L", the grass "G", and the soil "S". The rate of change of Livestock "$\dot{L}$" consists of a rate of new livestock "NF" (its value depends on the current profitability of the farm) minus a slaughtering rate "CU". The rate of change of grass "$\dot{G}$" is based on a primary production "PP", a grass decomposition rate "GD", and a grass consumption rate "GC". Lastly, the rate of change of soil "$\dot{S}$" is a function of two parameters intrinsically related to the region of the land (the weathering bedrock rate and the leaching rate), and the organic matter "OM", which is the degraded grass absorbed by the soil minus some soil erosion rate "SE" related the slope of the land and the eroding effect of the rain.

The farmer chooses the livestock level based on current costs per animal and current revenues per animal. When revenues are higher than costs, the farmer increases the level of livestock on the land. When costs are higher than revenues, they reduce the livestock levels. The revenues per animal are fixed (given by the market), and the costs per animal are based on a fixed amount plus the cost of the supplementary feed that the farmer needs to provide to the livestock (costs per animals are minimized when no supplementary feed is needed). The amount of the grass on the land at a given point in time depends on the amount of livestock on the land, the amount of grass on the land, and the levels of soil of the land.

All the equations are taken directly from Ibañez et al. (2007), which is the English version of Valderrama (2005). Capital letters represent variables, while lower case letters represent constants. Some values were omitted in Ibanez et al.'s model, but included in Valderrama's original dissertation. In this paper, I include all of Valderrama's original values. Figures 18, 19, 20 in the appendix contain all the parameters that they use, their meaning, their units of measure, and the corresponding values for the three sets of simulations (the numbers in brackets correspond to the my model's values).

Rate of Change of Livestock Equation ($\dot{L}$):

$$\dot{L} = NF - CU$$

The rate change of livestock "L" is based on the difference between the new females "NF" and the culling rate (slaughtering rate) \CU".

$$CU = \frac{L}{ubl}$$

The culling rate is a function based on the current level of livestock \L", and the useful life of a breeding female \ubl". It states that the animal population is composed so that the animal population \depreciates" gradually over time, instead of all at once.

$$NF = \left(\frac{uin}{UC}\right)^{nf1} \cdot CU$$

The new female variable is computed by a ratio between the the market price of a breeding female \$uin$" and the costs of maintaining that female \$UC$", to the power of a parameter \$nf1$". This parameter corresponds to the willingness of the farmer to change the herd size. If the \$uin/UC$" ratio is bigger than 1, current profits per animal are higher than current costs per animal, and the farmer increases the herd size. If the ratio is between zero and one, the farmer decreases the herd size (this model limits the decrease to values smaller or equal to \$CU$"). The \$NF$" is then multiplied by the culling rate, so that the change in livestock is proportional to the \depreciation" of the livestock.

$$UC = \frac{spr}{cec} \cdot MAX\left[0; uen - \left(FR \cdot gec\right)\right] + ouc$$

\$UC$" is the costs per animal. The first term of the equation represents the cost of feeding the animal. \$spr$" is the cost of feed, \$cec$" is the energy that the feed provides, \$uen$" is the energy that the animal requires to be optimally fed, \$FR$" is the functional response of the animal (how much the animal eats based on the available grass), and \$gec$" is the energy that the grass provides. The maximum function is required because, in this model, the animals can eat more than what they need, causing the farmer to buy a negative amount of feed. The second term \$ouc$" represents the other unspecified costs per animal.

The main takeaway from this equation is that when the animals are fully fed by the vast availability of the grass, the farmer does not have to bare other costs outside of the unspecified costs per animal. However, if the livestock is not able to fulfill its energy requirements with the grass on the land, the farmer buys the necessary supplementary feed to fulfill the animals' needs, increasing the costs per animal.

$$FR = xca\left[1 - EXP\left(-\frac{G}{fr1}\right)\right]$$

\$FR$" stands for functional response, and it models the animal's grass intake. \$xca$" is the maximum amount of grass that the animal can consume, \$G$" is the amount of grass available at any given point in time, and "$fr1$" is a functional response parameter that indicates how effective the animals are at finding the grass on the land.

The function for the rate of change of livestock can then be represented in terms of the relevant variables "L" (livestock), "G" (grass), and "S" (soil):

$$\dot{L} = \left(\frac{uin}{UC}\right)^{nf1} \frac{L}{ubl}\left(1 - \frac{L}{ubl}\right)$$

$$\dot{L} = \left(\frac{uin}{\frac{spr}{cec}\, MAX\left(0; uen - FR \cdot gec\right) + ouc}\right)^{nf1} \frac{L}{ubl}\left(1 - \frac{L}{ubl}\right)$$

$$\dot{L} = \left(\frac{uin}{\frac{spr}{cec}\, MAX\left[0; uen - \left\{xca\left[1 - EXP\left(-\frac{G}{fr1}\right)\right]gec\right\}\right] + ouc}\right)^{nf1} \frac{L}{ubl}\left(1 - \frac{L}{ubl}\right)$$

$$\dot{L} = \left[\left(\frac{uin}{\frac{spr}{cec}\, MAX\left[0; uen - \left\{xca\left[1 - EXP\left(-\frac{G}{fr1}\right)\right]gec\right\}\right] + ouc}\right)^{nf1} - 1\right]\frac{L}{ubl} \qquad (1)$$

This expression shows how the the grass variable "G" is the only variable that affects the sign of the rate of change of livestock, through its effect on the current costs per breeding female. The livestock variable "L" only serves the purpose of making the rate of change proportional to the current levels of livestock on the farm (it does not affect the sign of $\dot{L}$).

Rate of Change of Grass Equation ($\dot{G}$):

$$\dot{G} = PP - GD - GC$$

The change in grass is a function of its primary production "PP", the grass decomposition "GD", and the grass consumption "GC".

$$PP = GR \cdot G\left[1 - \frac{G}{GK}\right]$$

The primary production of grass is itself a function of the intrinsic growth capacity of the grass "GR", the amount of grass in the land at a given point in time "G", and the grass' carrying capacity "GK".

$$GR = xgr \quad SM$$

The intrinsic grass growth depends on the maximum potential grass growth rate \\$xgr$", and the soil-grass productivity multiplier \\$SM$".

$$GK = xgk \quad SM$$

The carrying capacity of the grass depends on the maximum carrying capacity of grass \`xgk", and the soil-grass productivity multiplier \\SM".

$$SM = 1 \quad EXP \left[\quad \frac{MAX(0; S \quad msg)}{sm1} \right]$$

SM is a function of the amount of soil available at a given point in time, where \\$msg$" is the minimum soil necessary for grass growth, and \\$sm1$" is an associated parameter. Extremely fertile land has a value of \\$SM$" equal to one, making \\$GK$" and \\$GR$" adopt their maximum values \\$xgk$", and \\$xgr$" (respectively). Deserted land has a SM value of zero, indicating that grass growth is no longer possible, since \\$GR$" and \\$GK$" would also be equal to zero.

The primary production of grass can then be expressed as:

$$PP = xgr \quad \left(1 \quad EXP \left[\quad \frac{MAX(0; S \quad msg)}{sm1} \right] \right) \quad G \quad \frac{xgr \quad G^2}{xgk}$$

One can assess that the primary production is quadratic in the grass variable from this expression.

$$GD = gdr \quad G$$

This function simply states that the grass decomposition is based on a decomposition rate \\gdr", which is a fraction of the current level of grass on the land.

$$GC = FR \quad L$$

The grass consumption is equal to how much grass an animal consumes times the amount of animals on the land.

The function for the rate of change of grass can then be represented in terms of the relevant variables "L" (livestock), "G" (grass), and "S" (soil):

$$\dot{G} = xgk \left\{ 1 - EXP\left[-\frac{MAX(0; S - msg)}{sm1} \right] \right\} G - \frac{xgr \cdot G^2}{xgk} - gdr \cdot G - L \left\{ xca \left[1 - EXP\left(-\frac{G}{fr1} \right) \right] \right\} \quad (2)$$

This equation shows the connection between the three variables. Grass, livestock, and soil affect the rate of change of grass. More soil always has a positive effect, more livestock always has a negative effect, and more grass can have either a positive or negative effect. Also, note that when grass is equal to zero, the rate of change of grass is also equal to zero, so grass never grows on the land again.

Rate of Change of Soil Equation ($\dot{S}$):

$$\dot{S} = bwr - lch + OM - SE$$

The rate of change in soil depends on the weathering rate of bedrock "bwr", the leaching rate "lch", the net soil organic matter "OM", and the soil erosion "SE".

$$OM = (1 - fga) \cdot gdr \cdot G$$

The organic matter is proportional to the grass decomposition "$gdr \cdot G$" ("GD"). "fga" indicates the fraction of the organic matter that is released into the atmosphere, and the rest is absorbed by the soil.

$$SE = sep \cdot \tan(lsl)^{1.6} \cdot SR^2$$

The soil erosion depends on a soil erosion parameter "sep", the land slope "lsl", and the surface runoff "SR". This function indicates that the soil erosion is greater in steeper lands with high levels of surface runoff.

$$SR = (arf - bsi) \cdot EXP\left[-\frac{G}{sr1} \right]$$

The surface runoff is the rate at which sediments are being flushed away from the land by the water from rain. The annual rainfall is indicated by "arf", the bare soil infiltration (the water that is being absorbed by the land) is "bsi", and "$sr1$" is a parameter associated with the soil runoff.

The soil erosion can be expressed as:

$$SE = sep \cdot \tan(lsl)^{1.6} \cdot (arf \cdot bsi)^2 \cdot EXP\left[-\frac{2G}{sr1}\right]$$

The function for the rate of change of soil can then be represented in terms of the relevant variables "L" (livestock), "G" (grass), and "S" (soil):

$$\dot{S} = bwr - lch + (1 - fga) \cdot gdr \cdot G - sep \cdot \tan(lsl)^{1.6} \cdot (arf \cdot bsi)^2 \cdot EXP\left[-\frac{2G}{sr1}\right] \qquad (3)$$

Note that this function only depends on the variable "G", which might be unrealistic. In this model, higher levels of grass always have a positive effect on the rate of change of soil. More grass increases the organic matter that is absorbed by the soil, and increases water absorption. The increase in water absorption reduces the surface runoff effect, which then decreases soil erosion.

The Profit Function (PH):

$$PH = \left(uin - UC\right) \cdot L - fch$$

PH stands for the profits per hectare. Using all the our previous equations, it can also be expressed as:

$$PH = \left\{uin - \frac{spr}{cec} \cdot MAX\left[0; uen - xca \cdot \left[1 - EXP\left(-\frac{G}{fr1}\right)\right]\right] - gec + ouc\right\} \cdot L - fch \qquad (4)$$

Valderrama uses this system of equations and parameter data from different farms in Spain, and similar lands across the world, to simulate the model. They simulate a cattle farm and a sheep farm in a Dehesa of Extremeña, and a goat farm in the Iberic South East. They find that farmers would deplete the land of the cattle farm and the sheep farm, and that the outcome of the goat farm is extremely sensitive to changes in the parameter values. The initial values that they use for the three variables of interest (livestock, grass, and soil) are also presented in the tables of the appendix (figures 18, 19, 20).

Valderrama does not claim that his findings are indeed what we would observe in reality. At the moment of their study, many of the parameters values had never been estimated in Spain, making the author recur to other studies done in similar regions of the world, or their own intuition and knowledge to fabricate them.

In this model, the only interaction between the farmer and the system is through the costs per animal. These costs enter indirectly into the "$\dot{L}$" function, through the new females variable "NF", and it implies that the farmer changes the level of livestock based on the current costs per animal. This restricts the farmer from making decisions such as lowering the livestock level (reducing present profits), letting the grass grow, and then reintroducing the livestock into the system. At first glance, this does not seem to be a issue of major concern, but it could potentially impact the lifetime profits of the farmers quite substantially.

In fact, once I replicate their simulations for all three sets of parameter values, I realize that the simulated farmers end up substantially decreasing their profits. This further illustrates my comments on the farmer being unrealistic. In the long run, two of their farmers end up sustaining levels of livestock that cost more than the revenues they generate. This happens because the rate of change of livestock "$\dot{L}$" is not well specified, and it restricts the farmer to slowly decrease the levels of livestock once all the grass has been depleted. The farmer is then "forced" to keep feeding their livestock with supplementary feed, generating them negative profits. Unfortunately, there are no parameter specifications nor mathematical transformations that can "fix" this issue. A new equation for the rate of change of livestock is then required.

Figure sets 1, 3, and 5 are extracted from Valderrama (2005), while Figure Sets 2, 4, and 6 are my own elaborations, which I compute and graph using the Jupyter Notebook for Python 3. The numbers in the x-axis of all the graphs correspond to time in years. The profit graphs were not presented by the original author, but I compute them using their equations for livestock (1), grass (2), soil (3), and profits (4), to illustrate that the conclusions that they achieve cannot be correct. It is obvious that these outcomes are inefficient even for a farmer that is only trying to maximize present profits. One would expect that the farmer would at least shutdown the farm once the profits hit the seemingly forever negative magnitudes that can be observed in the first two figure sets.

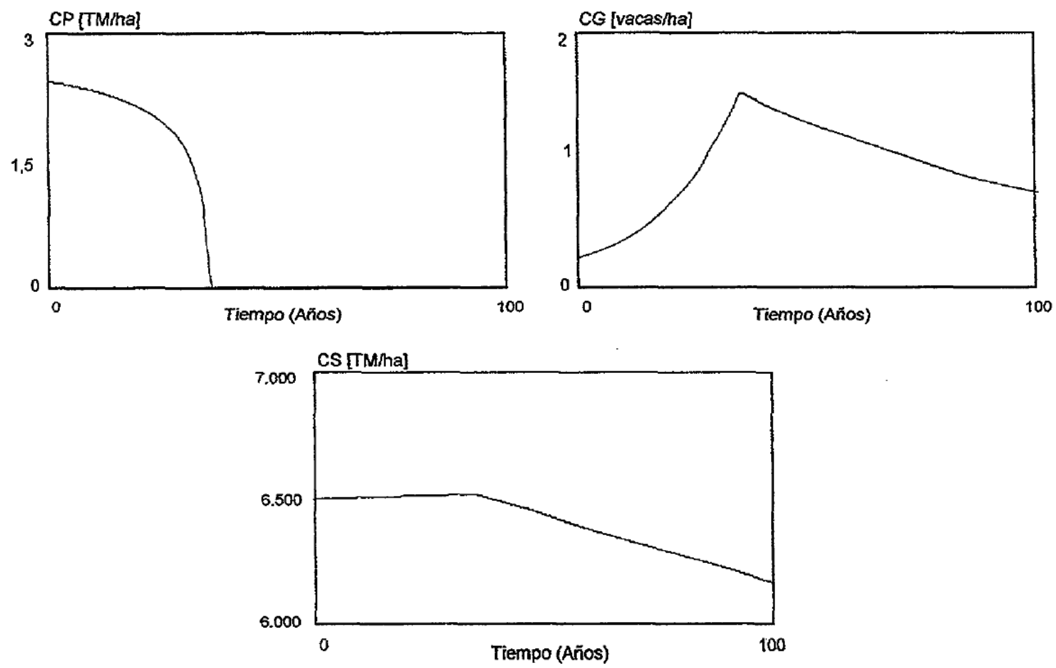

Figure 1: Cattle Farm (Valderrama 2005)

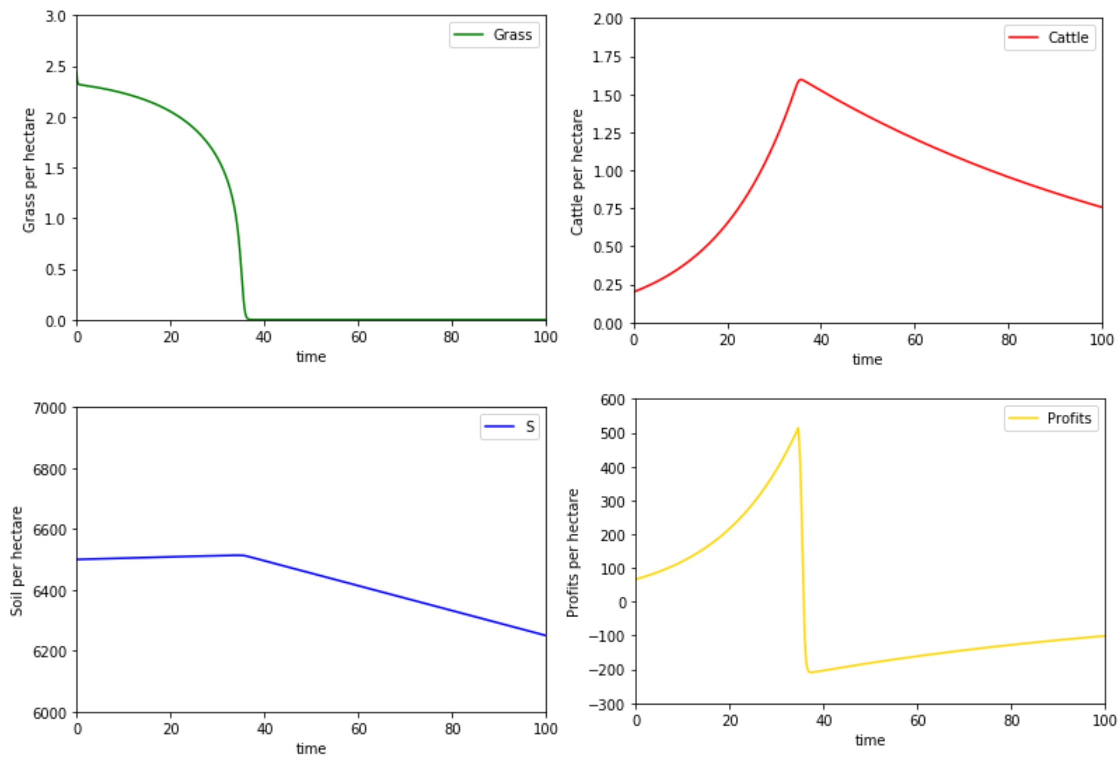

Figure 2: Cattle Farm (Own Elaboration)

Figure Sets 1 and 2 (Cattle Farm):

Figure Set 1 and Figure Set 2 show how I am able to fully replicate the cattle farm simulation. In this system, the farmer slowly depletes the land, reaching full depletion in around forty years. The pro t function demonstrates that, at the beginning of the simulation, the farmer makes positive pro ts, so they decide to increase the livestock level. Only once the system is fully depleted, the farmer's pro ts turn negative, as they have to fully feed the livestock with supplementary feed, and they start withdrawing the livestock from the land.

There are two main conclusions of this simulation. The rst one is that the behaviour of the farmer is truly myopic, as they always decide to increase their level of livestock when pro ts are positive, and always reduce livestock levels when pro ts are negative. The second one is based on the pro t graph in gure set 2. This graph shows that something in the model is not well speci ed. After the land is depleted, the modelled farmer follows a strategy that would cause them seemingly in nitely negative pro ts, and no economically rational farmer would ever decide to follow that strategy.

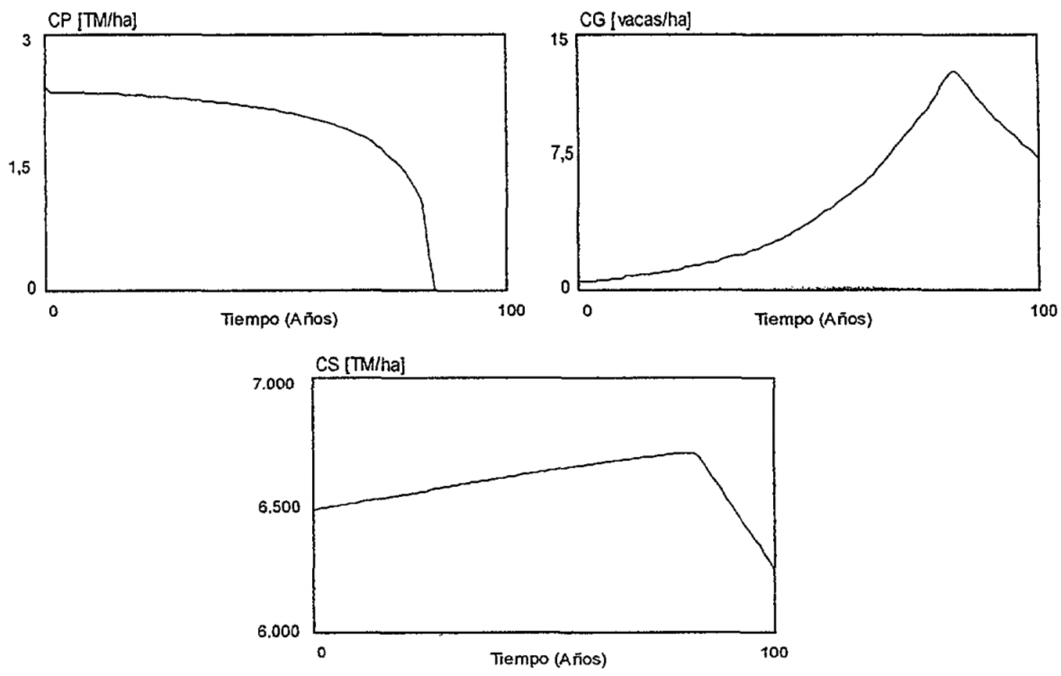

Figure 3: Sheep Farm (Valderrama 2005)

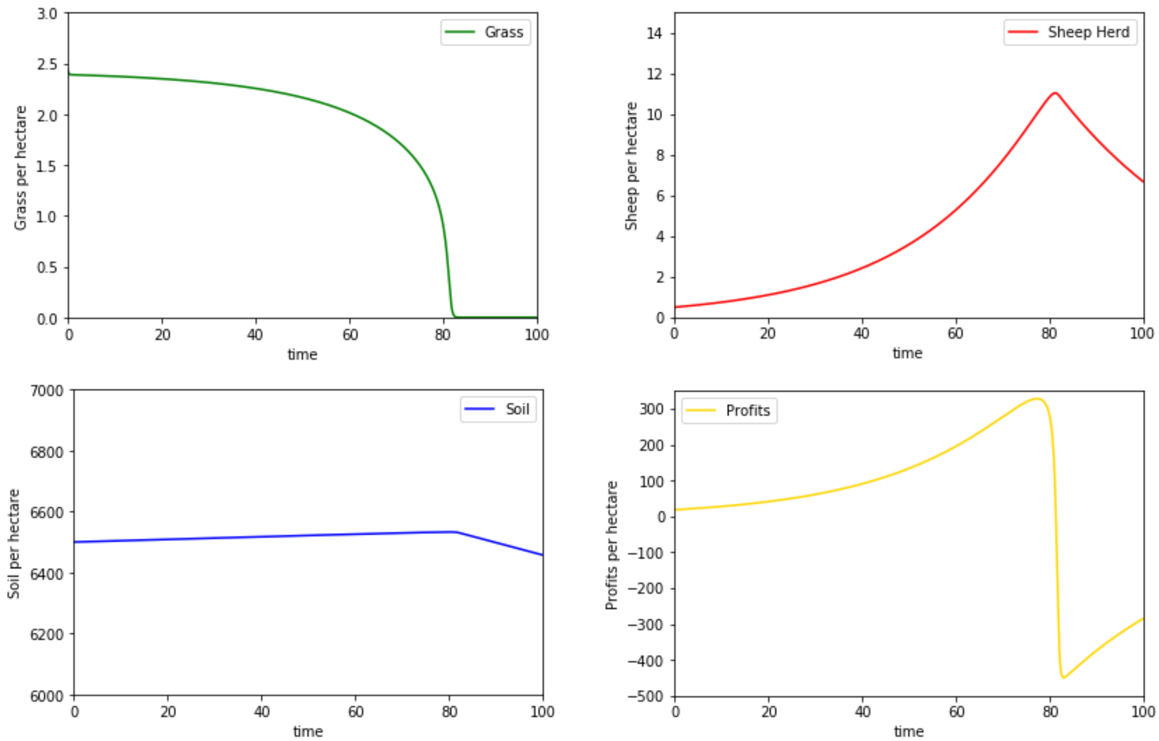

Figure 4: Sheep Farm (Own Elaboration)

Figure Sets 3 and 4 (Sheep Farm):

Even though this system takes more time to be fully degraded, the conclusions from these gures are the same as the ones from gure sets 1 and 2. The only thing that is worth mentioning is that the time path for soil presented by Valderrama must be incorrect. All the parameters that in uence the rate of change of soil are the same for these two sets simulations. So, once the level of grass is zero, the negative slope of the soil for the sheep farm should be the same as in the cattle farm. There is no mathematical way of explaining the di erent slope, so it must be a mistake. The correct soil path must be the one presented in the gure set 4.

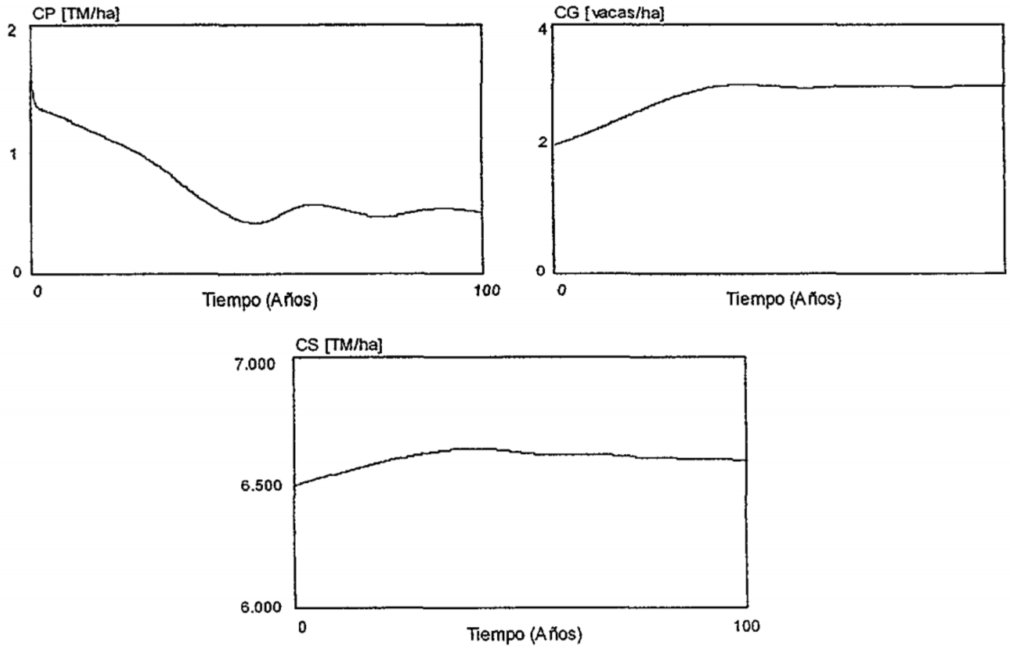

Figure 5: Goat Farm (Valderrama 2005)

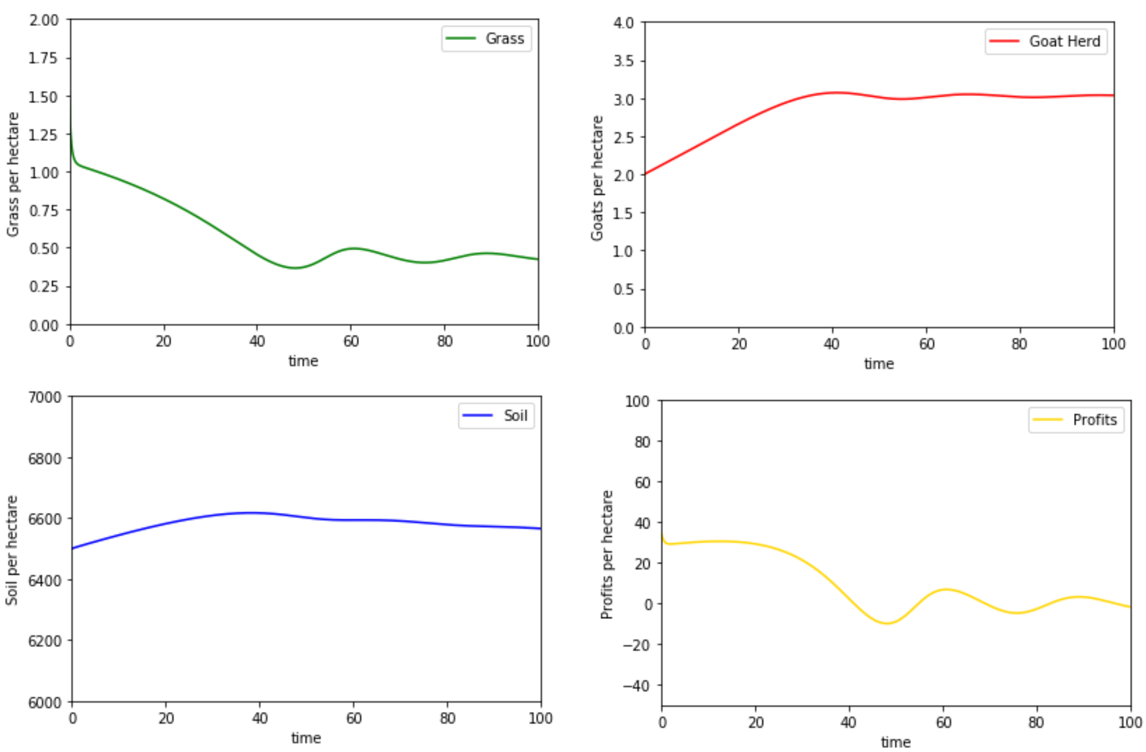

Figure 6: Goat Farm (Own Elaboration)

Figure Sets 5 and 6 (Goat Farm):

Finally, I can also fully replicate the goat farm graphs. This is the only simulation that does not end up in full depletion of the land, but there is still something to learn from the pro ts graph in gure set 6. The pro ts graph shows that the farmer starts with positive pro ts, so they decide to increase the level of livestock. However, this increase in the level of livestock does not generate an increase in pro ts, it in fact it decreases them. This occurs because the increased level of livestock consumes too much grass, forcing the farmer to provide supplementary feed to the goats. Again, it is clear that this model does not replicate true farmer behaviour, as the simulated farmer would intentionally lower their pro ts for nothing in return. An economically rational farmer would never choose to do this, unless they were truly myopic and were not aware of the consequences of their actions. The pro t graph in gure set 7 shows how simply keeping the herd size constant is a strictly superior strategy (higher pro ts, higher grass mass, and better soil) .

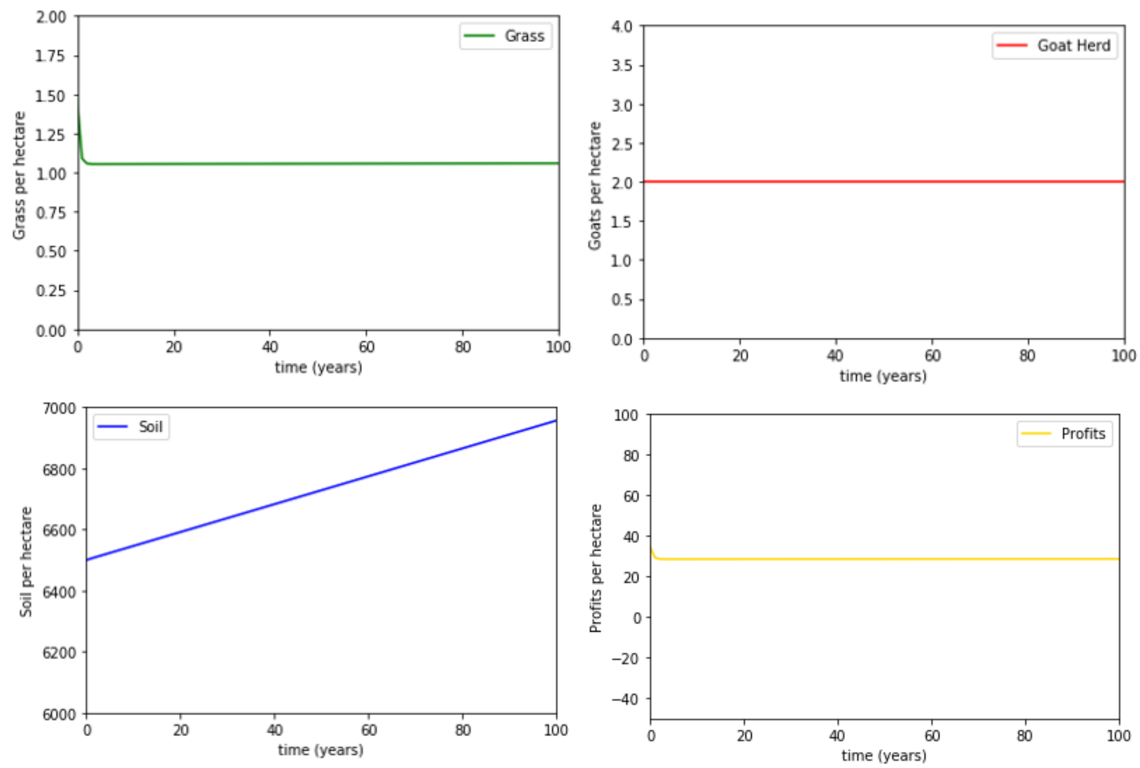

Figure 7: Goat Farm: Constant Herd Size (Own Elaboration)

3 The Forward Looking Model (^{f})

Motivated by the results in the previous section, I rede ne the rate of change of livestock (equation 1), which also requires a minor change to the pro t function (equation 4). I also make changes to the parameter values in the equation for the rate of change of grass (equation 2), so that the animals only eat as much as they need, and their functional response behaves as a constant.

New Rate of Change of Livestock ($\dot{L}^f$):

$$\dot{L}^f = A \quad CU \tag{1f}$$

The new rate of change of livestock consists of an explicit control variable \$A$", which stands for \new animals", and the culling rate \$CU$" of the previous model (\$L{=}ubl$"). The \$A$" variable can either be positive (the farmers buy new breeding females), or negative (the farmers sell breeding females).

New Rate of Change of Grass ($\dot{G}^f$):

$$\dot{G}^f = xgk \left(1 \quad EXP \left[\frac{MAX(0; S \quad msg)}{sm1} \right] \right) \quad G \frac{xgr \quad G^2}{xgk} \quad gdr \quad G \quad L \left(xca \left[1 \quad EXP \left(\frac{G}{fr1} \right) \right] \right) \tag{3f}$$

The mathematical form of the equation for the rate of change of grass remains the same, but I do make same changes to the parameter values for the simulations. In the \Original Model", given the parameters values for \$xca$", \$gec$", and \$uen$", cows are able to eat more than the energy they require, while sheep and goats cannot eat as much as they need. To x this issue, I de ne \$xca$" as a \$uen{=}gec$", which implies that the animal's maximum grass consumption equals their energy requirements over the energy provided by the grass. This change also implies that the \$MAX$" operator is no longer needed in the \$UC$" equation, since \$xca \quad FR \quad gec$" is never greater than \$uen$". The second x is for the \$fr1$" parameter, that describes how e cient the animals are at eating the grass available on the land. A high \$fr1$" value implies that there is enough grass on the farm for the animals to fully feed themselves, but they fail to do so. Based on Table 5 of Holechek (1988), the animal calorie intake is based solely on percentage of the animals weight, and this value corresponds to Valderrama's \$uen$" value (calorie intake required per animal). Therefore, I change the values of fr1 to be close to zero, to represent that the animals will always eat any amount of grass available on the land before needing to be fed supplementary feed. The functional response \$FR$" is then numerically a constant, but it still equals zero when there is no grass on the land. These changes to the \$xca$" and \$fr1$" make the animals only consume as much as they need, as long as there is su cient levels of grass on the land. Since fr1 is not zero, there is some ine ciency at very low levels of grass.

The New Profit Function (PH^f):

$$PH^f = uin \quad L \quad A \quad pa \quad \frac{spr}{cec} \left(uen \quad xca \quad 1 \quad EXP \quad \frac{G}{fr1} \quad ! \# \quad gec \right) \quad L + ouc \quad L \quad fch \qquad (4f)$$

The pro ts per hectare remain almost the same. I only add a new term to the equation, which is \$A \quad pa$". \A" is the new control variable, and it stands for the new animals bough/sold. The price of a new animal \$pa$" is chosen under my own criteria, as the previous research does not provide this number. Since prices do not really a ect the dynamics of the system, I choose an arbitrary price value equal to the annual revenues proceeding from an animal. This allows a farmer to make positive pro ts, as the animals generate revenues for as many years as their useful breeding life.

With these new equations I should be able to set up the optimal control problem, and solve it.

The Hamiltonian would be:

$$H = {}_1 \dot{L}^f + {}_2 \dot{G}^f + {}_3 \dot{S} + PH^f$$

Typically, one solves the optimal control problem by di erentiating the Hamiltonian with respect to the Variables \$A$", \$L$", \$G$", and \$S$", and the shadow prices \ $_1$", \ $_2$", and \ $_3$". These computations provide seven equations that represent the rst order conditions. The rst one (\$\frac{dH}{dA}$"), has the economic meaning that, in equilibrium, the marginal bene t from selling an extra animal has to be equal to the marginal cost of selling said animal. The marginal cost is the opportunity cost of the revenues forgone if the animal had been kept. The other FOC conditions represent the rate of change of shadow prices ($\dot{}_1$, $\dot{}_2$, and $\dot{}_3$), and the rate of change of the system of equations ($\dot{L}^f; \dot{G}^f; \dot{S}$), respectively. If the optimal control problem has an interior solution, one would simply search for the steady state of the system of equations, by setting all 7 equations to zero, and solving for \ $_1$", \ $_2$", \ $_3$", \$A$", \$L$", \$G$", and \$S$". Unfortunately, this system does not have a true interior solution (there is one equilibrium state, but it does not represent the solution to the optimal control problem). The reason behind the lack of a solution is that the livestock variable \$L$" and the control variable \$A$" only enter linearly in all equations, so the rst order conditions are not su cient for nding the solution that maximizes the sum of discounted pro ts. There is no other option than running simulations, based on educated guesses, to find an equivalent strategy that maximizes the discounted lifetime profits of the farmer. This method consists in comparing the outcomes of different grazing strategies, where the farmer either ends up depleting the land in some manner, or they reach some sort of steady state that they can sustain forever. This is obviously not ideal, but it is still a much better guess than some of the studies that I mention in the introduction. In this case, it is possible to fully observe how system interacts with the simulated grazing strategies, allowing for a much deeper understanding on what is happening to the land, what is driving that reaction, and whether the farmer would actually do such thing.

4 Results

After running various simulations, I conclude that there are three major sets of results corresponding to three different cases. Each case corresponds to a range of values of soil that yield the same dynamic behaviour. The first set of results corresponds to what I call "high levels of soil" (Case 1), the second set of results correspond to what "low levels of soil" (Case 2), and the third one corresponds to "irreversible desertification" (Case 3). The distinction between the three cases occurs because the rate of change of soil (equation 3) depends on the quantity of grass on the land "G", and the quantity of grass on the land also depends on the level of soil "S" in the land. The level of soil dictates how fast the grass grows (given by the intrinsic grass growth "GR"), and how much grass can be sustained on the land (given by carrying capacity of the grass "GK").

In Case 1, the optimal solution is choosing a grazing rate that consumes an amount of grass equal to the maximum grass production (call it "L "). I refer to the level of grass that maximizes grass growth as "G ". If "G " is lower than a certain critical level (call it "G^S"), then the soil quality decays forever, causing land degradation. These scenarios correspond to Case 2, and "L " is no longer optimal. The equation for finding "G^S" ($\frac{\dot{S}}{dG} = 0$) is not very interesting in its mathematical form, and it is very complex, so I will not present it in this paper. However, the "G^S" that is computed from this equation is of extreme importance for the analysis of the system. Finally, if the maximum amount of grass that the land can sustain (call it "G_{Max}") is below "G^S", then the land is always degraded, even when no livestock grazes it. This corresponds scenarios in Case 3.

All the figures in this section are computed with the parameter values corresponding to the cattle farm, but the same conclusions hold for the goat farm and the sheep farm.

4.1 Case 1: High Levels of Soil

For the "high levels of soil" simulations, the farmer maximizes their discounted lifetime pro ts by picking the livestock level that corresponds to the maximum grazing rate that the farm can sustain forever. This result holds for all three sets of values provided by Valderrama. To nd this result, I simulate many di erent grazing strategies, and nd the net present value associated with these strategies. By comparing the pro ts of all the simulation, I conclude that choosing a grazing rate level that equals the maximum grass growth rate, maximizes the discounted lifetime pro ts of the farmer. So, a long-term pro t maximizing farmer would always manipulate the livestock levels to reach the maximum grass growth rate as fast as possible, and then choose the livestock level that consumes an amount equal to this grass growth. In the appendix I show how to derive the grass level that maximizes the grass growth rate, and its associated optimal livestock level.

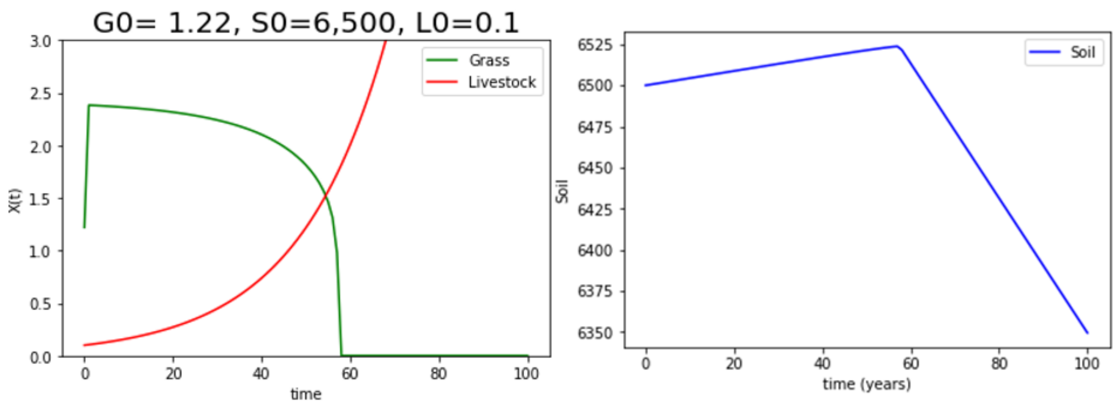

Figure 8: Evolution of the system where the farmer starts at a low livestock level of livestock, and increases the livestock level at a constant growth rate (full grass depletion occurs in 60 years and NPV of 1,511.7 euros/hectare).

Figure set 8 corresponds to a naive strategy. This farmer has very low pro ts at the beginning of the simulation, and then starts increasing the livestock at a constant growth rate. As I cannot predict when the livestock deplete the grass ex-ante, the livestock remains growing for computational simplicity. A real farmer would withdraw all livestock once the grass is fully depleted, so I compute the NPV using the positive pro ts only. This is nothing more than a benchmark simulation, to check that the results are consistent.

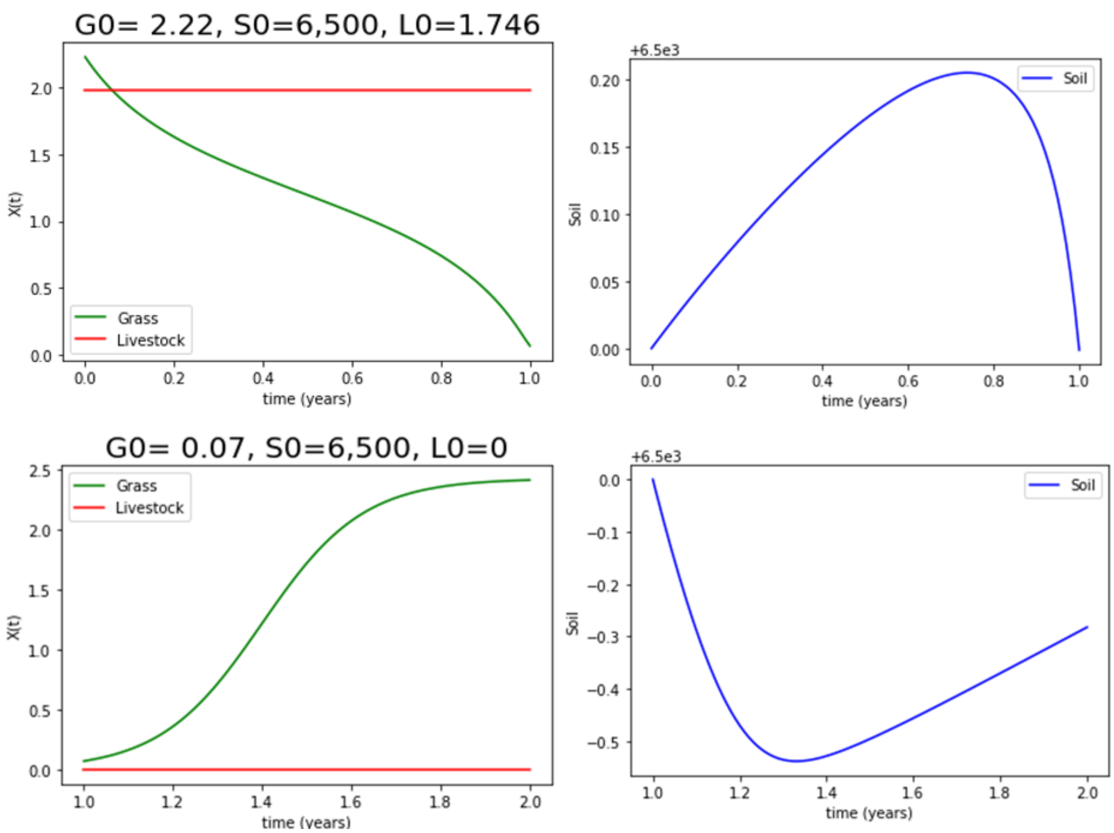

Figure 9: Evolution of the system with a yearly full grass consumption and a yearly full grass recovery (no land degradation and NPV of 5,309.5 euros/hectare).

Figure set 9 represents a grazing strategy where the farmer does a year of (almost) full grass consumption and a year of full grass recovery. The initial level of grass in this simulation corresponds to the ending value of the full year of recovery. I only present 2 years worth of simulations because I repeat the cycle 50 times, to generate 100 years worth of simulation, and then compute the NPV. Notice that the soil approximately returns to its initial value, so the land degradation is negligible. This strategy is strictly superior to the one presented in gure set 8.

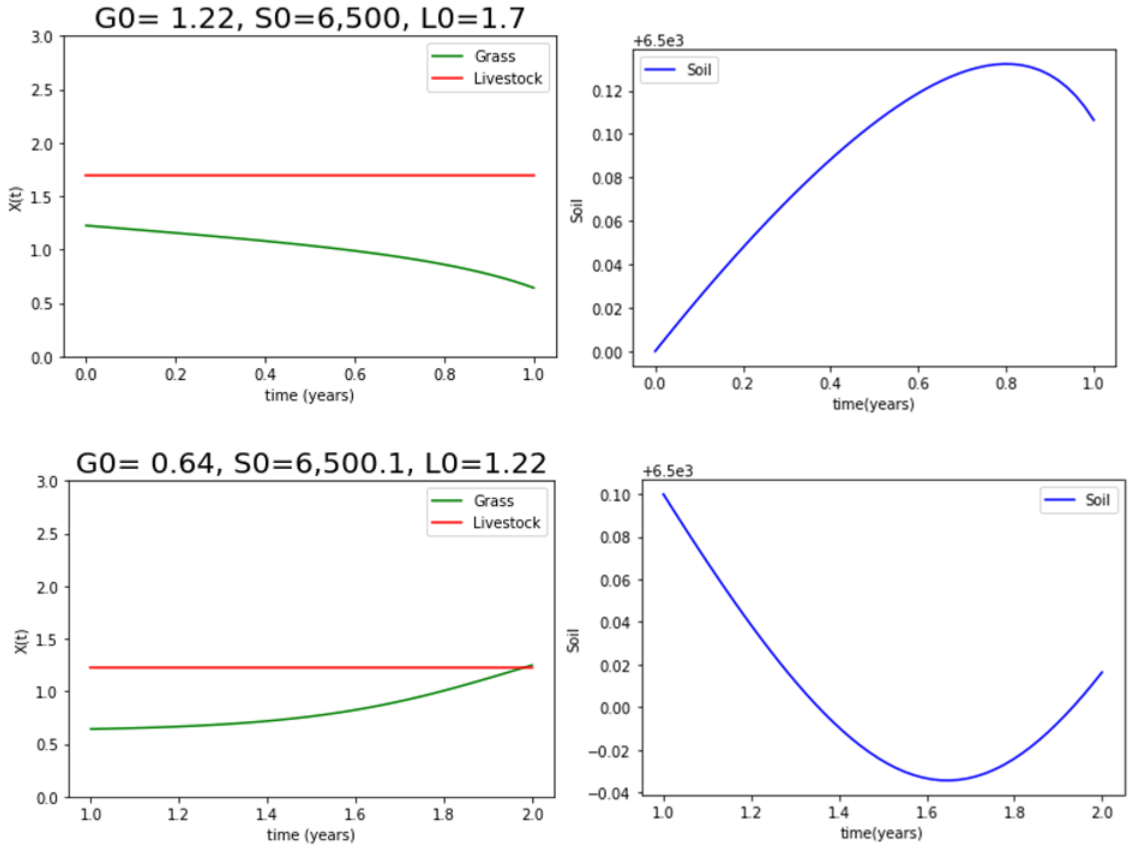

Figure 10: Evolution of the system under a year of high grazing pressure and a year low grazing pressure (no land degradation and NPV of 7,690 euros/hectare).

Figure set 10 represents a grazing strategy where the farmer does a more conservative cycle between a year of high-grazing rate and a year of low-grazing rate. The purpose of this cycle is to get full grass and soil recovery, so that the cycle is sustainable. Again, I only present 2 years worth of simulations because I repeat the cycle 50 time, to generate 100 years worth of simulation and then compute the NPV. This strategy is strictly superior to the one presented in gure set 9.

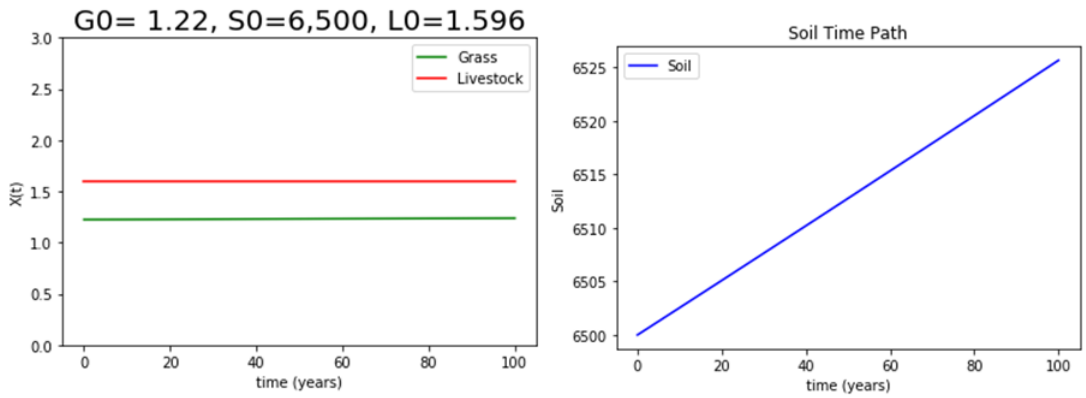

Figure 11: Evolution of the system under the maximum forever sustainable grazing rate (no land degradation and NPV of 8,379.9 euros/hectare).

Finally, gure set 11 represents the optimal grazing strategy, as this strategy maximizes the NPV for the initial conditions in this simulation. The farmer maximizes their discounted lifetime pro ts by choosing a level of livestock that only consumes the maximum grass growth. Under this scenario, the land is never degraded, and the farmer only has to buy enough livestock to keep the herd size constant.

4.2 Case 2: Low Levels of Soil

The second set of results correspond to the \low levels of soil" scenarios, where the steady state grazing strategy in gure set 11 is no longer optimal. In this case, the optimal grazing can only be computed numerically, but it always is in the farmer's best interest to deplete the land.

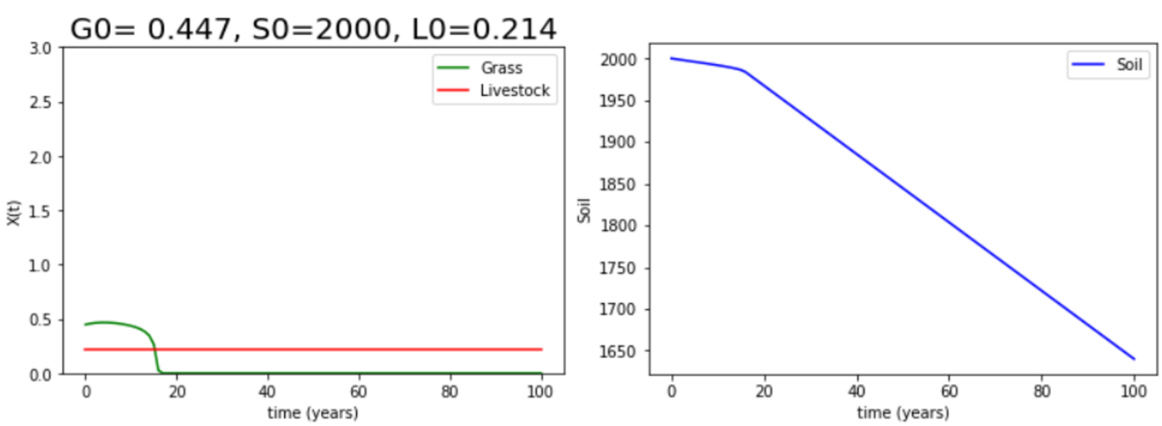

Figure 12: Evolution of Grass for a system with low initial levels of soil, using an initial grass that maximizes grass growth and an initial level of livestock that only consumes the grass growth (associated NPV: 586.09 euros/hectare)

Figure set 12 corresponds to a strategy where the initial grass maximizes grass growth for the initial level of soil. Then, the initial level of livestock is chosen so that consumption equals the grass growth, just as in the steady state strategy in gure 11. However, this strategy is no longer optimal. In this case, the steady state value of grass \$G$ " is below the \$G^S$" value, so the soil quality decades over time, and the land cannot sustain the livestock level \$L$ ".

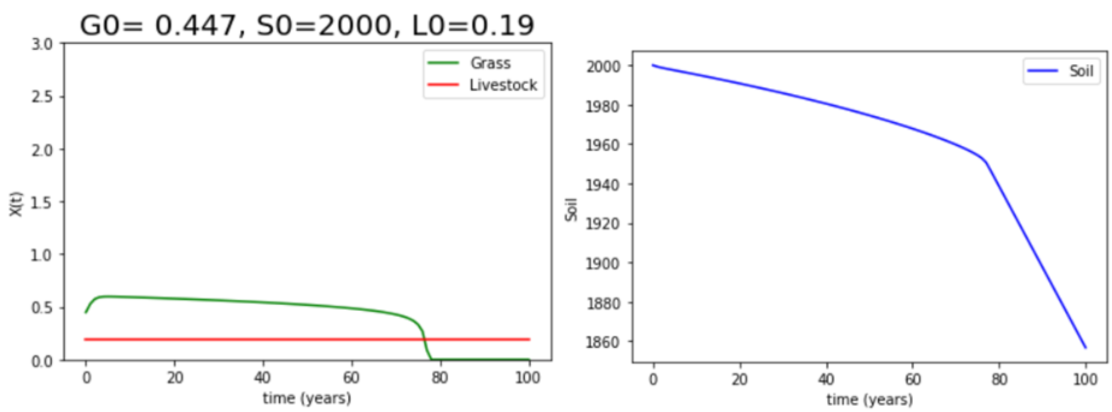

Figure 13: Evolution of Grass for a system with low initial levels of soil, using an initial grass that maximizes grass growth, and the initial level of livestock corresponds to the new optimal level (associated NPV: 957.87 euros/hectare).

Figure set 13 corresponds to a strategy where the initial grass is set to maximize grass growth for the initial level of soil. Then, the farmer chooses a level of livestock that consumes slightly less than the grass growth. This strategy generates a higher NPV than the one in gure 12, and my ndings indicate that this is the new optimal strategy, but I am not able to assess why this is optimal.

In gure sets 12 and 13 the livestock levels are sustained even after the depletion of the grass only for computational simplicity. I analyse the positive pro ts of the simulation when calculating the NPV, so I am assuming that the farmer shuts down the farm right before experiencing negative pro ts, even though the graphs do not show this behaviour.

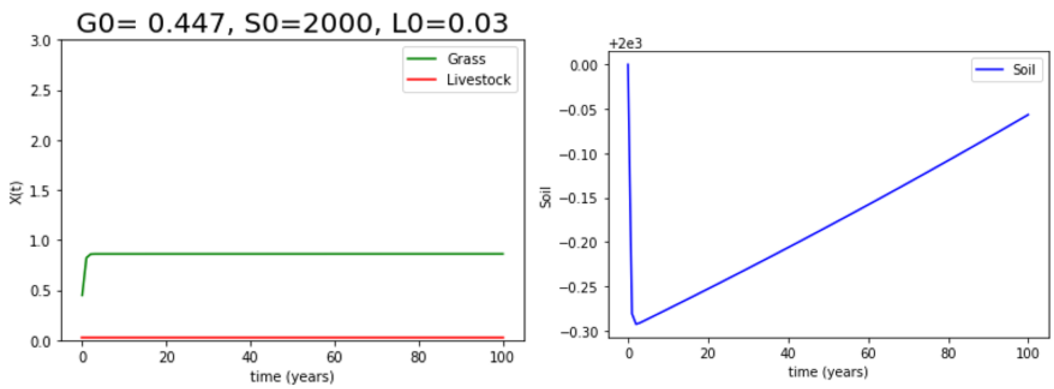

Figure 14: Evolution of Grass for a system with low initial levels of soil, using an initial grass that maximizes grass growth and a forever sustainable level of livestock (associated NPV: 153.65 euros/hectare).

Lastly, gure set 14 corresponds to the maximum forever sustainable livestock level in this low quality land. This set of graphs shows that the level of livestock that this farmer chooses does not consume all the grass growth. In this case, the farmer lets the grass grow beyond the level of grass that would maximize grass production, and in return, the soil levels recover. However, an economically rational farmer would never choose this strategy. The NPV associated with this strategy is far below the NPV associated with the other two strategies in gures 12 and 13. This implies that land degradation is optimal for low initial levels of soil.

4.3 Case 3: Irreversible Desertification

Finally, the "irreversible deserti cation" scenarios correspond to cases where even taking away all the livestock in the farm has no impact on the degradation of the land. In this scenario, the grass can never grow at levels high enough to reach \G^S", so the rate of change of soil is always negative, and the land degradation is imminent. An example of this scenario can be observed in gure set 15.

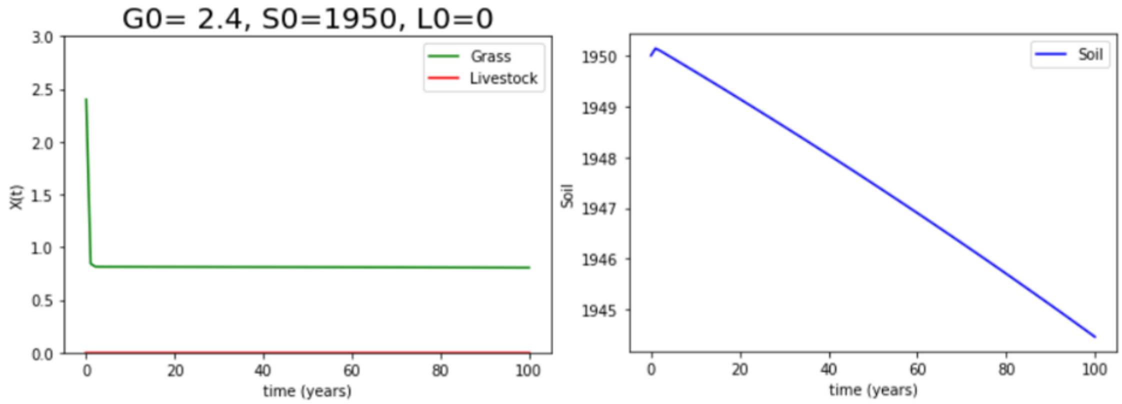

Figure 15: Evolution of Grass for a system with insu cient levels of soil, using a high level of initial grass and zero initial levels of livestock. Land degradation is imminent (associated NPV: 0 euros/hectare)

5 Discussion

There are several papers that discuss and criticize some of my modelling choices, and I am in full agreement with those critics. For example, Campbell et al. (2006), and Nasca et al. (2015), argue that agricultural research should not use models that rely on \one-size- ts-all" criteria. By ignoring the biological heterogeneity of the systems that are being studied, we might leave aside more e cient management techniques. This is specially relevant when looking at gure 17 in the appendix, which shows how sensitive the optimal steady state system in gure 11 is to changes in livestock levels. One should expect that the conclusions I derive from this dynamic analysis vary across di erent systems. Nonetheless, all my conclusions are speci c to the parameters for the lands that I study, and using my results for further generalization would be not be recommended.

Another crucial problem with my model is the exclusion of relevant variables, such as rain and rain variability. In the the system of di erential equations that I present, the annual rainfall variable only a ects the surface runo equation. My model does not take into account the direct e ect of rain on grass growth, which may or may not be relevant for the Spanish farms that I analyse, but it most de nitely a ects the livestock level decision making for most farmers. Yu et al. (2017), and Gherardi & Sala (2015) show how rain variability has a substantial negative impact on grass production. Therefore, a more realistic analysis would incorporate this interaction. In fact, Illius & O'Connor (1999) point out that in areas with highly variable climates in Africa, the long term grass productivity and land degradation risk are mostly determined by rainfall rather than the grazing levels chosen by the farmers.

Finally, variables tend to follow a stochastic behaviour, but my model is mostly deterministic. For example, Batabyal (2007) supports the claim that forage consumption by animals behaves as a random variable, instead of the deterministic behaviour that I adopt for this research. However, climate change is the most important stochastic process that I omit in my model. The negative effects that climate change have on livestock production have been studied quite substantially. Rojas-Downing et al. (2017) have published an extensive literature review of all the possible scenarios that the farmers might deal with in the upcoming decades, John et al. (2005) model the potential profit losses from different climate change outcomes in Western Australia, and Deressa et al. (2009) show the economic benefits that farmers can achieve from adopting different management strategies that take into account the effects of climate change on the land. Climate change is the single most difficult thing to model, as it causes most of the values of the elements in a model to follow a stochastic trend behaviour.

The reason for these comments is to further illustrate the complexity of understanding how biological systems behave, and my own limitations when it comes to assessing what the true optimal management strategy would be. While far from perfect, I believe that my model is a step in the right direction. My objective is simply to demonstrate the value that economists can add to the analysis of land degradation, without undermining the work of biologists. To reach any conclusions on how to act in response to the ongoing land degradation, it is fundamental to work with the best available information on the dynamics of the biological systems. Merging the two schools of knowledge (economics and biology) allows for a deeper comprehension on the underlying processes that are currently taking place in these lands. Societies, can then use this information to properly assess how to deal with the situation in their hands. So, I would like to encourage any biologist to criticize this model, further develop it, and work side to side with other economists to find the solution to this problem.

6 Conclusions

The conclusion from my analysis is that the optimal livestock level that the farmers choose is solely based on the biological dynamics of the system, and prices affect the overall profitability of the farm, but they do not affect the livestock levels chosen by the farmers. The conclusions with respect to the land degradation and government intervention vary across the different cases developed in the results section. As long as farmers are driven by the objective of maximizing the net present value of their lifetime profits, they never deplete their land in the high soil scenarios (Case 1), but land degradation does occur in low soil scenarios (Cases 2) and the imminent desertification scenarios (Case 3). With the current available information, I cannot judge which cases are responsible for most of the ongoing land degradation in the world.

6.1 Conclusions of Case 1

The Spanish ministry of agriculture published a national program to battle desertification where they included the creation of subsidies for farmers that keep their livestock levels below a fixed target (Valderrama, 2005). This program was primarily based on two UN publications (UNCED (1992), and UNCCD (1994)). The studies cited in these publication focus on biological trends only, so assessing whether the land degradation would follow its current trend, is not trivial. In fact, my results indicate that this subsidy might not provide any benefit to the Spanish society. If the Spanish farmers possess lands with "high" initial levels of soil, then these farmers would never deplete them, and the subsidy would be an unnecessary expenditure.

So, if the land degradation does indeed occur in lands with high volumes of soil, then the explanation lies outside of the constraints of the model. One possibility is the definition of land degradation that is being used, or how the land degradation is being measured. Sometimes, the reduction of the plant cover might be used to measure land degradation (Walpole, 1992). In the high level of soil cases that I study in this research, the farmer does reduce the plant cover, but the overall productive capacity of the land does not change. The only potential for government intervention would be in cases where there is a present negative externality. For example, if the reduction in plant cover affects the overall welfare of society. This reduction in plant cover could increase surface runoff, creating some floods in residential areas nearby. Not that many studies have looked into the effects of reduction in plant cover from grazing, but Rogger et al. (2017) have published a study that discusses the potential connection between floods and land use.

Another possible explanation could be lack of education from the farmers. Iosifides & Politidis (2005) do a qualitative study, based on conducting in-depth interviews with local livestock producers in western Lesvos, Greece, where they discover that most of the participants were not aware of the definition of "desertification". They also conclude that there is a serious problem with disseminating information about environmental problems, natural resource use, productive practices, and that there are no permanent structures of information and training on the environmental dimension of natural resource use in Lesvos. In this case, the forward looking farmers do not have all the information available about their land, so they cannot truly optimize their grazing management. At best, they would have use their own criteria to guess what is the ideal use of their land. In these cases, government intervention is also recommended. Investing in programs that measure the land conditions, education programs that train and inform the farmers about scientific findings, and facilitating communications between farmers and land experts, could potentially help these farmers better assess their grazing strategy. In these circumstances, government intervention would benefit both the farmers and the society, through sustainable food production and sustainable farming.

The last explanation is related with changes in discount rates and imperfect markets. In a perfect market, the price of the land would be equal to the highest net present value of the activities that can be performed on the land. In my analysis, NPV is maximized under the optimal forever sustainable grazing rate. So, if the farmers are in need of liquidity, but are not able to sell their farms for their true value (maybe due to asymmetric information of land quality), that would indicate a market imperfection that could cause land degradation. By not being able to sell their farm for their corresponding value, the farmers might want to deplete the land to maximize their present profits. In this case, the government could intervene by perfecting the market, so that the lands can transfer hands, and it is never depleted. In cases where the need for liquidity is due to political instability and massive migrations, the intervention of international bodies might be required.

6.2 Conclusions of Case 2

It could also be the case that the lands with high levels of soil are indeed never depleted, and the problem lies in the other two cases (lands with low levels soil and irreversible desertification). If this is indeed true, then the only thing that the government could implement are subsidies/tariffs based on the level of livestock that a farm sustains per hectare, which is exactly what the government of Spain did in 2005 (Valderrama, 2005). The farmer would then have an incentive to keep livestock levels that do not cause land degradation. The only policy improvement I would make would be to make the livestock target level contingent on the soil levels of the farms. Otherwise, the farmers who own lands with high levels of soil will receive free money (in the case of a subsidy), or unnecessarily restrict their the livestock production (in case of a tariff). Whether this measuring of the soil would be cost effective or not is beyond my knowledge.

6.3 Conclusions of Case 3

In the \imminent desertification" scenarios, the farmer cannot manipulate the grazing rate in any way to reverse the effects of land degradation. Since this degradation is independent from the grazing rate choice, I am not aware of any measures that governments can implement to stop these land dynamics, as the answer lies beyond the scope of this research. I suggest to further research these scenarios and land restoration techniques.

6.4 Final Comments

Finally, Valderrama mentions that some values that they use were estimated previously, but others were chosen arbitrarily. A very important value that was chosen arbitrarily was the initial level of soil in all the farms. If one were to be able to find this value, it would be possible to estimate the true optimal livestock grazing levels. Then, if the net present value of the lifetime profits that these grazing levels can achieve match the average cost of pasture-lands reported by the Government of Spain in 2005 (Gobierno de España, 2006), this would confirm that this model predicts realistic farmer behaviour. Since soil has never been measured, I would conclude that estimating the soil quality in regions that suffer from land degradation is of fundamental importance, and it should be the first measure to take.

7 Acknowledgements

I would like to start by thanking the entire economics department at the University of Victoria, and in particular, Dr. Marco Cozzi. I honestly think of myself as a very lucky individual for stumbling across such an excellent program with wonderful professors (almost by chance). All the merits go to the professors, who make sure that they are always available for students. All of you work harder than what any student would ever ask for, and I just want to let you know that your work does not go unnoticed. Thanks to all of you.

I would also like to mention that this thesis would have never been completed without the help of Dr. Daniel Rondeau. I really cannot under emphasize his contribution to this research. Not only he provided me with the tools and knowledge necessary for understanding and designing the model that I present in this study, but he also motivated me constantly to push myself everyday. Thanks to him I was able to go far beyond my own expectations, and achieve things I did not know I was capable of doing. Even if this does not end up being a great thesis, I could not be happier with this academic journey that we went through together.

Finally, to my mother and my father. Words will not do you justice, so I will simply like to say that all of this work is dedicated to both of you. Thank you.

References

Asner, G. P., Elmore, A. J., Olander, L. P., Martin, R. E., & Harris, A. T. (2004). Grazing systems, ecosystem responses, and global change. *Annu. Rev. Environ. Resour.*, *29*, 261–299.

Batabyal, A. A. (2007). Some statistical properties of total forage consumption by grazing animals on a rangeland. *Available at SSRN 1005201*.

Bowen, M., & Chudleigh, F. (2018). Grazing pressure, land condition, productivity and profitability of beef cattle grazing buffel grass pastures in the subtropics of australia: a modelling approach. *Animal Production Science*, *58*(8), 1451–1458.

Campbell, B. M., Gordon, I. J., Luckert, M. K., Petheram, L., & Vetter, S. (2006). In search of optimal stocking regimes in semi-arid grazing lands: one size does not t all. *Ecological Economics*, *60* (1), 75{85.

Deressa, T. T., Hassan, R. M., Ringler, C., Alemu, T., & Yesuf, M. (2009). Determinants of farmers' choice of adaptation methods to climate change in the nile basin of ethiopia. *Global environmental change*, *19* (2), 248{255.

Doran, J. W. (1996). Soil health and global sustainability. *Soil Quality is in the Hands of the Land Manager*, 45.

D'Odorico, P., Bhattachan, A., Davis, K. F., Ravi, S., & Runyan, C. W. (2013). Global deserti cation: drivers and feedbacks. *Advances in Water Resources*, *51*, 326{344.

FAO. (n.d.). *Animal production.* Retrieved 2019-04-16, from http://www.fao.org/animal-production/en/

Follett, R. F., & Reed, D. A. (2010). Soil carbon sequestration in grazing lands: societal bene ts and policy implications. *Rangeland Ecology & Management*, *63* (1), 4{15.

Galvin, K. A., Reid, R. S., Behnke, R., & Hobbs, N. T. (2008). Fragmentation in semi-arid and arid landscapes. *Consequences for Human and Natural Systems. Dordrecht, The Neth.: Springer*.

Gherardi, L. A., & Sala, O. E. (2015). Enhanced precipitation variability decreases grass-and increases shrub-productivity. *Proceedings of the National Academy of Sciences*, *112* (41), 12735{12740.

Gobierno de España. (2006). *Encuesta de precios de la tierra 2005.* Retrieved 2019-4-16, from https://www.mapa.gob.es/ca/estadistica/temas/estadisticas-agrarias/Encuesta_Precios_Tierra_2005_tcm34-104076.pdf

Holechek, J. L. (1988). An approach for setting the stocking rate. *Rangelands*, *10* (1), 10{14.

Hu aker, R. G., & Wilen, J. E. (1989). Dynamics of optimal stocking in plant/herbivore systems. *Natural Resource Modeling*, *3* (4), 553{575.

Hunt, L., McIvor, J., Grice, A., & Bray, S. (2014). Principles and guidelines for managing cattle grazing in the grazing lands of northern australia: stocking rates, pasture resting, prescribed re, paddock size and water points{a review. *The Rangeland Journal*, *36* (2), 105{119.

Ibañez, J., Mart nez, J., & Schnabel, S. (2007). Deserti cation due to overgrazing in a dynamic commercial livestock{grass{soil system. *Ecological Modelling*, *205* (3-4), 277{288.

Illius, A. W., & O'Connor, T. G. (1999). On the relevance of nonequilibrium concepts to arid and semiarid grazing systems. *Ecological applications*, *9* (3), 798{813.

Iosi des, T., & Politidis, T. (2005). Socio-economic dynamics, local development and deserti cation in western lesvos, greece. *Local Environment*, *10* (5), 487{499.

John, M., Pannell, D., & Kingwell, R. (2005). Climate change and the economics of farm management in the face of land degradation: dryland salinity in western australia. *Canadian Journal of Agricultural Economics/Revue canadienne d'agroeconomie*, *53* (4), 443{459.

MacLeod, N., Ash, A. J., & McIvor, J. G. (2004). An economic assessment of the impact of grazing land condition on livestock performance in tropical woodlands. *The Rangeland Journal*, *26* (1), 49{71.

Mangel, M., Talbot, L. M., Me e, G. K., Agardy, M. T., Alverson, D. L., Barlow, J., . . . others (1996). Principles for the conservation of wild living resources. *Ecological applications*, *6* (2), 338{362.

Nasca, J. A., Feldkamp, C., Arroquy, J. I., & Colombatto, D. (2015). E ciency and stability in subtropical beef cattle grazing systems in the northwest of argentina. *Agricultural Systems*, *133* , 85{96.

Noy-Meir, I. (1975). Stability of grazing systems: an application of predator-prey graphs. *The Journal of Ecology*, 459{481.

Pennsylvania State University. (2018, January). *Livestock grazing home study course.* https://extension.psu.edu/courses/livestock-grazing/grazing-economics/grazing-economics/pasture-versus-crop-land.

Rogger, M., Agnoletti, M., Alaoui, A., Bathurst, J., Bodner, G., Borga, M., . . . others (2017). Land use change impacts on oods at the catchment scale: Challenges and opportunities for future research. *Water resources research*, *53* (7), 5209{5219.

Rojas-Downing, M. M., Nejadhashemi, A. P., Harrigan, T., & Woznicki, S. A. (2017). Climate change and livestock: Impacts, adaptation, and mitigation. *Climate Risk Management*, *16* , 145{163.

Scholes, R., Montanarella, L., Brainich, E., Barger, N., ten Brink, B., Cantele, M., . . . others (2018). Ipbes (2018): Summary for policymakers of the assessment report on land degradation and restoration of the intergovernmental science-policy platform on biodiversity and ecosystem services.

Synodinos, A. D., Tietjen, B., Lohmann, D., & Jeltsch, F. (2018). The impact of inter-annual rainfall variability on african savannas changes with mean rainfall. *Journal of theoretical biology*, *437* , 92{100.

UN. (1997). *Glossary of environment statistics, studies in methods.* United Nations New York, NY.

UNCCD. (1994). *10. united nations convention to combat deserti cation in those countries experiencing serious drought and/or deserti cation, particularly in africa. paris, 14 october 1994.* Retrieved from https://treaties.un.org/pages/ViewDetails.aspx?src=TREATY&mtdsg_no=XXVII-10&chapter=27&clang=_en(2019/3/24)

UNCED. (1992). *Earth summit '92. in: The un conference on environment and development, rio de janeiro, brazil.* Retrieved from http://publications.gc.ca/collections/Collection-R/LoPBdP/BP/bp317-e.htm,https://treaties.un.org/pages/ViewDetails.aspx?src=IND\&mtdsg_no=XXVII-8\&chapter=27\&clang=_en(2019/3/24)

Valderrama, J. (2005). *Estudio de la deserti cacion por sobrepastoreo mediante un modelo de simulacion dinamica* (Unpublished doctoral dissertation). Agronomos.

Walpole, S. C. (1992). *The measurement of land degradation to assess losses in agricultural production* (Tech. Rep.).

White, B. (2000). A review of the economics of biological natural resources. *Journal of Agricultural Economics*, *51* (3), 419{462.

Williams, C. A., & Albertson, J. D. (2006). Dynamical e ects of the statistical structure of annual rainfall on dryland vegetation. *Global Change Biology*, *12* (5), 777{792.

Yu, K., Saha, M. V., & D'Odorico, P. (2017). The e ects of interannual rainfall variability on tree{grass composition along kalahari rainfall gradient. *Ecosystems*, *20* (5), 975{988.

8 Appendix

8.1 Computing the Forever sustainable Grazing Rate

To calculate the maximum forever sustainable grazing rate, I use a numerical approximation. Since the value of "$fr1$" is very low, "G" does not really affect the grass consumption by the livestock "GC", so this consumption is basically a constant times the amount of animals on the farm (the only purpose of "G" is to have a grass consumption equal to zero when there is no grass, and avoiding negative rates that would generate negative grass values). Then, by looking at figure 16, I can assess that the first order conditions are sufficient for finding the grass level "G^*" that maximizes grass growth rate.

$$\dot{G}^f = PP - GD - GC$$

$$\dot{G}^f = xgk\left(1 - EXP\left[-\frac{MAX(0;S-msg)}{sm1}\right]\right)G - \frac{xgr\,G^2}{xgk} - gdr\,G - L\,xca\left(1 - EXP\left[-\left(\frac{G}{fr1}\right)\right]\right)$$

$$\dot{G}^f = xgk\left(1 - EXP\left[-\frac{MAX(0;S-msg)}{sm1}\right]\right)G - \frac{xgr\,G^2}{xgk} - gdr\,G - L\,constant$$

$$\frac{d\dot{G}^f}{dG} = \frac{dPP}{dG} - \frac{GD}{dG} = 0$$

Solving for "G" provides the value of grass "G^*" that maximizes the new rate of change of grass function "$\dot{G}^f$", obtaining "$\dot{G}^f_{Max}$", when "L" is equal to zero (and "$GC = 0$"). I then compute "L^*" (the maximum sustainable level of livestock) by finding the level of livestock that generates an optimal consumption of grass that equals the maximum grass growth:

$$GC^* = FR \cdot L^* = \dot{G}^f_{Max}$$

Which implies that the rate of change of grass is equal to zero:

$$PP^* - GD^* - GC^* = \dot{G}^f = 0$$

Then, no matter what the initial conditions are (as long as their is enough soil on the land), the farmer always adjusts their livestock level as fast as possible, to either increase or decrease the grass level, reaching the optimal value \$G$ " that will yield the optimal rate of change in grass \$\dot{G}^f_{Max}$". If the farmer starts with too much grass, they will instantaneously increase the herd size to get rid of the excess grass, temporarily sustaining levels of livestock higher than the steady state value. And, if the farmer starts at a level of grass under the optimal level of grass \$G$ ", they will reduce the level of livestock to zero, let the grass recover as fast as possible, and then reintroduce an amount of livestock equal to \$L$ ".

Figure 16 illustrates that \$\dot{G}^f$" is concave in \$G$" for the relevant region. The value for \$\dot{G}^f_{Max}$" and the evolution path of the variable will depend on the parameter values of the system, and the initial value for soil.

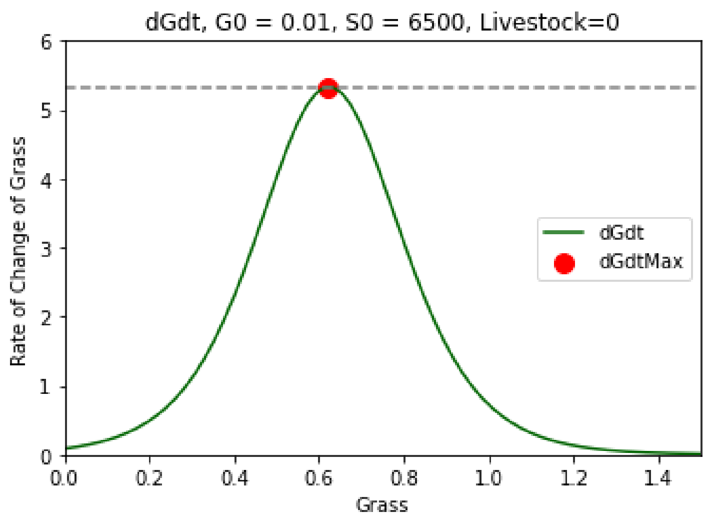

Figure 16: Evolution of the Rate of Change of Grass for the Cattle Farm. The chosen initial value for \$S$" a ects the value of \$dtG^f$" associated with \$G$", but the relevant region remains concave.

8.2 Sensitivity to Changes in Livestock of the Steady State in Figure 11

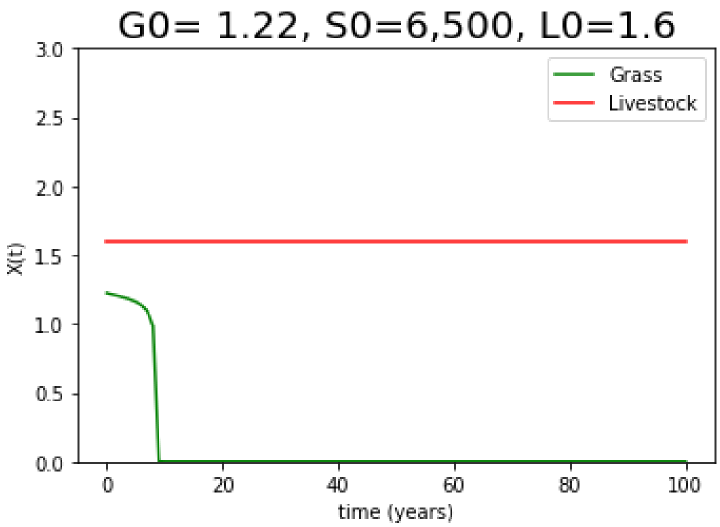

Figure 17: Evolution of the system under a slightly higher grazing rate than the optimal. Livestock levels remain constant for computational simplicity, but a real farmer would withdraw all livestock once the grass is fully depleted.

8.3 Tables with the parameter values of the simulations

Parameter Values for the Cattle Farm

Parameter	Value	Meaning
msg	1500	*Minimum volume of soil required for grass growth (tonne/hectare)*
sm1	1000	*Soil parameter (hectares/tonne)*
xgk	2.55	*Maximum intrinsic carrying capacity of grass (tonnes/hectare)*
xgr	9.26	*Maximum intrinsic growth capacity of grass (1/year)*
gdr	0.4	*Grass decomposition rate (1/year)*
sep	$1.752E-06$	*Soil erosion parameter (hectares*year/tonne)*
lsl	14.52	*Land slope (degrees)*
arf	5188	*Average annual rainfall (tonne/hectare)*
bsi	518.8	*Bare soil infiltration (10% of arf)*
sr1	0.68	*Soil runoff parameter (hectare/tonne)*
fga	0.85	*Fraction of organic matter released to the atmosphere*
bwr	0.3	*Annual weathering rate of bedrock (tonnes/hectare)*
lch	0	*Leaching rate (parameter omitted in Valderrama (2005)*
spr	195.27	*Price of supplementary feed (euros/tonne)*
cec	1020	*Energy content of concentrate (net energy of lactation/tonne)*
uen	2436	*Energy content required per animal (net energy of lactation/year)*
gec	730	*Energy content of grass (net energy of lactation/tonne)*
ouc	354.14	*Other unspecified costs per animal (euros/year)*
uin	687.2	*Annual revenues per animal (euros)*
xca	4.38 (uen/cec)	*Maximum consumption of grass per animal (tonnes/year)*
fr1	0.55 (0.05)	*Functional response parameter per animal (tonnes/hectare)*
fhc	0	*Fixed operating cost (no value provided)*
ubl	10	*Useful life of breeding females (years)*
nf1	0.7	*Parameter for new females (tonnes/hectare)*
pa	(687.2)	*Price of new animals (euros/animal)*
dep	(0.1)	*"Depreciation rate" (animals/year)*
p	(0.05)	*Discount rate*
G_o	2.44	*Initial value of Grass*
L_o	0.2	*Initial value of Livestock*
S_o	6500	*Initial value of Soil (value chosen arbitrarily)*

Figure 18: Parameter values for the cattle farm. Values in parenthesis correspond to the forward looking model.

Parameter Values for the Sheep Farm

Parameter	Value	Meaning
msg	1500	*Minimum volume of soil required for grass growth (tonne/hectare)*
sm1	1000	*Soil parameter (hectares/tonne)*
xgk	2.55	*Maximum intrinsic carrying capacity of grass (tonnes/hectare)*
xgr	9.26	*Maximum intrinsic growth capacity of grass (1/year)*
gdr	0.4	*Grass decomposition rate (1/year)*
sep	$1.752E-06$	*Soil erosion parameter (hectares*year/tonne)*
lsl	14.52	*Land slope (degrees)*
arf	5188	*Average annual rainfall (tonne/hectare)*
bsi	518.8	*Bare soil infiltration (10% of arf)*
sr1	0.68	*Soil runoff parameter (hectare/tonne)*
fga	0.85	*Fraction of organic matter released to the atmosphere*
bwr	0.3	*Annual weathering rate of bedrock (tonnes/hectare)*
lch	0	*Leaching rate (parameter omitted in Valderrama (2005)*
spr	195.27	*Price of supplementary feed (euros/tonne)*
cec	1020	*Energy content of concentrate (net energy of lactation/tonne)*
uen	753	*Energy content required per animal (net energy of lactation/year)*
gec	730	*Energy content of grass (net energy of lactation/tonne)*
ouc	43.12	*Other unspecified costs per animal (euros/year)*
uin	144.88	*Annual revenues per animal (euros)*
xca	0.58 (uen/cec)	*Maximum consumption of grass per animal (tonnes/year)*
fr1	0.5 (0.05)	*Functional response parameter per animal (tonnes/hectare)*
fhc	0	*Fixed operating cost (no value provided)*
ubl	6	*Useful life of breeding females (years)*
nf1	0.7	*Parameter for new females (tonnes/hectare)*
pa	(144.88)	*Price of new animals (euros/animal)*
dep	(0.167)	*"Depreciation rate" (animals/year)*
p	(0.05)	*Discount rate*
G_o	2.44	*Initial value of Grass*
L_o	0.5	*Initial value of Livestock*
S_o	6500	*Initial value of Soil (value chosen arbitrarily)*

Figure 19: Parameter values for the sheep farm. Values in parenthesis correspond to the forward looking model.