Autonomous Vehicles:
Modelling the Impact of their Adoption on Equilibrium Driving Behaviours

by

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A Thesis Submitted in Partial Fulfillment of the Requirements
for the Degree of Bachelor of Arts, Honours
in the Department of Economics
University of Victoria
April 2018

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Abstract

It has been claimed that autonomous vehicles (AVs) will make driving safer, by reducing human error. I provide a theoretical model that compares a world without AVs and one where AVs and human drivers share the road. It is a consumer choice model, where the individual chooses their driving speed and distance in response to the driving behaviours of the other individuals on the road. I find that the probability of an accident does not change with the introduction of AVs, despite an increase in average driving ability. The intuition behind this neutrality result is that as the average driving ability increases, the individuals respond by increasing their speed and distance to the point where there is no effect on their probability of an accident. I also provide a welfare analysis using compensating variation. I compare a given individual in each world and determine the change in wealth that would be required to make them indifferent between worlds. Under numerically simple examples, the introduction of AVs makes all individuals better off. However, this is not a general result. Under certain conditions, it arises that a given individual, who chooses to adopt in the world with AVs, was better off before they were introduced.

Special thanks to my supervisors, Linda and Peter – for your patience and guidance throughout this project, and for teaching me many things. I'm extremely grateful.

Table of Contents

1. Introduction **1**

2. Literature Review **3**

3. The Model **4**

4. The Model in a World with Autonomous Vehicles **13**

5. Compensating Variation **21**

6. Conclusion **23**

Works Cited **25**

Appendix **26**

Section 1 .... **26**
First order conditions for the model without AVs. 26
Section 4. .... **27**
First order conditions for the model with AVs. 27
Section 5. .... **28**
Average Compensating Variation 28

Figures **30**

Figure 1 – Probability Function 30
Figure 2 – Effect on speed from a change in number of cars 31
Figure 3 - Change in equilibrium speed (m=1) 32
Figure 4 - Change in equilibrium speed (m=1.25) 33
Figure 5 - Change in equilibrium speed (m=0.75) 34
Figure 6 - CV across individuals (m=1) 35
Figure 7 - CV across individuals (m=1.25) 36
Figure 8 - CV across individuals (m=0.75) 36

1. Introduction

Partially autonomous cars have been on the road for years, and fully autonomous vehicles (AVs) may be introduced in the next few. It has been claimed that AVs will decrease the time-cost of commuting by permitting that time to be used productively, because the occupant will no longer need to drive themselves (Litman, 2018). In Canada, 74% of daily commuters drive their cars to work, and the average trip is 25.4-29 minutes each way (Barr, 2013). Additionally, of those daily commuters, 94.4% of the vehicles are single occupant (Barr, 2013), which means that approximately 69% of Canadians are driving themselves to work each day, and foregoing potentially productive time. Human error was the cause of 94% of collisions in the United States between 2005-2007 (NHTSA, 2015), and it has been claimed that AVs will decrease the amount of collisions by eliminating human error (Schoettle & Sivak, 2015). Both claimed improvements are compelling reasons to adopt AVs, assuming they prove to be true, which will likely depend in part on the interactions between human drivers and AVs. Therefore, in this paper I provide a theoretical model to analyze the equilibrium outcomes of partial adoption on driving speeds, distances, and the probability of an accident.

In my model, consumers maximize their utility over distance driven, k, and a consumption good x. The individual gains utility from driving, but there is also a cost associated with any time spent in the car $\left(\frac{k}{s}\right)$, which decreases the amount of good x they can consume. There is therefore a trade-off between x and k. The individual's choice variables are their speed and their distance, which the use to maximize their expected utility, which is a function of their probability of being in an accident. This probability depends on the individual's choice variables, their innate driving ability, and the choices and abilities of the other drivers on the road. Intuitively, this is because if the individual is driving faster and further, they are more at risk of being in an accident, *ceteris paribus,* because they will have more exposure to hazards and they will have less time to react. Similarly, holding all else equal, if the other drivers on the road are also driving further and faster, then the individual is more at risk because they must share the road with these other drivers, who are more likely to cause a collision that could affect the individual. The same intuition holds for the effect on probability from the individual's ability and average ability. Higher skilled individuals are more capable at avoiding hazards and have lower odds of being in a collision, therefore a higher average ability should also decrease the probability of an accident, *ceteris paribus*. The nature of this model and the interaction between drivers means that it is a non-cooperative game across *n* individuals.

I solve for the equilibrium without AVs first, and then again with AVs and human drivers coexisting. I find that the probability of being in an accident is the same across all individuals in both worlds, and does not change with the introduction of AVs. Mathematically, this is because probability is independent of ability, which is the only idiosyncratic variable in both models. Intuitively, this is because as driving gets "safer", individuals adjust their speed and distance driven in a way that nullifies any potential increase in safety. The model solves for a new equilibrium with different speeds and distances, but with the same probability of an accident. This is a similar neutrality result to that of Anthony Downs, who proves "the fundamental law of highway congestion" in his seminal article *The Law of Peak-Hour Express-Way Congestion* (1962).

However, this neutrality result does not imply that the probability can't be affected by other changes to the model. The equilibrium probability function is dependent on one exogenous variable – the cost of an accident to the individual. Probability is monotonically decreasing in the cost of an accident, which means that if there is an exogenous decrease to the price of an accident, for example if the amount of insurance coverage increased, then the probability of an accident occurring would increase.

I find that the ability level of the marginal adopter, and therefore the fraction of adopters, is solely dependent on the driving ability of the AVs and how much of an increase in productivity they provide. In other words, as the AVs become more skilled, more individuals will adopt. Similarly, as the AVs allow for more productive use of time, more individuals will adopt.

Finally, I provide a welfare analysis through compensating variation. I consider a given individual before and after the introduction of AVs, and calculate the change in wealth required after introduction to make them indifferent between the two worlds. If the individual has been made better off by the introduction of AVs, then to equate their utility across worlds they would need to undergo a reduction in wealth in the world with AVs. The cardinality of compensating variation allows me to compare across individuals, and construct an average. In very simple numerical examples, all individuals are made better off by the introduction of AVs. However, in some cases there are individuals who are made worse off. In fact, there are permutations of the model in which an individual was better off before AVs were introduced, but after implementation they still choose to adopt.

2. Literature Review

Anthony Downs's article provides the framework for much of the intuition behind my model. He created this model as evidence against the effectiveness of improving peak-hour traffic congestion by investing in infrastructure. He claimed that the commuters were only interested in the duration of the commute, and so would choose the fastest route no matter what the condition of the roads. However, the time required to drive down a road is a function of its condition and the expected level of congestion. Because of this, Downs claimed that any attempt to improve one road, for example by widening it, would not decrease the level of congestion, as the improvement would draw in new drivers. He lists three reasons why there are new drivers: people who were previously using a different road will move to the renovated one, people who were travelling at off-peak times will shift to peak, and people who were using public transit will switch to driving their own cars (Downs, 1962). This same type of reaction takes place throughout my model, where a change in a variable does not affect the probability result in equilibrium, due to a reactionary shift in behaviour.

I model the adoption of AVs in a consumer choice under uncertainty framework, which results in a non-cooperative equilibrium. There are currently no similar models on this subject in the literature. However, there is a substantial body of work on technology adoption under production uncertainty in agricultural economics, which has similar qualities. In general, this field focuses on the adoption of a new farming technology in the face of uncertainty in the production of crops, and therefore profit. There are many papers on this topic, I will focus on one, entitled *Adoption and Diffusion of an Innovation under Uncertain Profitability* (Jensen, 1981), that is similar to my paper in its initial set-up, yet is approached in a different manner. Generally, in the agricultural economic literature the discussion centers around diffusion curves showing the overall adoption rate. In contrast, Jensen models the profit maximizing firms' decision process before they adopt a new form of technology; which is more based in micro decision theory (Jensen, 1981). He models this adoption decision in a learning system, where the firm is uncertain over the outcome of adoption; it could increase profits or not, and there is a fixed cost associated with adopting. The model has a time-component, and continues for infinite periods, to allow for the firm to gather more information and adjust its beliefs over the quality of the innovation. The firm's decision to adopt depends on a simple adoption rule; when the firm's current belief over the quality of the good exceeds some threshold quality level, the firm adopts. This threshold level is partially determined by the fixed cost of implementation and the quantity of revenue earned before and after implementation. When the requisite threshold level has not yet been reached, the firm does not adopt and waits to gather more information in the next period, *ad infinitum*.

Jensen takes a similar problem to my own, the adoption of a new technology, and models it in a very different way; my model does not include a time component, therefore neither does it allow for the updating of beliefs, among other differences. As such, our models solve for different results, and allow for very different analyses. His model examines the decisions behind the diffusion rate of an innovation, whereas mine models the change in equilibrium driving behaviours. My model was more suited to my research question, but it would be interesting to analyze the adoption of AVs through his approach as well.

3. The Model

The goal of this model is to analyze how various driving behaviours change as autonomous vehicles are introduced into the market, which I do by comparing two worlds. I start by constructing a model of driving behaviour in a world without AVs, then I alter the model to allow individuals to adopt autonomous vehicles if they wish. I solve for the equilibrium in each model and compare the following results: individual driving speed and distance driven, average driving speed and distance driven, standard deviation of speed, and the probability of an accident. Additionally, in the world with AVs I find the ability level of the marginal adopter and the fraction of drivers who adopt AVs.

To begin, I construct a utility maximizing problem in a world with only human drivers. I assume each individual is identical in all ways except for their driving ability (a_i), which is innate and perfectly observable to all individuals. There is one rider in each car, and each individual only has one car. There are no cars for hire; i.e. no taxis and no ride-share programs. There is a mass of n individuals, with ability levels that are uniformly distributed with lower bound a_l, upper bound a_u, and mean μ, where $a_u - \mu = \theta$ and $\mu - a_l = \theta$.

The individual is maximizing their utility over two variables: k and x, where k is the total distance they drive and x is consumption of non-transportation goods. The demand for k comes from an underlying valuation of transportation services, which allow the individual to enjoy other non-consumption goods, such as driving to visit a friend, or taking a road trip to the beach. x represents the sum of all non-transportation related goods and services the individual purchases. The amount of good x that the individual can consume depends on their wealth. When the individual spends time driving their car, they forfeit potential productive time, which is a cost to them in the form of lost wages and therefore reduces the amount of good x they can consume.

The individual's utility is:

$$u(x_i, k_i) = x_i + \beta \ln(k_i)$$

The quasi-linear form of this utility function permits closed-form solutions, which were not possible with Cobb-Douglas utility functions, and means that k is income neutral. The imposition of income-neutrality on distance implies that the individual's demand for distance driven is inelastic, and if their wealth changes, their demand for distance driven will not. In other words, k_i is a neutral good, not a normal good.

The individual's consumption of x_i is:

$$x_i = y_i - c\frac{k_i}{s_i} - \delta\omega$$

where y_i is the individual's wealth, *c* is the time cost of being in the car, s_i is the individual's speed, $\frac{k_i}{s_i}$ is the time spent in the car, ω is the cost of an accident, and δ is a Bernoulli random variable. δ equals *1* when the individual is in a collision and *0* when they are not. If the individual is in a collision they will incur costs; both from lost productive time and from any repair costs or health costs. This will reduce their consumption of x_i.

The value of δ depends on the individual's probability function, shown here in a simple form:

$$\pi_i = \frac{f_i}{1 + f_i}$$

where $\Pr(\delta = 1) = \pi_i$ and $\Pr(\delta = 0) = 1 - \pi_i$. π_i is bounded between 0 and 1, and is monotonically increasing in f_i, as shown in Figure 1. f_i is a function of the seven risk factors that affect individual *i*'s probability of being in a collision,

$$f_i = \frac{{s_i}^{\zeta_1}\, {k_i}^{\zeta_2}\, (n\kappa)^{\zeta_3}\, \bar{s}^{\zeta_4} \sigma^{\zeta_5}}{{a_i}^{\zeta_6}\, \nu^{\zeta_7}}.$$

Each risk factor has a ζ exponent to allow for flexibility in the model; each variable can have a different level of impact. The following paragraph is a description of each risk factor, the intuition behind its inclusion, and its effect on the individual's probability of being in a collision, if all other variables were held constant.

The first two factors are the individual's driving behaviours, which are their speed and distance, s_i and k_i. These are the choice variables in the model, and they both have a positive relationship with the individual's probability of collision. When the individual increases the distance they drive, they increase their exposure to potentially risky situations such as bad weather or poor road conditions. By increasing the speed with which they drive, the individual reduces the amount of time they have to react to any hazards they might encounter. The third and fourth factors, average speed and distance across individuals ($\bar{s}$ and κ), follow similar logic. If other drivers drive at higher speeds and for longer distances, then the individual's probability of being in an accident also increases. Higher average speed means the other drivers will have less time to react, and longer distances mean that the individual will interact with more cars per km. The fifth factor is the standard deviation of speed across individuals (σ). My model imposes the assumption of a single driving environment, for example a highway. Therefore, any variance in speed is due not to a difference in environment, but rather a difference in driving behaviour. Furthermore, the cars driving at different speeds will interact. If there is a wider distribution of speeds, with some individuals going excessively fast or slow, this will be reflected in the standard deviation of speed and will increase the probability of a collision. The intuition behind this is that it is easier to react safely to anticipated situations, so when all the cars on the road are driving at similar speeds the individual is less likely to come across an unusual situation and therefore will be less likely to have an accident. The last two factors are the individual's ability (a_i) and the average ability (ν), which both have negative relationships with f_i. An individual with higher driving ability (a_i) will be more capable of avoiding an accident, as they will commit fewer errors and react better to dangerous obstacles. This is true for all individuals, therefore if the average ability across individuals (ν) is higher, the probability of a collision is lower, *ceteris paribus.*

The individual's expected utility function is

$$EU = \pi_i \left(y_i - c_i \frac{k_i}{s_i} - \omega + \beta \ln (k_i)\right) + (1 - \pi_i) \left(y_i - c_i \frac{k_i}{s_i} + \beta \ln (k_i)\right),$$

which simplifies to

$$EU = y_i - c_i \frac{k_i}{s_i} + \beta \ln(k_i) - \pi_i \, \omega.$$

Where

$$\pi_i = \frac{{s_i}^{\zeta_1} \, {k_i}^{\zeta_2} \, (n\kappa)^{\zeta_3} \, \bar{s}^{\zeta_4} \sigma^{\zeta_5}}{{a_i}^{\zeta_6} \, \nu^{\zeta_7} \left(1 + \left(\frac{{s_i}^{\zeta_1} \, {k_i}^{\zeta_2} \, (n\kappa)^{\zeta_3} \, \bar{s}^{\zeta_4} \sigma^{\zeta_5}}{{a_i}^{\zeta_6} \, \nu^{\zeta_7}}\right)\right)}.$$

To find the individual's equilibrium competitive strategy, I solve the utility maximization problem with respect to k_i and s_i, which are the choice variables for individual *i*. I derive the first-order conditions,

$$\frac{\partial EU}{\partial s} = 0 \text{ and } \frac{\partial EU}{\partial k} = 0$$

(shown in full in the Appendix) and I solve the equations simultaneously. This yields four possible solutions, two of which are not real, one is a saddle point, and the last is a maximum. The equilibrium strategies which maximize utility are given by

$$\tilde{k}_i = \left[\frac{q\, \nu^{\zeta_7}\, a_i^{\zeta_6}}{2\, \sigma^{\zeta_5}\, \bar{s}^{\,\zeta_4} c^{\zeta_1}\, (n\kappa)^{\zeta_3}}\right]^{\frac{1}{\zeta_1+\zeta_2}}$$

and

$$\tilde{s}_i = \frac{(\zeta_1 + \zeta_2) c\, \tilde{k}_i}{\beta\, \zeta_1}$$

where q is a summary parameter

$$q = \frac{(\zeta_1 + \zeta_2)\omega - 2\beta - \sqrt{\omega^2(\zeta_1 + \zeta_2)^2 - 4\omega\beta(\zeta_1 + \zeta_2)}}{\beta^{(1-\zeta_1)} \left(\frac{\zeta_1 + \zeta_2}{\zeta_1}\right)^{\zeta_1}}$$

which is monotonically decreasing in omega. This implies that an exogenous increase in the cost of an accident will decrease q, and will in turn decrease equilibrium speed and distance – all other variables held constant.

As an aside, note that the ratio between the two variables is very simple:

$$\frac{\tilde{k}_i}{\tilde{s}_i} = \frac{\beta\, \zeta_1}{(\zeta_1 + \zeta_2)c}$$

This ratio is originally used in x_i and when I substitute it back in, it shows that consumption of x_i in each state is independent of ability, distance, and speed.
When $\delta = 1$,

$$\tilde{x}_i = y_i - \frac{\beta\, \zeta_1}{(\zeta_1 + \zeta_2)} - \omega$$

and when $\delta = 0$,

$$\tilde{x}_i = y_i - \frac{\beta\, \zeta_1}{(\zeta_1 + \zeta_2)}.$$

I now have the equilibrium best response functions for the individual's speed and distance. These strategies are functions of the average speed ($\bar{s}$), average distance (κ), and the standard deviation of speed (σ), which are all determined endogenously. In equilibrium, these variables must be consistent with the average and standard deviation across best response functions for all *n* individuals. In other words, to solve for the complete equilibrium, κ must equal the average $\tilde{k}_i$ across individuals, and the same must be true for average speed and standard deviation of speed. To ensure this consistency holds, I solve for the equilibrium averages and standard deviation by taking the expected value of individual *i's* best response functions. The best response functions only differ across individuals with respect to ability, a_i, as it is the only idiosyncratic variable. Therefore, to find the expected value of the entire function, I need only consider ability, which I manipulate into a new variable that is easier to evaluate,

$$a^z = a_i^{\frac{\zeta_6}{\zeta_1+\zeta_2}}.$$

Both of the individual's best response functions are linear in a^z, which is necessary for the use of an expectation operator. To construct the mean of a^z I take its expected value and call it $\propto$.

$$\propto = \int_{a_l}^{a_u} \frac{a^z}{(a_u - a_l)} da$$

Which yields

$$\propto = \frac{(\mu+\theta)^{z+1} - (\mu-\theta)^{z+1}}{2\theta(z+1)}$$

Then, to find the expected value of the entire best response functions for distance and speed, I multiply the remaining variables, which are equal across all individuals, by $\propto$.

$$s_{\propto} = \frac{\propto (\zeta_1+\zeta_2)c\left(\frac{q\mu^{\zeta_7}}{2\sigma^{\zeta_5}\bar{s}^{\zeta_4}c^{\zeta_1}(n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}}}{\beta\zeta_1}$$

$$k_{\propto} = \propto \left(\frac{q\mu^{\zeta_7}}{2\sigma^{\zeta_5}\bar{s}^{\zeta_4}c^{\zeta_1}(n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}}$$

To find the standard deviation of the best-response for speed, I first construct the variance of a^z (χ) from the following equation,

$$\chi = E(a^{2z}) - (\propto)^2,$$

which yields

$$\chi = \frac{(\mu+\theta)^{2z+1} - (\mu-\theta)^{2z+1}}{2\theta(2z+1)} - \frac{((\mu+\theta)^{z+1} - (\mu-\theta)^{z+1})^2}{4\theta^2(z+1)^2}.$$

From this, I use the variance characteristic $Var(cx) = c^2 Var(x)$, and multiply the individual's best-response function for speed by $\sqrt{\chi}$, which yields

$$s_\chi = \frac{\sqrt{\chi}(\zeta_1+\zeta_2)c\left(\frac{q\mu^{\zeta_7}}{2\sigma^{\zeta_5}\bar{s}^{\zeta_4}c^{\zeta_1}(n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}}}{\beta\zeta_1}.$$

As outlined above, the endogenous nature of the model requires consistency between the variables $\bar{s}$, κ and σ, and the average and standard deviation of the equilibrium responses. Now that I have found the average equilibrium strategies ($s_\propto$, $k_\propto$, and s_χ) I set them equal to the specified variables,

$$s_\propto = \bar{s}$$
$$k_\propto = \kappa$$
$$s_\chi = \sigma.$$

Solving simultaneously for the consistent equilibrium functions yields

$$\hat{\bar{s}} = \left(\frac{c^{\zeta_2+\zeta_3} \propto^{\zeta_1+\zeta_2+\zeta_5} q\mu^7 \left(\frac{\zeta_1+\zeta_2}{\zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3}}{2n^{\zeta_3}\beta^{\zeta_1+\zeta_2+\zeta_3}(\sqrt{\chi})^{\zeta_5}}\right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}$$

$$\hat{\kappa} = \left(\frac{\beta^{\zeta_4+\zeta_5} \propto^{\zeta_1+\zeta_2+\zeta_5} q\mu^7}{2n^{\zeta_3} c^{\zeta_4+\zeta_5+\zeta_1}(\sqrt{\chi})^{\zeta_5}\left(\frac{\zeta_1+\zeta_2}{\zeta_1}\right)^{\zeta_4+\zeta_5}}\right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}$$

$$\hat{\sigma} = \left(\frac{(\sqrt{\chi})^{\zeta_1+\zeta_2+\zeta_3+\zeta_4}\, q\mu^7 c^{\zeta_2+\zeta_3} \left(\frac{\zeta_1+\zeta_2}{\beta\zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3}}{2\propto^{\zeta_3+\zeta_4} n^{\zeta_3}} \right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}.$$

I then substitute these results back into the individual's best response functions to yield the consistent equilibrium strategies $\hat{k}_i$ and $\hat{s}_i$ for individual *i*. I then use $\hat{k}_i$ and $\hat{s}_i$ to calculate the individual's equilibrium utility ($\hat{V}_i$). The solutions for $\hat{k}_i$, $\hat{s}_i$, and $\hat{V}_i$ can be found in the Appendix; they are all closed-form, yet long and not immediately useful.

I solve for the equilibrium probability of an accident by substituting all the equilibrium results into the original probability function, which yields

$$\hat{\pi} = \frac{q(\zeta_1+\zeta_2)^{\zeta_1}}{2(\beta\zeta_1)^{\zeta_1}+q(\zeta_1+\zeta_2)^{\zeta_1}}.$$

It's interesting to note that this probability function is independent of ability and all the interaction zeta's, it is also not a function of any of the risk factors outlined above such as distance, speed or average ability. This means that the probability of being in a collision is the same across all individuals, and does not change when a risk factor does. This counterintuitive result is likely explained by a similar intuition to that of the "fundamental law of traffic congestion" (Downs, 1962). Individuals respond to an improvement in infrastructure, such as the widening of a roadway, by choosing to drive on that road instead of another, or taking their car instead of public transit, so that in equilibrium congestion remains unchanged. This explanation also applies to my model: when a risk factor changes, the drivers react by adjusting their speed and distance, which nets out to yield the same equilibrium probability outcome as before. An example of this is shown in Figure 2, where I reduce the density of cars by 50% and show the difference between equilibrium speeds, from before and after the change. A reduction in the number of cars would reduce the probability of an accident, *ceteris paribus*. However, in the new equilibrium, the probability necessarily remains the same and both the speed and distance driven are higher.

Additionally, the summary variable q reappears in this equation, and since it is decreasing in ω, this implies that an exogenous change to the cost of an accident would affect the probability of an accident in equilibrium.

4. The Model in a World with Autonomous Vehicles

All the previously stated assumptions on the model without autonomous cars still hold, and all the variables have the same definitions and restrictions. As outlined above, the adoption of AVs has been claimed to improve safety by reducing human error, and to reduce the time-cost of driving by allowing "drivers" to use their time productively. To capture the potential effects of these claimed improvements, I add a new ability variable *m* and a time-saving parameter ρ, where *m* is the driving ability of all AVs, which is determined exogenously and is perfectly observable.

The variable *m* replaces a_j in individual *j*'s probability function, where individual *j* is an adopter, as the individual is no longer responsible for driving the car and therefore their innate ability is no longer taken into consideration. I assume that all AVs are completely autonomous, i.e. the individual has no control over how the car reacts to situations and has no manual override. However, the individual is still allowed to choose the speed of the AV.

ρ is a time-savings parameter that appears in the *x* function to represent the possible reduction in time-cost of driving that an AV could provide. The occupant of the car, who previously would have been required to devote their time to driving, is now able to spend that time engaged in different activities. They could be work, or they could use that time to complete tasks they would have otherwise had to complete later in the day. In either case, the result is more time available for working, which increases the amount of good *x* the individual can consume, by increasing income. Note that all the agents in this world will adopt the AVs at different levels of *m*, corresponding to their respective ability levels. Further on, I solve for the level of ability of the marginal adopter and the fraction of adoption at equilibrium.

The utility function remains the same and the function for x_j is the same save for the addition of ρ, where

$$0 \leq \rho \leq 1,$$

so when $\rho = 1$, there is no reduction in the time-cost of driving, and when $\rho = 0$, there would no longer be any time-cost associated with driving.

$$u(x_j, k_j) = x_j + \beta \ln(k_j)$$
$$x_j = y_j - \rho c_j \frac{k_j}{s_j} - \delta_j \omega$$

The simplified expected utility for this problem is

$$EU = y_j - \rho\, c_j \frac{k_j}{s_j} + \beta \ln(k_j) - \pi_j\, \omega.$$

Where π_j is trivially the same as in the model without AVs, the risk factors all have the same definitions and effects on π_j as they did on π_i, however, now a_j (the ability of individual *j,* an adopter) has been replaced with m, which is the driving ability of AVs.

$$\pi_j = \frac{s_j^{\zeta_1}\, k_j^{\zeta_2}\, (n\kappa)^{\zeta_3}\, \bar{s}^{\,\zeta_4} \sigma^{\zeta_5}}{m^{\zeta_6}\, \nu^{\zeta_7} \left(1 + \left(\frac{s_j^{\zeta_1}\, k_j^{\zeta_2}\, (n\kappa)^{\zeta_3}\, \bar{s}^{\,\zeta_4} \sigma^{\zeta_5}}{m^{\zeta_6}\, \nu^{\zeta_7}}\right)\right)}$$

I find the equilibrium competitive strategies of k_j and s_j in the same manner as I did for k_i and s_i in the model without AVs. I solve for the first-order conditions (FOCs) by differentiating expected utility with respect to speed and distance. The FOCs can be seen in full in the Appendix. Solving the FOCs simultaneously yields

$$\tilde{k}_j = \frac{q\, \nu^{\zeta_7}\, m^{\zeta_6}}{2\, \sigma^{\zeta_5}\, \bar{s}^{\,\zeta_4} (\rho c)^{\zeta_1}\, (n\kappa)^{\zeta_3}}^{\frac{1}{\zeta_1 + \zeta_2}}$$

and

$$\tilde{s}_j = \frac{(\zeta_1 + \zeta_2) \rho\, c\, \widetilde{k_j}}{\beta\, \zeta_1},$$

which are very similar to those from the original model. This is to be expected, as I only changed the model slightly by including m and ρ, which are the only differences in these best response functions.

As before, the model's equilibrium requires consistency between the variables in the probability function and the averages across equilibrium strategies. Currently, I have the non-cooperative solutions for the individual as a function of the variables which are not yet in equilibrium: average speed, average distance, and standard deviation speed. To find the average equilibrium strategies, I follow the same process as before. However, there are both adopters and non-adopters on the road, and as the two groups differ in the distribution of ability, I must deal with them separately. First, I will consider the adopters. In this model, the adopters' innate driving ability is irrelevant to their equilibrium speed and distance, as they are now passengers in AVs with equal driving levels, m. Therefore, I construct a variable from m in which both best-response functions are linear (A^z), to facilitate the calculation of their means.

$$A = m\,\rho^{\frac{\zeta_2}{\zeta_6}}$$

and

$$z = \frac{\zeta_6}{\zeta_1+\zeta_2}.$$

Note that the construction of A^z mirrors that of the variable a^z which I constructed previously in the human-only model. Because of this, replacing a^z with A^z in the best response function of s_i, from the human-only model, yields the best response function for s_j, the individual's speed in the model with AVs. Using this relationship, I find the equilibrium competitive strategies of this model for a second time, now expressed in a way that will be more useful in the coming calculations, because it means that the expressions for $\tilde{s}$ in both models are of the same form except for a^z and A^z, which are the variables I use to find the averages. In other words, after I find the expected values of a^z and A^z, I multiply both expressions for $\tilde{s}$ by the same coefficient. The new forms of the equilibrium strategies are

$$\tilde{s}_j = \frac{(\zeta_1 + \zeta_2)\, c \left(\frac{q\, v^{\zeta_7}\, A^{\zeta_6}}{2\, \sigma^{\zeta_5}\, \bar{s}^{\,\zeta_4} c^{\zeta_1}\, (n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}}}{\beta\, \zeta_1}$$

and

$$\tilde{k}_j = \frac{\beta\, \zeta_1\, \tilde{s}_j}{(\zeta_1 + \zeta_2)\rho\, c}.$$

As mentioned above, I must account for the two types of drivers in the world, as simply taking the expected value of these functions and using that to find the equilibrium functions for the risk variables would exclude the best-responses of the non-adopters and would therefore be incorrect.

I construct the equation for $\bar{a}$, the ability of the marginal non-adopter, by first constructing the fraction of non-adopters ϕ. Recall the distribution of ability across individuals is uniform, with a lower bound a_l, an upper bound a_u, and a mean μ, where $a_u - \mu = \theta$. Then, imposing $\bar{a}$ as the ability of the marginal non-adopter, the fraction of drivers who adopt is equal to phi,

$$\phi = \frac{\bar{a} - a_l}{a_u - a_l},$$

which simplifies to

$$\phi = \frac{\bar{a} - \mu + \theta}{2\theta},$$

and the marginal non-adopter's ability is

$$\bar{a} = 2\phi\theta + \mu - \theta.$$

This means that only drivers with $a < \bar{a}$ will adopt an AV. The average ability of the non-adopters is then

$$a_{ave} = \frac{a_u + \bar{a}}{2} = \phi\theta + \mu$$

and the average overall ability is

$$v = \phi m + (1 - \phi)(\phi\theta + \mu).$$

The expression of the fraction of adoption (ϕ) allows me to construct weighted averages across the two groups of individuals; adopters and non-adopters. To derive the weighted mean of A^z and a^z, named $\propto_1$, I first find the expected value of a^z for the non-adopters ($\propto_h$) and then I find expected value of A^z for the adopters ($\propto_m$). I then use the fraction of adoption derived above to sum the two $\propto' \, s$, weighted by $(1-\phi)$ and ϕ respectively.

$$\propto_h = \int_{\bar{a}}^{a_u} \frac{a^z}{a_u - \bar{a}} da$$

Which equal

$$\propto_h = \frac{(\mu+\theta)^{(z+1)} - (2\phi\theta+\mu-\theta)^{(z+1)}}{\theta(z+1)(1-\phi)},$$

and

$$\propto_m = \int_{a_l}^{\bar{a}} \frac{A^z}{a_l - \bar{a}} da,$$

which is simply

$$\propto_m = A^z.$$

Therefore, overall expected value is

$$\propto_1 = \phi \propto_m + (1-\phi) \propto_h,$$

where ϕ is the fraction of adopters and (1- ϕ) is the fraction of non-adopters. This expands to:

$$\propto_1 = \phi A^z + \frac{(\mu+\theta)^{(z+1)} - (2\phi\theta+\mu-\theta)^{(z+1)}}{2\theta(z+1)}$$

The average best response of speed is then found in the same manner as before; I take the individual's best response function and multiply its endogenous variables by the expected value of the rearranged exogenous variable, which yields the expected value of the whole function,

$$\tilde{s}_{\propto} = \frac{\propto_1(\zeta_1+\zeta_2)\, c \left(\frac{q\, v^{\zeta_7}}{2\, \sigma^{\zeta_5}\, \bar{s}^{\zeta_4} c^{\zeta_1}\, (n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}}}{\beta\, \zeta_1}.$$

Evaluated at *A =1* and *a* = 1, the best response functions of speed for the non-adopters and the adopters are equivalent, due to the method of their construction. This is what allowed me to use the method outlined above. However, evaluated at *A =1* and *a* = 1, the best response functions of distance are not equivalent across adopters and non-adopters, so I must find the average best responses for each group separately, and then sum them together per their weights. Recall $\propto_m$ and $\propto_h$ are the expected values of A^z and a^z respectively.

$$\tilde{k}_{\propto} = \phi \propto_m \frac{1}{\rho}\left(\frac{q\,\nu^{\zeta_7}}{2\,\sigma^{\zeta_5}\,\bar{s}^{\,\zeta_4}c^{\zeta_1}\,(n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}} + (1-\phi)\propto_h\left(\frac{q\,\nu^{\zeta_7}}{2\,\sigma^{\zeta_5}\,\bar{s}^{\,\zeta_4}c^{\zeta_1}\,(n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}}$$

This simplifies to:

$$\tilde{k}_{\propto 2} = \propto_2\left(\frac{q\,\nu^{\zeta_7}}{2\,\sigma^{\zeta_5}\,\bar{s}^{\,\zeta_4}c^{\zeta_1}\,(n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}}$$

where

$$\propto_2 = \frac{\phi \propto_m}{\rho} + (1-\phi)\propto_h$$

The variance is constructed using the same method as in Section 2

$$\mathrm{X} = [(\,\phi \propto_m{}^2 + (1-\phi)\propto_h{}^2) - \propto^2],$$

which expands to

$$\mathrm{X} = -\left(\phi A^z + \frac{(\mu+\theta)^{z+1}-(2\phi\theta+\mu-\theta)^{z+1}}{2\theta(z+1)}\right)^2 + \phi A^{2z} + \frac{(\mu+\theta)^{2z+1}-(2\phi\theta+\mu-\theta)^{2z+1}}{2\theta(2z+1)}.$$

Now that I have the standard deviation of the exogenous variable $\propto_2$, I find the standard deviation of the best response function by multiplying the remaining endogenous variables in the function by $\sqrt{\mathrm{X}}$. This yields the expression

$$\tilde{s}_{\mathrm{X}} = \frac{\sqrt{\mathrm{X}}\,(\zeta_1+\zeta_2)c\left(\frac{q\,\nu^{\zeta_7}}{2\,\sigma^{\zeta_5}\,\bar{s}^{\,\zeta_4}c^{\zeta_1}\,(n\kappa)^{\zeta_3}}\right)^{\frac{1}{\zeta_1+\zeta_2}}}{\beta\zeta_1}.$$

As in the model without AVs, the equilibrium requires consistency across the average variables and the standard deviation variable, ($\bar{s}$, κ, and σ) that are factors in the probability function, with the averages and standard deviation across all individuals' equilibrium strategies. Now that I have the expressions for $\tilde{s}_{\propto}$, $\tilde{k}_{\propto}$, and $\tilde{s}_{\mathrm{X}}$ I can set them equal to their respective variables and solve simultaneously for the consistent equilibrium expressions. Again, to have consistency in the equilibrium the following equations must hold true

$$\tilde{s}_{\propto} = \bar{s}$$

$$\tilde{k}_{\propto} = \kappa$$

$$\tilde{s}_{\mathrm{X}} = \sigma.$$

I set them equal and solve simultaneously to yield

$$\hat{\hat{s}} = \left(\frac{c^{\zeta_2+\zeta_3} \propto_1^{\zeta_1+\zeta_2+\zeta_3+\zeta_5} qv^7 \left(\frac{\zeta_1 + \zeta_2}{\zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3}}{2n^{\zeta_3}\beta^{\zeta_1+\zeta_2+\zeta_3}(\sqrt{\mathrm{X}})^{\zeta_5} \propto_2^{\zeta_3}} \right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}$$

$$\hat{\kappa} = \left(\frac{\beta^{\zeta_4+\zeta_5} \propto_2^{\zeta_1+\zeta_2+\zeta_4+\zeta_5} qv^7}{2n^{\zeta_3} c^{\zeta_4+\zeta_5+\zeta_1}(\sqrt{\mathrm{X}})^{\zeta_5} \propto_1^{\zeta_4} \left(\frac{\zeta_1 + \zeta_2}{\zeta_1}\right)^{\zeta_4+\zeta_5}} \right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}$$

$$\hat{\sigma} = \left(\frac{(\sqrt{\mathrm{X}})^{\zeta_1+\zeta_2+\zeta_3+\zeta_4} qv^7 c^{\zeta_2+\zeta_3}\left(\frac{\zeta_1+\zeta_2}{\beta\zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3}}{2\propto_2^{\zeta_3}\propto_1^{\zeta_4} n^{\zeta_3}} \right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}.$$

These are the equilibrium functions, in a world with AVs, for average speed, average distance, and standard deviation of speed, which satisfy the consistency requirement.

I solve for equilibrium $\hat{s}_h, \hat{k}_h, \hat{s}_m$, and $\hat{k}_m$ by substituting these equilibrium results back into the original best response functions. This yields closed form solutions which can be found in the Appendix. I solve for equilibrium probability of an accident by substitution, which gives

$$\hat{\pi} = \frac{q(\zeta_1+\zeta_2)^{\zeta_1}}{2(\beta\zeta_1)^{\zeta_1}+q(\zeta_1+\zeta_2)^{\zeta_1}}.$$

This is the same as the equilibrium probability of an accident before autonomous cars were introduced,

$$\hat{\pi}_h = \frac{q(\zeta_1+\zeta_2)^{\zeta_1}}{2(\beta\zeta_1)^{\zeta_1}+q(\zeta_1+\zeta_2)^{\zeta_1}}.$$

This result is unsurprising in this model, because in the first model I showed that the equilibrium probability function was independent of ability, which is the primary variable of interest, and was the main difference between models (the only other change being the inclusion of ρ). This result indicates that the level of the AVs' driving ability (m) has no effect on the equilibrium probability function. The intuition behind this is the same as before, when any risk factor changes, the individuals respond by adapting their speed and distance to the new environment. The lowest ability drivers adopt first, so as soon as there is any level of adoption the average ability increases, and in response the equilibrium average speed and average distance increase, and the variance across speed decreases. In Figures 3, 4, and 5, I demonstrate the changes in equilibrium after the adoption of AVs, at varying levels of m.

Using the equilibrium results derived above, I solve for a given individual's equilibrium utility as an adopter and a non-adopter ($\hat{V}_m$, $\hat{V}_h$), and use this to determine the ability of the marginal adopter, by solving for $\bar{a}$ from

$$\hat{V}_m - \hat{V}_h = 0.$$

The resulting expression, named $\hat{a}$ for consistent notation, is

$$\hat{a} = \frac{m}{\rho^{\frac{\zeta_1}{\zeta_6}} e^{\frac{\zeta_1+\zeta_2}{\zeta_6 \beta}}}.$$

In other words, when the individual's ability is just equal to this expression, they will be indifferent between driving their own car or being driven in an AV. Individuals with ability lower than this threshold are better off (have higher equilibrium utility) when they are in an AV instead of driving themselves, and will therefore adopt. Conversely, an individual whose ability is higher than this threshold will have higher equilibrium utility when they are driving themselves, and will not adopt.

Note that $\hat{a}$ is independent of any interaction $\zeta's$ so eliminating the effect of any of the risk factors by setting their exponent to zero will not change this equilibrium result. It is however, a function of AVs' driving ability (m) and the time-cost scaling parameter (ρ). This result supports the initial intuition that two of the most important factors in the decision to adopt are the capability of the AVs and the amount of time the individual can save by no longer having to drive themselves. When the ability of the AVs increases, so does the adoption threshold, all else held equal. An increase in ρ decreases the adoption threshold, which is to be expected, as ρ is a time-savings parameter bounded between 0 and 1, where $\rho = 1$ means that there is no productivity improvement over driving oneself.

Finally, I also construct the equilibrium fraction of adoption,

$$\hat{\phi} = \frac{(\hat{a} - a_l)}{(a_u - a_l)}$$

which expands to

$$\hat{\phi} = \frac{m\rho^{\frac{-\zeta_1}{\zeta_6}} e^{\frac{-(\zeta_1+\zeta_2)}{\zeta_6 \beta}} - \mu + \theta}{2\theta}.$$

Which necessarily implies the same results and intuition as the equilibrium adoption threshold.

5. Compensating Variation

In this section I undertake a simple comparison of a given individual's equilibrium utilities across models, and the compensating variation required to equate them, to examine the AVs' impact on welfare. The compensating variation is the change in income the individual would need to undergo to equate their utility in the two different scenarios. As it is a quantifiable measure of the effect of the new outcome on the individual, it can be used to compare across agents and construct an average, which gives an idea of the effect on the population in general.

I undergo the same process twice, once for adopters and once for non-adopters. For a given non-adopter, I substitute in $(y - cv)$ for y into the non-adopter equilibrium utility function ($\hat{V}_h$) from the model with AVs. I then set the new $\hat{V}_h$ equal to the individual's equilibrium utility in the model without AVs ($\hat{V}$) and solve for the variable cv. I do the same thing for a given adopter, but using their respective equilibrium utility function $\hat{V}_m$.

The resulting equations are

$$cvNA = \frac{\beta \ln\left(\frac{\left(\frac{\nu}{\mu}\right)^{\zeta_7}\left(\frac{\chi}{\mathrm{X}}\right)^{\frac{\zeta_5}{2}} \propto^{\zeta_3+\zeta_4}}{\propto_1^{\zeta_4}\propto_2^{\zeta_3}}\right)}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}$$

and

$$cvA = \frac{\beta\, \zeta_6 \ln\left(\frac{1}{a_i}\right)}{\zeta_1+\zeta_2} + cvNA.$$

Where *cvA* is the compensating variation for adopters and *cvNA* is the compensating variation for non-adopters. Necessarily, for the marginal adopter they are equal.

I average these across individuals,

$$CV = \int_{a_l}^{\tilde{a}} cvA\, da + \int_{\tilde{a}}^{a_u} cvNA\, da,$$

and the result is a general expression for average compensating variation, which can be found in the Appendix. This average CV can be examined in different equilibria, resulting from different specifications of AVs' driving ability, *m*. I examine a simple example, roughly calibrated using data from the US (Turner, Duranton, & Couture, 2016), with all zetas and ρ equal to 1. I examine the effects on *cv* as *m* changes, and the results are shown graphically in Figures 6, 7, and 8. As *m* increases, the number of agents who adopt increases, and the average CV does as well. This implies that for higher ability AVs, on average individuals needs to undergo a larger reduction in income to remain at the same level of utility as before AVs were introduced, which means they are better off. As expected, the lowest ability individual has the highest *cv,* as they are the ones who stand to gain the most from this new technology.

It is of interest to note that it is possible for a given adopter to be worse off in the world with AVs. That is, they are still of a low enough ability level that in the model with AVs, they choose to adopt, but to equate their utility across worlds they would need to be paid a sum in the world with AVs. As such, I cannot claim that AVs will definitely make individuals better off.

6. Conclusion

This model provides a basic framework, and there is a lot of room for permutation. Future work on this model could include tailoring it to fit some aspects of the real world more closely. For example, in the case that AVs continue to be bound by speed limits, there would realistically be an upper limit on speed, which is currently missing from my model. I would like to examine how the inclusion of this changes my equilibrium results. An examination of the effects of speed's variance would reflect the possible benefits that might come from the technology of "fleeting" (Litman, 2018). This type of technology connects the AVs wirelessly so they can communicate their planned actions before they undertake them, which has been claimed to increase the maximum speed at which the AVs can drive safely, and decrease the necessary space between them. Finally, I would like to examine the effect of including a price premium on the AVs. It seems probable that these cars will be priced higher than an average new car, and this could have effects on the equilibrium outcomes as well.

This model describes how individuals alter their driving behaviour (speed and distance driven) in response to changes in the world around them. I find that before the adoption of AVs, the probability of getting into an accident is the same across all individuals, as it is independent of ability. This also holds once AVs have been introduced, and is therefore a neutrality result. An increase in the average driving ability, which is the result of introducing AVs, causes individuals to change their driving behaviours such that there is no effect on probability in equilibrium. This type of neutrality result has been seen a few times in the past, and has been discussed at length in Downs's *The Law of Peak-Hour Express-Way Congestion* (1962).

However, this neutrality result does not mean that there is no possible way to change the probability outcome. An exogenous increase to the cost of an accident would decrease the probability in equilibrium, due to its relationship with the summary variable q. This holds in both models, with and without AVs, and implies that a more effective way to reduce the probability of accidents is to price them, instead of improving the technology.

Other important results are that as AV driving ability *m* increases, the number of agents who adopt increases, equilibrium average speed and distance do as well, and the variance in speed decreases. Additionally, exogenous changes in the time-saving parameter ρ increases the number of agents who adopt. The implications of the results are that as AVs get "better" at driving, more people will adopt them and they will drive further and faster to the point where their probability of being in an accident remains the same. Also, as the AVs provide more time-saving benefits, more people will adopt them.

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Appendix

Section 1

First order conditions for the model without AVs.

$$\frac{\partial EU_i}{\partial k_i} = -\frac{c}{s_i} + \frac{\beta}{k_i} - \frac{s_i^{\zeta_1} k_i^{\zeta_2} \zeta_2 \, \sigma^{\zeta_5} \, \bar{s}^{\,\zeta_4} (n\kappa)^{\zeta_3} \omega}{k_i a_i^{\zeta_6} \nu^{\zeta_7} \left(1 + \frac{s_i^{\zeta_1} k_i^{\zeta_2} (n\kappa)^{\zeta_3} \bar{s}^{\zeta_4} \sigma^{\zeta_5}}{a_i^{\zeta_6} \nu^{\zeta_7}}\right)} + \frac{\omega \zeta_2}{k_i} \left(\frac{s_i^{\zeta_1} k_i^{\zeta_2} \sigma^{\zeta_5} \, \bar{s}^{\,\zeta_4} (n\kappa)^{\zeta_3}}{a_i^{\zeta_6} \nu^{\zeta_7} \left(1 + \frac{s_i^{\zeta_1} k_i^{\zeta_2} (n\kappa)^{\zeta_3} \bar{s}^{\zeta_4} \sigma^{\zeta_5}}{a_i^{\zeta_6} \nu^{\zeta_7}}\right)} \right)^2 = 0$$

$$\frac{\partial EU_i}{\partial s_i} = \frac{c \, k_i}{s_i^2} - \frac{s_i^{\zeta_1} k_i^{\zeta_2} \zeta_1 \, \sigma^{\zeta_5} \, \bar{s}^{\,\zeta_4} (n\kappa)^{\zeta_3} \omega}{s_i \, a_i^{\zeta_6} \nu^{\zeta_7} \left(1 + \frac{s_i^{\zeta_1} k_i^{\zeta_2} (n\kappa)^{\zeta_3} \bar{s}^{\zeta_4} \sigma^{\zeta_5}}{a_i^{\zeta_6} \nu^{\zeta_7}}\right)} + \frac{\omega \zeta_1}{s_i} \left(\frac{s_i^{\zeta_1} k_i^{\zeta_2} \sigma^{\zeta_5} \, \bar{s}^{\,\zeta_4} (n\kappa)^{\zeta_3}}{a_i^{\zeta_6} \nu^{\zeta_7} \left(1 + \frac{s_i^{\zeta_1} k_i^{\zeta_2} (n\kappa)^{\zeta_3} \bar{s}^{\zeta_4} \sigma^{\zeta_5}}{a_i^{\zeta_6} \nu^{\zeta_7}}\right)} \right)^2 = 0$$

Equilibrium results for $\hat{k}_i$, $\hat{s}_i$ and $\hat{V}_i$

$$\hat{s}_i = \frac{(\zeta_1 + \zeta_2) c \; 2^{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5} \left(q \, a_i^{\zeta_6} \nu^{\zeta_7} \left(\left(\chi^{\frac{1}{2}(\zeta_1+\zeta_2+\zeta_3+\zeta_4)} q \, \mu^{\zeta_7} c^{\zeta_2+\zeta_3} \left(\frac{\zeta_1+\zeta_2}{\beta \zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3} \propto^{-\zeta_3-\zeta_4} n^{-\zeta_3} \right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}} \right)^{-\zeta_5} \right)^{\frac{1}{\zeta_1+\zeta_2}}}{1}$$

$$* \frac{\left(\left(n^{-\zeta_3} c^{\zeta_2+\zeta_3} \, \beta^{-\zeta_1-\zeta_2-\zeta_3} \propto^{\zeta_1+\zeta_2+\zeta_5} \, q \, \mu^{\zeta_7} \chi^{\frac{-\zeta_5}{2}} \left(\frac{\zeta_1+\zeta_2}{\zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3} \right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}} \right)^{\frac{-\zeta_4}{\zeta_1+\zeta_2}}}{\beta \zeta_1}$$

$$* \frac{c^{\frac{-\zeta_1}{\zeta_1+\zeta_2}} \left(\left(n^{-\zeta_3} c^{-\zeta_1-\zeta_4-\zeta_5} \, \beta^{\zeta_4+\zeta_5} \propto^{\zeta_1+\zeta_2+\zeta_5} q \, \mu^{\zeta_7} \chi^{\frac{-\zeta_5}{2}} \left(\frac{\zeta_1+\zeta_2}{\zeta_1}\right)^{-\zeta_4-\zeta_5} \right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}} n \right)^{\frac{-\zeta_3}{\zeta_1+\zeta_2}}}{1}$$

$$\hat{k}_i = 2^{-\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}\left(q\, a_i^{\zeta_6}\nu^{\zeta_7}\left(\left(\chi^{\frac{1}{2}(\zeta_1+\zeta_2+\zeta_3+\zeta_4)}q\,\mu^{\zeta_7}c^{\zeta_2+\zeta_3}\left(\frac{\zeta_1+\zeta_2}{\beta\zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3} \propto^{-\zeta_3-\zeta_4} n^{-\zeta_3}\right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}\right)^{-\zeta_5}\right)^{\frac{1}{\zeta_1+\zeta_2}}$$

$$* \left(2^{-\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}\left(n^{-\zeta_3}c^{\zeta_2+\zeta_3}\,\beta^{-\zeta_1-\zeta_2-\zeta_3} \propto^{\zeta_1+\zeta_2+\zeta_5} q\,\mu^{\zeta_7}\chi^{\frac{-\zeta_5}{2}}\left(\frac{\zeta_1+\zeta_2}{\zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3}\right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}\right)^{\frac{-\zeta_4}{\zeta_1+\zeta_2}}$$

$$* c^{\frac{-\zeta_1}{\zeta_1+\zeta_2}}\left(2^{-\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}}\left(n^{-\zeta_3}c^{-\zeta_1-\zeta_4-\zeta_5}\,\beta^{\zeta_4+\zeta_5} \propto^{\zeta_1+\zeta_2+\zeta_5} q\,\mu^{\zeta_7}\chi^{\frac{-\zeta_5}{2}}\left(\frac{\zeta_1+\zeta_2}{\zeta_1}\right)^{-\zeta_4-\zeta_5}\right)^{\frac{1}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}} n\right)^{\frac{-\zeta_3}{\zeta_1+\zeta_2}}$$

$$\hat{V}_i = y - \frac{\beta\zeta_1}{\zeta_1+\zeta_2} + \beta\ln\left(\hat{k}_i\right) - \frac{q(\zeta_1+\zeta_2)^{\zeta_1}}{2(\beta\zeta_1)^{\zeta_1}+q(\zeta_1+\zeta_2)^{\zeta_1}} * \omega$$

Where $\frac{q(\zeta_1+\zeta_2)^{\zeta_1}}{2(\beta\zeta_1)^{\zeta_1}+q(\zeta_1+\zeta_2)^{\zeta_1}}$ is probability in equilibrium ($\hat{\pi}_i$).

Section 4.

First order conditions for the model with AVs.

$$\frac{\partial EU_j}{\partial k_j} = -\frac{\rho c}{s_j} + \frac{\beta}{k_j} - \frac{{s_j}^{\zeta_1}\,{k_j}^{\zeta_2}\zeta_2\,\sigma^{\zeta_5}\,\bar{s}^{\,\zeta_4}(n\kappa)^{\zeta_3}\omega}{k_j m^{\zeta_6}\nu^{\zeta_7}\left(1+\frac{{s_j}^{\zeta_1}\,{k_j}^{\zeta_2}\,(n\kappa)^{\zeta_3}\,\bar{s}^{\,\zeta_4}\sigma^{\zeta_5}}{m^{\zeta_6}\,\nu^{\zeta_7}}\right)} + \frac{\omega\zeta_2}{k_j}\left(\frac{{s_j}^{\zeta_1}\,{k_j}^{\zeta_2}\,\sigma^{\zeta_5}\,\bar{s}^{\,\zeta_4}(n\kappa)^{\zeta_3}}{m^{\zeta_6}\nu^{\zeta_7}\left(1+\frac{{s_j}^{\zeta_1}\,{k_j}^{\zeta_2}\,(n\kappa)^{\zeta_3}\,\bar{s}^{\,\zeta_4}\sigma^{\zeta_5}}{m^{\zeta_6}\,\nu^{\zeta_7}}\right)}\right)^2 = 0$$

$$\frac{\partial EU_j}{\partial s_j} = \frac{\rho\, c\, k_j}{{s_j}^2} - \frac{{s_j}^{\zeta_1}\,{k_j}^{\zeta_2}\zeta_1\,\sigma^{\zeta_5}\,\bar{s}^{\,\zeta_4}(n\kappa)^{\zeta_3}\omega}{s_j\, m^{\zeta_6}\nu^{\zeta_7}\left(1+\frac{{s_j}^{\zeta_1}\,{k_j}^{\zeta_2}\,(n\kappa)^{\zeta_3}\,\bar{s}^{\,\zeta_4}\sigma^{\zeta_5}}{m^{\zeta_6}\,\nu^{\zeta_7}}\right)} + \frac{\omega\zeta_1}{s_j}\left(\frac{{s_j}^{\zeta_1}\,{k_j}^{\zeta_2}\,\sigma^{\zeta_5}\,\bar{s}^{\,\zeta_4}(n\kappa)^{\zeta_3}}{m^{\zeta_6}\nu^{\zeta_7}\left(1+\frac{{s_j}^{\zeta_1}\,{k_j}^{\zeta_2}\,(n\kappa)^{\zeta_3}\,\bar{s}^{\,\zeta_4}\sigma^{\zeta_5}}{m^{\zeta_6}\,\nu^{\zeta_7}}\right)}\right)^2 = 0$$

Equilibrium functions in model with AVs.

$$\hat{k}_m = \frac{1}{\rho}\left(\left(\frac{q\nu^{\zeta_7}A^{\zeta_6}}{2\left(e^{-\frac{2\zeta_1\ln(c)+2\zeta_4\ln\left(\frac{\propto_1 c(\zeta_1+\zeta_2)}{\beta\zeta_1 n\propto_2}\right)-\ln\left(\left(\frac{c(\zeta_1+\zeta_2)}{\beta\zeta_1 n\propto_2}\right)^2 \mathrm{X}\right)(\zeta_1+\zeta_2+\zeta_3+\zeta_4)+2\ln\left(\frac{1}{n\propto_2}\right)(\zeta_1+\zeta_2)+2\ln(2)-2\ln\left(q\nu^{\zeta_7}\right)}{2(\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5)}}\right)^{\zeta_5}}\right)\mathrm{I} * \mathrm{II}\right)^{\frac{1}{\zeta_1+\zeta_2}}$$

where

$$\mathrm{I} = \left(\frac{c^{\zeta_2+\zeta_3} \propto_1{}^{\zeta_1+\zeta_2+\zeta_3+\zeta_5} \; q\, \nu^{\zeta_7} \left(\frac{\zeta_1+\zeta_2}{\zeta_1}\right)^{\zeta_1+\zeta_2+\zeta_3}}{2n^{\zeta_3}\beta^{\zeta_1+\zeta_2+\zeta_3}\left(\sqrt{\mathrm{X}}\right)^{\zeta_5} \propto_2{}^{\zeta_3}} \right)^{\frac{\zeta_4}{\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5}} \qquad c^{\zeta_1}$$

and

$$\mathrm{II} = \left(n2^{-\frac{1}{\zeta_1+\zeta_2}} \left(q\nu^{\zeta_7} c^{-\zeta_1}\, e^{Y}\right)^{\frac{1}{\zeta_1+\zeta_2}} \propto_2 \right)^{\zeta_3}$$

for Y

$$\begin{aligned} Y = {} & \frac{1}{2(\zeta_1+\zeta_2+\zeta_3+\zeta_4+\zeta_5)} \Bigg(-2\zeta_4 ln\left(\frac{\propto_1 c(\zeta_1+\zeta_2)}{\beta\zeta_1 n \propto_2}\right)(\zeta_1+\zeta_2) \\ & - \zeta_5 \ln\left(\mathrm{X}\left(\frac{c(\zeta_1+\zeta_2)}{\beta\zeta_1 n \propto_2}\right)^2\right)(\zeta_1+\zeta_2) + 2\ln\left(\frac{1}{n\propto_2}\right)(\zeta_1\zeta_3+\zeta_1\zeta_4+\zeta_1\zeta_5+\zeta_2\zeta_3+\zeta_2\zeta_4+\zeta_2\zeta_5) \\ & - 2\ln\left(q\nu^{\zeta_7}c^{-\zeta_1}\right)(\zeta_4+\zeta_5) + 2\ln(2)(\zeta_3+\zeta_4+\zeta_5)\Bigg) \end{aligned}$$

$$\hat{s}_m = \frac{\rho c(\zeta_1+\zeta_2)}{\beta\zeta_1}\hat{k}_m$$

$$\hat{k}_h = \rho\hat{k}_m$$

$$\hat{s}_h = \frac{c(\zeta_1+\zeta_2)}{\beta\zeta_1}\hat{k}_h$$

Equilibrium utility for adopters and non-adopters:

$$\hat{V}_m = y - \frac{\beta\,\zeta_1}{\zeta_1+\zeta_2} + \beta\ln(\hat{k}_m) - \frac{q(\zeta_1+\zeta_2)^{\zeta_1}}{2(\beta\zeta_1)^{\zeta_1}+q(\zeta_1+\zeta_2)^{\zeta_1}} * \omega$$

$$\hat{V}_h = y - \frac{\beta\,\zeta_1}{\zeta_1+\zeta_2} + \beta\ln(\hat{k}_h) - \frac{q(\zeta_1+\zeta_2)^{\zeta_1}}{2(\beta\zeta_1)^{\zeta_1}+q(\zeta_1+\zeta_2)^{\zeta_1}} * \omega$$

Section 5.

Average Compensating Variation

$$CV = -\frac{1}{\zeta_1+\zeta_2}\left(\left(\rho^{\frac{-\zeta_1}{\zeta_6}} m\beta e^{\frac{-(\zeta_1+\zeta_2)}{\zeta_6\beta}}\left(ln(\rho)\,\zeta_1 - ln\left(\rho^{\frac{\zeta_1}{\zeta_6}}\right)\zeta_6 - \zeta_6\right)\right) + \ln\left(\frac{m}{\mu-\theta}\right)\beta\zeta_6(\mu-\theta) - \ln(\rho)\,\beta\zeta_1(\mu+\theta) + \beta\zeta_6(\mu-\theta) + (\zeta_1+\zeta_2)(-2\theta cvH - \mu + \theta)\right)$$

This has been left as a function of *cvH*, as it would otherwise be unreadable due to its length.

Figures

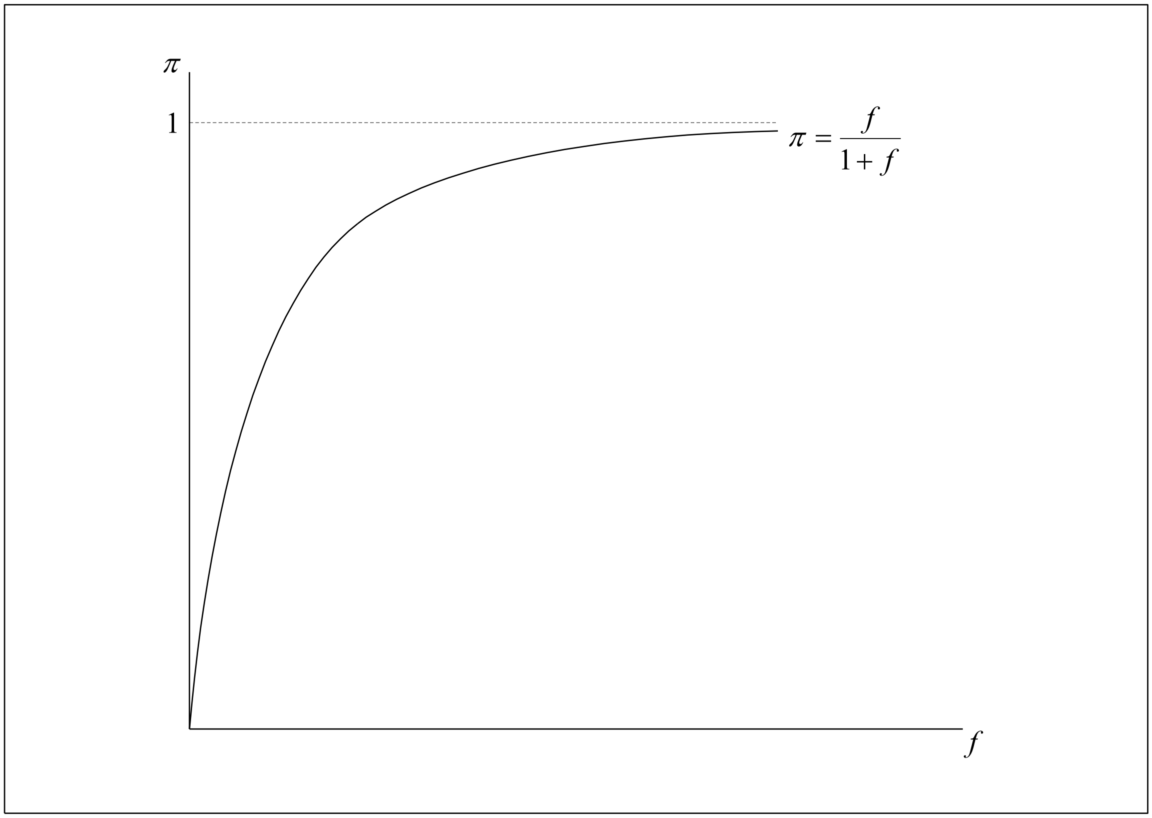

Figure 1 – Probability Function

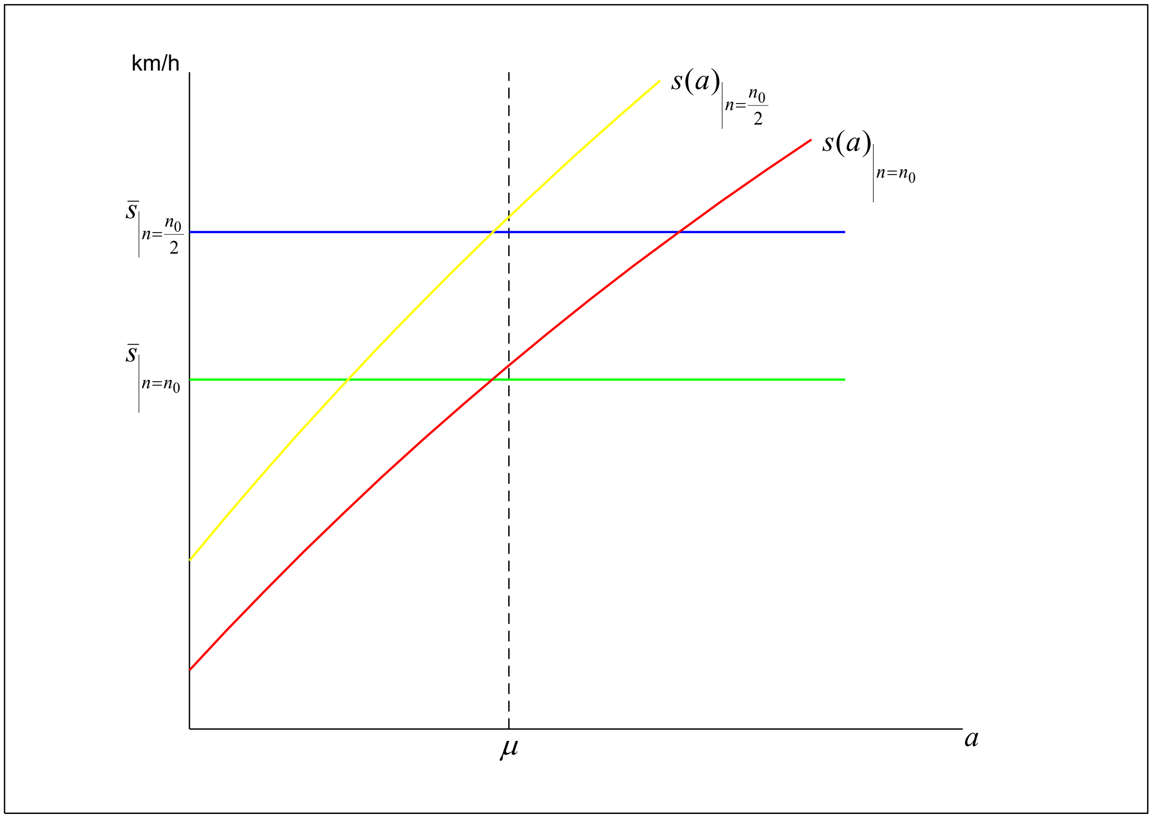

Figure 2 – Effect on speed from a change in number of cars

This graph demonstrates the effect of a decrease in n by 50%. Individual speed changes in a_i (a on the axis), which is the innate ability of each individual. I impose the assumption that ability is uniformly distributed across individuals. Individual speed at equilibrium shifts up from the red sloped curve to the yellow sloped curve after the reduction in n. As a result, average speed across all individuals also shifts up in the new equilibrium, from the green horizontal line to the blue one. Due to the symmetry of the model, these same results are observed in individual distance and average distance.

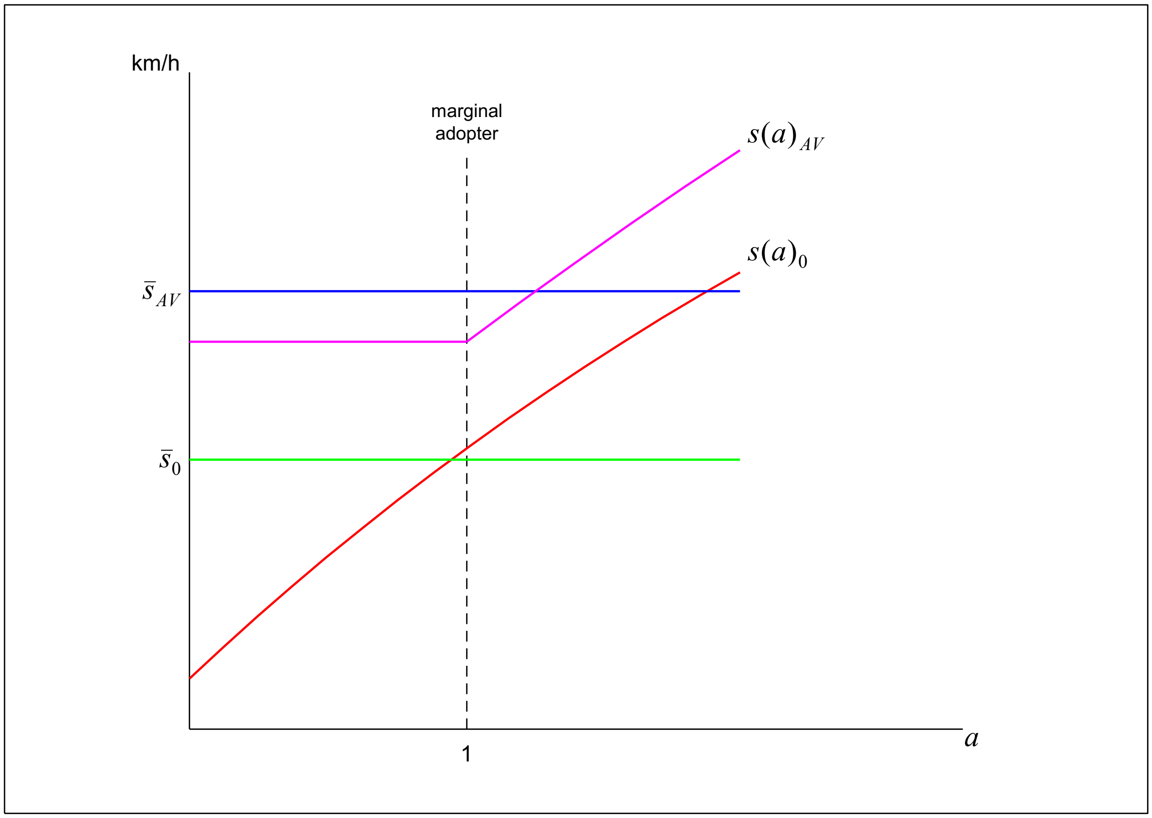

Figure 3 - Change in equilibrium speed (m=1)

In this example, the AVs have $m=1$ and $\rho=1$, which means they offer no time-cost savings. Ability ranges from 0.5-1.5, so m is in the middle of the distribution. The red curve is speed across individuals when there are no AVs present, and the magenta curve is speed across individuals once AVs are in the model. The kink in the magenta line occurs at $a=1$, which is the ability level of the marginal adopter in this simple example. The two horizontal lines represent the average speed in each model, where the higher one is the model with AVs.

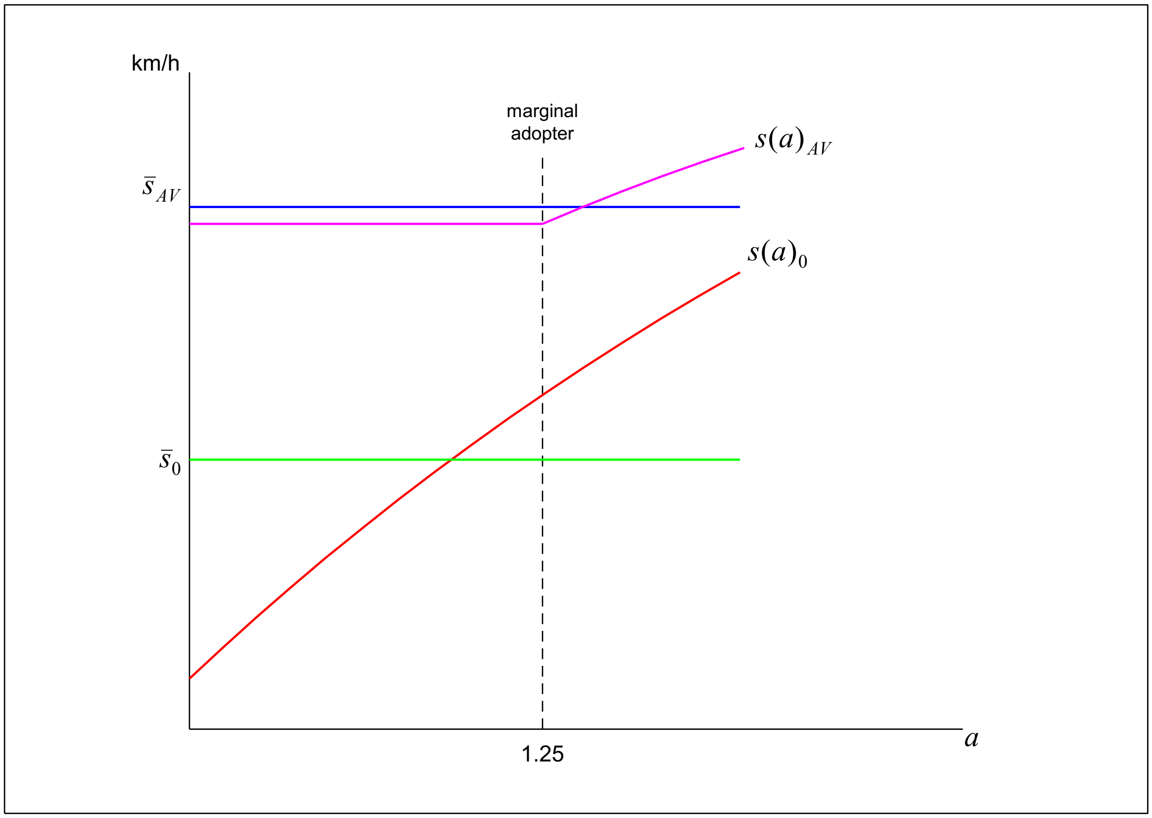

Figure 4 - Change in equilibrium speed (m=1.25)

This figure shows the same example as in Figure 3, but this time with AVs' ability higher than the midpoint of human ability. As seen in the difference between the two graphs, the high *m* leads to higher speeds.

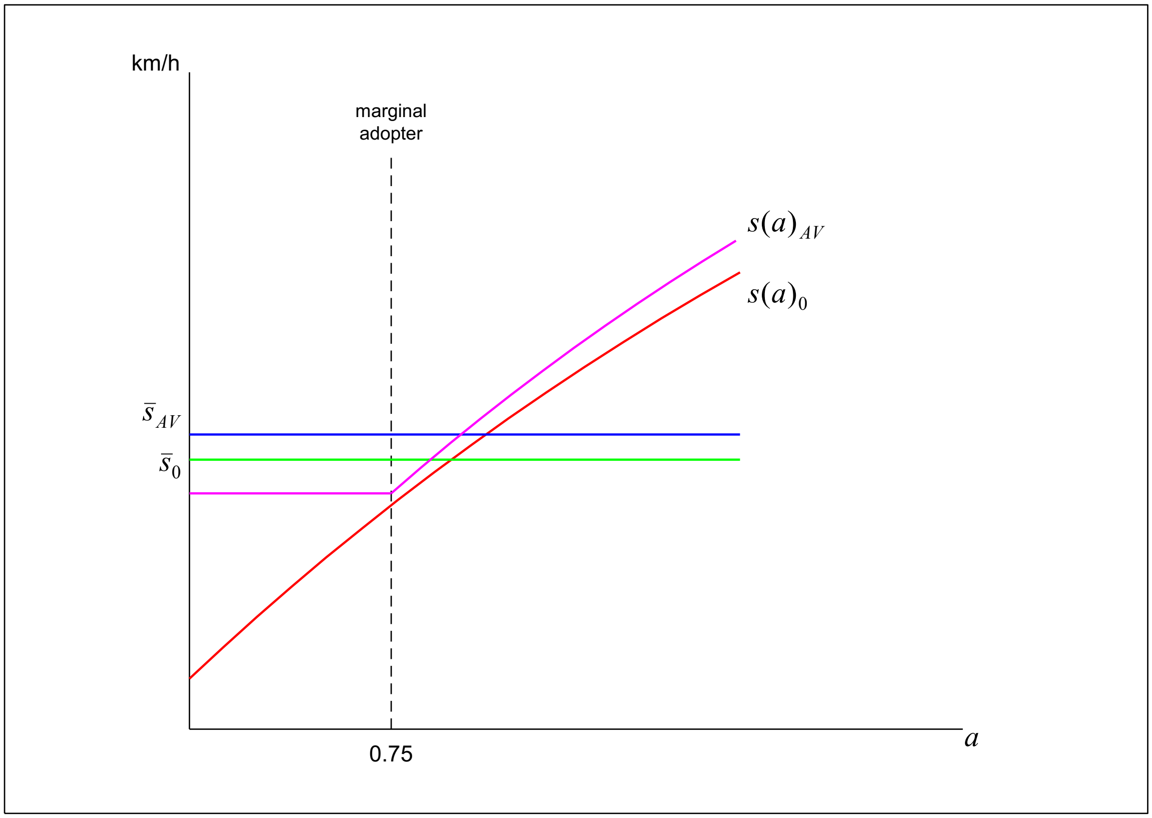

Figure 5 - Change in equilibrium speed (m=0.75)

In this case, the ability of the AVs was below the midpoint of the distribution of human ability at *m=0.75*. There is still an increase in average and individual speed, it is just of a smaller magnitude.

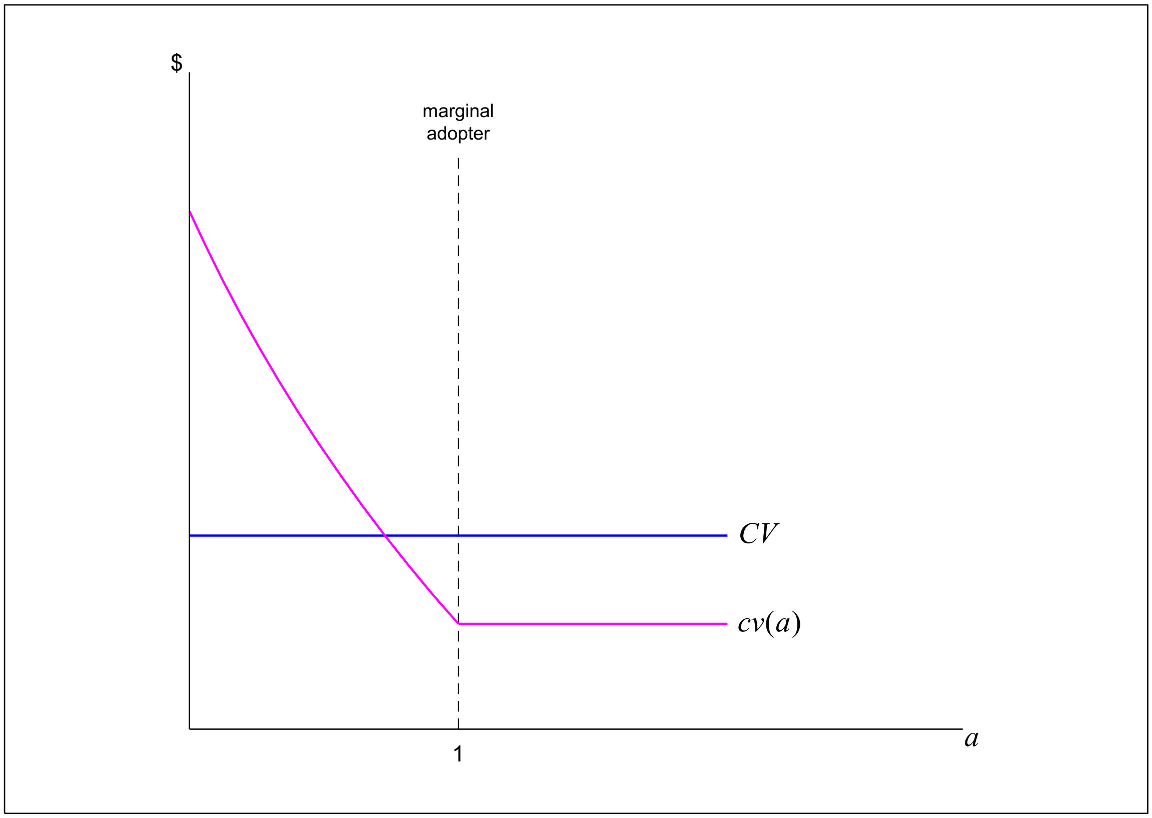

Figure 6 - CV across individuals (m=1)

In this figure, as in the following two, the blue line represents the average CV across all individuals, and the purple line shows the *cv* for each individual, as they vary across ability. For the adopters it changes with their ability level, but for the non-adopters it is constant. As expected, the individual with the lowest ability has the highest *cv* as they gain the most from adoption. In this simple calibration, the *cv'*s are always positive, which means that for every level of *m* all individuals are better off in the world with AVs, but this is not a general result.

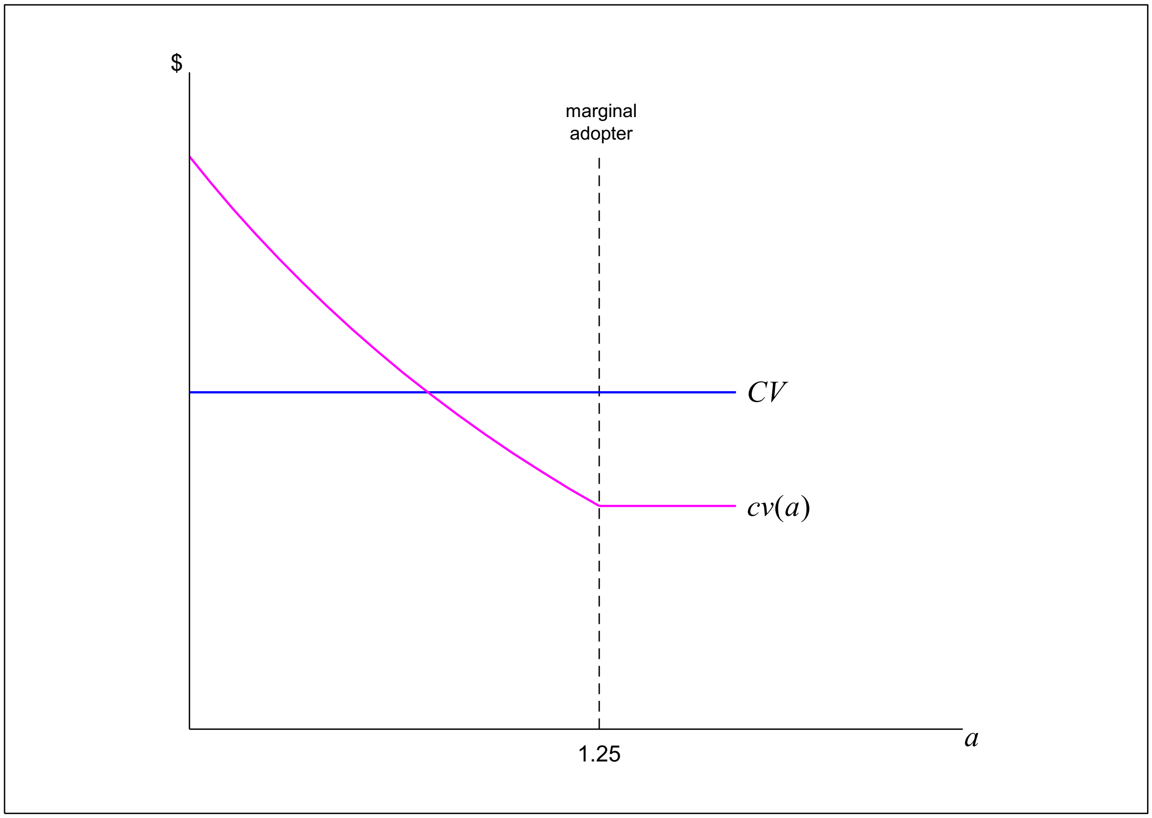

Figure 7 - CV across individuals (m=1.25)

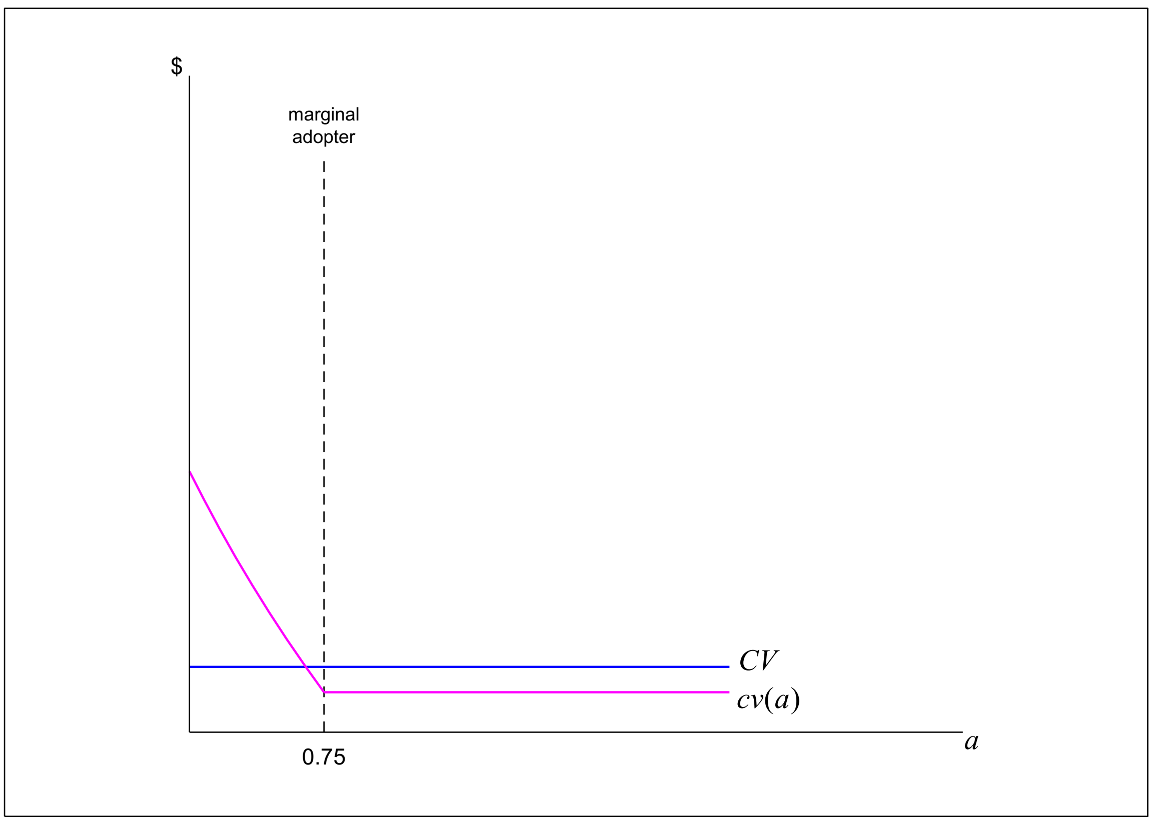

Figure 8 - CV across individuals (m=0.75)