The Effect of the Term Structure of Interest Rates on the Probability of Bank Runs

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Abstract

This paper examine the relationship between real interest rate term structure and the probability of a bank run by extending Adao and Temzelides' (1998) model, which proves that a mixed strategy equilibrium exists, where a bank run occurs with a stochastic probability. By studying how changes in interest rate term structure lead to changes in the mixed strategy equilibrium, this paper concludes that although a bank faces a higher risk of a bank failure when its investment return rises, it is possible for that bank to avoid the increased likelihood of a bank run by making the right adjustment in its deposit interest rate term structure.

1. Introduction

Bank runs occur when depositors fear that a bank may become insolvent and begin to withdraw their deposits rapidly to minimize possible losses. A bank is bankrupt when it can no longer satisfy depositors' wanting to withdraw their demand deposits. Furthermore, through a process of contagion a single bank failure may trigger other bank failures when depositors panic. Bank runs were once a prominent feature during the Great Depression. Though they are not as common now days, they are still with us. During March 2010, a total of twenty banks failed in U.S. (FDIC, 2010), and the loss amounted to billions of dollar. While it is important for a government to design rescue plans for failed banks, preventing bank runs is certainly another important issue, and understanding the factors that affect the probability of a bank run would help a government to design policies to minimize the likelihood of a bank run.

In the existing bank run literature, the various factors that determine the probability of a bank run include bank fundamentals, signals regarding the health of a bank received by the depositors, and interest rate term structure. While many papers attempt to study ways to measure the health of a bank and how signals regarding bank fundamentals can influence the likelihood of a bank run, very few studies focus on the relationship between the interest rate term structure and the probability of a bank run.

The research question of this paper is to determine how likelihood of a bank run is affected by changes in (i) the term structure of investment interest rates, and (ii) the term structure of deposit interest rates. In my attempt to answer this research question, I extend Adao and Temzelides' (1998) model, which is a generalization of Diamond and Dybvig's (1983) classic banking model. Whereas Diamond and Dybvig show that a sunspot can trigger the pure strategy bank-run equilibrium, Adao and Temzelides' (1998) extend Diamond and Dybvig's model and prove that a mixed strategy equilibrium exists, where a bank run occurs with a stochastic probability. Unlike the sunspot generated bank run equilibrium, the mix strategy equilibrium does not rely on exogenous factors that are hard to explain. Therefore, Adao and Temzelides' (1998) model is arguably the first model that should be examined when attempting to explain the probability of bank runs. They do not examine how changes in bank interest rates and asset returns affect the probability of bank run or, more generally, the equilibrium distribution of deposit payoffs. In my paper, I concentrate on how bank interest rates affect the probability of a bank run, though I also look at the asset returns and the overall distribution of payoffs.

The main finding of this paper is that although a bank faces a higher risk of a bank failure when its investment return rises, it is possible for that bank to avoid the increased likelihood of a bank run by making the right adjustment in its deposit interest rate term structure.

This paper proceeds as follows. Section 2 provides a brief literature review, which outlines several bank run models and relevant empirical studies, while section 3 details the basic model for the case of 3 agents. Section 4 describes the mixed strategy equilibrium, and section 5 examines how the equilibrium is affect by exogenous changes in the term structure of interest rates. Section 6 generalizes the results in several ways, including looking at the case of more than 3 agents. Section 7 concludes.

2. Literature Review

In this section, I briefly summarize some papers that are relevant to my topic; the literature review outlines several empirical studies and bank run models.

2.1 Empirical Studies

While the empirical studies link bank failures with interest rates, they are not designed to answer my research questions. Arena (2007) shows that banks which failed during the late 1990s in East Asia had lower loans and deposit interest rates than banks that did not fail. On the other hand, failed banks in Latin America during the same period had higher loans and deposit interest rates than banks that survived. Von Hagen and Ho (2007) use a logit estimation model in an attempt to find indicators that would identify a banking crisis, and they conclude that real interest rates have a negative effect on the likelihood of bank failures from 1980 to 2001. Kraft and Galac (2007) not only associate high deposit interest rates with bank failures in Croatia from 1998 to 1999, but also claim that deposit interest rates can be used as a predictor for bank failures. Kraft and Galac (2007) hypothesize that a risk-loving bank is more likely to offer high deposit rates to compensate for their risky lending behavior, and the high rates force other less risk-loving banks to also raise their deposit rates to maintain competitiveness. They also find that before the crisis period, banks with high deposit rates experienced rapid growth in deposits; however, after the crisis period, customers learned to associate banks that offer high deposit rates with bank failures and those banks experience slower deposit growth than banks with lower deposit rates.

Although the empirical studies show that there is certainly a relationship between real interest rate term structure and bank failures, they produce conflicting results.

Moreover, it is likely that some of those studies fail to control the effect on bank runs from business cycles. Therefore, the empirical studies do not provide satisfying answers to my research questions.

2.2 Theoretical Bank Run Models

While several bank run models exist in the literature, Diamond and Dybvig's (1983) bank run model is the most prominent. This section briefly outlines Diamond and Dybvig's model and its two extensions, models by Goldstein and Pauzner (2005), and Adao and Temzelides (1998).

Diamond and Dybvig (1983) construct a bank run model, which demonstrates that: (1) By issuing demand deposits, banks transform illiquid assets into liquid assets and provide the optimal risk sharing among consumers who prefer consumption at different periods. (2) The demand deposit contract has a bank-run equilibrium, where all depositors withdraw, including those that are not initially worried about bank failures. (3) Bank runs lead to real loss, since even healthy banks can fail. Diamond and Dybvig's model has three periods (period 0, 1, and 2), and a single homogeneous asset. The bank in the model has access to productive technology which gives it a return of 1 if it chooses to liquidate the asset at the end of period 1; if it waits patiently and liquidates the asset at the end of period 2, the return is R, which is greater than 1.

The bank is assumed to survive for only three periods and will liquidate its entire asset at the end of the period 2. Diamond and Dybvig define depositors who prefer consumption in period 1 as type 1 agents, while type 2 agents are those who would rather consume in period 2. At period 0, all agents are identical and a consumption shock determines their types in period 1.The bank is assumed to act as a social planner, and its aim is to optimize the ex-ante consumptions of both types of agents in each period. Knowing the proportion of type 1 and type 2 agents in period 0, the bank is able to construct a deposit contract, which pays r_1 in the end of period 1 and r_2 in the end of period 2. The table below summarizes each agent type's cash flow under the deposit contract.

Table 1. Deposit Contract

	Cash Flow at period 0	Cash Flow at period 1	Cash Flow at period 2
Agent Type			
Type 1 (Impatient)	-1	r_1	0
Type 2 (Patient)	-1	0	r_2

In the Diamond and Dybvig's (1983) model, type 2 agents trigger a bank run by randomly misrepresenting their types and withdrawing early. When type 2 agents withdraw early, it is possible that a bank cannot fully satisfy depositor's demands, and all depositors would rush to withdraw to avoid losing their assets. Therefore, two pure strategy Nash equilibriums exist in this model: the optimal equilibrium and the bank run equilibrium. The optimal equilibrium happens where only type 1 agents withdraw at period 1, and this equilibrium provides optimal risk sharing. The bank run equilibrium occurs when the bank's resource is exhausted in the first period from the agents' attempt to withdraw.

Goldstein and Pauzner (2005) study a modified version of Diamond and Dybvig's model, where the fundamentals of the economy uniquely determine the likelihood of a bank run. Their study focuses on the relationship between the banking contract and the probability of a bank run. The underlying assumptions in Goldstein and Pauzner's paper are stochastic fundamentals of the economy and one-sided strategic complementarities. Given the stochastic fundamentals, depositors observe only noisy signals about the fundamentals, and a type 2 agent would withdraw early if the signal received is below some critical value. While a type 2 agent's incentive to withdraw early does not increase monotonically with the number of other type 2 agents withdrawing early, the incentive to withdraw does increase as long as the number of agents who withdraw is small enough that a type 2 agent would still prefer to wait. If the critical signal threshold, below which a type 2 agent would choose to withdraw early to is θ^*, Goldstein and Pauzner conclude that θ^* increases when r_1 increases. Furthermore, they claim that the optimal r_1 that leads to the lowest probability of a bank run would have to be larger than 1 but less than the r_1 specified under the deposit contract in Diamond and Dybvig's model.

Based on the Diamond-Dybvig bank run model, Adao and Temzelides (1998) present their model with a two-staged game, where at the first stage of the game, agents decide whether to deposit in the bank or not, and once they deposit and proceed to the second stage of the game, those who happen to be type 2 agents would need to decide whether to withdraw early and misrepresent their types or to wait patiently. Adao and Temzelides prove that when agents are only allowed to play pure strategies, the only robust sequential equilibrium under the logic of forward induction is for all agents to deposit in the bank and represent their types truthfully. They also prove that when agents are allowed to play mixed strategies, there exists an equilibrium involving a bank run with positive possibility. Furthermore, they show that even when a bank run can occur, agents would still choose to deposit in the bank ex ante.

Since Diamond and Dybvig's model (1983) does not allow agents to play mixed strategies, it is not possible to investigate the relationship between interest rate term structure and the likelihood of a bank run within their framework. Goldstein and Pauzner's more sophisticated model (2005) allows for discussions on the probability of a bank run by introducing a critical signal threshold θ^*, below which type 2 agents would decide to withdraw early. Although it is tempting to conclude that the likelihood of a bank run increases with r_1 based on one of Goldstein and Pauzner's propositions, which states that θ^* increases when r_1 increases, the argument is not compatible with my research questions. In Goldstein and Pauzner's model, a change in r_1 is exogenous; θ^* increases when r_1 increases, holding all other variables constant. With the higher θ^*, an economy would need to maintain better fundamentals for type 2 agents to wait patiently and the probability of a bank run is thus increased. However, in my paper, a change in r_1 is in most cases accompanied by a change in r_2, and it is unknown how θ^* would change when both rates change. Adao and Temzelides' paper (1998) proves that there is a way to discuss the likelihood of a bank run without introducing a sun spot into the model, although their model does not have a conclusive comment on whether an increase in R would lead to increased likelihood of a bank run.

3. Three-Agent Model

In my analysis, I follow Adao and Temzelides' (1998) notation and some of Diamond and Dybvig's (1983) assumptions, which state that agents are *ex ante* homogenous at period 0, and agents at period 1 experience a consumption shock that determines their types. Type 1 agents want to consume immediately in period 1; whereas, type 2 agents would be willing to wait and withdraw in period 2 if they can get a higher rate of interest by waiting. Adao and Temzelides (1998) prove that there exists a mixed strategy equilibrium, where each type 2 agent chooses to withdraw early with a positive possibility p. The aim of my paper is to examine the changes in p caused by changes in the term structure of investment interest rates, and the term structure of deposit interest rates. This section is an outline of the three-agent model, where the population contains one type 1 agent and two type 2 agents.

3.1 Notation

The exogenous parameters are the following:

N: Number of total depositors

n: Number of type 2 agents; $N - n$ is the number of type 1 agents.

$t = (N\text{-}n)/N$: The proportion of type 1 agents; $1\text{-}t = n/N$ is the proportion of type 2 agents.

$R>1$: Return on long-term assets liquidated at the end of the second period; when the bank chooses to liquidate its assets at anytime other than the end of the second period, the return is 1.

The endogenous variables are:

r_1: Return paid out by the bank at the end of the first period.

r_2: Return paid out by the bank at the end of the second period.

p_i: The probability for the i^{th} type 2 agent withdrawing early.

3.2 Assumptions

For my analysis, I employ assumptions produced by Adao and Temzelides (1998), and an outline of the assumptions is provided below.

(1) Constraints:

$(t)(r_1) + (1\text{-}t)(r_2)/R = 1$

$1 < r_1 < r_2 < R$

The first constraint is the resource constraint when type 1 agents withdraw in period 1 and type 2 in period 2; the second constraint is the incentive compatibility condition necessary for type 2 agents to be willing to wait patiently till the second period. For simplicity, there is no liquidation cost, so assets liquidated in period 1 yield return 1. Type 2 agents who choose to withdraw early reinvest their returns at rate 1, the return given by storage technology.

(2) Distribution of payments

When demand in the first period exhausts the bank's resources, each agent in line receives an average of the bank's resources. If the resources are not exhausted in the first period, agents who choose to withdraw in the second period receives the remaining resources.

(3) The rationality assumption

All depositors are assumed to be rational and forward looking.

3.3 Game Tree

The game tree for the three-agent model involves only the two type 2 agents, players P1 and P2. In the game tree, each player chooses either to withdraw early or to wait, and the players make their decisions simultaneously. Figure 1 is the game tree from P1's perspective while Figure 2 shows the game tree from P2's perspective. Nodes a, b, c, and d in Figure 1 denote P1's possible destinations, and nodes e, f, g, and h in Figure 2 denote P2's possible destinations. The payoff function at a particular node is attached at the bottom of that node.

Figure 1

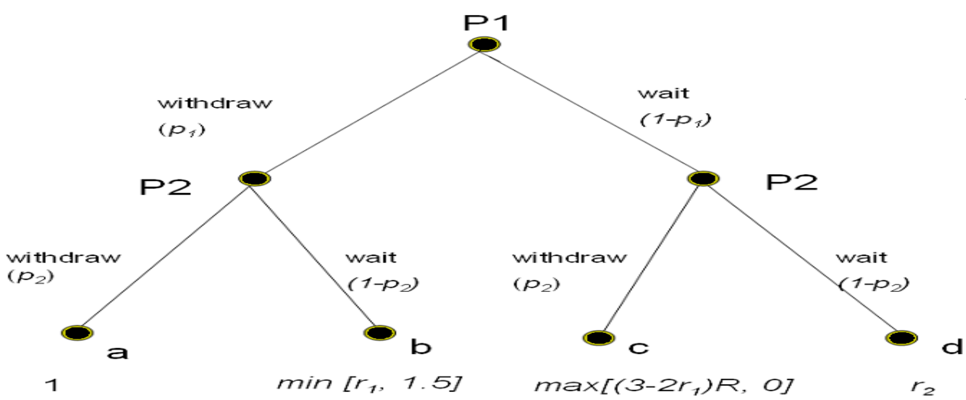
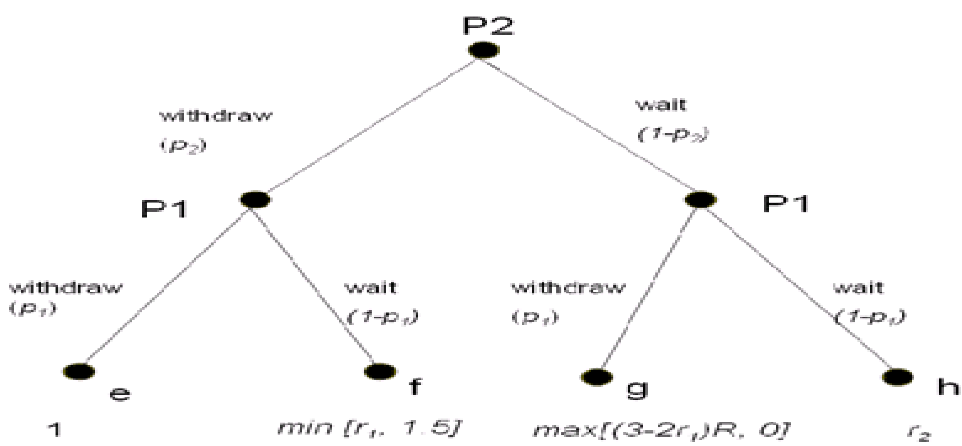


Figure 2



4. Mixed Strategies and Equilibrium – Three-Agent Model

According to the definition of mixed strategy equilibrium, the equilibrium occurs when the agent is indifferent between withdrawing and waiting. Agents make their choices based solely on their payoff functions. Thus, the mixed strategy equilibrium in this game is where the payoff for waiting equals the payoff for withdrawing.

4.1 Individual Expected Payoffs

When the player P1 chooses to withdraw, he/she arrives at two possible nodes in Figure 1, node *a* or node *b*. If the agent arrives at node *a*, both players P1 and P2 choose to withdraw; the bank's resource is exhausted when both players withdraw, and the payoff at node *a* is an average of the bank's total resource. Therefore, at node *a*, the payoff is 1. At node *b*, player P1 decides to withdraw while player P2 waits. P1 gets r_1 if the bank's resource is not exhausted; however, if the bank cannot fulfill its promised payment of r_1, it would divide the resource between P1 and the other type 1 agent. The payoff function at node *b* is $min\ [r_1, 1.5]$. When P1 decides to withdraw, the expected payoff function comes down to:

$$(1-p_2)\ min\ [r_1, 1.5] + (p_2) \quad (1)$$

Similarly, when player P1 decides to wait, he/she can arrive at either node *c* or *d*. At node *c*, P1 waits and P2 withdraws; P1 would obtain the remaining of the bank's resource, $(3\text{-}2r_1)R$, if the resource is not fully exhausted. However, it is also possible for P1 to get 0 when the resource is fully exhausted. The payoff at node *c* is therefore $max[(3\text{-}2r_1)R,\ 0]$. At node *d*, both agents P1 and P2 wait, and the payoff is simply r_2. The expected payoff function when P1 waits is thus:

$$(1\text{-}p_2)\ (r_2) + (p_2)\{\ max[3 - 2r_1]\ R,\ 0\}\} \qquad (2)$$

By equating equations (1) and (2), I can solve for the probability p_2. Similarly, from player P2's perspective, the expected payoff functions from withdrawing and waiting are:

$$(1\text{-}p_1)\ \ min\ \ [r_1,\ \ 3/2]\ \ +\ \ (p_1) \qquad (3)$$

$$(1\text{-}p_1)\ \ (r_2)\ \ +\ \ (p_1)\ \ \max\ \ \{[3\ \ -\ \ 2r_1]\ \ R,\ \ 0\} \qquad (4)$$

Thus, the probability p_1 can be solved by equating equations (3) and (4).

4.2 Symmetric Mixed Strategy Equilibrium

Since the symmetric mixed strategy equilibrium for both players occurs when $p_1 = p_2 = p$, then *p* is the solution of:

$$(1-p) \min \{r_1, 1.5\} + (p) = (1-p)(r_2) + (p) \max \{[3 - 2r_1]R, 0\}$$

The solutions for p depend on the value of r_1. When r_1 is less than 1.5, the solution is:

$$p = \frac{(r_2 - r_1)}{(1 - r_1 + r_2 - 3R + 2r_1 R)} \tag{5}$$

When r_1 is equal to or greater than 1.5, the solution is:

$$p = \frac{(r_2 - 1.5)}{(r_2 - 0.5)} \tag{6}$$

Clearly for equation (5), the probability is always interior, and p takes a value between 0 and 1. For equation (6), the probability is only between 0 and 1 when:

$$R < \frac{1}{(3 - 2r_1)} \tag{7}$$

R is greater than 1 by assumption, and this constraint is tight when r_1 is greater than 1 and approaches 1. When constraint (7) is not satisfied, the solution of p takes a value equal to or greater than 1. Since the probability is bounded above by 1, p simply equals to 1 in this case. This implies that any type 2 agent would choose to play the pure strategy of always withdrawing early. However, p is never 0. Thus, bank run equilibrium is always possible, independent of the value of r_1. The following proposition summarizes the equilibrium.

Proposition 1: A symmetric mixed strategy equilibrium exists where there is always a positive probability p of a bank run. The particular equilibrium probability depends on the value of r_1 as follows:

(i) $r_1 < 1.5$.

If $R < \frac{1}{(3-2r_1)}$, then the probability is given by equation (5), $p = \frac{(r_2 - r_1)}{(1 - r_1 + r_2 - 3R + 2r_1R)}$

If $R \geq \frac{1}{(3-2r_1)}$, a bank run occurs with certainty, and $p = 1$.

(ii) $r_1 \geq 1.5$

The probability is given by equation (6), $0 < p = \frac{(r_2 - 1.5)}{(r_2 - 0.5)} < 1$.

5. The Term Structure of Interest Rates and the Probability of a Bank Run

As mentioned before, I intend to investigate changes in the probability of a bank run caused by changes in (i) the term structure of investment interest rates, and (ii) the term structure of deposit interest rates. In my analysis, a change in the term structure of investment interest rates is viewed as a change in R, while a change in the term structure of deposit interest rates is represented by a change in r_1 and r_2, holding R constant. To make the discussions easier to follow, I produce two sub-sections, which focuses on each scenario.

5.1 Term Structure of Investment Interest Rates and the Likelihood of a Bank Run

When there is an increase in *R*, the bank could choose to alter its deposit interest rates in any way, as long as the rates satisfy the resource constraint and the incentive compatibility condition. Since there are two sets of solutions to *p*, I will investigate how an increase in *R* changes *p* using both sets of solutions.

Case 1: $r_1 < 1.5$, $R < \frac{1}{(3-2r_1)}$, and $p = \frac{(r_2 - r_1)}{(1 - r_1 + r_2 - 3R - 0.5r_1R)}$

It is possible to derive from the resource constraint that:

$$r_2 = R(1.5 - 0.5r_1) \quad (8)$$

Therefore, the solution of *p* when r_1 <1.5 can be rewritten by substituting equation (8) into equation (5). The new form of the solution is:

$$p = \frac{(1.5R - 0.5r_1R - r_1)}{(1 - r_1 - 1.5R - 1.5\, r_1R)} \quad (9)$$

A total differentiation of equation (9) yields:

$$dp = \frac{dr_1(3R^2 - 0.5R + 1) + dR(2r_1^2 - 0.5r_1 + 1.5)}{(1 - r_1 - 1.5R - 1.5\, r_1R)^2} \quad (10)$$

When both R and r_1 increase, the numerator of equation (10) is always greater than 0; this indicates that the probability of a bank run increases whenever R and r_1 increase. Furthermore, when R increases and r_1 stays constant, p still increases. However, when R increases and r_1 decreases, the effect on p is ambiguous, depending on the magnitude of the change.

Case 2**:** $r_1 \geq 1.5$ **,** and $R < \frac{1}{(3-2r_1)}$, then $p = \frac{(r_2 \text{ - } 1.5)}{(r_2 \text{ - } 0.5)}$ In this scenario, the probability p is solely determined by the value of r_2. Differentiating equation (6) in respect to r_2 yields:

$$\frac{dp}{dr_2} = \frac{1}{(r_2 - 0.5)^2} > 0 \tag{11}$$

An increase in r_2 always increases p, regardless of the value of R and r_1, and vice versa.

When a bank optimizes agent's ex-ante consumption in each period, as it does in Diamond and Dibvig's (1983) model, it is necessary for the bank to increase both r_1 and r_2. In both case 1 and case 2, an increase in r_1 and r_2 would increase p unambiguously. When a bank does not optimize agents' consumption, an increase in R does not necessarily increase p; in case 1, a bank could avoid the increase in p by lowering r_1, while it could achieve the same in case 2 by reducing r_2. The following two propositions summarize the results in this analysis.

Proposition 2: If a bank is optimizing agents' utility of consumption in each period, an increase in R increases the *ex-ante* likelihood of a bank run.

Proposition 3: If a bank is not optimizing consumers' utility, an increase in R does not necessarily increase the *ex-ante* probability of a bank run.

5.2 Term structure of deposit interest rates and the likelihood of a bank run

In this analysis, I change the term structure of deposit interest rates by changing the value of r_1 and r_2, holding R constant. Like section 5.1; the analysis is performed on two sets of solutions of p.

Case 1: $r1 < 1.5$, $R < \frac{1}{(3-2r_1)}$, and $p = \frac{(r_2 - r_1)}{(1 - r_1 + r_2 - 3R - 0.5r_1R)}$

As proved in section 5.1, the solution of p can be rewritten as equation (9); differentiating equation (9) in respect to r_1 would yield:

$$\frac{dp}{dr_1} = \frac{(2.5R + 2R^2 - 1)}{(1 - r_1 - 1.5R - 1.5r_1R)^2} \qquad (12)$$

Since equation (12) is always positive by assumption, an increase in r_1 accompanied by a decrease in r_2 always leads to an increase in p, and vice versa.

Case 2: $r_1 \geq 1.5$, and $R < \frac{1}{(3-2r_1)}$

As discussed before, p is determined only by r_2, and an increase in r_2, accompanied by a decrease in r_1, increases p. An increase in r_1 has different effect on p in case 1 and in case 2, and the proposition below summarizes this finding.

Proposition 4: Holding R constant, an increase in r_1 has ambiguous effect on the likelihood of a bank run in a 3-agent model; the effect depends on the parameter value of r_1.

6. Extensions

In this section, I extend the basic three-agent model in two ways. The first extended model is the three-agent model with the sequential service constraint, whereas the second extended model deals with N agents.

6.1 Three-agent model with sequential service constraint

In the above three-agent model, the bank does not follow the sequential service constraint as it does in Diamond and Dybvig's (1983) model. The sequential service constraint does not allow the bank to pay out demanded deposits to two agents at the same time. The assumption in the above three-agent model specifies that the bank must pay out an average of its resource to each agent when the resource is exhausted in the first period, and this assumption violates the sequential service constraint. With the sequential service constraint in place, the bank pays out demanded deposit in the first-come-first-serve basis, and the probability p of a type 2 agent withdrawing early is obtained by solving 2 equations, depending on the parameter value of r_1.

When $r_1 < 1.5$, p is the solution of:

$$(1-p)r_1 + (\tfrac{2}{3})p(r_1) + (\tfrac{1}{3})p(3-2r_1) = (1-p)r_2 + p(3-2r_1)R \qquad (13)$$[1]

When $r_1 \geq 1.5$, p is the solution of:

$$(\tfrac{1}{2})(1-p)(r_1) + (\tfrac{1}{2})(1-p)(3-r_1) + (\tfrac{1}{3})(p)(r_1) + (\tfrac{1}{3})(p)(3-r_1) = (1-p)(r_2) \qquad (14)$$[2]

Therefore, when $r_1 < 1.5$, p is:

$$p = \frac{(r_2 - r_1)}{(1 - r_1 + r_2 - 3R + 2r_1R)} \qquad (15)$$

When $r_1 \geq 1.5$, p is:

$$p = \frac{(r_2 - 1.5)}{(r_2 - 0.5)} \qquad (16)$$

The two sets of solutions, equation (15) and (16), are same as equation (5) and (6). This indicates that in the three-agent model, adding the sequential service constraint does not alter p, and Propositions 1, 2, and 3 < or is it Propositions 1-4 ??> still hold in the extended model.

[1] See Appendix I
[2] See Appendix I

Proposition 5: Proposition 1, 2, and 3 are independent of the sequential service constraint

6.2 *N*-agent model

When the bank has *N* depositors, the probability *p* of a type 2 agent withdrawing early is obtained by solving:

$$\sum_{m=0}^{n-1}\{(\frac{(n-1)!}{m!(n-m-1)!})p^{m}(1-p)^{(n-m-1)}\min(r_1,\frac{N}{N-n+m+1})\}$$
$$=(1-p)^{(n-1)}(r_2)+\sum_{m=1}^{n-1}\{(\frac{(n-1)!}{m!(n-m-1)!})p^{m}(1-p)^{(n-m-1)}\max[(\frac{n-mr_1}{n-m})R,0]\} \quad (17)^{3}$$

, where *N* is the total number of depositors, *n* is the number of type 2 agents, and *(N-n)* is the number of type 1 agents.

Since equation (17) does not have an analytical solution, I leave it open to simulations, and I encourage fellow economists to perform analysis on the *N*-agent model.

7. Conclusion

By constructing and analyzing the three-agent model, I conclude that when the term structure of investment interest rates is altered due to an increase in a bank's investment return, the probability of a bank run is increased in most cases. However, a

[3] See Appendix II

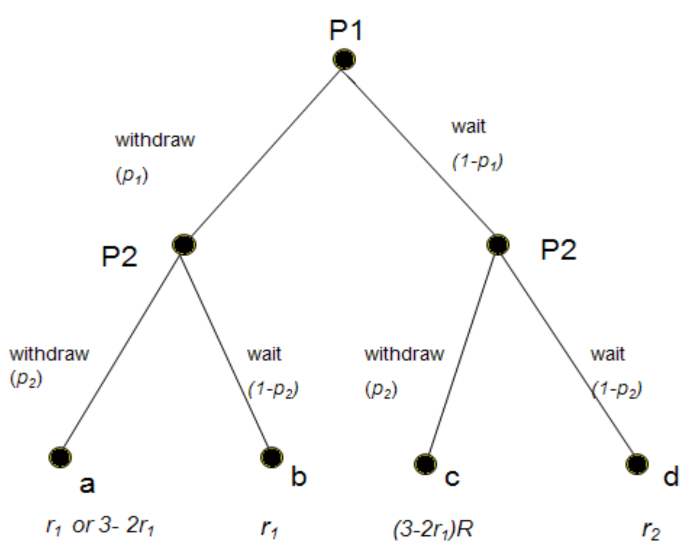
bank might be able to avoid the increased chance of a bank failure by adjusting its deposit interest rates. Furthermore, when a local bank increases its short-term deposit interest rates, it faces a risk of increased probability of a bank run.

Appendix I- Three-agent model with sequential service constraint

When r_1<1.5, the player P1 faces the game tree in Figure 3. At node *a*, P1, P2 and the other type 1 agent would withdraw; P1 obtains r_1 or ($3\text{-}2r_1$) depending on P1's place in line. P1 would get r_1 if he/she is one of the first two agents in line, and P1 gets ($3\text{-}2r_1$) when he/she is the last agent in line. At node *b*, P1 and the type 1 agent choose to withdraw, and P1 obtains r_1 for certain. P1 gets ($3\text{-}2r_1$)R at node *c*, after the type 1 agent and P2 withdraw; at node *d*, P1 receives the amount specified in the deposit contract. Since P2 also faces the same game tree, the probability *p* of a type 2 agent withdrawing is the solution of:

$$(1-p)r_1 + (\tfrac{2}{3})p(r_1) + (\tfrac{1}{3})p(3-2r_1) = (1-p)r_2 + p(3-2r_1)R$$

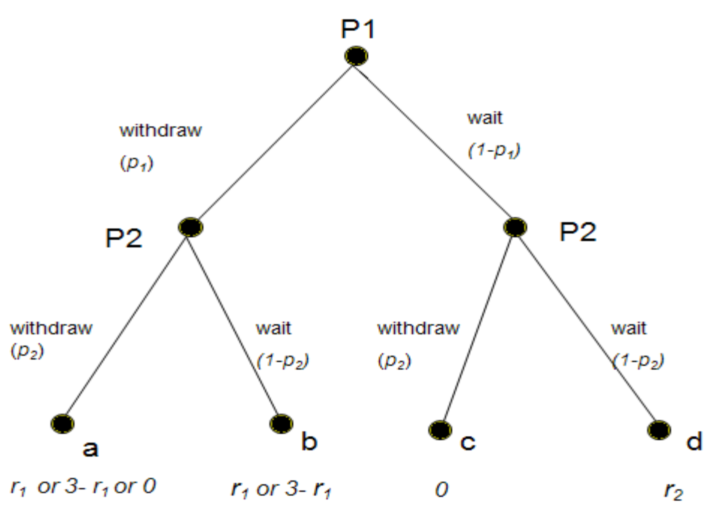
Figure 3



When $r_1 \geqq 1.5$, the player P1 faces the game tree in figure 4. At node *a*, both P1, P2, and the other type 1 agent decide to withdraw, and P1 can obtain r_1 if he/she is the first agent in line. If P1 is the second agent in line, he/she gets $(3\text{-}r_1)$; when P1 is the last agent in line, he/she would get no return. At node *b*, both P1 and the other type 1 agent withdraw, and P1 can get r_1 or $(3\text{-}r_1)$. At node *c*, the type 1 agent and P2 choose to withdraw while P1 waits, and P1 does not get any return because the bank would not have sufficient funds. At node *d*, P1 receives the amount specified in the deposit contract. Since P2 also faces the same game tree, the probability *p* of a type 2 agent withdrawing is the solution of:

$$(\tfrac{1}{2})(1-p)(r_1)+(\tfrac{1}{2})(1-p)(3-r_1)+(\tfrac{1}{3})(p)(r_1)+(\tfrac{1}{3})(p)(3-r_1)=(1-p)(r_2)$$

Figure 4



Appendix II: *N*-agent model

In the N-agent model with N depositors and n type 2 agent, any type 2 agent's expected payoff from withdrawing is:

$$\sum_{m=0}^{n-1}\{(\frac{(n-1)!}{m!(n-m-1)!})p^m(1-p)^{(n-m-1)}\min(r_1,\frac{N}{N-n+m+1})\} \tag{18}$$

In equation (18), m represents the number of type 2 agents who decide to withdraw early. When all type 2 agents wait, m is 0, and the payoff is the minimum of r_1 or $(\frac{N}{N-n+1})$ with probability $(1-p)^{(n-1)}$. Since the number of type 2 agents who choose to withdraw is between 0 to (n-1), any type 2 agent's expected payoff from withdrawing is the sum of the product of each combination and its probability of occurring. When N =3 and n=2, equation (18) converges to equation (1) for the player P1.

When a type 2 agent decides to wait in the N-agent model, his/her expected payoff is:

$$(1-p)^{(n-1)}(r_2)+\sum_{m=1}^{n-1}\{(\frac{(n-1)!}{m!(n-m-1)!})p^m(1-p)^{(n-m-1)}\max[(\frac{n-mr_1}{n-m})R,0]\} \tag{19}$$

While m in equation (19) again represents the number of type 2 agents who choose to withdraw, the first term in the equation is the expected payoff when all type 2 agents decide to wait. In that scenario, any type 2 agent would receive r_2, the amount specified in the deposit contract, with probability $(1-p)^{(n-1)}$. In all other scenarios, at least one type 2 agent decides to withdraw, and *m* is therefore any number between 1 and (*n-1*). The second term in equation (19) is again the sum of the product of each combination and its probability of occurring. In the three-agent model where *N*=3 and *n*=2, equation (19) simplifies to equation (2) for the player P1.

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