

Intergenerational Trade and Altruism in a Climate Change Model

by

Duncan Whyte

A Thesis Submitted in Partial Fulfillment of the Requirements for the
Degree of Bachelor of Sciences, Honours
in the Department of Economics
University of Victoria
April 2017

Supervised by Dr. Peter Kennedy

for

Dr. Chris Auld, Honours co-advisor

Dr. Merwan Engineer, Honours co-advisor

Abstract

This paper examines various outcomes of an overlapping generations model with intergenerational trade, an evolving climate, and intergenerational altruism. I show that optimal outcomes exist in both the non-cooperative framework and the social planner framework. I then compare the optimal choices between the non-cooperative equilibrium framework and the socially optimal social planner framework. There are negative externalities associated with the outcome from the non-cooperative equilibrium, and therefore even with altruistic preferences, agents do not achieve the socially optimal outcome.

Keywords: intergenerational trade, overlapping generations, intergenerational altruism, climate change

Special Thanks

I wholeheartedly thank my Honours supervisor Dr. Peter Kennedy for his immense patience and support. I owe many thanks to Dr. Merwan Engineer, Dr. Chris Auld, and to my friends, family, and loved ones. I could not have done it without you.

Table of Contents

1. Introduction.....	3
2. Review of Related Literature.....	6
3. The Model.....	9
4. Equilibrium with Trade but No Altruism.....	12
5. Equilibrium with Altruism but No Trade.....	15
6. Equilibrium with Trade and Altruism.....	18
7. Pareto Efficiency.....	27
8. Conclusion.....	30
References.....	33
Figures.....	35

1. INTRODUCTION

The environment is a growing concern in today's world, but with global population projected to reach 9.7 billion by the year 2050 (United Nations, 2015), better care must be taken with regards to the climate to ensure human existence. When considering an environmental issue, there are many factors that must be accounted for if good policy is to be designed. Climate change is fundamentally an intergenerational matter due to the inertial lag between emissions and the impact on the climate. The inertial lag is estimated to be between 25 and 50 years (Hansen et al., 2005), though not without uncertainty. Therefore, using a generation-length lag is suitable for modeling.

The presence of the lag means that policy must be forward looking and that our motivation for adopting policies, concerning reductions in emissions, are not for the prevention of impacts on the current generation, but for mitigating the impacts felt by the next generation. Consequently, there is always a trade-off between today's consumption and the effects on tomorrow's generation based on the effect on the environment.

Altruism is not often a parameter in economic models, but has long-since been a part of economics. Adam Smith (1790) wrote in *The Theory of Moral Sentiments*

How selfish soever man may be supposed, there are evidently some principles in his nature, which interest him in the fortune of others, and render their happiness necessary to him, though he derives nothing from it, except the pleasure of seeing it. (p. 11)

Agents may not need to see others' pleasure in order for themselves to receive pleasure; so long as they believe that others will be happy, that could be enough for them. In a climate change scenario, the benefits from a reduction in emissions are not felt until the next generation, and for many, this means not being able to witness that benefit. However, the agent may still derive

happiness from such a reduction in emissions solely from the knowledge or assumption that someday in the future, someone will benefit.

In this paper, I compare the non-cooperative equilibrium with the socially optimal outcome using dynamic programming. The process of solving will be adapted from Kennedy (1994) and Zietz (2007). This paper takes an existing model of non-cooperative behaviour with intergenerational altruism (Kennedy, 1994) and builds upon it in three ways. First, the model used in this paper recasts the resource stock in terms of climate services, where emissions damage the quality of climate services. Second, it introduces an overlapping generations structure, as opposed to the sequential generation structure used in the original model. Third, it introduces intergenerational trade into the model. The third extension of the original model is the most important and the most novel because it creates a link between current agents and future climate states, even in the absence of altruism.

The model used in this paper is an overlapping generations (OLG) model with intergenerational altruism and intergenerational trade. In my model, each agent in the model lives for two periods. In their first period, they are Young and they produce output. In their second period, the agent is Old and no longer produces output. Agents must consume to live, and therefore for the Young agent to live when they are Old they must not consume all they produce, but instead sell a portion of their production to the Old agents in the current time-period.

I assume that agents are identical and that their preferences can be simply stated as a function of what they consume today, what they consume in the next period, and the utility of the next generation. The productivity of any individual in a given time period is determined by two weighted factors: the climate services of that period, and the resource use of that period. Decisions regarding the resource use will result in a corresponding amount of emissions. That

level of emissions will affect the climate services for the next period. The fact that any agent's resource use decision affects the climate services for the next generation, and thusly, the productivity of the next generation, is the defining problem in this model.

For the non-cooperative equilibrium outcomes, agents optimally choose their consumption, but where they gain utility from higher levels of consumption, they also negatively affect their utility by adversely affecting the utility of the next generation. Every agent simultaneously chooses a resource-use rule that is a best-response to every other agent's' best-response resource-use rule. The equilibrium outcome is a Nash Equilibrium, where every agent chooses the identical resource-use rule in the current generation and over all generations. The solution to the social planner's problem is to find what the socially optimal resource-use rule for a representative agent is, and here, every agent is identical, so what is the optimal resource-use rule for every agent. This resource-use rule states that the agent should use an amount of resource in the current period equal to the current climate services multiplied by a set of parameters that stay constant over time. The outcome from the non-cooperative equilibrium is not pareto-efficient due to the presence of negative externalities. These externalities take three distinct forms. The first externality is a well-known result from Cournot-Nash problems, where the competitive equilibrium is not socially optimal. The second is a negative production externality that results in emissions that affect everyone in the following generation. The third externality is from individuals not fully accounting for the impacts on others of a future price change as result of consumption decisions today. Therefore, even with altruistic preferences, the competitive equilibrium does not achieve the socially optimal outcome. This means there is a role for policy to correct the market failure.

The rest of the paper is organized as follows. Section 2 reviews the body of relevant literature and situates my contribution relative to that literature. Section 3 describes the model characteristics. Section 4 analyzes the equilibrium outcome with intergenerational trade, but no altruism. Section 5 details the equilibrium outcome with altruism, but without trade. Section 6 characterizes the equilibrium outcome with altruism and trade. Section 7 analyzes the Pareto Optimal outcome with altruism and trade, and compares the results to the equilibrium outcome. Section 8 provides some concluding remarks. The Appendix contains technical details of the model.

2. RELATED LITERATURE

My model has four key characteristics that underpin all of the analysis: overlapping generations (OLG), intergenerational trade, intergenerational altruism, and an evolving climate system that acts with a lag. The majority of the existing literature on climate change does not include the time dimension. In dynamic models, there is typically either an infinitely-lived agent framework, or seldom, intergenerational altruism among finitely-lived agents. The differences between the results with a finitely-lived agent OLG model and an infinitely-lived agent (ILA) model are minor. Barro (1974) shows that with the appropriate assumptions on altruism and transfer methods, finitely-lived agents in the OLG model act as though they are infinitely-lived, so long as there exists an operative chain of intergenerational transfers. Furthermore, Stephan, Müller-Fürstenberger, and Previdoli (1997), compare the ILA model with the intergenerational altruism among a finitely-lived agent OLG model and find insignificant differences of results between the two methods. However, Schneider, Traeger, and Winkler (2012) show the OLG

approach and ILA approach generate equivalent observational results, but the OLG method allows for disentanglement of intertemporal aspects from intergenerational considerations. Thereby, increasing the degree of transparency with regards to policy implications. Schneider et al. (2012), also note that the altruistic transfer motive required for Barro's (1974) result is weak and that the consumption paths of agents are driven by their lifecycle planning. In my model, I use a finitely-lived agent model with overlapping generations, instead of an ILA model. I do so because agents that have infinitely long lives does not represent reality, and because it explains some of the issues faced within a generation.

Preferences that change over time in OLG models have been shown to have merit (Wendner, 2002; de La Croix, 1996). However, in my model, I assume that agents have log-linear preferences that are identical and stable within and across generations. In my model, the altruism parameter exists as a weight parameter on the utility of future generations. Ramsey (1928) claims that discounting future utility is unethical and that one's position in time should not result in a greater weight than future generations. This is a normative statement. I do not wish to analyse the ethical position of that argument, but instead propose a practical response. With utility measured equally across generations, policy proposals would result in consumption coming to a standstill. No one would want to affect the next generation in any negative way because the current generation placed a weight on the utility of the future generation equal to their own. The decision for what course of action to take with regards to a climate change problem in a democratic country is a democratic vote. Those eligible to vote are those who are currently living, not those who someday in the future will be facing the consequences. In reality, there is clearly some positive weight assigned to the value of future generations, otherwise there would be no incentive to create policies that reduce or change consumption decisions today that

aim to limit the effects of climate change. Utility of future generations matter, but not with a one-to-one ratio to utility of today's generation.

Intergenerational trade is not a new concept in economics and has been discussed for many years. In my model, the Young generation does not have money and must trade with the Old generation in order to receive money, and thus be able to trade and consume in the next generation. All Young agents accept the Old agents' money, which is consistent with Samuelson's (1958) concept as money as a social contrivance.

Solow (1974, 1986) both address intergenerational issues; however, they do so in a finite resource stock context and not in a climate change framework, which is what is addressed in this paper. John and Pecchenino (1994) use an OLG model where they have consumption negatively affecting the utility of the future generations through the effect on the climate, which acts with a lag of one generation; however, they do not include intergenerational altruism. Their result is that the climate acts as a long-lived public good, and they find that altruistic contributions to the public good is achieved by intergenerational transfers. Jouvét, Michel, and Vidal (2000) use an OLG model to characterize equilibria with intergenerational altruism and a pollution parameter. They find that the private competitive outcome, even with altruistic individuals, does not reach the socially optimal outcome, due to agents' incomplete internalization of all externalities. This is consistent with my results. Due to the presence of the previously mentioned externalities in my model, there remains a role for policy to aid in reaching the socially optimal outcome.

My model is an adaptation from the intergenerational altruism model from Kennedy (1994), where I change the situation from a resource extraction context, to an evolving climate services context. My model also adds OLG framework instead of the sequential generation model. Finally, my model incorporates intergenerational trade, which creates a connection

between agents of the current generation and agents of future generations, even without altruistic preferences.

3. THE MODEL

Modelling is a challenge of balancing tractability and parsimony. In my model, I attempt to create a realistic representation with some simplifying assumptions that allow for its solvability. In my modelled economy, there is an infinite sequence of overlapping generation, each comprising of n agents. Agents in my model are identical, and live for two periods. I assume no population growth; an agent born in period t is Young in period t and is Old in period $t+1$. All Young agents in period t have the utility function $u(c_t, c_{t+1}, u_{t+1})$ defined over their own consumption when Young c_t , their consumption when Old c_{t+1} , and the utility of the future generation u_{t+1} . This representation of intergenerational altruism, takes the form of non-paternalistic altruism. Non-paternalistic altruism is utility derived from the utility of future generations, which differs from paternalistic altruism where agents derive utility from the consumption of future generations. Here I do not assume that the utility of agent k derived from the utility of future generations are solely from the utility of agent k 's direct heir. I assume a specific functional form for the representative agent, agent k in period t , which is amenable to the closed-form solution:

$$(1) \quad u_{k,t} = \log(c_{k,t}) + \beta \log(c_{k,t+1}) + \alpha u_{t+1}$$

Where $\beta \in [0,1)$ reflects agent k 's intertemporal discount factor on future consumption or their private rate of time preference. Each generation correctly assumes that the preferences of future generations will be identical to their own.

The production by agent k in period t takes the following form:

$$(2) \quad y_{k,t} = x_{k,t}^{\phi} S_t^{\theta}$$

Where $x_{k,t}$ is the resource use of agent k in period t and S_t is the climate services in period t , which is an endogenous variable. There is implicitly a single unit of labour that agents supply inelastically. I assume that $\theta > 0$ and $\phi > 0$ for solvability, which are the relative productivity of climate services and the relative productivity of an agent's resource use, respectively. I believe that the relative productivity of climate services is greater than the relative productivity of resource use (i.e. $\theta > \phi$). Agents are not productive in period $t+1$, and therefore do not produce when they are Old. In addition, the good produced cannot be stored.

Agent k in period t creates emissions as a result from their production in period t , which takes the following form:

$$(3) \quad e_{k,t} = \gamma x_{k,t}$$

Where the units of resource use only partially result in emissions by the scaling and conversion parameter, $\gamma > 0$.

Aggregate emissions in period t take the following form:

$$(4) \quad E_t = \gamma X_t$$

Where

$$(5) \quad X_t = \sum_{i=1}^n x_{i,t}$$

The evolution of climate services takes the following form:

$$(6) \quad S_{t+1} = (S_t - E_t^{\delta})(1 + \omega)$$

Where $\omega > 0$ is the rate of the climate's ability to regenerate. The scaling parameter on aggregate emissions (δ) is the relative effect emissions have on the climate services.

With the environment of the model established, I will now describe the solution method for the non-cooperative equilibria. The following three sections, although different specifications of the original model, follow the same solution path. The first step is solving for the optimal intertemporal consumption decision of a representative agent. The optimal consumption decisions of the agent are integrated into a value function of consumption.

The second step is solving for the production choice under rational expectations. The representative agent correctly anticipates that the utility of future generations (u_{t+i}) is determined by the equilibrium production in period $t+i$. All of which depends on the climate services in that period. However, the representative agent can only influence the climate services for the immediately following period, but the evolution of the climate services depends on the stream of equilibrium outcomes, of which the agent has no direct control.

The third step is to posit a time-invariant resource-use rule for the representative agent in period t :

$$(7) \quad x_{k,t} = rS_t^{1/\delta}$$

This posited rule is the best-response function for a representative agent. It is worth noting that there may exist other resource-use rules that result in equilibria. Furthermore, the equilibrium outcomes from my model are not necessarily unique.

The fourth and final step is solving the symmetric equilibrium. Imposing symmetry on the resource-use rule requires that the result from the previous step is the same across all other agents. The solution is a value for r that does not depend on the stock of climate services.

4. EQUILIBRIUM WITH TRADE BUT NO ALTRUISM

Modelling the economy with trade but without altruism is a special case condition of the first model, where the weight parameter on the utility of future generations for agent k is set to zero. Agents produce when they are Young, but when Old, they no longer produce any output; furthermore, the goods produced cannot be stored. A representative agent in period t saves an amount q_t when Young and sells that amount to the Old in the current generation at price p_t .

Thus, the agent receives money income:

$$(8) \quad m_{k,t} = p_t q_{k,t}$$

Where m_t is the money that the agent receives and it is storable. Money in this model functions similar to units of money whose nominal value may change, but the quantity in aggregate remains fixed. Money is universally accepted at market prices. The amount of money in equilibrium is supplied by an institution to initial (time period zero) generation of amount equal to M_0 . The agent's consumption when Young takes the following form:

$$(9) \quad c_{k,t} = y_{k,t} - q_{k,t}$$

When the agent is Old, they use their money to purchase the consumption good from the Young agents in the next period. The price of the consumption good in period $t+1$ is p_{t+1} . Thus, their consumption when old takes the following form:

$$(10) \quad c_{k,t+1} = \frac{m_{k,t}}{p_{t+1}} = \frac{p_t}{p_{t+1}} q_{k,t}$$

Where the agent's future consumption depends on the relative prices of today and prices in the next period.

Solving the problem begins with the consumption choice problem, which is as follows:

$$(11) \quad \max_{q_{k,t}} \log(y_{k,t} - q_{k,t}) + \beta \log\left(\frac{p_t}{p_{t+1}} q_{k,t}\right)$$

This has the solution:

$$(12) \quad q_{k,t} = \frac{\beta y_{k,t}}{1+\beta}$$

This implies consumption levels:

$$(13) \quad c_{k,t} = \frac{y_{k,t}}{1+\beta}$$

$$(14) \quad c_{k,t+1} = \frac{p_t \beta y_{k,t}}{p_{t+1}(1+\beta)}$$

The indirect utility function for this agent is as follows:

$$(15) \quad v_{k,t} = \log\left(\frac{y_{k,t}}{1+\beta}\right) + \beta \log\left(\frac{p_t \beta y_{k,t}}{p_{t+1}(1+\beta)}\right)$$

After solving the consumer's intertemporal consumption choice problem, the next step is solving the production choice under rational expectations. The agent recognizes that the price in the next period (p_{t+1}) will be determined by the equilibrium production in period $t+1$, and that this is in turn determined by the equilibrium climate services in that period. The agent also recognizes that their production decision affects the climate services in the next period S_{t+1} , particularly:

$$(16) \quad S_{t+1} = (S_t - (\gamma x_{k,t} + \gamma X_{-k,t})^\delta)(1 + \omega)$$

Where $x_{k,t}$ is the resource use of agent k and $X_{-k,t}$ is the resource use of all other agents (not k). In a symmetric equilibrium, per capita resource use in period t by all agents other than agent k is as follows:

$$(17) \quad x_{j,t} = \frac{X_{-k,t}}{n-1} \quad \text{for } j \neq k$$

Thus, per capita income in period t for all agents other than agent k is as follows:

$$(18) \quad y_{j,t} = \left(\frac{X_{-k,t}}{n-1}\right)^\delta S_t^\theta \quad \text{for } j \neq k$$

The per capita consumption of these agents in period $t+1$ is as follows:

$$(19) \quad c_{j,t+1} = \frac{p_t \beta y_{j,t}}{p_{t+1}(1+\beta)} \quad \text{for } j \neq k$$

Aggregate consumption by Old agents in period $t+1$ is given by the following:

$$(20) \quad C_{O,t+1} = c_{k,t+1} + \sum_{j \neq k}^{n-1} c_{j,t+1}$$

In equilibrium, aggregate consumption by the Old in period $t+1$ must be equal to the aggregate savings of the Young in period $t+1$. Those aggregate savings are as follows:

$$(21) \quad Q_{t+1} = \frac{n(\beta y_{t+1})}{1+\beta}$$

Where y_{t+1} is per capita income for the Young in period $t+1$, which in turn is equal to:

$$(22) \quad y_{t+1} = x_{t+1}^\phi S_{t+1}^\theta$$

Where x_{t+1} is per capita resource use by the Young in period $t+1$, and S_{t+1} is given by equation

(16). Equilibrium prices must satisfy the following:

$$(23) \quad C_{O,t+1} = Q_{t+1}$$

Solving for p_{t+1} yields the anticipated equilibrium price in period $t+1$:

$$(24) \quad p_{t+1} = \frac{p_t \left(x_{k,t}^\phi + (n-1) \left(\frac{x_{-k,t}}{n-1} \right)^\phi \right) S_t^\theta}{n x_{t+1}^\phi \left((S_t - (\gamma x_{k,t} + \gamma x_{-k,t}))^\delta (1+\omega) \right)^\theta}$$

The choice problem for agent k is to maximize the value function from equation (15) subject to equation (24). The consumption decisions of an agent depend partly on future prices, and this creates a connection between the agents of today and the agents in the future. Therefore, there is intergenerational connection even in the absence of altruistic preferences.

The next step is positing an equilibrium resource-use rule:

$$(25) \quad x_{k,t} = r_{k,t} S_t^{1/\delta}$$

Then impose an analogous rule for all Young agents in period $t+1$:

$$(26) \quad x_{t+1} = r_{t+1} S_{t+1}^{1/\delta}$$

Now there are forms for x_{t+1} and x_t which are substituted into the choice problem, and then the choice problem is maximized with respect to $r_{k,t}$. The result is a best-response function for agent

k ; it identifies the privately optimal choice of $x_{k,t}$ in response to the anticipated equilibrium values of x_{t+1} and $X_{-k,t}$. There is no closed-form solution to this best-response function.

The final step is solving the symmetric equilibrium. In the symmetric equilibrium:

$$(27) \quad X_{-k,t} = (n-1)x_{k,t}$$

And

$$(28) \quad r_{t+1} = r_t$$

By imposing these conditions, the first-order conditions now have closed-form solutions:

$$(29) \quad \hat{x}_t = \hat{r} S_t^{1/\delta}$$

Where

$$(30) \quad \hat{r} = \frac{\phi(n+\beta(n-1))\left(\frac{1}{n\gamma}\right)^{\frac{1}{\delta}}}{n\phi(1+\beta)+\beta\theta\delta}$$

Crucially, this takes the form of the posited resource-use rule. Thus, this rule is an equilibrium rule.

5. EQUILIBRIUM WITH ALTRUISM BUT NO TRADE

The result from the previous model where there was intergenerational trade, but no intergeneration altruism, showed that there was a connection across generations through the affect on prices. Now I am considering the special case of the original model where there is some positive weight on the altruism parameter. I also assume storability of the good produced and remove intergenerational trade from the model. There remains a connection across generations, but now the mechanism is the altruistic preferences, not the prices. I analyze the model in a like fashion as originally presented in Section 3, and I examine for potential equilibria using the posited resource-use rule from (7).

The model setup is consistent with functions (1) through (6). Solving the intertemporal consumption choice with storable goods means that the agent's consumption choices are independent of their production choice. The agent's consumption choice problem is as follows:

$$(31) \quad \max_{c_{k,t}} \log(c_{k,t}) + \beta \log(c_{k,t+1})$$

Subject to

$$(32) \quad c_{k,t+1} = y_{k,t} - c_{k,t}$$

In this special case, the optimal consumption choice for agent k in period t is as follows:

$$(33) \quad c_{k,t} = \frac{y_{k,t}}{1+\beta}$$

And for consumption in period $t+1$:

$$(34) \quad c_{k,t+1} = \frac{\beta y_{k,t}}{1+\beta}$$

The resulting value function of consumption is as follows:

$$(35) \quad v_{k,t} = (1 + \beta) \log\left(\frac{y_{k,t}}{1+\beta}\right) + \beta \log(\beta)$$

The next step is solving for the production choice under rational expectations. The representative agent recognizes that u_{t+i} will be determined by the equilibrium production in period $t+i$, and that this in turn will depend on the equilibrium climate services in that period. The agent recognizes that their production choice in period t affects S_{t+1} , but the agent has no direct effect on the evolution of climate services beyond that point. The agent correctly anticipates that

$$(36) \quad \hat{u}_{t+1} = (1 + \beta) \log\left(\frac{\hat{y}_{t+1}}{1+\beta}\right) + \beta \log(\beta) + \alpha \hat{u}_{t+2}$$

Where $\hat{y}_{k,t+1} = \hat{y}_{t+1}$ by the assumption of identical agents. By using recursive substitution, equation (35) can be equivalently written as the following:

$$(37) \quad \hat{u}_{t+1} = \sum_{i=1}^{\infty} \alpha^{i-1} ((1 + \beta) \log \left(\frac{\hat{y}_{t+i}}{1+\beta} \right) + \beta \log(\beta))$$

Agent k 's production choice problem is as follows:

$$(38) \quad \max_{x_{k,t}} (1 + \beta) \log \left(\frac{x_{k,t}^{\phi} S_t^{\theta}}{1+\beta} \right) + \beta \log(\beta) + \alpha \hat{u}_{t+1}$$

Subject to

$$(39) \quad \sum_{i=1}^{\infty} \alpha^{i-1} \left((1 + \beta) \log \left(\frac{\hat{x}_{t+i}^{\phi} \hat{S}_{t+i}^{\theta}}{1+\beta} \right) + \beta \log(\beta) \right)$$

$$(40) \quad S_{t+1} = (S_t - (\gamma x_{k,t} + \gamma X_{-k,t}))^{\delta} (1 + \omega)$$

$$(41) \quad S_{t+i} = (S_{t+i-1} - (n\gamma x_{t+i-1}))^{\delta} (1 + \omega) \quad \forall i \geq 2$$

The difference between (40) and (41) was discussed at the beginning of this section, namely that the agent has a direct effect on the climate services for the immediately following period, but the agent only has an indirect effect on evolution of climate services in subsequent periods.

Using the original resource-use rule from equation (7), but cast forward in time and as an equilibrium condition (no longer agent k 's rule), the equation is as follows:

$$(42) \quad \hat{x}_{t+i} = r S_{t+i}^{1/\delta}$$

Equation (42) is substituted into equation (39) and the result is substituted into the agent's choice problem from equation (38), which yields

$$(43) \quad \max_{x_{k,t}} (1 + \beta) \log \left(\frac{x_{k,t}^{\phi} S_t^{\theta}}{1+\beta} \right) + \beta \log(\beta) + \sum_{i=1}^{\infty} \alpha^{i-1} \left((1 + \beta) \log \left(\frac{\hat{x}_{t+i}^{\phi} \hat{S}_{t+i}^{\theta}}{1+\beta} \right) + \beta \log(\beta) \right)$$

Subject to

$$(44) \quad S_{t+1} = (S_t - (\gamma x_{k,t} + \gamma X_{-k,t}))^{\delta} (1 + \omega)$$

$$(45) \quad S_{t+i} = S_{t+i-1} (1 - (n\gamma r)^{\delta}) (1 + \omega) \quad \forall i \geq 2$$

Equation (45) can be rewritten as

$$(46) \quad S_{t+i} = S_{t+1} \left((1 - (n\gamma r)^\delta)(1 + \omega) \right)^{i-1}$$

Making this substitution into equation (42) and collecting terms yields a relatively simple problem:

$$(47) \quad \max_{x_{k,t}} (1 + \beta) \phi \log(x_{k,t}) + \theta \log(S_t) + \left(\frac{\alpha(1+\beta)}{1-\alpha} \right) \left(\theta + \frac{\phi}{\delta} \right) \log(S_{t+1}) + A$$

Where A is a collection of terms that are time-invariant.

The first-order condition for this problem is a best-response function for agent k ; it identifies the privately optimal choice of $x_{k,t}$ in response to the anticipated equilibrium value of $X_{-k,t}$. There is no closed-form solution for this best-response function. However, in the symmetric equilibrium

$$(48) \quad X_{-k,t} = (n - 1)x_{k,t}$$

By imposing that condition on the first-order condition, there is a solvable close-form solution:

$$(49) \quad \hat{x}_t = \hat{r} S_t^{1/\delta}$$

Where

$$(50) \quad \hat{r} = \frac{1}{n\gamma} \left(\frac{(1-\alpha)\phi n}{\alpha\theta\delta + \phi(\alpha + n(1-\alpha))} \right)^{\frac{1}{\delta}}$$

This takes the form of the original posited resource-use rule from equation (7). Thus, this rule is an equilibrium rule.

6. EQUILIBRIUM WITH ALTRUISM AND INTERGENERATIONAL TRADE

Equilibrium Control Rule

This section covers both individual aspects of the previous two models. Now the model has intergenerational trade and intergenerational altruism. This section illustrates how the first

two models analyzed are merely special cases of this more complete model. The underlying set up of equations such as the production function and preferences, i.e. equations (1) through (6) are maintained. In addition, the to-be posited equilibrium control rule is as represented in equation (7). The first step in solving the model begins with the intertemporal consumption choices. The representative agent k saves an amount $q_{k,t}$ when Young in period t . The agent will then sell this amount to the Old in period t at the current period prices p_t . The proceeding steps are identical to the steps in Section 4. Using equations (8) through (14), the resulting value function for consumption is identical to equation (15). Equation (15) can be simply rewritten as

$$(51) \quad v_{c_{k,t}} = (1 + \beta) \log \left(\frac{y_{k,t}}{1 + \beta} \right) - \beta \log \left(\frac{p_{t+1}}{\beta p_t} \right)$$

The indirect utility function takes the form

$$(52) \quad v_{k,t} = (1 + \beta) \log \left(\frac{y_{k,t}}{1 + \beta} \right) - \beta \log \left(\frac{p_{t+1}}{\beta p_t} \right) + \alpha u_{t+1}$$

Solving for the rational expectations over future outcomes involves the approach used in Section 5; however, the addition of intergenerational trade into the model results in changes of observed outcomes. The agent correctly anticipates that the incomes of future agents—and the utility of those agents—are determined by future resource-use choices, future prices, and future climate services. In particular, the agent correctly anticipates that

$$(53) \quad u_{t+1} = (1 + \beta) \log \left(\frac{y_{t+1}}{1 + \beta} \right) - \beta \log \left(\frac{p_{t+2}}{\beta p_{t+1}} \right) + \alpha u_{t+2}$$

Where $y_{t+1} = x_{t+1}^\phi S_{t+1}^\theta$ from equation (2).

By recursive substitution I get

$$(54) \quad u_{t+1} = \sum_{i=1}^{\infty} \alpha^{i-1} \left((1 + \beta) \log \left(\frac{x_{t+i}^\phi S_{t+i}^\theta}{1 + \beta} \right) - \beta \log \left(\frac{p_{t+1+i}}{\beta p_{t+i}} \right) \right)$$

The agent also correctly anticipates the evolution of climate services as a function of the agent's own resource use and of all other agents' resource use (current and future). The anticipated evolution is consistent with the previously anticipated equations (40) and (41) from Section 5.

$$(40) \quad S_{t+1} = (S_t - (\gamma x_{k,t} + \gamma X_{-k,t})^\delta)(1 + \omega)$$

$$(41) \quad S_{t+i} = (S_{t+i-1} - (n\gamma x_{t+i-1})^\delta)(1 + \omega) \quad \forall i \geq 2$$

Similar to the approach in Section 4, the agent correctly anticipates the future prices determined in equilibrium by future production and consumption choices. All equations (17) through (23) are the same as in this section. The equilibrium-satisfying anticipated equation for prices in period $t+1$ can be rewritten as

$$(55) \quad \frac{p_{t+1}}{p_t} = \frac{\left(x_{k,t}^\phi + (n-1)\left(\frac{X_{-k,t}}{n-1}\right)^\phi\right) S_t^\theta}{n x_{t+1}^\phi S_{t+1}^\theta}$$

Where the equilibrium prices are shown as a price ratio.

Equilibrium prices in periods after $t+1$ are determined by the equilibrium choices of future agents. In particular, agents who are Young in period $t+1+i$ produce

$$(56) \quad y_{t+1+i} = x_{t+1+i}^\phi S_{t+1+i}^\theta$$

Using the savings rule from equation (12), I can calculate aggregate savings by these agents:

$$(57) \quad Q_{t+1+i} = \frac{n(\beta y_{t+1+i})}{1+\beta}$$

Agents who are Old in period $t+1+i$ produced

$$(58) \quad y_{t+i} = x_{t+i}^\phi S_{t+i}^\theta$$

when they were Young in period $t+i$. Using the consumption rule from equation (13), I can calculate aggregate consumption of Old agents in period $t+1+i$:

$$(59) \quad C_{O,t+1+i} = n \left(\frac{\beta p_{t+i} y_{t+i}}{(1+\beta) p_{t+1+i}} \right)$$

In equilibrium,

$$(60) \quad C_{O,t+1+i} = Q_{t+1+i}$$

Solving for p_{t+1+i} and dividing by p_{t+i} yields the anticipated price ratio in period $t+1+i$:

$$(61) \quad \frac{p_{t+1+i}}{p_{t+i}} = \frac{x_{t+i}^\phi S_{t+i}^\theta}{x_{t+1+i}^\phi S_{t+1+i}^\theta} \quad \forall i \geq 1$$

The next step is for agent k to posit a resource-use rule for all other agents. I restrict attention to a time-invariant rule of the form

$$(62) \quad x_{j,t+i} = r S_{t+i}^{1/\delta} \quad \forall i$$

I then show that a rule of the same form is a best-response for agent k , and therefore a rule of this form is an equilibrium rule. However, this is only one equilibrium rule of many that are possible; furthermore, the equilibrium I identify may not be unique itself. If all agents other than agent k in period t use the candidate rule, then

$$(63) \quad X_{-k,t} = (n-1)r S_t^{\frac{1}{\delta}}$$

and the price ratio from equation (55) reduces to

$$(64) \quad \frac{p_{t+1}}{p_t} = \frac{x_{k,t}^\phi S_t^\theta + (n-1)r^\phi S_t^\theta}{nr^\phi S_{t+1}^{\frac{\phi}{\delta} + \theta}}$$

Similarly, the price ratio from equation (61) reduces to

$$(65) \quad \frac{p_{t+1+i}}{p_{t+i}} = \left(\frac{S_{t+i}^\theta}{S_{t+1+i}^\theta} \right)^{\frac{\phi}{\delta} + \theta} \quad \forall i \geq 1$$

Moreover, the evolution of climate services as a function of future agents' resource use from equation (41) reduces to

$$(66) \quad S_{t+i} = \rho S_{t+i-1} \quad \forall i \geq 2$$

Where

$$(67) \quad \rho = (1 - (nyr)^\delta)(1 + \omega)$$

And by using recursive substitution, equation (66) can be written as

$$(68) \quad S_{t+i} = \rho^{i-1} S_{t+1} \quad \forall i \geq 2$$

Making these substitutions into the anticipated path of future agents' utility from equation (54) yields

$$(69) \quad u_{t+1} = \sum_{i=1}^{\infty} \alpha^{i-1} \left((1 + \beta) \log \left(\frac{r^{\phi} (\rho^{i-1} S_{t+1})^{\frac{\phi}{\delta} + \theta}}{1 + \beta} \right) - \beta \log \left(\frac{\rho^{-\left(\frac{\phi}{\delta} + \theta\right)}}{\beta} \right) \right)$$

This reduces to a very simple form that is linear in $\log(S_{t+1})$. In particular, it is straightforward to show that

$$(70) \quad u_{t+1} = \frac{(1+\beta)(\phi+\delta\theta)}{(1-\alpha)\delta} \log(S_{t+1}) + \Omega$$

Where Ω is a collection of terms that are independent of S_{t+1} and $x_{k,t}$. The resource-use problem for agent k now reduces to a very simple two-period problem:

$$(71) \quad \max_{x_{k,t}} (1 + \beta) \log \left(\frac{x_{k,t}^{\phi} S_t^{\theta}}{1 + \beta} \right) - \beta \log \left(\frac{x_{k,t}^{\phi} S_t^{\theta} + (n-1)r^{\phi} S_t^{\theta}}{\beta n r^{\phi} S_{t+1}^{\frac{\phi}{\delta} + \theta}} \right) + \alpha \left(\frac{(1+\beta)(\phi+\delta\theta)}{(1-\alpha)\delta} \log(S_{t+1}) + \Omega \right)$$

Subject to

$$(72) \quad S_{t+1} = \left(S_t - \left(\gamma x_{k,t} + \gamma(n-1)r S_t^{\frac{1}{\delta}} \right)^{\delta} \right) (1 + \omega)$$

The first-order condition for this problem is a best-response function for agent k ; it identifies the privately optimal choice of $x_{k,t}$ in response to the anticipated equilibrium value of r . There is no closed-form solution for this best-response function.

However, imposing symmetry on the resource-use rule requires the solution to the first-order condition from equation (71) to take the form:

$$(73) \quad x_{k,t} = r S_t^{1/\delta}$$

Imposing this requirement on $x_{k,t}$ in the first-order condition allows for its solving for a closed-form:

$$(74) \quad \hat{x}_t = \hat{r} S_t^{1/\delta}$$

Where

$$(75) \quad \hat{r} = \frac{1}{n\gamma} \left(\frac{(1-\alpha)\phi(n+\beta(n-1))}{\alpha(\theta\delta - \phi(n+1)(1+\beta)) + n\phi(1+\beta) + \beta\theta\delta} \right)^{\frac{1}{\delta}}$$

Critically, \hat{r} is independent of S_t and therefore equation (73) takes the form of the posited resource-use rule. Thus, the posited rule is a symmetric equilibrium rule.

Comparative Statics

The previous two sections were special cases of this more complete model, where the equilibrium control rule from Section 4 had the discount factor on future consumption (β) equal to zero, and Section 5 had the altruism parameter (α) equal to zero. Upon finding a rational expectations equilibrium for the complete model, I look deeper into the dynamics of this more interesting problem. The variable of interest is the climate services, and I show how changes in parameters affect its evolution path. First, I mention some key considerations about the climate services variable. Careful readers may notice that without any resource use in the model, climate services will increase without bound to positive infinity. With the relative environmental impact parameter $\gamma > 0$, the only way that resource use in the model can be zero is when no agents produce any output and they subsequently die, ending the existence of humans. However, climate services only have an interpretation with respect to humans in the model; it is a measure of how the environmental conditions affect the productivity of agents, nothing else. I assume that there exists an upper bound on climate services that is sufficiently large so that it is never reached when there are agents in the model. In addition, there is a lower bound on climate

services equal to zero. Crucially, if the stock of climate services ever reaches zero, for whatever reason, the variable will never regenerate. Thus, climate services equal to zero is a steady state condition of the variable.

The most critical of parameter changes is with respect to the altruism parameter, α . For a set of given values for the evolution equation of climate services, there are three critical values for α that determine the evolutionary path

$$(76) \quad \text{Threshold } \alpha = -\frac{\beta\delta\omega\theta - \beta n\phi + \beta\omega\phi - n\phi}{\beta n\phi + \delta\omega\theta - \beta\theta + n\phi + \omega\phi}$$

There are several negative externalities present in my model that will cause the non-cooperative equilibrium to be lower (in most cases) than the social planner's outcome. However, there are conditions in which non-cooperative equilibrium yields a more efficient outcome than the social planner's outcome, but I will explain this further in a later section.

When there is only one agent ($N=1$) in the model, one of the negative externalities is no longer present. Namely, the negative production externality that exists among agents within a generation. If there is only one agent in the model, then there is no one that this particular externality affects. Thus, the utility of all agents must be higher, with all else held constant. Continuing with the case of only one agent in the model, I calculate the threshold value for α to be equal to 0.47. This threshold value of α ensures that the climate services variable remains in a steady state permanently. I then show how slight deviations in α from its steady-state value cause the evolution path of climate services to differ (see Figure 1). What we can see from Figure 1 is that over the time span of 50 generations, the differing values of α only cause slight deviations from the steady state.

Next, I see how the threshold value of α changes when I add a second agent to the model. As shown in Figure 2, when there are two agents in the model, the new threshold value for α is

equal to 0.78. When the original threshold value for α is used in the model with two agents, the stock of climate services rapidly approaches zero (within 10 generations). The effect of adding the second agent to the model has reintroduced the negative externality that affects agents within a generation.

Looking further at this more interesting case in which the externality is present, Figure 3 plots the new threshold value for α , as well as small deviations from that threshold value. The effect from the slight deviations from the steady state affect the path of climate services in a relatively minimal way. A value for α equal to 0.78 will result in a stable state for climate services, which stays at its initial value before any human effects ($S_0 = 1000$). A value for α greater than 0.78, say 0.80, will result in the climate stock increasing forever. A value for α lower than 0.78, say 0.75, will result in the climate stock decreasing steadily, and the stock of climate services reaching approximately zero in 50 generations or 1000-2000 years. However, if those deviations from the threshold value are increased further, then the dynamics change drastically (see Figure 4). With a value for α equal to 0.65, the climate services stock falls to approximately zero within 20 generations or 400-800 years.

As can be seen, if the level of α is even a few more percentage points off from the steady state rate, then climate services either diverges from the steady state or converges to zero at a much greater rate. Interestingly, the evolution path of the climate stock does not depend on the value of the relative rate of emissions from resource use (γ).

Clearly, the rate of which we are altruistic towards future generations matters, and it matters a great deal. These results indicate the fragility of the environmental system and that if the rate at which we care about future generations is not at a certain threshold, and then the stock of climate services may fall to zero and be permanently irrecoverable. As well as speaking of the

terror of collapsing the stock of climate services, caring more about the wellbeing of future generations can have serious positive implications. These results are generated with equilibrium values and it is a troubling thought that even with altruistic agents, the evolutionary path of climate services converges to zero and the stock collapses, thereby resulting in the extinction of the human population.

I then plot the required threshold rate of altruism against key parameters to understand their interactions better. Figure 5 shows how the threshold value of α decreases as the relative damage of emissions (δ) increases. The reason for this result is due to $0 < E_t < 1$ from equation (6)

$$(6) \quad S_{t+1} = (S_t - E_t^\delta)(1 + \omega)$$

Figure 6 plots the threshold value of α against the discount parameter on future consumption.

The discount factor $0 < \beta < 1$, does not greatly affect the threshold value for α .

Finally, Figure 7 plots the threshold value of α against the number of agents. As more agents are incorporated into the model, the previously mentioned externality present among agents within a generation gets continually more severe. This is clear from how the threshold value of α asymptotically approaches one with as little as ten agents.

The threshold value for α that results in a steady state for the climate services variable is relatively insensitive to the value of β , but is highly sensitive to other parameters such as the number of agents and the relative damage of emissions. The overall effect of the value of α is highly dependent on the surrounding parametric conditions. However, given a set of parameter, even a slightly different value for α from the given threshold value will result in climate services diverging monotonically away from its original steady state. This divergence can take one of two paths: either increasing indefinitely or collapsing to zero. Clearly, whatever the value for α is will have serious implications.

7. PARETO EFFICIENCY

Pareto efficiency is a concept developed by Vilfredo Pareto in the late 19th century, which represents a state of optimal allocation of resources such that no any one individual can be made better off without making at least one other worse off. Because all agents in my model are identical, maximizing the utility of a representative agent within each generation will achieve the first best Pareto efficient allocation. This is also interpreted as the socially optimal outcome.

In this section, I derive the socially optimal outcome, and then compare the privately optimal outcome with the socially optimal outcome. There are options for which method to use when determining the socially optimal outcome. In my paper, I consider the method of assuming intergenerational cooperation. Thus, arriving at the intergenerational coordination equilibrium. The alternative method of solution is where there is a dictator who chooses the resource use for all agents, such that they maximize the utility of a representative agent. This basis for my paper was that I believe there to be some degree of altruism present in the world around us. I do not believe that a dictatorship, in which everyone's consumption choices are determined externally, is a reality. However, although there is not perfect intergenerational cooperation in the world outside of this model, I believe it a more plausible outcome than the former.

In this paper, I look at what equilibrium control rule the Social Planner would choose if they could force agents to act accordingly. Here, I consider what the equilibrium outcome is with agents forced to coordinate.

The solution to this problem begins by posing an equilibrium resource-use rule consistent with equation (7). Recall,

$$(7) \quad x_{k,t} = rS_t^{1/\delta}$$

The key difference with the solution method from the non-cooperative equilibrium is that where the non-cooperative equilibrium maximizes the value function with respect to the representative's resource use ($x_{k,t}$), the social planner's problem maximizes the value function with respect to the time-invariant control rule (r).

Recall the resource-use problem for agent k , but now maximized with respect to r :

$$(71) \quad \max_r (1 + \beta) \log\left(\frac{x_{k,t}^\phi S_t^\theta}{1 + \beta}\right) - \beta \log\left(\frac{x_{k,t}^\phi S_t^\theta + (n-1)r^\phi S_t^\theta}{\beta n r^\phi S_{t+1}^{\frac{\phi}{\delta} + \theta}}\right) + \alpha \left(\frac{(1+\beta)(\phi + \delta\theta)}{(1-\alpha)\delta}\right) \log(S_{t+1}) + \Omega$$

Subject to

$$(7) \quad x_{k,t} = r S_t^{1/\delta}$$

And

$$(72) \quad S_{t+1} = \left(S_t - \left(\gamma x_{k,t} + \gamma(n-1)r S_t^{\frac{1}{\delta}}\right)^\delta\right) (1 + \omega)$$

The resulting first order condition is independent from the climate service in period t (S_t). The result is a utility-maximizing control rule that does not vary across time. Thereby, satisfying the condition of the posited equilibrium rule.

$$(77) \quad x_t^* = r^* S_t^{1/\delta}$$

Where

$$(78) \quad r^* = \frac{1}{n\gamma} \left(\frac{(\alpha-1)\phi(\beta+1)}{\alpha\beta\phi - \alpha\delta\theta - \beta\theta\delta - 2\beta\phi - \phi}\right)^{\frac{1}{\delta}}$$

Critically, r^* is independent of S_t and therefore equation (77) takes the form of the posited resource-use rule. Thus, the posited rule is a symmetric equilibrium rule.

The critical question is how this outcome compares to the outcome from the non-cooperative equilibrium. Recall the equilibrium control rule from the non-cooperative equilibrium (NCE):

$$(75) \quad \hat{r} = \frac{1}{n\gamma} \left(\frac{(1-\alpha)\phi(n+\beta(n-1))}{\alpha(\theta\delta - \phi(n+1)(1+\beta)) + n\phi(1+\beta) + \beta\theta\delta} \right)^{\frac{1}{\delta}}$$

Taking a ratio of the equilibrium control rules from NCE and the intergenerational coordination equilibrium (ICE), I obtain

$$(79) \quad R = \left(\frac{(\theta\delta\alpha + \phi - \phi\beta\alpha + 2\beta + \beta\theta\delta)(-\beta + n + n\beta)}{(1+\beta)(\phi n + \theta\delta\alpha + \beta\theta\delta + \phi n\beta + \phi\alpha + \phi\beta\alpha - \phi n\alpha - \phi n\beta\alpha)} \right)^{\frac{1}{\delta}}$$

Both control rules are equal when their ratio equals one. Looking at the population size in a given generation (n) that achieves this equality gives

$$(80) \quad \bar{n} = \frac{2\beta+1}{1+\beta}$$

This critical value for n (where $0 < \beta < 1$) is strictly above one, but below two. Thus, there is an intergenerational externality even when n equals one. In particular, R from equation (79) is less than one when n equals one. This intergenerational externality for when n equals one, says that there is not enough resource use. The current generation would like the next generation to produce more, thereby giving the current generation higher consumption when Old (through the reduction in future prices). However, with the number of agents being greater than or equal to two, the ICE control rule, R is greater than one. This means that private optimization leads to over production and is socially inefficient when compared with the ICE outcome. Therefore, the intragenerational externality quickly overpowers the intergenerational one, as the number of agents surpasses two.

Figure 8 compares the utility paths of both the NCE and the ICE outcomes for when there is a single agent in the model. The relationship shows that the ICE outcome does not Pareto

dominate the NCE outcome. Although the current period agent is better off under ICE, all future agents are worse off under the ICE outcome when compared to the NCE. This is because when there is one agent, the social planner effectively chooses prices to the representative agent's benefit, and consequently to the detriment of future agents.

However, when the number of agents is greater than one, the relationship is much different. Figure 9 shows that for $n > 1$, the ICE outcome Pareto dominates the NCE for current and all future agents. We would expect this outcome, as the result from the intergenerational coordination equilibrium is the socially optimal outcome. The correction of the externality generates enough benefit to future agents to more than offset the loss from the price manipulation.

8. CONCLUSION

In this paper, I created an overlapping generations model economy that combined three key features: intergenerational trade, non-paternalistic altruistic preferences, and an evolving climate services stock. Intergenerational trade and altruism in my model both serve as connections to future generations. Intergenerational trade, through the direct interaction as well as through the effect on future prices, connects agents across generations. Agents with altruistic preferences care about the utility of future generations. The altruism parameter (α) plays a very significant role in determining the evolution of climate services, and hence the utility of future agents. For a given set of parametric conditions, there will be a unique value for α that maintains the climate services stock at its steady state. Slight deviations from this value will have dramatic

effects and will either cause the climate services stock to increase indefinitely or collapse to zero if the value is higher or lower, respectively.

In this model economy, I showed that even with altruistic preferences, agents acting optimally do not achieve the socially optimal outcome. There are three sources of externalities in my model. There are two negative intergenerational externalities in the non-cooperative equilibrium. The first being that, privately, agents do not fully account for the utility of future generations because they have an altruism parameter that is smaller than one but larger than zero. A second negative externality exists when agents acting optimally, fail to produce enough goods. As noted by Marglin (1963), if current generation individuals care about the well-being of individuals of future generations, then the well-being of those future agents has the properties of a public good for the agents of the current generation. Therefore, resulting in under-provision for those future agents. Producing more goods would drive down the price, thereby increasing the amount of consumption the Old agents in that generation could consume. The third externality affects agents within a generation. This externality is due to all agents optimizing their own consumption with respect to how the negative effect on future generations affect solely themselves. Where in actuality, all agents feel this negative effect on future generations within a generation. The relationship between the externalities depends on the number of agents in the model. The intragenerational externality swamps the intergenerational externality when there are two or more agents. Therefore, the externality that affects agents within a generation is more problematic and the more effective target for increasing efficiency.

The best strategy for policy to increase efficiency is to aid in the coordination of agents within a given generation. This is a powerful result, as democratic governments already have a method in place of coordinating individuals through the voting process. This is not meant to have

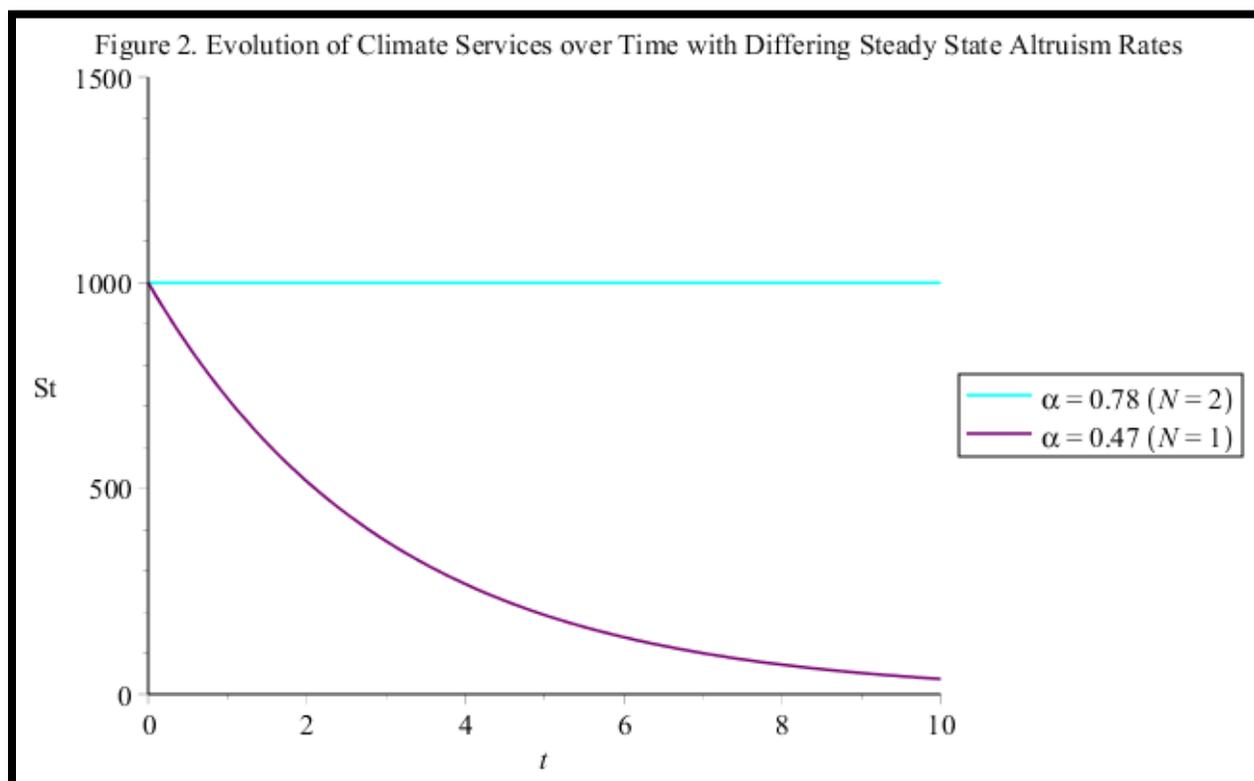
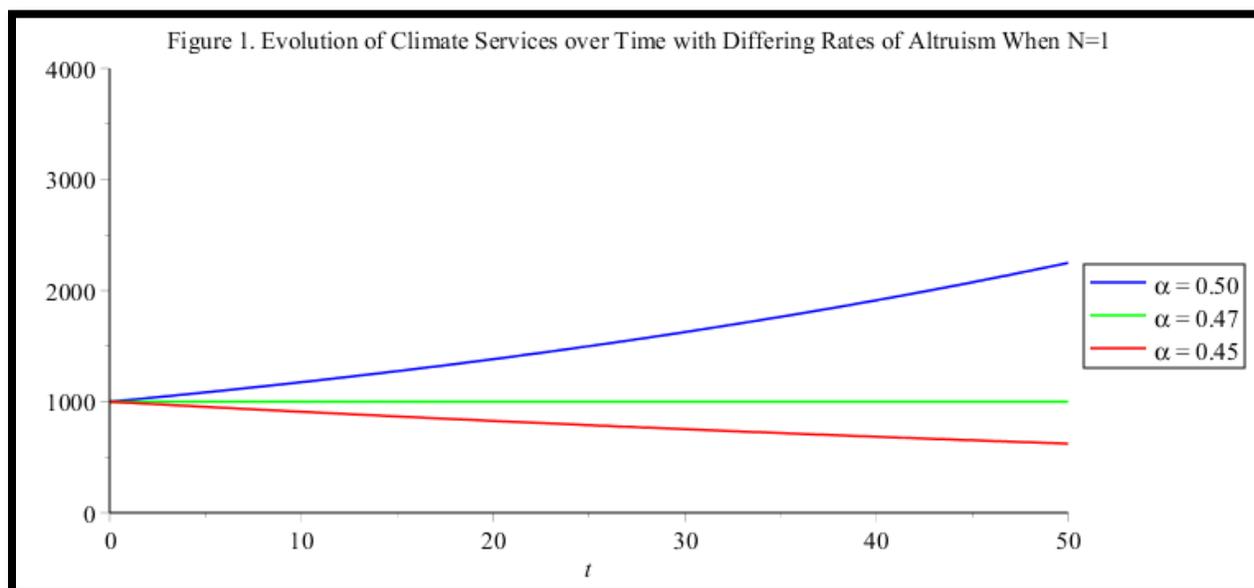
been an exercise in direct guidance of policy, but instead an illustration of the nature of how altruism, trade, and the climate interact. This paper argues that a stronger influence exists among individuals within a generation when considering how best to achieve a socially optimal outcome.

REFERENCES

- Barro, R. J. (1974). Are government bonds net wealth?. *Journal of political economy*, 82(6), 1095-1117.
- de La Croix, D. (1996). The dynamics of bequeathed tastes. *Economics Letters*, 53(1), 89-96.
- Hansen, J., Nazarenko, L., Ruedy, R., Sato, M., Willis, J., Del Genio, A., & Novakov, T. (2005). Earth's energy imbalance: Confirmation and implications. *science*, 308(5727), 1431-1435.
- John, A., & Pecchenino, R. (1994). An overlapping generations model of growth and the environment. *The Economic Journal*, 1393-1410.
- Jouvet, P. A., Michel, P., & Vidal, J. P. (2000). Intergenerational altruism and the environment. *The Scandinavian Journal of Economics*, 102(1), 135-150.
- Kennedy, P. (1994) an unpublished manuscript.
- Marglin, S. A. (1963). The social rate of discount and the optimal rate of investment. *The Quarterly Journal of Economics*, 95-111.
- Nash, J. (1951). Non-cooperative games. *Annals of mathematics*, 286-295.
- Ramsey, F. P. (1928). A mathematical theory of saving. *The economic journal*, 38(152), 543-559.
- Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of political economy*, 66(6), 467-482.
- Schneider, M. T., Traeger, C. P., & Winkler, R. (2012). Trading off generations: infinitely lived agent versus OLG.
- Smith, A. (1790). *The Theory of Moral Sentiments*. Republished in the series Cambridge Texts in the History of Philosophy, edited by Knud Haakonssen (2002).

- Solow, R. M. (1974). Intergenerational equity and exhaustible resources. *The review of economic studies*, 41, 29-45.
- Solow, R. M. (1986). On the intergenerational allocation of natural resources. *The Scandinavian Journal of Economics*, 141-149.
- Stephan, G., Müller-Fürstenberger, G., & Previdoli, P. (1997). Overlapping generations or infinitely-lived agents: intergenerational altruism and the economics of global warming. *Environmental and Resource Economics*, 10(1), 27-40.
- Stern, N. H. (2007). *The economics of climate change: the Stern review*. Cambridge University press.
- UN, (2015) https://esa.un.org/unpd/wpp/publications/files/key_findings_wpp_2015.pdf
- Wendner, R. (2002). Capital accumulation and habit formation. *Economics Bulletin*, 4(7), 1-10.
- Zietz, J. (2007). Dynamic programming: An introduction by example. *The Journal of Economic Education*, 38(2), 165-186.

Figures



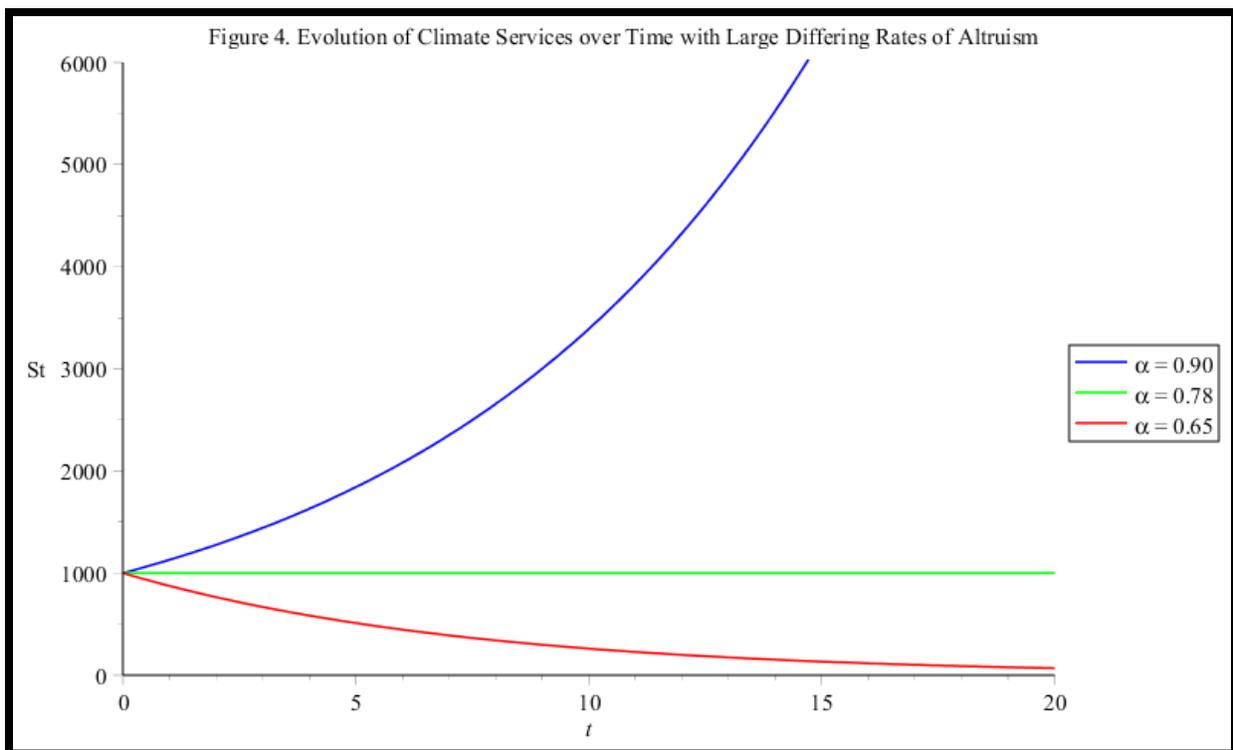
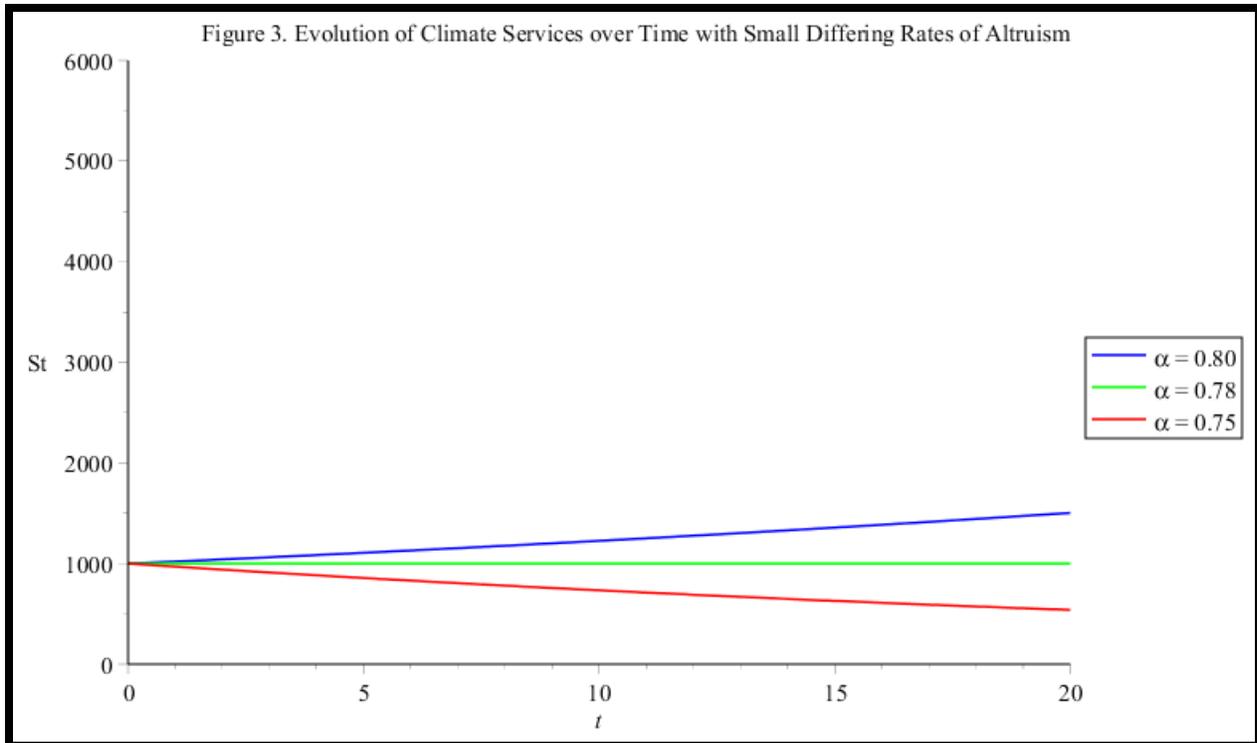


Figure 5. The Threshold Rate of Altruism (α) that Keeps Climate Services in Steady State with Differing Levels of Relative Damage from Emissions (δ)

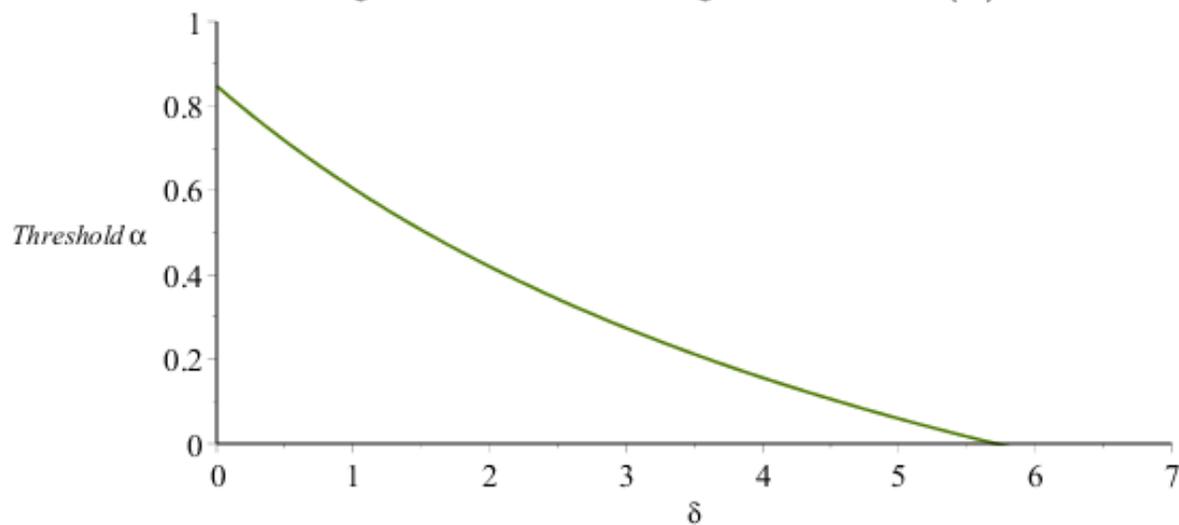


Figure 6. The Threshold Rate of Altruism (α) that Keeps Climate Services in Steady State with Differing Discount Rates on Future Consumption (β)

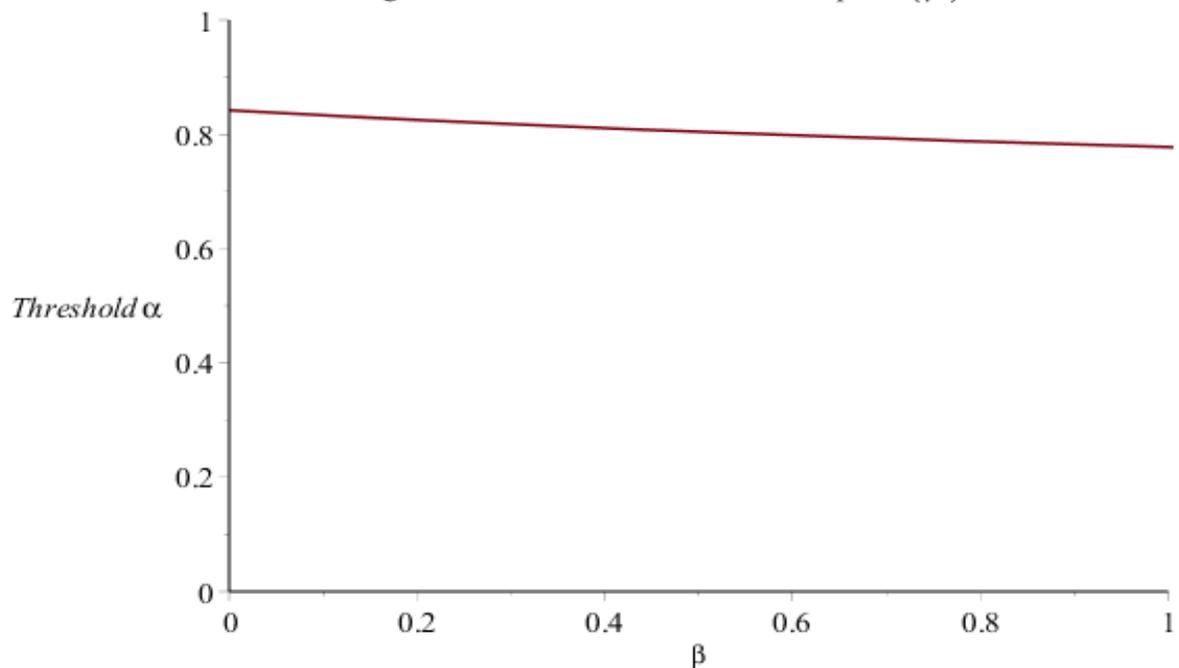


Figure 7. The Threshold Rate of Altruism (α) that Keeps Climate Services in Steady State with Differing Number of Agents (n)

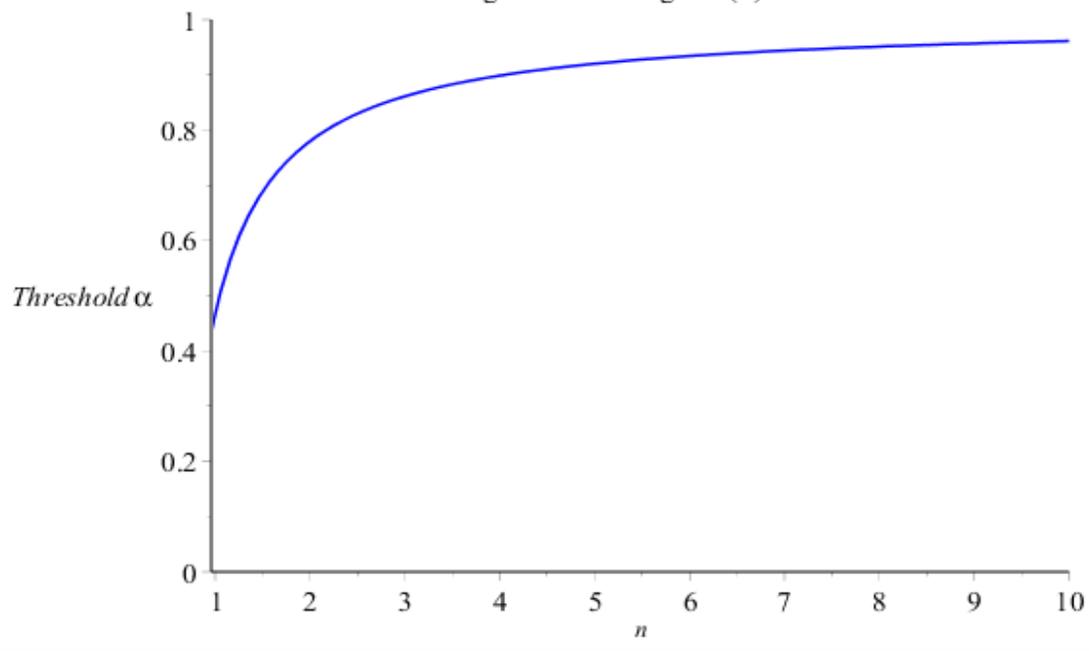


Figure 8. Comparing the Evolution of Utility Between the Non-cooperative Equilibrium (NCE) and the Intergenerational Coordination Equilibrium (ICE) With $N=1$

