Managing Urban Deer

Daniel Rondeau and Jon M. Conrad

Conflicts are emerging between humans and wildlife populations adaptable to the high density of humans found in urban and suburban areas. In response to these threats, animal control programs are typically designed with the objective of establishing and maintaining a stable population. This article challenges this view by studying the management of urban deer in Irondequoit, NY. Pulsing controls can be more efficient than steady-state regimes under a wide range of conditions in both deterministic and stochastic environments, but potential gains can be dissipated by management constraints. The effect of citizen opposition to lethal control methods is also investigated.

Key words: deer, dynamic programming, pulsing, stochastic simulations, wildlife control.

> *When deer populations are regulated by cars, the costs are shared by everyone through their insurance premiums and these costs are significant. Any other method of deer control is safer, more humane and more cost-effective.*
>
> Curtis, p. 8

Bear, elk, wolf, coyote, cougar, beaver, geese, alligator, bison, porcupine, and deer are just a few of the North American animal species that are protected, due to their amenity values, but are also the subject of population control measures put in place to avoid unacceptable levels of damage to human health and property (Rondeau). Aside from the sporadic removal of problem animals by wildlife management agencies, regulated hunting and trapping are the most common form of active management for these potentially problematic species.

Traditional wildlife management approaches, however, are not easily implemented in populated areas where there is growing unease over conflicts between humans and white-tailed deer. One hundred years after reaching its historical population low of approximately 100,000 animals (McCabe and McCabe, 1997; Winter), it is now estimated that between sixteen and twenty million white-tailed deer browse within the continental United States. White-tailed deer are involved in 1.5 million car collisions per year, killing between 100 and 200 motorists annually and causing in excess of $2 billion in damage (Winter). Adaptable to suburban environments and lacking natural predators, deer browse vegetable gardens, destroy ornamental plants, and are instrumental to the persistence of endemic levels of Lyme's disease in the Northeast. Although many city dwellers enjoy the sight of a deer in their backyard, the level of damage they cause has led to the implementation of animal control programs by many local governments.

In this article, we study the management of a deer herd in Irondequoit, NY, a suburb of Rochester. The town of Irondequoit is located on the shores of Lake Ontario, in Monroe County. It is a predominantly residential, suburban area of 43 km^2 with a stable (human) population of approximately 53,000 (U.S. Bureau of Census). By the late 1800s the white-tailed deer had been extirpated from this part of New York State. Re-colonization of the area by deer drifting north occurred slowly. Deer hunting resumed in Monroe County in 1945, although no deer were present in Irondequoit at that time (Hauber). The first reports of car collisions and complaints of deer-damaged orchards were made in 1974. By 1976, hunting was allowed in most of Monroe County but a by-law prohibiting the discharge of firearms and bows within the town's boundaries allowed for the continued growth of the Irondequoit herd. The number of deer-vehicle accidents (DVAs) in Irondequoit grew steadily

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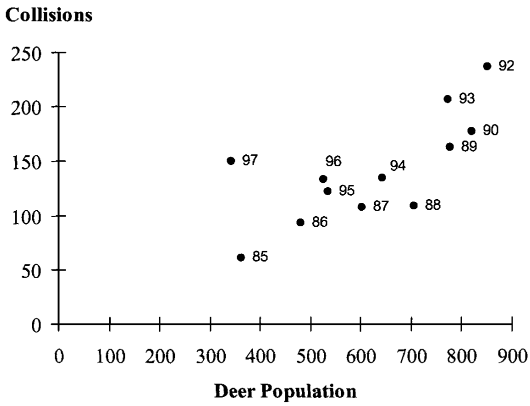

Figure 1. Deer population and deer-vehicle accidents, 1985–97

from 61 accidents in 1985 to 237 collisions in 1992 (figure 1).

In response to the growing number of DVAs and complaints, a citizen task force (CTF) was formed in 1992 to advise town, county, and state officials on deer management issues. The CTF consisted of eleven members and included representatives of animal welfare and conservation groups, as well as citizens advocating a sharp reduction in the number of deer. The mandate given the CTF was first, to establish an acceptable number of animals for the town of Irondequoit and second, to recommend a method of population control (Curtis). The CTF ultimately recommended a density of eight to ten deer per km^2 of "suitable habitat." This contrasts sharply with the population of 850 animals (a density of twenty deer for *each* km^2 of territory) estimated in the winter of 1993 by Nielsen, Porter, and Underwood.

After considering the feasibility and costs of alternative removal options, the CTF recommended that a culling program be initiated with the goal of reaching the recommended density within five years. From 1993 to 1998, a total of 712 deer have been removed from the herd by sharpshooters using rifles. Designated areas are baited with corn for several consecutive weeks in early winter to attract deer and create a feeding habit. Sharpshooters can then move in at night, firing on feeding deer from elevated platforms. This is a typical approach to the reduction of urban deer herds. Thus far, only law enforcement officers have acted as sharpshooters and the culling activities have taken place in Durand-Eastman Park, a natural area boasting the best deer habitat in Irondequoit.

Although the near consensus of the CTF was a great achievement, the mandate it was given and the questions asked of its members may have been too narrowly defined to permit a search for an optimal solution. Asking the CTF to identify an "ideal population target" first, and only subsequently, to investigate and recommend a method to achieve this target ignores control costs in the determination of an optimal management regime. This approach also presupposes that a steady-state deer population is desirable in the long run. The analysis presented herein demonstrates the suboptimality of the steady-state objective adopted for the management of the Irondequoit deer herd. We identify a pulsing control solution in which the population is culled intermittently, challenging the commonly held view that wildlife populations are best managed with the objective of maintaining a steady-state population.

The potential advantages of pulsing in the harvesting of renewable resources has been noted by Clark (1973), Pope, and Hannesson for certain commercial fisheries. Clark (1990) also demonstrates the possible optimality of pulsing when marginal fishing costs are decreasing with the size of the catch. This situation shares similarities with the decreasing marginal costs observed here, although the problem studied by Clark does not contain a dynamic effect of harvesting on costs.

There is also a long literature on agricultural pests and the threshold population that should trigger pesticide applications (a review is presented by Saphores). This vein of research produces broadly defined pulsing policies resulting typically from the large fixed costs of pesticide application, prescribed pesticide dosage and full coverage of the affected field. These situations differ substantially from the context of this article. In earlier work on generalized pest control methods, Jaquette (1972, 1974) found that under a set of somewhat restrictive assumptions, it is optimal to manage certain pest populations that vary stochastically over time under a pulsing policy.[1]

This article establishes a link between the renewable resource and pest control literature by extending Jacquette's results to situations in which stock growth is deterministic, nonlinear and stock dependent, and where the animal population generates both social costs and

[1] Discontinuous control policies such as pulsing have also been studied in the context of inventory management (Arrow, Karlin, and Scarf, Scarf) and advertising (Mesak and Darrat).

benefits. We identify a dynamic cost effect, resembling the dynamics at work in the fisheries studied by Hannesson, as the primary cause for the efficiency gains in pulsing.

The article is organized as follows. In the next section a basic deterministic model of the Irondequoit deer herd is specified. Then the model is calibrated. The next section presents the results of the base model as well as an exploration of the causes of pulsing, and an analysis of the importance of constraints on harvesting in urban areas. The model is then extended in two directions. First, we introduce the possibility that culling deer may produce disutility for some individuals. We briefly analyze the surprising implications of this likely scenario for both lethal and nonlethal control methods. Second, the deer population is subjected to random shocks. In this stochastic environment, the properties of the deterministic pulsing rule and five other management policies are compared. In final remarks, we summarize our main findings and reflect on the applicability of these results to other wildlife control problems and nonlethal methods.

A Model of Deer Management for Irondequoit

By seeking the advice of community members, state, county, and town officials sought to balance the damage caused by deer overabundance against the recreational and other nonconsumptive benefits of deer (e.g., wildlife observation, amenity value) and the costs of controlling the population. We therefore postulate that town officials seek to maximize the welfare provided by the deer herd to the community, by choosing a sequence of animal removals $\{Y_t\}$ to solve

$$(1)\qquad \underset{\{y_t\}}{\text{Maximize}} \sum_{t=0}^{\infty} \rho^t \left[\frac{B(X_{t-1})}{\rho} + \left(p - \frac{c}{X_{t-1}} \right) Y_t - SC \right]$$

$$\text{Subject to } X_t - X_{t-1} = F(X_{t-1}) - Y_t \left[1 + \frac{F(X_{t-1})}{X_{t-1}} \right]$$

$$Y_t \leq \alpha X_{t-1}$$

$$SC = \begin{cases} 0 & \text{if} \quad Y_t = 0 \\ q > 0 & \text{if} \quad Y_t > 0 \end{cases}$$

$$X_{-1} \text{ given}$$

$$B(X_{-1}) = 0$$

where t is an index of time denoting years, X_t is the number of animals in the population at year t, Y_t is the number of animals removed in a culling operation in year t, ρ is the annual discount factor, p is the value derived from consuming the venison (deer meat) of culled deer, α is the proportion of the herd X_t that can physically be removed in year t, SC is set-up costs if management actions are taken in year t ($0 or $q if culling takes place), $B(X_t)$ is the net benefits derived from population X_t in year t. It is assumed that $B(X)$ is a strictly concave, single-peaked function reaching its maximum at $X = \bar{X}$ with $B_x > 0$ for $X \in [0, \bar{X})$, $B_x(\bar{X}) = 0$ and $B_x < 0$ for $X \in (\bar{X}, \infty)$, c a variable cost parameter, $F(X_{t-1})$ represents the net change in the animal population occurring without deliberate human intervention between periods $t-1$ and t.

The particular timing of the events adopted for this model reflects the typical approach to deer culling. A population estimate is obtained in early winter (X_{t-1}), after which culling takes place. Since a proportion of the animals removed during the winter are pregnant females, the effective impact of culling on the herd's population in the next period is assumed equal to $Y_t[1 + F(X_{t-1})/X_{t-1}]$.[2] Thus, the net change in the number of deer between years $t-1$ and t is given by subtracting this "effective take" from the natural growth $F(X_{t-1})$. The resulting population generates benefits and damage measured by $B(X_t)$. This timing is consistent with greater opportunities to see deer in the spring and fall seasons, increased damage to personal gardens in spring, summer and fall, and the occurrence of roughly 52% of all DVAs between September and December (Irondequoit Deer Action Committee).

The function $B(X)$ is a parabola that measures both the positive benefits and negative costs of the deer herd. For a stock lower than $\bar{X}$, the net benefits from the stock are increasing, whereas for stocks exceeding this critical value they are decreasing.[3]

Safety concerns severely restrict how and where population control activities can take place. Because only part of the territory can be covered by culling operations, a constraint exists on the number of animals that can be taken

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[2] Because $F(X)$ measures annual mortality (for instance, high mortality of young fawns) as well as fecundity, this assumption slightly underestimates the number of embryos taken by the cull and the effectiveness of winter hunting.

[3] Recent efforts to measure benefit functions for wildlife and habitat support the assumption of decreasing marginal benefits. See, for example, Rollins and Lyke, or Layton, Brown, and Plummer.

in a given year. Based on discussions with biologists, it is assumed that a constant proportion (α) of the stock resides in and around the areas where culling takes place.

Historical data from Irondequoit suggest that culling costs are linear in Y and decreasing in X (details are presented below). This implies that the marginal cost of harvesting a deer is determined entirely by the stock at the beginning of the culling operation. This cost structure is an artifact of the bait and shoot program. The deer's reliance on feeding stations is greater when population densities are high and natural foods more difficult to find. In addition, regardless of the number of animals feeding at the station on a given night (which diminishes intra-season as the number of deer is brought down), only one deer can typically be taken per night at a given station. This is because the killing of a deer introduces noise, blood, and human scent to the bait station. After a kill, sharpshooters move to another baiting station and only return to the sight of a kill on the following night. The combination of (1) density-dependent reliance on bait stations; (2) feeding habits that constantly brings deer to bait stations; and (3) the absence of deer at stations following a kill give rise to nearly constant intra-seasonal marginal costs of culling determined primarily by the stock at the beginning of the winter. This relationship is reflected in the data presented in the next section and motivates our choice of cost function. Because of the linearity of the cost function, the model admits the possibility of a "bang-bang" control solution whereby the wildlife manager adjusts the stock as rapidly as possible to bring it to its eventual steady state.[4] This management strategy is not unlike the recommendation of the Irondequoit CTF and reflects the general tendency to design urban deer control programs around a fixed population target (Jordan et al.).

Unfortunately, the presence of set-up costs and stock-dependent variable culling costs violates the sufficient conditions for the optimality of bang-bang solutions presented by Spence and Starrett.[5] The practical implication is that the popular most rapid approach to a steady state may be suboptimal. Everything else being equal, set-up costs can induce pulsing—where the herd is managed in cycles of alternating years of natural growth and culling—because they introduce economies of scale in harvesting. Independently of set-up costs, stock-dependent marginal costs can also induce pulsing because reducing the stock increases future marginal costs. If this dynamic effect is sufficiently strong, allowing the stock to grow before proceeding with large-scale culling can increase the effectiveness of sharpshooters and reduce the total variable cost of culling, over time.

Because the problem of interest for this article has an objective function linear in the control variable, is nonmonotone in the state (and therefore the stock has a shadow price which can be either positive or negative), has a nonlinear law of motion for the stock, and decreasing marginal costs in X, an analytical solution remains elusive. We therefore proceed empirically to gain insight into critical relationships, parameter values, and form of the solution.

Model Calibration

Biology

In 1993, Nielsen, Porter and Underwood estimated the Irondequoit deer population at 850 animals. They later concluded that the annual change in the level of the Irondequoit deer herd is given by $F(X) = 0.5703X(1-X/858)$, where X is the pre-culling population and $K = 858$ is the area-carrying capacity. Although the empirical estimation of the parameters of the growth function could technically embody density-dependent migration, the law of motion does not explicitly account for migrations between Irondequoit and adjacent deer-populated areas. This is readily seen from the fact that a zero population is stationary. Although in-migration of deer into Irondequoit is possible, the geography of Irondequoit greatly limits the free flow of animals. The town is surrounded on three sides by water (two of them being Lake Ontario) and on the fourth by the highly urbanized center of Rochester. This explains why Irondequoit was the last area of N.Y. State to be re-colonized by deer

[4] It can be shown that the steady state of the bang-bang control for the unconstrained problem ($\alpha = 1$) is the stock X that solves

$$\delta = F_x - \left(\frac{F(X)}{X + F(X)}\right)\left(\frac{XF_x(X) - F(X)}{X}\right) + \frac{B_x}{\rho}\left(\frac{X + F(X)}{(p - c/X)X}\right) + \frac{cF(X)}{X^2(p - c/X)}$$

[5] Set-up costs violate the quasi-concavity requirements of the theorem and stock-dependent variable costs violate the separability requirements. See Spence and Starrett for details.

(Hauber), and why we do not account for possible density-dependent diffusion rates in the model we develop. Accordingly, we write the law of motion for the stock as

$$(2)\qquad X_t - X_{t-1} = F(X_{t-1}) - Y_t\left[1 + \frac{F(X_{t-1})}{X_{t-1}}\right] = 0.5703X_{t-1}\left(1 - \frac{X_{t-1}}{858}\right) - Y_t \times \left(1.5703 - \frac{0.5703X_{t-1}}{858}\right).$$

Stock Benefits and Damage

Benefits stemming from the existence of free roaming deer in suburban areas include wildlife observation and the nonuse benefits accruing to those who have preferences for living in an environment that retains natural features. Whereas no direct data exist on the magnitude of such benefits to the residents of Irondequoit, the deliberations and final recommendations of the CTF provide some insights.

Recall that the CTF first selected an ideal deer density by weighting only the benefits and damage of alternative deer populations. Because the task force ignored management costs in determining this target, we adopt the view that its recommendation corresponds to the maximum ($\bar{X}$) of the net benefit function $B(X)$. The most conservative interpretation of the CTF recommendation translates into an objective deer population of forty-five to fifty-five animals. However, because of uncertainties surrounding the amount of "suitable habitat," the recommendation is generally interpreted as a mandate to reduce the population to 10–20% of the 1993 level (Curtis, Hauber, personal communications). In what follows, we assume a target population of $\bar{X} = 100$ animals (11.8% of the 1993 population).

Damage from Car Collisions

Despite a slight decrease in human population, the number of DVAs grew steadily over the 1985–97 period, exceeding 100 collisions per year since 1987 and reaching a high of 237 in 1992. Figure 1 illustrates the relationship between the estimated number of deer-vehicle accidents (DVA_t) and the estimated population in the fall of that year (when most collisions occur), after culling and new births have taken place. The risk of collision associated with a marginal deer was estimated via a constrained linear model. A dummy variable identifies observations prior to 1991 to account for a change of methodology in recording DVAs. The estimated relationship is $\text{DVA}_t = -35.418\ \text{Dummy}_t + 0.2605X_t$. The t-statistics for the two parameters are, respectively, −1.893 and 11.651. The F-statistic for the regression is 9.068 and 80.3% of the total variance found in the data is explained by the model.

A study of insurance claims resulting from DVAs in Tompkins County, NY during 1988 (Decker, Loconti-Lee, and Connelly) indicates vehicle damage averaging $1,904 per collision (constant dollars of 1998 including deductible). This number is consistent with the figure of $2,000 cited by the AAA Foundation for Traffic Safety (AAA). However, the value of insurance claims overstates true damage to vehicles because the existence of deductibles in most automobile insurance contracts imply that minor collisions are less likely to result in a claim. On the other hand, insurance claims do not account for medical costs nor for the loss of productivity resulting from personal injuries. Neither do they account for the potential loss of human life. While no human deaths have yet been attributed to DVAs in Irondequoit, 5–6% of collisions in a neighboring county do result in personal injuries (Decker, Loconti-Lee, and Connelly). In the absence of additional data, we limit the estimate of damage to property losses. Multiplying the constant probability of collision per deer (0.2605) by the value of claims per collision ($1,904) puts the expected annual damage per marginal deer at $495.

Damage to Vegetation and Property

In a survey of Western New York residents, 17% of homeowners reported property damage caused by deer in the previous year (Sayre and Decker). Losses estimated at $203 (dollars of 1998) per damaged property translate into an average loss over all residences of approximately $34.50. Assuming similar losses from a herd at carrying capacity in Irondequoit, the damage caused by deer to the 20,941 residential and small commercial properties of the town (Town of Irondequoit Taxes and Assessment Division—personal communication) would amount to $720,000 per year, or an average damage of $842 per deer per year. Adding this figure to marginal losses from DVAs yields an estimate of total marginal damage at

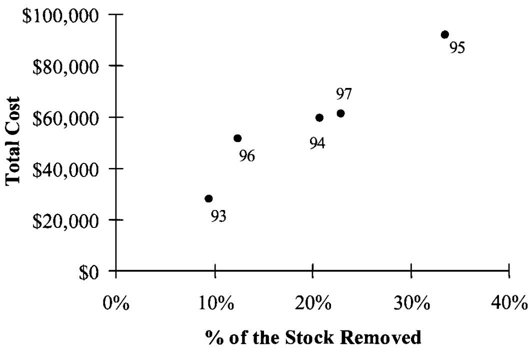

Figure 2. Culling costs and proportion of the stock removed, 1993–97

$X = 858$) of $-\$1{,}337$. On this basis we set $m = B_x(K) = -\$1{,}300$.

Net Benefits Function

We assume that net benefits can be represented by a function of the Gompertz family

$$(3) \qquad B(X) = \begin{cases} 0 & \text{if} \quad X = 0 \\ a\,X\,\mathrm{Ln}(b/X) & \text{if} \quad X > 0 \end{cases}$$

where $a > 0$, $b > 0$. This particular functional form is strictly concave and meets the assumptions of the model ($B_x > 0$ for $X < \bar{X}$, $B_x < 0$ for $X > \bar{X}$). Based on the data developed above we solve the system of equations $B_x(\bar{X}) = 0$ and $B_x(K) = m$ and simplify to obtain $a = 604.81$ and $b = 271.83$. Accordingly, a deer herd of 100 animals would procure annual net benefits to the community of $B(100) = \$60{,}481$ or an average of approximately \$1.11 per resident. Similarly, culling the last deer from Irondequoit would result in losses to the community of approximately $B(1) - B(0) = \$3{,}390$.

Culling Costs

Figure 2 illustrates the relationship between the cost of culling and the proportion of the stock harvested.[6] Harvesting costs vary from \$28,448 for the removal of eighty deer from the initial population of 850 in 1993 to \$91,900 for the removal of 215 deer from a stock of 643 in 1995. Based on these limited data, the cost of culling is taken to be $C\,(X_{t-1}, Y_t) = \$7{,}763 + \$231{,}192\,(Y_t/X_{t-1})$. This accounts for known set-up costs averaging $q = \$7{,}763$ per year. Regressing variable costs on the proportion of the stock taken yields $c = \$231{,}192$ ($t = 5.206$). This parameter can be interpreted as the total variable cost that would have to be incurred if the entire herd was removed in a single year using the current technology. As will be shown, the main results of this article are insensitive to wide variations in q and c.

Value of Venison Meat (p)

An average of 28.8 lbs. of stew meat per deer harvested has been donated to charitable organizations since 1993. Given the availability of close substitutes, a replacement cost of \$2.49 per pound is imputed to set $p = \$72$ per deer.

Proportion of the Herd That can be Removed in a Given Year (α)

Aerial population surveys place approximately one-third of the Irondequoit herd in Durand-Eastman park (where culling takes place) during winter months (Hauber, Porter, personal communications). Based on this indication and accounting for the fact that an estimated 33.4% of the total population was removed in 1995, we initially set $\alpha = 0.35$. The effect of this constraint is studied extensively below.

Initial Condition and Discount Rate

An initial population (X_{-1}) of 331 animals and a real annual discount rate $\delta = 4\%$ [$\rho = 1/(1 + \delta) = 0.9615$] are adopted. Results for $\delta = 0$ and $\delta = 0.08$ are also reported.

Results

The solution to (1) is obtained by recursively solving the Bellman equation

$$(4) \qquad V(t, X_{t-1}) = \underset{Y_t}{\mathrm{Max}} \left\{ \rho^t \left[\frac{B(X_{t-1})}{\rho} + \left(p - \frac{c}{X_{t-1}} \right) Y_t - FC \right] \right\} + \rho^{t+1} V(t+1, X_t)$$

subject to the constraints in (1) and with the added terminal condition $V(T+1,\ X_t) = 0$. The present value function $V(t, X)$ represents the maximum achievable net present value of starting period t with a stock X given the action Y, under the assumption that the remainder of the management program will be optimal given this period's choice of control level. A

[6] Costs incurred once (e.g., purchase of rifles, a helicopter flyover in 1996, carcass inspection (for research purposes only), meat inspection costs in 1993 (no longer required)) have been removed to provide a basis to predict future culling costs.

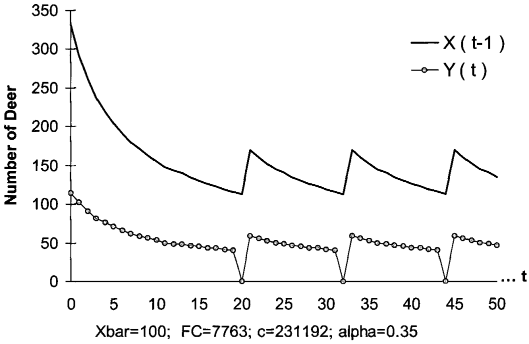

Figure 3. Optimal management of the Irondequoit deer herd ($\alpha = 0.35$)

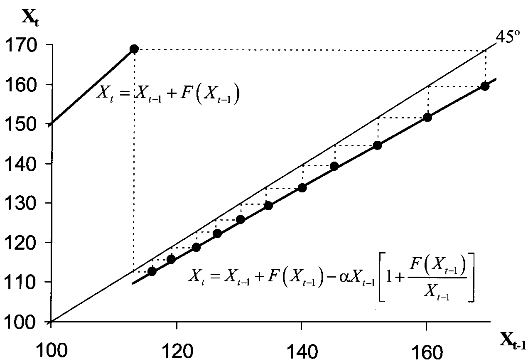

Figure 4. Phase diagram ($\alpha = 0.35$)

finite time horizon $T = 70$ years was imposed, although duplication of some simulations with $T = 100$ confirmed that the results reported below extend to longer time horizons and are likely stationary solutions that solve the infinite horizon problem.[7] In order to solve this problem numerically via dynamic programming, it was discretized, first by rounding the state variable before each new calculation of the value function, and second, by imposing that Y_t be an integer in the interval $[0, \alpha X_{t-1}]$.

The Irondequoit Scenario

The optimal program for the management of the Irondequoit deer herd is illustrated in figures 3 and 4. During an initial adjustment phase, the stock level is brought down from 331 to 113 animals, at which point a stable cycle of natural growth and culling is established. Because of the constraint on harvesting and the rapid natural regeneration of the deer population, the initial adjustment phase lasts nineteen years. By then, the restriction on the stock that can be harvested results in an effective take barely greater than the natural growth of the stock and only very small population reductions can be achieved by pursuing additional culling. Once the population reaches 113 animals, it is allowed to grow naturally for a single year. This returns the stock to 169 animals and marks the beginning of a stationary management cycle.[8] This stable cycle consists of eleven consecutive years of culling, at the constrained rate of $0.35X_{t-1}$, followed by a single year in which no harvesting takes place. This optimal harvesting rule can be written formally as

$$(5) \qquad Y_t^* = \begin{cases} 0 & \text{if } X_{t-1} \le X^* \\ \alpha X_{t-1} & \text{if } X_{t-1} > X^*. \end{cases}$$

For stocks smaller than or equal to X^*,[9] the optimal strategy is to let the stock grow at its natural rate. For stocks above X^*, the maximum number of animals allowed by the constraint on harvesting are removed. In the first case, the stock transition equation giving next year's population is simply $X_t = X_{t-1} + F(X_{t-1})$. When culling takes place, the transition equation is the full law of motion: $X_t = X_{t-1} + F(X_{t-1}) - \alpha X_{t-1}[1 + F(X_{t-1})/X_{t-1}]$. These two components of the stock transition form the boundaries of the cycle illustrated in the phase diagram of figure 4. The dotted trajectory is constructed from an initial condition on X_{t-1}, using the policy function to determine the stock in the following period and projecting this stock back on the horizontal axis using the 45-degree line. Following the dotted line clockwise yields the cyclical portion of the time path found in figure 3.

The deer population of Irondequoit is never brought down to the target of 100 animals because it is simply too expensive to do so, given the severe constraint on harvesting. Over the twelve years of the optimal cycle, an average of 136 deer roam the community, causing on average thirty-five car collisions per year, but

[7] *Mathematica*™ 4.0 programs developed in the course of this research can be found at http://web.uvic.ca/~rondeau/fileshare.htm

[8] Strictly speaking, applying (equation 10) to continuous variables X and Y yields chaotic trajectories whereby no value of X is ever exactly repeated (differences, however, are only detected at the third or fourth decimal level). Cycle stability always results when harvesting levels are constrained to integer values. The reader interested in a discussion of deterministic chaos as it applies to the management of animal populations can consult Grafton and Silva-Échenique.

[9] The critical stock X^* is determined empirically by iterating the initial condition X_{-1} until the point where increasing the initial stock by one unit causes the period zero harvesting to go from zero to the positive amount Y^*. For the baseline calibration, $X^* = 113$.

Table 1. Characteristics of Optimal Steady-State Cycles

α	X* ($\tilde{x}$, X^*)	Total Number of Years	No. of Years with Culling	Minimum Population	Maximum Population	Amplitude	Mean Population	NPV of Infinite Optimal Program[a]	NPV of MRAP to $\bar{X}$[a]
0.35	113	12	11	113	169	56	135.58	−$833,634	−$894,567
0.40	109	9	7	103	163	60	130.22	−$663,669	−$784,985
0.50	117	7	4	92	170	78	128.14	−$435,647	−$726,966
0.70	125	3	1	70	160	90	112.33	−$74,363	−$680,108
0.90	166	11	2	26	211	185	93.90	$229,266	−$659,023
1.00	(18,181)	7	1	18	223	205	90.14	$284,543	−$659,023

[a] Includes the benefits and costs of the adjustment phase.

as many as forty-four collisions when the population reaches its maximum.

The net present value of the optimally managed herd is −$833,000.[10] In contrast, the option recommended by the CTF would require culling for twenty-four years to stabilize the population, and has a NPV of −$894,000. The bang-bang solution (described in footnote 4) has a steady-state population of 126 animals and an NPV of −$846,000. Finally, refraining from managing the herd and allowing it to reach carrying capacity would have a net present value of −$14.3 million. Thus, whereas the optimal cycle only offers marginal improvements in welfare relative to the CTF approach, both control methods yield substantially greater benefits than laissez-faire.

Foregoing culling on certain years has several consequences. No culling costs need to be incurred on the off years, resulting in direct savings for the community. Refraining from culling also results in an increase in the number of deer, which in turn produces greater viewing opportunities and recreational benefits, but raises the number of car collisions and the rate of property damage. Most importantly, allowing the population to increase reduces the marginal cost of future culling through higher population density. Pulsing is the mechanism that allows the community to capitalize on naturally diminishing marginal costs of harvesting over time. Over the length of the cycle, the cost savings from pulsing outweigh the lower benefits received in years of low populations and the higher damage incurred in years of more abundant wildlife.

[10] Increasing T does not modify the stable cycle but only increases the number of repetitions that are observed. Thus, we are confident that the stable cycles are stationary solutions to the infinite horizon problem. The NPV of programs reported in the article are the sum of (*a*) the NPV for the adjustment period and (*b*) the NPV of an infinite number of repetitions of the stable cycle.

Relaxing the Constraint on the Territory Covered by Culling (α)

Relaxing the constraint (α) on the proportion of the herd that can be taken on a given year increases the speed and efficiency of the initial population adjustment phase for both cyclical and most rapid approach path MRAP management regimes (the constraint is binding in both cases). However, more flexible culling confers additional gains only under pulsing. If α is larger than the natural growth rate at the chosen steady state, the constraint will not be binding in equilibrium and relaxing it therefore, has no additional value. In contrast, increasing α relaxes a constraint that binds on the culling years of a cyclical management regime. The resulting fine tuning of cyclical harvesting programs translates into increased amplitude of cycles, reduced mean population, systematic reduction of the minimum stock, and a reduction of the optimal frequency of culling from eleven out of twelve years for $\alpha = 0.35$ to once every seven years for $\alpha = 1$. These results are summarized in table 1.

In terms of the phase diagram of figure 4, increasing α changes the point of discontinuity (X^*) of the transition equation. More importantly, the lower branch corresponding to active culling years is rotated down. These modifications can create complex sequences of harvesting and natural growth. For instance, with $\alpha = 0.5$, culling takes place on years 2, 4, 6 and 7 of a seven-year cycle. With greater flexibility afforded by an increased α, the population can effectively be reduced to levels lower than the CTF target. The community is then in a position to take advantage of additional years when $B(X)$ is increasing (early in the cycle) and c/X is decreasing (throughout). It can also delay the rapid increase in damage to vegetation and property, and avoid set-up costs.

The general solution to the problem is obtained when $\alpha = 1$. The policy function differs slightly from equation 3. For any $\alpha > 0.94$, the optimal policy is given by the rule $(\tilde{x}, X^*) = (18, 181)$, whereby the herd is allowed to grow provided it does not exceed 181 animals, but is reduced to $\tilde{x} = 18$ as soon as it exceeds X^*. With $\alpha = 1$, it is technically feasible but not optimal to exterminate the herd. Under this optimal management rule, a herd averaging ninety animals over a seven-year cycle produces a positive NPV of $284,000.

In Irondequoit, current restrictions on culling significantly reduce the flexibility of wildlife managers and dissipate most potential gains from pulsing. Yet, other areas frequented by deer exist where safe culling could take place (Hauber, personal communication). Extending the culling operation to these areas would afford greater latitude to wildlife managers and increase the gains from adopting cyclical management.

The Root Cause of Pulsing

Set-up costs have little effect on the solution. Without a constraint on the scope of culling ($\alpha = 1$) removing all set-up costs (setting $q = 0$) produces an optimal policy $(\tilde{x}, X^*) = (18, 176)$. Although X^* is reduced by the removal of set-up costs, the resulting optimal cycle is identical in all other respects to the optimal cycle with set-up costs of $7,763. Increasing set-up costs to $25,000 produces the policy rule $(\tilde{x}, X^*) = (18, 182)$ but the same optimal cycle once again. It takes an increase in set-up costs to $33,000 to extend the cycle to eight years under the policy $(\tilde{x}, X^*) = (11, 183)$. The longer cycle is obtained entirely through a reduction in $\tilde{x}$.

With culling costs averaging $534 per deer over the period 1993–97, the Irondequoit operation is more expensive than other similar bait-and-shoot programs where typical average costs vary from $200 to $400 per deer (Revkin; Jones and Whitham; Drummond; Stradtmann et al.). This difference is likely due to the high level of safety precautions taken in Irondequoit (e.g., two police officers per station, but only one shooter, security perimeter around the park, etc.). Nevertheless, even in the absence of set-up costs, cyclical culling continues to be optimal with a variable cost parameter as small as 1/8 of its current value. The reduction in costs required to eliminate pulsing would bring the average historical cost per deer removed in Irondequoit to approximately $60 per head. This is more than three times smaller than the lowest level reported in the literature for any bait and shoot operation. It therefore seems likely that the potential efficiency gains associated with cyclical herd reduction programs extend beyond Irondequoit to many other communities facing urban deer conflicts.

Discount Rate

Variations in the discount rate between 0% and 8% do not affect the qualitative properties of the solution regardless of the presence of a constraint on harvesting. Reducing the discount rate extends the length of optimal cycles and reduces their respective perigees. A lower discount rate gives future events a greater impact on the NPV of the optimal program. In a cyclical management regime, the distant future (years preceding the next culling) are characterized by larger stocks and marginal animals that impose net welfare costs. The community can trade away some of the future costs either by culling at an earlier future date, or by increasing the length of the cycle through more drastic reductions of the population when culling takes place. The empirical evidence from Irondequoit indicates that this second strategy is more beneficial. The lower population initially provides fewer benefits to the community, but these losses are more than compensated by the benefits obtained by delaying high damage rates associated with larger populations, and increasing the interval before culling is again necessary.

The Social Cost of Lethal Controls: The Bambi "In My Back Yard" Effect

Even in situations where most people agree that a decrease in wildlife population is desirable, many may disagree as to how to achieve the reduction. While culling may be the most economical, lethal methods may impose a welfare loss on individuals who, for ethical reasons, oppose such action. A survey of 890 Irondequoit residents conducted in 1998 (five years after the first cull), indicates that while 64.5% of respondents supported a reduction of the deer population, significant proportions of respondents considered lethal techniques "not at all acceptable": 27% disapprove of bow hunting, 34% are against the current bait and shoot approach, 55% reject tranquilizer capture followed by lethal injections and 56% find live trapping and killing unacceptable. Overall, only 24.3% of the population considers lethal techniques to be the most appropriate control method (Lauber and Knuth). In Irondequoit and other communities, court injunctions have been sought, bait stations have been spiked with gasoline and other products that repel deer, demonstrations and vigils have been held and even bomb threats have been made to prevent the killing of deer (Revkin, Hauber, personal communication). The presence of opposition to lethal methods implies that the model developed thus far may fail to account for welfare losses associated with different management methods. Objecting to the killing of deer comes in part from those who are forced to forego the benefits and enjoyment they would otherwise receive from a more abundant deer population. In this case, the opposition is simply the expression by individuals that they stand to receive reduced benefits because of a lower deer population. These losses are readily accounted for by the current model.

Others individuals, however, may suffer additional losses resulting directly from the actual killing of animals. In a striking example, an anonymous resident of Cayuga Heights, NY, who vehemently opposed the use of lethal controls for the village's overabundant deer herd, agreed to partially fund a sterilization experiment whereby does will be netted, anesthetized, surgically sterilized, and released back into the village. The personal cost to this resident may be in the neighborhood of \$100,000.[11]

In terms of the model, it can be postulated that the killing of deer imposes disutility on individuals that can be accounted for by subtracting a term $d(Y_t)$ from the objective function. One can think of this term as a fixed or variable disutility cost. In the case of a fixed disutility cost, the number of animals culled beyond the first one does not matter and the disutility takes the form

$$(6) \qquad d(Y_t) = \begin{cases} 0 & \text{if} \quad Y_t = 0 \\ D & \text{if} \quad Y_t > 0 \end{cases}$$

where $D > 0$ is the fixed level of disutility.

On the other hand, the disutility may be variable, increasing with the number of deer culled: $\partial D(\bullet)/\partial Y_t > 0$. For either formulation, it is important to note that the disutility of killing deer acts in the same way as an increase in the cost of deer removal. As we have observed before, increasing the cost of deer removal increases the length of constrained cycles and reduces (rather than increases) the minimum and mean population levels over the cycle.

An even stronger result can emerge if $\alpha = 1$. For the unconstrained model, we find that subtracting a fixed disutility parameter $D \geq$ \$266,000 (an average disutility of \$5.02 per Irondequoit resident) makes it optimal to entirely eradicate the herd. Similarly, subtracting a linear disutility term $d(Y_t) = 3472Y_t$ also leads to the surprising result that the herd should be completely eliminated. Formally, the solution in both of these cases takes the form $(0, X^*)$. Although the application of this policy function results in a steady-state population of zero, it is clearly a pulsing solution because eradication occurs at $t = 0$ only if $X > X^*$. For smaller initial stocks, the population is allowed to grow until it exceeds X^*.

This eradication after a single pulse result holds as long as $X^* < 858$ in the optimal regime $(0, X^*)$. For a larger value of X^* the threshold population is never reached due to the carrying capacity constraint and no management of the population whatsoever should ever be undertaken. For no management to be optimal, it is necessary that the cost of eradicating the herd exceed the net present cost of a free ranging deer population in Irondequoit (\$14.3 million). This calls for fixed disutility costs of approximately \$14 million or constant marginal disutility of \$43,000 per deer, two disutility levels that seem unlikely to be observed in reality.

The welfare losses of those opposed to the lethal removal of animals do not alter the main result that pulsing is optimal, but it has the rather perverse effect of making it optimal to remove more, or perhaps even all deer, rather than fewer. Though surprising, this result is quite intuitive. Keeping a positive population results in the eventual recovery of the stock, and with it, the need for future culling and the additional disutility it entails. It follows that eradicating the herd entirely is desirable because it reduces the future welfare losses of those who object to lethal control methods.

The disutility associated with lethal control increases the relative attractiveness of more expensive nonlethal management techniques. At present, live capture/relocation and contraception are the only two viable nonlethal methods available, although, both have significant shortcomings.[12] To the extent that live capture and relocation can be differentiated

[11] Cayuga Heights is a village adjacent to Cornell University where plans to control the local deer herd via sterilization are being finalized.

[12] Live capture and relocation, often suggested by the public as an acceptable management option, is quite stressful for the deer

from culling only by the levels of fixed and variable costs or disutility they impose, we can predict with some confidence that its efficient application would take the form of pulsing (or complete relocation of the herd). However, the same argument cannot be made with the same confidence in the case of fertility control. Although the dynamic cost effect is likely to remain an important factor in favor of pulsing, contraceptive methods imply potentially significant modifications to the law of motion characterizing annual population changes. Additional work is therefore required before firm conclusions can be reached on the optimality of cyclical contraception campaigns.

Pulsing Under Uncertainty

In this section, we compare the efficiency properties of six deterministic population control rules in the context of deer population subject to idiosyncratic shocks. Although there is no presumption that any of the rules investigated is the optimal solution to a fully stochastic model, this exercise reinforces the conclusion that pulse control outperforms other commonly adopted approaches to urban deer management.

In what follows, it is assumed that population biology follows the stochastic process

$$(7) \quad X_t - X_{t-1} = F(X_{t-1}) - Y_t \times \left[1 + \frac{F(X_{t-1})}{X_{t-1}}\right] + \sigma z_t X_{t-1}$$

where σX_{t-1} measures the susceptibility of the stock to a random shock $z \sim N(0, 1)$. As σ increases, the population becomes more susceptible to random events. Because z is symmetrically distributed around zero, the expected and deterministic growth of the stock are equal (i.e., the deterministic population function is the certainty equivalent of equation (7)).

One thousand simulations ($n = 1$ to 1000) were conducted for each of three values of α (0.35, 0.4, 0.5) and each of six management regimes. For each simulation, one hundred normal deviates, $z_{n,t}$, ($t = 0$ to 99) were drawn, forming a set of 100,000 simulation and time-specific random numbers. The same set of random deviates was used to simulate population dynamics over a one-hundred-year period for each of the six management policies and values of α. Consequently, observed differences in performance are strictly attributable to changes in culling regime. The six regimes, compared using $\sigma = 0.25$ are:

1. The "CTF" regime: the deer population is returned to the CTF objective $\bar{X} = 100$ as rapidly as possible, based on the expectation that the random shock has a mean value of zero. This policy is formalized by the rule

$$(8) \quad Y_t = \begin{cases} 0 & \text{if} \quad X_{t-1} + F(X_{t-1}) \leq \bar{X} \\ \alpha X_{t-1} & \text{if} \quad (1-\alpha)[X_{t-1} + F(X_{t-1})] > \bar{X} \\ \dfrac{X_{t-1}[X_{t-1} + F(X_{t-1}) - \bar{X}]}{X_{t-1} + F(X_{t-1})} & \text{otherwise.} \end{cases}$$

2. The "MRAP" policy: the population is returned to the bang-bang steady state of 126 based on the expectation that the random shock takes a value of zero. This policy is obtained by replacing $\bar{X}$ by 126 in equation (8).
3. The "X^*" policy: the optimal deterministic cyclical policy found in the previous section [$X^*(\alpha = 0.35) = 113$; $X^*(\alpha = 0.40) = 109$; and $X^*(\alpha = 0.50) = 117$].
4. The "Mode $\hat{X}$" regime: the cyclical policy where X^* is replaced by the barrier $\hat{X}$ that most often results in the maximization of NPV over the 1000 simulations. That is, X^* is replaced by the mode of the distribution $\{\hat{X} = X\text{:NPV}(\hat{X}) \geq \text{NPV}(X)\ \forall\ X \in [X_{\min}, X_{\max}]\}$, where $X_{\min}$ and $X_{\max}$ are arbitrary boundaries of a search determined after a limited pretest (details below).
5. The "EV max" $\hat{X}$ regime: the cyclical policy where X^* is the barrier that empirically maximizes the expected NPV of the deer herd over all 1000 simulations.
6. "The benchmark": the full information cyclical policy, where X^* is chosen separately for each simulation as the barrier that maximizes the NPV of the particular program.

and actually results in high mortality rates (up to 85%) among the relocated deer (O'Bryan and McCullough). There is also typically little or no good habitat available (i.e., not already occupied by deer) where animals can be relocated. For these reasons, wildlife management agencies are reluctant to issue live capture and relocation permits. The administration of contraceptives to female deer is currently an experimental method. Its effectiveness remains unproven at the population level and its use requires site-specific approval from the Food and Drug Administration (Warren, White, and Lance; Muller, Warren, and Evans). Deer must be treated annually, increasing control costs. Since treated venison is not yet known to be safe for human consumption, fertility treatment may require the closure of hunting grounds adjacent to urban and suburban control areas and result in lost hunting opportunities.

The policies defined in (4), (5), and (6) make use of the results of an empirical search for the barrier $\hat{X}$, (equivalent to X^* in (5)), that maximizes the NPV of each individual simulation. Each possible value of $X^* \in [1, 179]$ for $\alpha = 0.35$ and $\alpha = 0.40$, and $X^* \in [28, 205]$ for $\alpha = 0.5$ was applied to each of the 1000 simulations and the resulting NPV of the one-hundred-year program was recorded. After these computations, the $\hat{X}$ of regime (4) was determined by choosing the barrier that most often provides the greatest NPV; in regime (5) $\hat{X}$ is the barrier with the highest mean NPV. Management regime (6) selects the barrier with the highest NPV specific to each simulation. This policy cannot be applied in reality because it makes use of unknown information about future random events but serves as a benchmark.

The probability distributions for optimal barriers over 1000 simulations for each value of α are presented in figure 5. Both the 0.35 and 0.4 distributions are centered to the left of their corresponding deterministic X^*, suggesting that whereas the random shocks are symmetrically distributed around zero; positive and negative shocks have asymmetrical effects on the expected NPV of management programs. Large increases in population resulting from positive shocks are more damaging than negative shocks of the same magnitude.

With a stringent constraint, the wildlife manager cannot effectively counter the effects of unexpected population growth. In the deterministic analysis, a tight constraint simply makes it inefficient to pursue population reductions when the natural rate of growth of the population approaches α. In the stochastic context, positive shocks bring, on average, the prospect of a long period of high damage rates and costly population control. On the other hand, negative shocks lower the benefits from the herd but postpone the necessity of culling. Knowing that the response to future positive shocks is severely constrained, a wise wildlife manager can take advantage of randomly occurring low stocks by decreasing the threshold that triggers population controls and intentionally keeping the stock at a lower level.

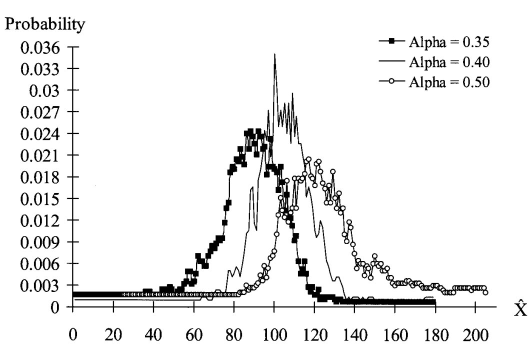

Figure 5. Probability distributions for the optimal barrier $\hat{X}$

This effect disappears when α reaches 0.5. The distribution for $\hat{X}$ is centered exactly on the optimal deterministic rule $\hat{X} = X^* = 117$. Although overall welfare could still be improved if it was possible to further relax the constraint α, the proportion of the stock that can be taken in a given year is sufficiently high to ensure effective reactions to positive population shocks.

Table 2 presents descriptive and summary statistics on the performance of each management policy. We find that with $\alpha = 0.35$, the expected NPV of pulsing is not statistically different from the value of the CTF policy ($t = 1.394$) and MRAP rules ($t = 1.193$). The barrier that most often maximizes welfare is 88 whereas the barrier that maximizes expected NPV is 86. Both measures are much smaller than the deterministic barrier of 113. For both $\alpha = 0.4$ and $\alpha = 0.5$, the performance of the X^* rule is not statistically different from the expected value of the mode or EV max rules ($t < 0.229$). Yet, all three cyclical policies have significantly greater expected NPV than maintaining the stock at either of the steady states prescribed by the CTF or MRAP regimes ($t > 5.1$). For $\alpha = 0.5$, the $X^* = 117$ barrier is also the mode of the distribution of optimal barriers. The expected NPV of this policy is not significantly different from the EV max policy ($t = -0.0337$).

It is worth noting that increasing the scope of culling operations to gain access to 40% or 50% of the herd produces highly significant increases in welfare. These increases are accomplished through a reduction in the number of years in which culling takes place and an increase in the number of animals removed in culling years. As α goes from 0.35 to 0.5, the frequency of culling under the EV max policy decreases from 84.9% to 50.5% of years, and the average kill per operation increases from fifty-five to seventy-nine animals. Increasing α also decreases the average annual stock under all policies.

Overall, the results reinforce the view that pulsing is a preferable management policy.

Table 2. Descriptive and Summary Statistics of Alternative Population Control Regimes in a Stochastic Environment

α	0.35	0.40	0.50
CTF policy ($\bar{X} = 100$)			
Steady-state objective	100	100	100
Mean NPV	−$906,656	−$700,797	−$552,439
S.D. of NPV	$447,830	$291,506	$190,147
Mean stock level	141.1	117.1	102.6
Proportion of years with culling	95.1%	92.3%	88.1%
Average kill on culling years	48.1	42.9	40.5
FOC MRAP (Steady state = 126)			
Steady-state objective	126	126	126
Mean NPV	−$923,166	−$692,818	−$525,930
S.D. of NPV	$455,010	$302,757	$200,613
Mean stock level	161.2	139.3	125.5
Proportion of years with culling	94.2%	91.5%	87.6%
Average kill on culling years	54.3	50.2	48.3
Deterministic Policy (X^*)			
X^*	113	109	117
Mean NPV	−$911,261	−$622,399	−$350,689
S.D. of NPV	$465,548	$314,680	$208,510
Mean stock level	169.6	139.6	118.4
Proportion of years with culling	81.5%	71.0%	50.4%
Average kill on culling years	65.2	64.7	79.6
Mode $\hat{X}$			
Mode of the distribution	88	100	117
Mean NPV	−$878,964	−$620,139	−$350,689
S.D. of NPV	$456,971	$310,305	$208,510
Mean stock level	146.3	130.8	118.4
Proportion of years with culling	84.6%	72.1%	50.4%
Average kill on culling years	55.7	60.3	79.6
Expected NPV maximizer			
EV maximizer	86	105	116
Mean NPV	−$878,455	−$619,187	−$350,375
S.D. of NPV	$456,768	$312,085	$208,774
Mean stock level	144.4	135.9	117.5
Proportion of years with culling	84.9%	71.6%	50.5%
Average kill on culling years	54.9	62.8	78.9
Optimal barrier policy			
Mean $\hat{X}$	85.8	102.1	122.3
S.D.	27.0	24.7	34.1
Mean NPV of optimal barrier	−$855,806	−$594,474	−$310,059
Mean stock level	147.2	133.3	119.4
Proportion of years with culling	84.0%	71.0%	49.3
Average kill on culling years	56.0	61.7	80.6

More importantly, they reassert the importance of relaxing the binding constraint on the proportion of the herd that can be harvested in a single culling operation.

Concluding Remarks

The results suggest that managing urban animal populations for a steady state can lead to inefficiencies and reduced community welfare. When constrained by the urban setting in which animal control activities must take place, the optimal management of the Irondequoit deer herd dictates that authorities harvest as many animals as safety constraints will allow, when (and only when) the stock exceeds an endogenously determined threshold level X^*. This constrained policy is a specific case of the unconstrained $(\tilde{x}, X^*)$ regime in which the animal population is immediately reduced to $\tilde{x}$ whenever it exceeds X^*. This result was shown to be optimal for a wide range of policy-relevant parameter values. It empirically extends Jacquette's results to situations where the animal population provides benefits in addition to imposing damage, and to cases where nonlinear and stock-dependent growth is either deterministic or disturbed by random shocks.

The negative impact of culling on future harvesting costs is the primary cause of optimal pulsing. In wildlife management situations where a herd is confined and a significant portion of the stock can be removed in a single season, the dynamic cost effect can easily create intertemporal economies of scale making pulsing optimal. For larger resource pools such as fisheries, pulsing has been historically viewed as a technical curiosity (Clark, 1990) despite Hannesson's empirical finding that a fishing fleet harvesting from several distinct fish stocks could increase its profits by cyclically depleting individual stocks and allowing them to recover. Although it remains technically difficult in many fisheries to harvest a large proportion of the fish stock in a given year, recent decades have brought dramatic improvements in fishing technology and increased the capacity of fishers to rapidly deplete fish stocks. This potentially exacerbates the dynamic cost effect and increases the likelihood that pulsing may now be optimal in more instances than earlier thought.

The ability of wildlife managers to harvest a large proportion (if not all) of a well-defined wildlife population confined to an urban area or a specific and accessible natural area, the presence of fixed set-up costs, and an inverse relationship between the population level and the marginal cost of harvesting are the central factors making pulsing superior to traditional steady-state management. However, the fact that the marginal cost of harvesting remained constant throughout the culling period (despite real-time declines in the population) is an important feature giving rise to pulsing in Irondequoit. This feature is largely attributable to the baiting strategy used in the culling of deer. Pulsing is therefore less likely to be optimal in situations where fixed costs are low (conferring little economies of scale), or when harvesting instantaneously results in an increase in marginal harvesting costs (making little difference whether the next animal is taken from a population that was high or low at the beginning of the culling period). Therefore, pulsing is less likely to offer advantages when animals are elusive and cannot easily and effectively be rounded up or baited (e.g., coyote, alligator, raccoon). On the other hand, it is more likely to be efficient when the animals are in a well-defined or confined area, and where fixed set-up or disutility costs are a significant portion of total control costs.

The extent to which a herd of animals is isolated from neighboring herds may also play an important role in defining which policy pulsing, or MRAP is more efficient. In Irondequoit, geographical barriers prevent rapid in-migration by other deer in the aftermath of culling and extend the benefits of a large cull over several years. In the absence of such barriers, deer from adjacent herds may migrate to the management area to take advantage of the sudden decrease in deer density that follows culling. In this situation, benefit maximization may require annual culling.

When compared to steady-state regimes, the magnitude of the welfare gains that can be achieved with cyclical management depends critically on the constraint on harvesting (α) imposed by patterns of land development and safety concerns. The more stringent the constraint, the smaller the ability of managers to rapidly reduce the population and the smaller will be the advantage of pulsing over alternative approaches. In Irondequoit, the current level of restrictions make for modest potential gains from pulsing. The decision to restrict culling to the Durand-Eastman park was made principally for political reasons and other areas exist where culling could be safely conducted. Expanding the operation to those areas would allow a partial relaxation of the α constraint, increase the number of deer removed per cull, reduce the frequency of culling, and increase the net value of the herd to Irondequoit residents. In general, however, more complex situations where the human population, wildlife population, benefits and damage vary systematically across a landscape, and where culling operations in different areas affect parts of the herd differently than others, it may be necessary to specify a more sophisticated spatial model of deer management.

Two other phenomena could eliminate the potential advantages of cyclical harvesting. First is the presence of transactions costs associated with switching from culling on a certain year to not-culling the next or vice versa. Such costs may take the form of strong public opposition to the resumption or interruption of culling, or from the need to debate the issues every time new approvals or budgets must be sought in order to resume culling. Sufficiently large switching costs could eliminate the gains from cyclical management and return the optimality to a steady-state regime. In continuous time problems, such transactions costs have been shown to eliminate chattering (Romer).[13]

Finally, the disutility associated with lethal control methods also holds the potential to eliminate pulsing, but only if the disutility is sufficiently large to make unabated natural population growth a better alternative than culling. Interestingly, we have found that introducing a wide range of disutility costs results in optimal constrained cycles with lower minimum and average populations and, in the unconstrained model, they may lead to a single pulse to extirpate the population. This strategy efficiently eliminates the future disutility of culling. Although nonlethal methods appear to be considerably more expensive, their adoption may be warranted in many instances for ethical reasons. Yet, their high set-up costs and the likelihood of stock-dependent variable costs preserve for nonlethal methods the conditions that give rise to optimal pulsing. Considerations should therefore be given to pulsing in the development of both lethal and nonlethal wildlife management plans.

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[13] Chattering is an infinitely small pulse taking place at infinitely small intervals in certain types of continuous-time dynamic problems. Problems in which chattering emerges do not actually have a mathematical solution since a smaller and more rapid chatter can always be found. Transactions costs may eliminate the chatter and restore the existence of a finite pulsing or steady state solution. See Romer for details.

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