

## MARKET POWER, PRICE ADJUSTMENT, AND INFLATION\*

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We study a monetary search economy in which endogenous fluctuations in market power driven by changes in consumers' search intensity determine the extent of price adjustment to movements in productivity and the money growth rate. A calibrated version of the economy exhibits countercyclical fluctuations in markups and is consistent with the observed incomplete response of nominal prices to cost movements associated with productivity fluctuations and to changes in the money growth rate. Furthermore, a higher average rate of inflation results in a lower average markup and increases the sensitivity of prices to fluctuations in either productivity or money growth.

### 1. INTRODUCTION

In this article, we study a monetary search economy in which endogenous fluctuations in market power driven by changes in consumers' search intensity determine the extent to which real and nominal prices adjust to random fluctuations in productivity and the money growth rate. A calibrated version of our economy displays both incomplete pass-through of changes in production costs to prices and incomplete adjustment of nominal prices to changes in the money growth rate. The economy also exhibits lower average markups and increasing price sensitivity to both productivity and money growth fluctuations the higher is the average rate of inflation.

Empirical studies suggest that the responsiveness of nominal prices to various shocks is incomplete, varies over time, and is positively related to the average rate of inflation. Taylor (2000), for example, argues that the response of nominal prices to changes in costs has declined with the rate of inflation over time in the United States and other developed countries. Similarly, Gagnon (2009) documents a positive relationship between inflation and the magnitude of price changes for individual goods in Mexico, whereas Nakamura and Steinsson (2008) observe a positive relationship between inflation and the frequency of price increases in the United States. Leibtag et al. (2007) document incomplete pass-through of cost changes to coffee prices in the United States. In addition, Choudhri and Hakura (2006), Campa and Goldberg (2005), Devereux and Yetman (2002), and others present evidence that pass-through of nominal exchange rate movements to consumer prices is increasing in the average rate of inflation.

A large literature explores the effects of incomplete price adjustment in models with explicit nominal rigidities.<sup>2</sup> For the most part, the source of nominal rigidity is of secondary concern in this literature—price changes are typically assumed to be subject to costs and/or frequency

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<sup>2</sup> Some important sources that present key results and describe the literature are Goodfriend and King (1997), Clarida et al. (1999), and Woodford (2003).

limitations. In contrast, we focus on the economic forces that determine the extent of price adjustment to shocks. Similarly, whereas most of this literature places little emphasis on the trend rate of inflation, focusing on dynamics in a neighborhood of a constant (often zero) inflation steady-state, we consider explicitly the effect of the average rate of inflation on the degree of market power in the economy and the resulting responsiveness of prices to shocks.

We embed the price-posting structure of Burdett and Judd (1983) in a general equilibrium environment similar to models of Shi (1999) and Head and Shi (2003) in which money functions as a medium of exchange in the tradition of Kiyotaki and Wright (1993). Head and Kumar (2005) study the welfare costs of trend inflation in a similar but nonstochastic environment. In their model, the degree of price dispersion in equilibrium depends on the average rate of inflation. In this article, uncertainty is introduced, and our focus is on the response of prices to random shocks. Here, both the average degree of price dispersion and its response to shocks are key factors determining both market power and price adjustment in equilibrium.

In our economy, the adjustment of prices to shocks is determined by the combination of two opposing effects. First, for a fixed degree of search intensity by consumers, an increase of either production costs or the money growth rate is passed through differentially to consumer prices by sellers pricing in different regions of the price distribution, resulting in greater dispersion of prices. Second, increased dispersion raises the gains to search, inducing a larger fraction of buyers to observe more than one price. This reduces sellers' market power and limits the extent to which prices rise. The adjustment of prices to either type of shock is incomplete if the response of search intensity and market power is sufficiently strong.

The relative strength of the conflicting forces depends on the average rate of inflation through its effect on market power. This mechanism is plausible, as several studies have presented evidence that the average markup is negatively related to the trend rate of inflation (Gali and Gertler, 1999; Banerjee et al., 2007; and others). In our economy, when inflation is low, a relatively large fraction of buyers observes only a single price. In this case, an increase of either costs or money growth generates a large increase in price dispersion and thus induces a strong increase in search intensity. The resulting reduction in sellers' market power substantially limits the adjustment of prices in response to these shocks, causing the effect of the shock on the average price level to be relatively small. As the rate of trend inflation rises, the share of buyers observing more than one price rises and the average markup falls. A given shock has a smaller effect on price dispersion and so the response of search intensity diminishes. As a result, prices are more responsive to either type of shock than at lower rates of inflation.

Quantitatively, in our calibration we find that the pass-through of cost shocks is of a degree broadly consistent with the empirical studies noted above. Similarly, nominal price adjustment to monetary shocks is incomplete and the extent of price adjustment to either type of shock is positively related to the average rate of inflation. Prices respond asymmetrically to a change in either costs or money growth, with increases resulting in larger responses than reductions. The economy also predicts a countercyclical average markup, another finding consistent with previous empirical work.

The relationship between average inflation and the extent of price adjustment in response to shocks is also studied in the literature on state-contingent pricing. We obtain results similar in several respects to those of this literature in spite of the fact that we impose no exogenous nominal rigidity. For example, state-contingent pricing models with menu costs (e.g., Dotsey et al., 1999) predict the price level to be more responsive to shocks at higher inflation, as a larger share of firms will find it profitable to change prices in a given period the higher the rate of inflation. Also, asymmetric responses of prices to positive and negative shocks are predicted by the model of Devereux and Siu (2007).

Our work is also related to research focusing on price adjustments in environments with search frictions. In a search economy with menu costs, Benabou (1988, 1992) finds a negative relationship between inflation and the degree of market power. Craig and Rocheteau (2008) consider the implications of menu costs for the welfare costs of inflation in a search model in which fiat money is essential. They do not, however, consider the adjustment of prices to shocks.

Eden (1994) considers the adjustment of prices to monetary shocks in a model of uncertain and sequential trade. The mechanism by which price stickiness is generated in his model differs from ours, and changes in expected inflation have no effect on real prices. Alessandria (2009) uses a nonmonetary model with a similar form of price determination to study international price differentials. Search intensity of the type we study, however, is constant in his model.

The remainder of the article is organized as follows: Section 2 describes the environment and defines a symmetric Markov monetary equilibrium. Section 3 describes qualitatively the responses of prices to shocks in equilibrium and considers an example with a small number of states. Section 4 presents the calibration and main quantitative results and considers the robustness of these findings to changes in several parameters. Section 5 discusses implications of the results for future work and concludes.

## 2. THE ECONOMY

We extend the economy considered by Head and Kumar (2005) in a number of ways—in particular to allow for aggregate uncertainty. We abbreviate the exposition of technical results that were previously proved in that paper. The Appendix provides details on some of the more complicated extensions required.

*2.1. The Environment.* Time is discrete. There are  $H \geq 3$  different types of households, and there are unit measures of households of each type. A type  $h$  household produces good  $h$  and derives utility from consumption of good  $h + 1$ , modulo  $H$ . Each household is comprised of unit measures of two different types of members: “buyers” and “sellers.” Individual household members do not have independent preferences but rather share equally in household utility.<sup>3</sup>

Members of a representative type  $h$  household who are *sellers* produce good  $h$  in period  $t$  at marginal disutility  $\phi_t > 0$  utils per unit. Production costs are stochastic with  $\phi_t \in \mathcal{P}$ , a finite set. Using  $y_t$  to denote the total quantity of good  $h$  produced by all sellers from this household in period  $t$ , the household’s total period disutility from production is equal to  $\phi_t y_t$ .

Members of this household who are *buyers* observe random numbers of price quotes and may purchase good  $h + 1$  at the lowest price that they observe individually. Let  $q_{kt}$  denote the measure of the household’s buyers who observe  $k \in \{0, \dots, K\}$  price quotes at time  $t$ . The household chooses the probabilities with which buyers observe different numbers of quotes. Since the household contains a unit measure of buyers, the probability of an individual buyer observing  $k$  prices is equivalent to the measure of a household’s buyers who observe  $k$  prices. For each price quote observed, the household pays an information or search cost of  $\mu$  utils. Thus, the household’s total disutility of search in period  $t$  is equal to  $\mu \sum_{k=0}^K kq_{kt}$ .<sup>4</sup>

A representative household seeks to maximize the expected discounted sum of its period utility over an infinite horizon.

$$(1) \quad U = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( u(c_t) - \phi_t y_t - \mu \sum_{k=0}^K kq_{kt} \right) \right].$$

Here,  $u(c_t)$  denotes consumption utility where  $c_t$  is the total purchases of good  $h + 1$  by the household’s buyers. We assume that  $u(\cdot)$  is strictly increasing and strictly concave with  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ .

Since a type  $h$  household produces good  $h$  and consumes good  $h + 1$ , a double coincidence of wants between members of any two households is impossible. Moreover, it is assumed that

<sup>3</sup> We denote the economy-wide per household level of variable  $x$  with a tilde,  $\tilde{x}$ , and the individual household level without a tilde. Also, we use uppercase to denote nominal variables and lowercase to denote “real” variables, by which we mean nominal values divided by the per household money stock.

<sup>4</sup> The maximum number of price quotes observed,  $K$ , is unimportant, as we will show later. We may think of  $K$  as being chosen by the household or of the household as setting  $q_{kt} = 0$  for all  $k \geq K$  and for all  $t$ .

households of a given type are indistinguishable and that individual household members are anonymous and cannot be relocated in the future following an exchange. Since consumption goods are nonstorable, direct exchanges of goods cannot be mutually beneficial. Instead, exchange is facilitated by the existence of perfectly durable and intrinsically worthless *fiat money*. A type  $h$  household may acquire fiat money by having its producers sell output to buyers of type  $h - 1$  households. This money may then be exchanged for consumption good  $h + 1$  by the household's own buyers in a future period.

In the initial period ( $t = 0$ ) each household is endowed with  $\tilde{M}_0$  units of fiat money. The average money stock across households at time  $t$  is denoted  $\tilde{M}_t$ . At the beginning of each period  $t \geq 1$  each household receives a lump-sum transfer,  $(\gamma_t - 1)\tilde{M}_{t-1}$ , of new units of money from a monetary authority with no purpose other than to change the stock of money over time. The gross growth rate of the average money stock is denoted

$$(2) \quad \gamma_t = \frac{\tilde{M}_t}{\tilde{M}_{t-1}},$$

and  $\gamma_t \in \mathcal{G}$ , a finite set.

Let  $\sigma_t \equiv (\phi_t, \gamma_t)$  be the vector of stochastic variables. We assume that  $\sigma_t$  evolves via a discrete Markov chain with

$$(3) \quad \pi_{jl,km} \equiv \text{Prob}\{\sigma_{t+1} = (\phi_k, \gamma_m) \mid \sigma_t = (\phi_j, \gamma_l)\} \quad \forall t, t + 1; \quad \sigma_t, \sigma_{t+1} \in \mathcal{S},$$

where  $\mathcal{S} \equiv \mathcal{P} \times \mathcal{G}$ . In each period then, the state is given by  $\sigma_t$  and the average money stock,  $\tilde{M}_t$ .

**2.2. The Trading Session.** In describing the optimization problem of a representative household (of any type), it is useful to begin with exchange within a period. At the beginning of period  $t$ , a representative household observes the state of the economy,  $(\tilde{M}_t, \sigma_t)$ , and has post-transfer individual money holdings  $M_t$ .<sup>5</sup> The household chooses the probabilities with which each buyer observes different numbers of price quotes,  $\mathbf{q}_t \equiv \{q_{0t}, \dots, q_{Kt}\}$ . Buyers and sellers then divide for a *trading session* where fiat money is exchanged for goods. We assume that it is not until this trading session begins that the exact number of quotes observed by individual buyers is known. As a result, households have no incentive to treat their members asymmetrically; they distribute money holdings equally to all buyers and issue the same instructions to all buyers and to all sellers.<sup>6</sup>

In the trading session, sellers post prices and buyers decide whether or not to purchase at the posted price. Since trading begins after  $\mathbf{q}_t$  is chosen, we treat the measures of buyers observing particular numbers of price quotes as fixed and return to their determination when we consider households' dynamic optimization problems below. Following trading, buyers and sellers reconvene and the household consumes the goods purchased by its buyers. The sellers' revenue (in fiat money) and any remaining money unspent by the buyers are pooled and carried into the next period. They are then augmented with transfer  $(\gamma_{t+1} - 1)\tilde{M}_t$  to become  $M_{t+1}$ .

With the measures of buyers observing different numbers of prices fixed, the mechanism by which buyers and sellers are matched is similar to the "noisy search" process of Burdett and Judd (1983). Households know the distribution of prices offered by sellers, but individual buyers may purchase only at a price they are quoted by a specific seller in a particular period.<sup>7</sup> Let the distribution of nominal prices posted by sellers of the appropriate type at time  $t$  be described by the cumulative distribution function (*c.d.f.*)  $\tilde{F}_t(P_t)$  on support  $\tilde{\mathcal{F}}_t$ . Given  $\tilde{F}_t(P_t)$ ,

<sup>5</sup> In this subsection, we suppress the economy state vector, as it remains fixed throughout the period.

<sup>6</sup> The optimality of equal treatment of symmetric members by the household is addressed by Peterson and Shi (2004). Here we treat it as an assumption.

<sup>7</sup> We assume that buyers cannot return to sellers from whom they have purchased in the past and instead draw new price quotes from the distribution each period.

the *c.d.f.* of the distribution of the *lowest* price quote received by a buyer at time  $t$  is given by

$$(4) \quad J_t(P_t) = \sum_{k=0}^K \left( \frac{q_{kt}}{1 - q_{0t}} \right) (1 - [1 - \tilde{F}_t(P_t)]^k) \quad \forall P_t \in \tilde{\mathcal{F}}_t.$$

Buyers in a representative household who purchase do so at the lowest price they observe, spending  $X_t(P_t)$  when they pay nominal price  $P_t$ . Each buyer is constrained to spend no more than the money distributed to them at the beginning of the session by the household. Thus, each buyer faces the following expenditure constraint:

$$(5) \quad X_t(P_t) \leq M_t \quad \forall P_t.$$

Because the household contains a continuum of symmetric buyers, it faces no uncertainty with regard to its overall trading opportunities in the trading session of the current period. Realized household consumption purchases in this period are then

$$(6) \quad c_t = (1 - q_{0t}) \int_{\tilde{\mathcal{F}}_t} \frac{X_t(P_t)}{P_t} dJ_t(P_t).$$

Individual sellers produce to meet the demand of the buyers who observe their price and wish to purchase. The expected quantity of goods sold in the current period trading session for a representative seller who posts  $P_t$  is given by

$$(7) \quad y_t(P_t) = \underbrace{\left[ \frac{\hat{X}_t(P_t)}{P_t} \right]}_{\text{quantity per sale}} \underbrace{\left[ \sum_{k=0}^K k \hat{q}_{kt} [1 - \hat{F}_t(P_t)]^{k-1} \right]}_{\text{expected number of sales}}.$$

Here  $\hat{X}_t(P_t)$  is the belief of the household regarding the spending rule of its prospective customers,  $\hat{q}_{kt}$  is their belief regarding the average measure of those buyers observing  $k$  prices, and  $\hat{F}_t(P_t)$  is their belief regarding the distribution of prices posted by their competitors. The expected number of sales in (7) equals the number of observations of the seller's price multiplied by the probability that it is the lowest price observed. Since the household contains a continuum of sellers, it faces no uncertainty with regard to its total sales in the current trading session. These are given by

$$(8) \quad y_t = \int_{\mathcal{F}_t} y_t(P_t) dF_t(P_t),$$

where  $F_t(P_t)$  is the distribution of prices posted by the household's sellers and  $\mathcal{F}_t$  is its support.

Using (6)–(8) we can write the law of motion for a household's money holdings.

$$(9) \quad M_{t+1} = M_t - \underbrace{(1 - q_{0t}) \int_{\tilde{\mathcal{F}}_t} X_t(P_t) dJ_t(P_t)}_{\text{expenditure}} + \underbrace{\int_{\mathcal{F}_t} P_t y_t(P_t) dF_t(p_t)}_{\text{sales revenue}} + \underbrace{(\gamma_{t+1} - 1) \tilde{M}_t}_{\text{transfer}}.$$

A representative household's money holdings going into next period's trading session are  $M_t$  minus the amount spent by its buyers this period plus its sellers' receipts of money plus the transfer received at the beginning of the next period.

We may think of a household as issuing instructions,  $X_t(P_t)$  and  $F_t(P_t)$ , to its buyers and sellers, respectively. The household's gain to having a buyer exchange  $X_t(P_t)$  units of currency for consumption is given by its marginal utility of current consumption,  $u'(c_t)$ , times the quantity

of consumption good purchased,  $X_t(P_t)/P_t$ . The household's cost of this exchange is the number of currency units given up,  $X_t(P_t)$ , times the marginal value of a unit of money in the trading session of the next period, which we denote  $\omega_t$ . Note that  $\omega_t$  is the value to the household of relaxing constraint (9) marginally. Hence, a household's reservation price equals  $u'(c_t)/\omega_t$ .

Since individual buyers are small and the household may not reallocate money balances between them once the trading session has begun, it is optimal for buyers to spend their entire money holdings if the lowest price they observe is below the reservation price and to return with money holdings unspent otherwise.

$$(10) \quad X_t(P_t) = \begin{cases} M_t & \text{for } P_t \leq \frac{u'(c_t)}{\omega_t}, \\ 0 & \text{otherwise,} \end{cases}$$

where it is understood that  $P_t$  is the lowest observed price.<sup>8</sup>

Next, consider price posting by a household's sellers. The expected return measured in utils to the household from having a seller post price  $P_t$  is

$$(11) \quad r_t(P_t) = \underbrace{\left[ \omega_t \hat{X}_t(P_t) - \phi_t \frac{\hat{X}_t(P_t)}{P_t} \right]}_{\text{return per sale}} \underbrace{\sum_{k=0}^K k \hat{q}_{kt} [1 - \hat{F}_t(P_t)]^{k-1}}_{\text{expected number of sales}}.$$

The first term in (11) is the utility value of the currency units obtained minus the disutility of production. Here it is clear that the return to posting a price lower than the marginal cost price,  $P_t^* \equiv \phi_t/\omega_t$ , is negative, and the household will instruct no seller to do so. In addition, the return for posting a price at which no buyer would buy is zero. A household optimizes by instructing its sellers to post prices such that

$$(12) \quad P_t \in \underset{P_t}{\operatorname{argmax}} r_t(P_t) \equiv \mathcal{F}_t.$$

We represent the household's overall price-posting strategy with the distribution of posted prices,  $F_t(P_t)$  on support  $\mathcal{F}_t$ .<sup>9</sup>

**2.3. Dynamic Optimization.** To this point, we have focused on trading within a period, holding fixed the probabilities of a representative household's buyers observing different numbers of prices and taking as given the household's marginal value of a unit of money. We now consider the determination of these variables, assuming throughout that households employ Markov strategies.

At time  $t$ , the state for a representative household is  $(M_t, \tilde{M}_t, \sigma_t)$ . We represent the dynamic optimization problem of such a household by the following Bellman equation:

$$(13) \quad V_t(M_t, \tilde{M}_t, \sigma_t) = \max_{q_t, M_{t+1}, X_t(P_t), F_t(P_t), \mathcal{F}_t} \left\{ u(c_t) - \phi_t y_t - \mu \sum_{k=0}^K k q_{kt} \right. \\ \left. + \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \pi(\sigma_{t+1}, \sigma_t) V_{t+1}(M_{t+1}, \tilde{M}_{t+1}, \sigma_{t+1}) \right\},$$

subject to: (4)–(9),  $q_{kt} \geq 0 \quad \forall \quad k = 0, \dots, K, \quad M_{t+1} \geq 0, \quad X_t(P_t) \geq 0$ , and  $\sum_{k=0}^K q_{kt} = 1$ .

<sup>8</sup> Note that (10) is an application of Lemma 1 in Head and Kumar (2005).

<sup>9</sup> Any configuration of price posting by individual sellers consistent with this distribution will generate the same overall return to the household. For example, all sellers could draw randomly from  $\mathcal{F}_t$  each period. Alternatively, the household could minimize the share of sellers changing their price each period. We discuss the implications of different such rules for price dynamics in Section 4.

The household takes as given the actions of others,  $\hat{y}_t(P_t; \tilde{M}_t, \sigma_t)$ ,  $\hat{X}_t(P_t; \tilde{M}_t, \sigma_t)$ , and  $\hat{q}_t(\tilde{M}_t, \sigma_t)$ ; as well as the distributions of prices posted by both its competitors (households of type  $h$ ) and by producers of its preferred good (type  $h + 1$ ). The value function is written here as time varying because it depends on the distributions of nominal prices, which may be expected to change over time as the money stock grows.

The first-order conditions associated with choice of  $q_{kt}$  for  $k = 0, \dots, K$  are given by

$$(14) \quad u'(c_t)c_t^k \leq \mu k + \xi_t(M_t, \tilde{M}_t, \sigma_t) \quad q_{kt} \geq 0 \quad q_{kt}[u'(c_t)c_t^k - \mu k - \xi_t(M_t, \tilde{M}_t, \sigma_t)] = 0,$$

where  $\xi_t(M_t, \tilde{M}_t, \sigma_t)$  is the multiplier associated with the requirement that the  $q_{kt}$ 's sum to one. Here  $c_t^k$  is the quantity of the consumption good purchased by buyers who observe exactly  $k$  prices and is given by

$$(15) \quad c_t^0 = 0, \quad c_t^k = M_t \int_{\tilde{F}_t} \frac{1}{P_t} dJ_t^k(P_t) \quad \forall k \geq 1,$$

where  $J_t^k(P_t) = 1 - [1 - \tilde{F}_t(P_t)]^k$ . Note that we have made use of the buyers' optimal expenditure rule, (10), in this derivation.

The first-order condition for  $M_{t+1}$  is given by

$$(16) \quad \omega_t(M_t, \tilde{M}_t, \sigma_t) = \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \pi(\sigma_{t+1}, \sigma_t) \left[ \frac{\partial V_{t+1}(M_{t+1}, \tilde{M}_{t+1}, \sigma_{t+1})}{\partial M_{t+1}} \right].$$

For expenditure,  $X_t(P_t)$ , assuming the nonnegativity constraint is slack, we have

$$(17) \quad (1 - q_{0t}) \left( \frac{u'(c_t)}{P_t} - \omega_t(M_t, \tilde{M}_t, \sigma_t) \right) - \lambda_t(P_t; M_t, \tilde{M}_t, \sigma_t) = 0 \quad \forall P_t,$$

where  $\lambda_t(P_t; M_t, \tilde{M}_t, \sigma_t)$  is the Lagrange multiplier on a buyer's expenditure constraint, (5). Finally, we have the envelope condition

$$(18) \quad \frac{\partial V_t(M_t, \tilde{M}_t, \sigma_t)}{\partial M_t} = \int_{\tilde{F}_t} \lambda_t(P_t; M_t, \tilde{M}_t, \sigma_t) dJ_t(P_t) + \omega_t(M_t, \tilde{M}_t, \sigma_t).$$

Equations (14)–(18), together with the buyers' expenditure rule, (10), and the requirement that  $\tilde{F}_t$  satisfy (12), characterize the household's optimal behavior conditional on its money holdings,  $M_t$ , the aggregate state,  $(\tilde{M}_t, \sigma_t)$ , and its beliefs regarding the actions of other households.

**2.4. Equilibrium.** We consider only equilibria that are symmetric and Markov. By symmetric, we mean that in equilibrium all households choose the same probabilities for their buyers to observe different numbers of price quotes, the same distribution of posted prices, and that all have the same consumption, money holdings, and valuation of money.

The equilibria we consider are Markov in that quantities, price quote probabilities, and the distributions of *real prices* (measured as nominal prices divided by average money stock) are time-invariant functions of the aggregate state. We will denote the Markov equilibrium distributions of real prices and their supports with the same notation as those for nominal prices but without a time subscript, i.e.,  $\tilde{F}(\cdot)$ ,  $F(\cdot)$ ,  $J(\cdot)$ ,  $\tilde{\mathcal{F}}(\cdot)$ , and  $\mathcal{F}(\cdot)$ . In this Markov setting, we drop the time subscript where possible and use a prime ( $'$ ) to denote the value of a variable in the next period.

We begin by deriving certain properties that such an equilibrium must have, assuming that one exists. In this article, we do not establish the existence of an equilibrium formally. Rather, we confirm existence by computing equilibria for parameterized versions of the economy.<sup>10</sup>

<sup>10</sup> For a similar economy with no aggregate uncertainty (i.e., in which both costs and money growth are constant), Head and Kumar (2005) establish formally the existence of an equilibrium of the type considered here. Their arguments

If in a symmetric monetary equilibrium (SME) all nominal posted prices at time  $t$  are proportional to  $\tilde{M}_t$ , then there exist distributions of *real* posted prices, which depend only on the state. These distributions are characterized by the following supports and conditional *c.d.f.s*:

$$(19) \quad \tilde{\mathcal{F}}(\sigma) \equiv \{p \equiv P_t / \tilde{M}_t \quad \forall P_t \in \tilde{\mathcal{F}}_t; \quad \forall t \mid \sigma_t = \sigma\}$$

$$(20) \quad \tilde{F}(p \mid \sigma) = \tilde{F}_t(p \tilde{M}_t) \quad \forall p \in \tilde{\mathcal{F}}(\sigma), \quad \forall t \mid \sigma_t = \sigma.$$

We define  $F(p \mid \sigma)$  and  $\mathcal{F}(\sigma)$  similarly. If conditional distributions satisfying (19) exist, then we may think of buyers as observing *real* price quotes and define corresponding conditional distributions of lowest real prices observed in a manner analogous to (4).

$$(21) \quad J(p \mid \sigma) = \sum_{k=0}^K \left( \frac{q_k(\sigma)}{1 - q_0(\sigma)} \right) [1 - [1 - \tilde{F}(p \mid \sigma)]^k].$$

Similarly, if the distributions of posted and transactions prices are time invariant, conditional on  $\sigma$ , then households' nominal money holdings,  $M_t$ , expenditure rule for buyers,  $X_t(P_t)$ , and the support of a household's sellers' posted prices,  $\mathcal{F}_t$ , may be divided by the average money stock to obtain time-invariant conditional *real* counterparts:  $m(\sigma)$ ,  $x(p \mid \sigma)$ , and  $\mathcal{F}(\sigma)$ .

We then have the following definition:

**DEFINITION.** An SME is a collection of time-invariant, individual household choices,  $\mathbf{q}(\sigma)$ ,  $m'(\sigma)$ ,  $x(p \mid \sigma)$ ,  $F(p \mid \sigma)$ ; average expenditure rules  $\tilde{x}(p \mid \sigma)$  and probabilities  $\tilde{\mathbf{q}}(\sigma)$ ; and conditional distributions of posted prices,  $\tilde{F}(p \mid \sigma)$ , such that

- (i) Taking as given the distributions of posted prices,  $\tilde{F}(p \mid \sigma)$ , the average expenditure rule,  $\tilde{x}(p \mid \sigma)$ , and probabilities of buyers observing different numbers of price quotes,  $\tilde{\mathbf{q}}(\sigma)$ , a representative household chooses  $\mathbf{q}_t = \mathbf{q}(\sigma)$ ,  $M_{t+1} = m'(\sigma)\tilde{M}_{t+1}$ ,  $X_t(P_t) = x(p \mid \sigma)\tilde{M}_t$ , and distribution  $F_t(P_t) = F(p \mid \sigma)$  to satisfy the household Bellman equation, (13).
- (ii) Individual choices equal average quantities:  $\mathbf{q}(\sigma) = \tilde{\mathbf{q}}(\sigma)$ ,  $x(p \mid \sigma) = \tilde{x}(p \mid \sigma)$ ,  $F(p \mid \sigma) = \tilde{F}(p \mid \sigma)$ , and individual household money holdings equal the average money stock:  $m(\sigma) = 1$ .
- (iii) Money has value in all states: For all  $\sigma \in \mathcal{S}$ ,  $\tilde{F}(p \mid \sigma) > 0$  for some  $p < \infty$ .

In characterizing an SME for this economy, a key quantity is the sequence of households' marginal valuations of money,  $\{\tilde{\omega}_t\}_{t=0}^{\infty}$ , as this determines the returns to sellers and buyers from transacting at a particular price at a particular point in time. Returning to the household optimization problem and combining (16)–(18), we have

$$(22) \quad \omega_t(M_t, \tilde{M}_t, \sigma_t) = \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \pi(\sigma_{t+1}, \sigma_t) \left[ [1 - q_0(\sigma_{t+1})] u'(c(\sigma_{t+1})) \int_{\tilde{\mathcal{F}}_t} \frac{1}{P_{t+1}} dJ_{t+1}(P_{t+1}) \right. \\ \left. + q_0(\sigma_{t+1}) \omega_{t+1}(M_{t+1}, \tilde{M}_{t+1}, \sigma_{t+1}) \right].$$

In an SME, (6) and (10) imply that average consumption must satisfy

$$(23) \quad \tilde{c}(\sigma_t) = [1 - \tilde{q}_0(\sigma_t)] \tilde{M}_t \int_{\tilde{\mathcal{F}}_t} \frac{1}{P_t} d\tilde{J}_t(P_t) \quad \forall t.$$

may be extended to our stochastic economy by exploiting the continuity of consumption in the parameters governing costs and money growth. We do not do so here, however, because this entails imposing complicated and specific (and economically uninteresting) parameter restrictions governing the degree of variation in costs and money growth across states. In our numerical experiments, we find that equilibria of the type we consider do indeed exist for a wide range of parameters.

Thus, in an SME (22)–(23) implies, for all  $t$ ,

$$(24) \quad \tilde{\omega}_t(\tilde{M}_t, \sigma_t) = \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \pi(\sigma_{t+1}, \sigma_t) \left[ u'(\tilde{c}(\sigma_{t+1})) \frac{\tilde{c}(\sigma_{t+1})}{\tilde{M}_{t+1}} + \tilde{q}_0(\sigma_{t+1}) \tilde{\omega}_{t+1}(\tilde{M}_{t+1}, \sigma_{t+1}) \right].$$

We define the marginal value of a real currency unit,  $\tilde{\Omega}(\sigma) \equiv \tilde{\omega}_t(\tilde{M}_t, \sigma_t) \tilde{M}_t$ , for  $\sigma = \sigma_t$ . and derive<sup>11</sup>

$$(25) \quad \tilde{\Omega}(\sigma) = \beta \sum_{\sigma' \in \mathcal{S}} \frac{\pi(\sigma', \sigma)}{\gamma'} [u'(\tilde{c}(\sigma')) \tilde{c}(\sigma') + \tilde{q}_0(\sigma') \tilde{\Omega}(\sigma')] \quad \forall \sigma \in \mathcal{S}.$$

An SME is associated with a collection of state-contingent values,  $\tilde{\Omega}(\sigma)$ ,  $\sigma \in \mathcal{S}$ .

We now describe briefly several properties of the nonstochastic economy studied by Head and Kumar (2005) that extend to the stochastic environment studied here. Details of the required extensions are in the Appendix. First, here, as established in Proposition 1 (p. 542) of Head and Kumar (2005), if the SME is characterized by some buyers observing one price whereas others observe more than one price, then the distribution of posted prices will exhibit price dispersion necessarily. Second, if an SME exists, then it will be characterized by positive measures of buyers observing exactly one and two prices only, in all states. This is an application of Proposition 8 (Corollary 2, p. 554) of Head and Kumar (2005). Thus, we may associate an SME with probabilities  $\tilde{q}(\sigma)$  of a buyer observing a single price in each state. In equilibrium, this will equal the measure of buyers observing one price with the remaining buyers all observing two prices. Taking these two properties together, we see that any SME must exhibit price dispersion in all states.<sup>12</sup>

Head and Kumar (2005) prove that an SME exists and derive the c.d.f. of posted prices for that equilibrium. Although in this article we do not prove existence formally, it is straightforward to show that if an SME exists, the distribution of real posted prices has the same form as in their nonstochastic economy. For all  $\sigma$ , these distributions are characterized by

$$(26) \quad \tilde{F}(p | \sigma) = \frac{\left[ \tilde{\Omega}(\sigma) - \frac{\phi}{p} \right] [2 - \tilde{q}(\sigma)] - \left[ 1 - \frac{\phi}{u'[\tilde{c}(\sigma)]} \right] \tilde{\Omega}(\sigma) \tilde{q}(\sigma)}{\left[ \tilde{\Omega}(\sigma) - \frac{\phi}{p} \right] 2[1 - \tilde{q}(\sigma)]}$$

with connected supports,  $\tilde{\mathcal{F}}(\sigma) = [p_\ell(\sigma), p_u(\sigma)]$ , where

$$(27) \quad p_\ell(\sigma) = \frac{[2 - \tilde{q}(\sigma)] \phi p_u(\sigma)}{2[1 - \tilde{q}(\sigma)] \tilde{\Omega} p_u(\sigma) + \tilde{q}(\sigma) \phi} \quad \text{and} \quad p_u(\sigma) = \frac{u'[\tilde{c}(\sigma)]}{\tilde{\Omega}(\sigma)}.$$

Using (25)–(26), it is straightforward to derive expressions for the conditional densities of both posted and transactions prices and to show that they are monotonically decreasing in all states.

<sup>11</sup>  $\tilde{\Omega}(\sigma)$  may be thought of as marginal value of a currency unit normalized by the number of such units in circulation.

<sup>12</sup> This result differs from that of Burdett and Judd (1983), who find that there is always an equilibrium in which all buyers observe exactly one price and all sellers charge the monopoly price. For a discussion of the reasons for this difference between their economy and ours, see the Appendix.

Finally, we can show that the optimal choice of  $q(\sigma)$  for an individual household is similar to that derived in Head and Kumar (2005).

$$(28) \quad q(\sigma) = \begin{cases} 0 & \text{if } \mu < \mu_L(\sigma) \equiv u'(c^2(\sigma))[c^2(\sigma) - c^1(\sigma)] \\ \frac{u'^{-1}\left(\frac{\mu}{c^2(\sigma) - c^1(\sigma)}\right) - c^2(\sigma)}{c^1(\sigma) - c^2(\sigma)} & \text{if } \mu_L(\sigma) \leq \mu \leq \mu_H(\sigma) \\ 1 & \text{if } \mu > \mu_H(\sigma) \equiv u'(c^1(\sigma))[c^2(\sigma) - c^1(\sigma)], \end{cases}$$

where  $c^k(\sigma)$  for  $k = 1, 2$  are defined in (15). In (28)  $\mu_L(\sigma)$  and  $\mu_H(\sigma)$  are state contingent cutoff levels for search costs. In state  $\sigma$ , if the search cost is *below*  $\mu_L(\sigma)$ , then the household will choose to have *all* of its buyers observe more than one price (i.e.,  $q(\sigma) = 0$ ). Similarly, if  $\mu > \mu_H(\sigma)$ , the household will choose to have *no* buyer observe a second quote (i.e.,  $q(\sigma) = 1$ ). As discussed above, both of these cases are inconsistent with the existence of an SME, so we require  $\mu \in [\mu_L(\sigma), \mu_H(\sigma)]$  for all  $\sigma$ .

From (28) it is clear that in order for an SME to exist, the search cost parameter,  $\mu$ , must be consistent with an interior value for  $q(\sigma)$  in all states, i.e.,  $\mu \in [\bar{\mu}_L, \underline{\mu}_H]$ , where  $\bar{\mu}_L$  and  $\underline{\mu}_H$  denote the maximal lower and minimal upper cutoff levels for search costs across states. Similarly, existence requires that expected inflation exceeds the discount factor,  $\beta$ , in all states. These requirements restrict the range of variation in both  $\phi$  and  $\gamma$  across states that are not general as they depend upon the other parameters of the economy and on functional forms. For this reason, we do not approach the existence of equilibrium formally. We find, however, that these restrictions are not binding in our benchmark calibration or in any of the parameterizations of the economy we consider in our robustness checks. Like Head and Kumar (2005), we also do not prove that the SME is unique. For any of the parameterizations that we consider, however, we are unable to find numerically more than one equilibrium.

### 3. EQUILIBRIUM PRICE RESPONSES

We now consider qualitatively the behavior of equilibrium prices and markups in response to stochastic fluctuations in productivity and the growth rate of the money stock.

3.1. *Definitions.* We define the *real price level* in state  $\sigma_t$  in an SME as the average real transaction price.<sup>13</sup>

$$(29) \quad \bar{p}(\sigma_t) \equiv \int_{\mathcal{F}(\sigma_t)} p(\sigma_t) dJ(p | \sigma_t).$$

Similarly, we define the *nominal price level* at time  $t$  (in state  $\sigma_t$ ) as the average nominal transaction price.

$$(30) \quad \bar{P}_t \equiv \int_{\mathcal{F}_t} P_t dJ_t(P_t) = \tilde{M}_t \bar{p}(\sigma_t).$$

The nominal price level is not stationary because of money growth and thus is written as a function of time. We define the *inflation rate* as the net growth rate of the nominal price level.

$$(31) \quad I_t \equiv \left( \frac{\bar{P}_t - \bar{P}_{t-1}}{\bar{P}_{t-1}} \right) \times 100.$$

<sup>13</sup> We focus on *transactions* instead of *posted* prices. We do this because changes in the former more accurately signal the quantitative effects of shocks on output, consumption, and welfare. Average transactions and posted prices respond similarly to both productivity and money growth shocks.

*Real* and *nominal marginal cost*, respectively, in state  $\sigma_t$  are given by

$$(32) \quad mc(\sigma_t) \equiv \frac{\phi_t}{\Omega(\sigma_t)} \quad \text{and} \quad MC_t \equiv mc(\sigma_t) \tilde{M}_t.$$

Finally, we denote the *markup* at time  $t$  as

$$(33) \quad MU_t \equiv \left( \frac{\bar{P}_t}{MC_t} - 1 \right) \times 100.$$

Consider first movements in the nominal price level in response to changes in nominal marginal costs caused by changes in productivity,  $\phi$ , only.<sup>14</sup> We define the following measure of *cost pass-through* between state  $\sigma_{t-1} = (\phi_i, \gamma_k)$  and state  $\sigma_t = (\phi_j, \gamma_k)$ , which share the same money creation rate,  $\gamma_k$ :

$$(34) \quad \theta_{ik,jk} \equiv \left[ \frac{\frac{\bar{P}_t}{\bar{P}_{t-1}} - 1 - (\gamma_k - 1)}{\frac{MC_t}{MC_{t-1}} - 1 - (\gamma_k - 1)} \right] = \left[ \frac{\frac{\bar{p}(\phi_j, \gamma_k) - 1}{\bar{p}(\phi_i, \gamma_k)} - 1}{\frac{mc(\phi_j, \gamma_k) - 1}{mc(\phi_i, \gamma_k)} - 1} \right].$$

Now, when  $\theta_{ik,jk} = 1$ , we say that cost pass-through is *complete*. This occurs when the markup does not change in response to a movement from  $\phi_i$  to  $\phi_j$  holding  $\gamma_k$  constant. If  $\theta_{ik,jk} < 1$ , we say that cost pass-through is *incomplete*. In this case, the markup necessarily moves in the opposite direction of the change in  $\phi$ . Finally, pass-through is said to be *more than complete* when  $\theta_{ik,jk} > 1$ .

Typically, we are interested in the average rate of cost pass-through in equilibrium, which we denote  $\bar{\theta}$ . We measure this by weighting the  $\theta_{ik,jk}$ 's by the frequencies of possible changes in  $\phi$ , conditional on a change occurring *without* a simultaneous change in the money creation rate:

$$(35) \quad \bar{\theta} = \sum_{k \in \mathcal{G}} \sum_{i \in \mathcal{P}} \bar{\pi}_{ik} \left[ \frac{\sum_{j \in \mathcal{P}, j \neq i} \pi_{ik,jk} \theta_{ik,jk}}{\sum_{h \in \mathcal{P}, h \neq i} \pi_{ik,hk}} \right],$$

where  $\bar{\pi}_{ik}$  is the unconditional probability with which state  $(\phi_i, \gamma_k)$  occurs.

We also introduce the following measure of the *real price elasticity* to quantify the extent to which fluctuations in monetary growth affect real transaction prices:

$$(36) \quad \chi_t \equiv \frac{\left[ \frac{\bar{p}(\sigma_t) - \bar{p}(\sigma_{t-1})}{\bar{p}(\sigma_{t-1})} \right]}{\left[ \frac{\gamma_t - \gamma_{t-1}}{\gamma_{t-1}} \right]}.$$

Note that  $\chi_t$  is defined only when  $\gamma$  changes. If changes in the money growth rate have no effect on real prices (i.e.,  $\chi_t = 0$ ) then nominal prices change in proportion to the rate of money creation and inflation must equal net money growth. If  $\chi_t < 0$ , then movements in the inflation rate are smaller than the underlying change in the growth rate of money and we say that nominal price

<sup>14</sup> From (32) it can be seen that in our economy nominal marginal costs change over time due both to exogenous movements in  $\phi$  and to changes in  $\Omega(\sigma)$  that result from the response of households' willingness to produce in exchange for money when either  $\phi$  or  $\gamma$  changes. We focus on pass-through of cost changes during state transitions in which *only*  $\phi$  changes because these are comparable to the exogenous cost changes examined in the empirical literature on exchange rate and cost pass-through.

adjustment is incomplete (or nominal prices are “sticky”). In this case, a stochastic increase in the money creation rate lowers real prices and raises consumption. In contrast, if  $\chi_t > 0$  nominal price adjustment is more than complete, and an increase in the money growth rate raises real prices and lowers consumption. Below, we report the average real price elasticity, denoted  $\bar{\chi}$ , which is calculated in an analogous manner to  $\bar{\theta}$  in equation (35).

3.2. *An Illustrative Numerical Example.* We now present a numerical example to illustrate briefly the mechanisms that lead to incomplete cost pass-through and nominal price stickiness in equilibrium. This example is meant to be illustrative only; we turn to quantitative analysis of the economy in the next section. We restrict attention to constant relative risk-aversion preferences.

$$(37) \quad u(c) = \frac{c^{1-\alpha} - 1}{1-\alpha} \quad \alpha \geq 0,$$

For this example, we set  $\alpha = 1.5$ ,  $\beta = 0.99$ ,  $\mu = 0.00293$ ,  $\bar{\gamma} = 1.012$ , and  $\bar{\phi} = 0.1$ . These values imply that, in the nonstochastic economy, we have approximately 82% of households observing one price, an average markup of transaction prices over marginal cost equal to 10%, and annual inflation equal to 4.89%. We first examine movements in  $\phi$  alone so as to focus on cost pass-through.

Suppose that  $\phi$  follows a two-state Markov process with  $\phi \in \{\phi_L, \phi_H\} \equiv \{0.095, 0.105\}$ , whereas  $\gamma$  is fixed at  $\bar{\gamma}$ . Let the transition matrix for  $\phi$  be symmetric with persistence parameter equal to 0.95 (the persistence of productivity shocks in Cooley and Hansen, 1989). In this example, cost pass-through for an increase in  $\phi$  from  $\phi_L$  to  $\phi_H$  equals 0.53 whereas pass-through for a fall in  $\phi$  from  $\phi_H$  to  $\phi_L$  equals 0.51. Thus, cost pass-through is incomplete as prices change by less than marginal cost in response to movements in  $\phi$ .

In order to understand why pass-through is incomplete, consider a transitory increase in  $\phi$  from  $\phi_L$  to  $\phi_H$ . First, we isolate the effect of an increase in  $\phi$  on buyers' and sellers' willingness to purchase and produce at a given price by examining price and cost responses in an economy in which search intensity is fixed at its equilibrium level in the low  $\phi$  state, i.e., at  $q_L = 0.86$ . The average real transaction price and marginal cost responses in such an economy are depicted in Panel A of Figure 1, which graphs the growth rates of these variables when  $\phi$  temporarily increases to  $\phi_H$  in period two. This figure shows that in response to this increase, the average price increases by 7.71% whereas marginal cost increases by only 6.70%. So, with fixed search intensity cost pass-through is more than complete.

The increase in the average price is associated with an increase in price dispersion as measured by either  $p_u/p_l$  (which rises by 0.24%) or by the coefficient of variation of the posted prices (which increases by 7.5%). This increase in dispersion emanates from properties of the price distribution. In particular, since the density of posted prices is monotonically decreasing, the households' potential expected loss in sales as a result of a price increase by an individual seller is decreasing in the posted price. Thus, the household increases its relatively high prices by more than its relatively low ones, and price dispersion increases.

Increased price dispersion raises the gains to search as it increases the ratio of expected consumption by buyers observing two prices to those of buyers observing only one. In response, households increase search intensity by lowering the fraction of buyers who observe only one price to  $q_H = 0.79$ . From (26), we see that sellers respond to this reduction in market power by posting lower prices with a higher probability, lowering the average real transaction price and reducing cost pass-through. Panel B of Figure 2 depicts the full price and marginal cost response to the transitory increase in  $\phi$  and shows that cost pass-through is incomplete as a result of the search intensity response.

Finally, note that incomplete pass-through is associated with a reduction in the average markup when  $\phi$  rises. Of course, the increase in  $\phi$  also generates a reduction in output, implying that cost shocks in our economy generate pro-cyclical movements in the average markup.

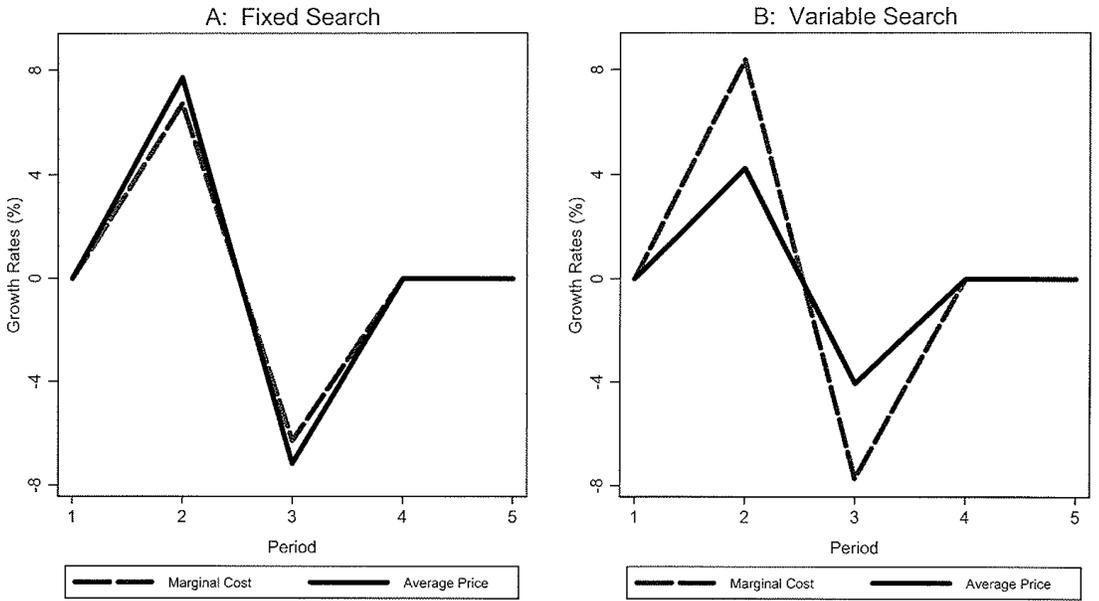


FIGURE 1

TRANSITORY COST INCREASE

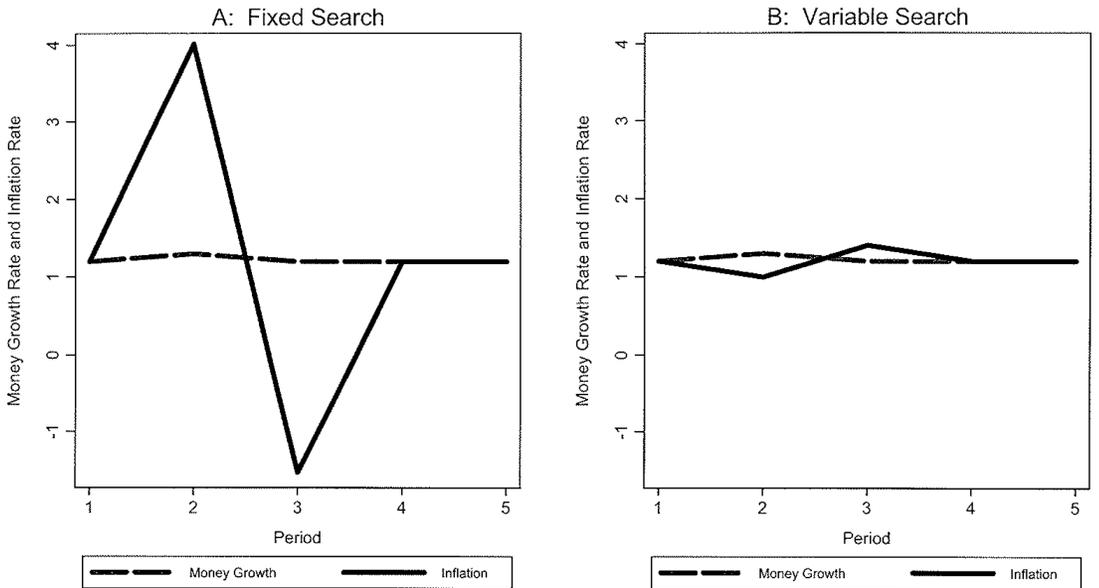


FIGURE 2

TRANSITORY MONEY GROWTH INCREASE

We now examine the effects of movements in the money growth rate in this example. Suppose that  $\gamma$  follows a two-state Markov process with  $\gamma \in \{\gamma_L, \gamma_H\} \equiv \{1.011, 1.013\}$ , whereas  $\phi$  is fixed at  $\bar{\phi}$ . Let the transition matrix for  $\gamma$  be symmetric with persistence parameter equal to 0.5 (also taken from Cooley and Hansen, 1989). In this example, the real price elasticity when  $\gamma$  increases from  $\gamma_L$  to  $\gamma_H$  equals  $-1.51$ , whereas a fall in  $\gamma$  generates a real price elasticity of  $-1.52$ , implying that nominal prices are “sticky” in that inflation does not fully adjust to the change in the money supply.

In order to demonstrate why this occurs, we again separate the responses of prices with search intensity fixed from the full general equilibrium response. Consider an increase in  $\gamma$  from  $\gamma_L$  to  $\gamma_H$  with search intensity fixed at  $q_L = 0.83$ . The increase in  $\gamma$  raises expected future money growth, lowering the marginal value of money,  $\Omega$ . This causes sellers to post higher prices, increasing the average nominal transaction price by 4.0%. Since the net increase in the money supply is 1.3%, the average real transaction price rises by 2.7%. Hence, in the absence of a search intensity response, nominal price adjustment due to an increase in the money growth rate is *more than* complete.<sup>15</sup> Panel A of Figure 2 illustrates this effect by depicting a transitory rise in the net money growth rate in period two and the resulting relatively large increase in inflation when search intensity is held fixed.

For similar reasons as in the case of a cost increase (discussed above), the response of prices with search intensity held fixed results in increased price dispersion ( $p_u/p_\ell$  rises by 0.61% and the coefficient of variation of posted prices by 19.22%). As in the case of a cost increase, this raises the return to search and induces households to increase search intensity. Sellers post relatively low prices with higher probability, reducing the adjustment of the nominal price level. Panel B of Figure 2 depicts the full equilibrium response of inflation to the transitory increase in  $\gamma$  and demonstrates that the average nominal transaction price rises by only 1.0% in response to a net increase in the money supply of 1.3%. In this case, the average real transaction price falls.

Finally, note that when  $\gamma$  rises from  $\gamma_L$  to  $\gamma_H$ , the average markup falls whereas output rises due to the reduction in the average real transaction price. Thus, money growth shocks in our economy may generate countercyclical movements in the average markup as a result of the response of search intensity.

#### 4. QUANTITATIVE ANALYSIS

In the previous section, we used a simple numerical example to demonstrate that endogenous search intensity is the mechanism generating both incomplete cost pass-through and incomplete nominal price adjustment in response to changes in the growth rate of money. In order to analyze these effects quantitatively, we now consider a calibrated version of our economy. We begin with a benchmark calibration and then investigate the robustness of our findings to changes in economy parameters.

**4.1. Benchmark Calibration.** We set the discount factor,  $\beta$ , equal to 0.99, consistent with an annual real interest rate equal to 4% when each period is taken to represent one quarter. We use the utility function given by equation (37) and set  $\alpha = 1.5$ , a value within the range typically used in calibrated macroeconomic models. We consider variation in both of these parameters in our analysis of robustness below.

Given  $\beta$  and  $\alpha$ , we choose the search cost parameter to achieve an average markup of transactions prices over marginal cost of 10%. This value for the markup is within the range of markups estimated for the U.S. economy (see Basu and Fernald, 1997, or Bowman, 2003, for example). Given that we normalize the average value of the cost parameter to 0.1, this requires a value of  $\mu \approx 0.0028$ .<sup>16</sup> In the sensitivity analysis below, we consider both different average markups and different levels of search cost.

We now turn to the parameters that govern the stochastic process for costs and money growth in our economy. We use M1 in the United States as our measure of the money supply from 1959 to 2007 at quarterly frequencies. The quarterly gross growth rate of M1, our measure of

<sup>15</sup> This is similar to the effect of a persistent money growth shock in a flexible price real business cycle model with money introduced either directly into the utility function or through a cash-in-advance constraint as in Cooley and Hansen (1989).

<sup>16</sup> This implies that the marginal search cost is 2.8% of marginal production cost. This value seems not unreasonable given the results of Hong and Shum (2006), who estimate marginal search costs in a structural model of nonsequential search based on the model of Burdett and Judd (1983). Their estimates imply average search costs ranging from 2.64 to 5.52% of marginal selling costs.

TABLE 1  
BENCHMARK PARAMETERS AND RESULTS

<i>Parameter Values</i>	
$\beta = 0.99$	Discount Factor
$\alpha = 1.5$	Coefficient of Relative Risk Aversion
$\mu = 0.0028275$	Search Cost
$\bar{\gamma} = 1.012$	Mean of Money Growth Rate
$\sigma_\gamma = 0.0136$	Standard Deviation of Money Growth Rate
$\rho_\gamma = 0.45$	Autocorrelation of Money Growth Rate
$\bar{\phi} = 0.1$	Mean of Production Disutility
$\sigma_\phi = 0.0011$	Standard Deviation of Production Disutility
$\rho_\phi = 0.70$	Autocorrelation of Production Disutility
$\rho_{\gamma,\phi} = -0.08$	Contemporaneous Correlation of Money Growth Rate and Production Disutility
<i>Results</i>	
$\bar{\theta} = 0.21$	Average Cost Pass-Through
$\bar{\chi} = -1.26$	Average Real Price Elasticity
$\bar{q} = 0.81$	Average Fraction of Buyers Observing One Price
$\bar{MU} = 10\%$	Average Markup
$\bar{I} = 4.98\%$	Average Annual Inflation Rate
$\sigma_I = 1.56\%$	Standard Deviation of Quarterly Inflation
$\rho_I = 0.24$	Autocorrelation of Quarterly Inflation
$\rho_{\gamma,mu} = -0.90$	Correlation between Output and Markup

$\gamma_t$ , over this time period ranged between 0.982 and 1.069 with an average growth rate equal to 1.012 (implying an annual average net growth rate of 4.92%). We measure productivity,  $1/\phi$ , as seasonally adjusted quarterly real average output per hour from 1959 to 2007. In particular, we calculate  $\phi_t$ , as the inverse of output per hour in the data, normalize the mean of the series to 0.1, and, following the procedure employed by Shimer (2005), Hornstein et al. (2005), and Hagedorn and Manovskii (2008), use deviations from Hodrick–Prescott trend with smoothing parameter 1600.

We estimate the following continuous-valued process for the evolution of  $\sigma_t \equiv (\gamma_t, \phi_t)$ ,

$$(38) \quad \sigma_t = A\sigma_{t-1} + B\epsilon_t,$$

as a restricted first-order vector auto-regression with  $\epsilon \sim N(0, I)$ . We then approximate this continuous process with a discrete 49-state Markov chain using the methods of Tauchen (1986). The approximation includes seven evenly spaced values for both  $\gamma$  and  $\phi$  and an estimate of the transition matrix,  $\pi$ . Details of the data, the VAR results, and the discrete approximation are given in the Appendix. Table 1 contains parameters (upper panel) and results (lower panel) for the benchmark calibration.<sup>17</sup>

**4.2. Results.** We focus first on our measures of price responses to shocks in the benchmark economy. Average cost pass-through to nominal transaction prices is incomplete with  $\bar{\theta} = 0.21$ . Although not shown in the table, we also find that pass-through is proportionally larger for cost increases than for cost reductions. For example, cost pass-through resulting from a one standard deviation increase in  $\phi$  equals 0.21 whereas pass-through from a similar decrease in  $\phi$  equals only 0.18. The magnitude of a cost shock also matters: pass-through is increasing in the magnitude of a cost increase but decreasing in the magnitude of a cost decrease.

The fairly low level of average cost pass-through exhibited by our benchmark economy is consistent with estimates of short-run and long-run exchange rate pass-through to the CPI in

<sup>17</sup> The results reported for  $\bar{\theta}$  and  $\bar{\chi}$  are the weighted average of these variables across all relevant state transitions. The results for  $\bar{q}$  and  $\bar{MU}$  are the weighted average of these variables across all possible states. The values for  $\bar{I}$ ,  $\sigma_I$ ,  $\rho_I$ , and  $\rho_{\gamma,mu}$  are averages across 10,000 simulations, each of length 195 periods. The respective standard deviations of these variables across the 10,000 trials were 0.60, 0.17, 0.04, and 0.001, respectively.

the literature.<sup>18</sup> For example, Choudhri and Hakura (2006) estimate average short-run pass-through equal to 0.04 for countries with average inflation rates less than 10%. They also report an average long-run pass-through estimate of 0.16. Gagnon and Ihrig (2004) estimate average long-run pass-through in their sample of 20 countries to be 0.23, Campa and Goldberg (2005) estimate average pass-through to be 0.46 in the short-run and 0.64 in the long-run, and Bailliu and Fujii (2004) report estimates of 0.08 and 0.16 for short- and long-run average pass-through, respectively. Our results are also consistent with the estimates of Leibtag et al. (2007), who estimate long-run pass-through of the cost of coffee beans to retail coffee prices in the United States at around 0.25. Finally, our finding that cost pass-through is larger for cost increases than for decreases is consistent with the findings of Aguiar and Santana (2002) for commodity cost changes in Brazil and with the results of Kinnucan and Forker (1987) for dairy products in the United States.

The benchmark economy generates a negative value for the average real price elasticity equal to  $\bar{\chi} = -1.26$ , indicating incomplete responses of nominal prices to changes in the money growth rate, or “sticky prices.” As with cost pass-through, the real price elasticity varies depending on the size and direction of the change in the money growth rate. In particular, the real price response (and, therefore, output response) is smaller for increases in money growth than it is for reductions, with  $\chi = -1.22$  for a one standard deviation increase in  $\gamma$  (with an output elasticity of 1.24) and  $\chi = -1.36$  for a similar size reduction (with an output elasticity of 1.34). Furthermore, output responses are decreasing in the magnitude of money growth increases but increasing in the magnitude of reductions.

Our finding of asymmetric responses to monetary shocks is consistent with several empirical studies that examine the relationship between output and monetary policy shocks. Many papers suggest that positive money shocks have smaller real effects than do negative shocks, which is consistent with our economy. See, for example, Cover (1992), Devereux and Siu (2007), Ravn and Sola (2004), and Weise (1999). These papers and others also suggest that large shocks are associated with smaller output multipliers than smaller shocks. This is true for our economy in the case of positive shocks to the money growth rate.

Asymmetric responses to either cost or money growth shocks emanate from the effect of the fraction of buyers observing a single price,  $q$ , on the shape of the distribution of prices and hence on the returns to search. For example, consider a cost shock. As noted above, an increase in costs causes price dispersion to increase, search intensity to rise, and  $q$  to fall. This causes the distribution of posted prices to become more concentrated at relatively low prices. This in turn mitigates the effect of the cost shock on the gains to search, since the relatively high prices, which increase by a relatively large amount, occur with lower frequency as  $q$  falls. In contrast, a reduction in costs raises  $q$ , an effect that increases the frequency with which sellers post higher prices and enhances the effect of the cost shock on the gains to search. The result of this process is a weaker search intensity response (and more pass-through) for a cost increase than for a cost reduction.

As described in the previous section, our economy is potentially consistent with any cyclical relationship between output and the average markup, depending on the relative importance of cost and money growth shocks. For our benchmark calibration, the average markup is strongly countercyclical. Evidence that the average markup is countercyclical has been presented by many authors (see Rotemberg and Woodford, 1999, for a survey). Below we consider the robustness of this result to changes in several parameters.

Our economy cannot, however, account for aggregate inflation dynamics. Although the average inflation rate equals the average growth rate of money, period-by-period changes in inflation are endogenous, as they depend on the responses of prices to movements in  $\phi$  and  $\gamma$ . At quarterly frequencies, the percent standard deviation and autocorrelation of inflation in our benchmark economy are 1.56% and 0.24, respectively. CPI inflation in U.S. data has a percent standard

<sup>18</sup> In our economy, short-run and long-run cost pass-through are equal as all price adjustment occurs in the period of the cost change.

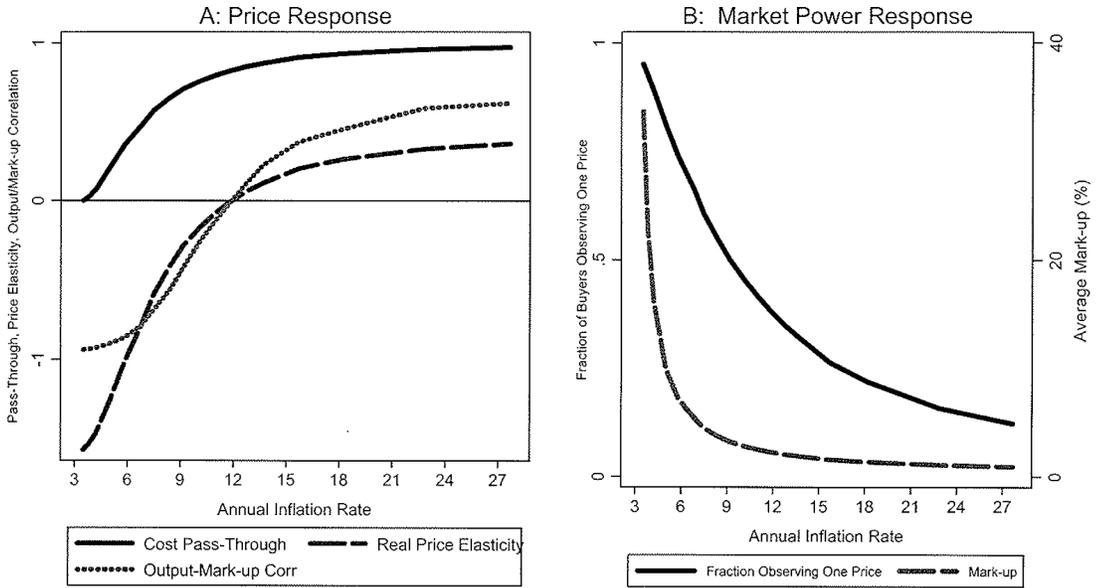


FIGURE 3  
 VARIATION IN INFLATION

deviation of 0.80% and autocorrelation of 0.69 over the time period used to estimate the joint process for costs and money growth. Insufficient inflation persistence in the model is unsurprising, as the economy has neither capital accumulation nor any exogenously imposed nominal rigidity that might be expected to generate persistence. We are considering these issues in separate research.

Although the economy cannot replicate the dynamics of inflation, its predictions for the frequency of individual price changes are not unreasonable. As noted above, we can imagine households ensuring that their sellers post the required distribution of prices in each state in a number of different ways. Suppose that households act so as to minimize the number of sellers who change their prices from one period to the next.<sup>19</sup> In this case, in the benchmark economy on average 42% of sellers change their price each period (quarter). This fraction is roughly consistent with evidence on individual price changes presented by Nakamura and Steinsson (2008).

4.3. *Average Inflation and Equilibrium Price Responses.* We now consider the relationships between price responses and the average rate of inflation. We do so by scaling the gross money growth rates in the data by a factor varying between 0.996 and 1.05. We then re-estimate equation (38) using the scaled data and construct a discrete approximation of the resulting continuous state process as described above. Panel A of Figure 3 graphs average cost pass-through, the average real price elasticity, and the average correlation between output and the markup as a function of the average annual rate of inflation.

The figure illustrates that cost pass-through is increasing in the average rate of inflation at a decreasing rate. This finding is consistent with a number of empirical studies that have examined the empirical relationship between inflation and exchange rate pass-through to goods prices. Campa and Goldberg (2005), Choudhri and Hakura (2006), Devereux and Yetman (2002), and Gagnon and Ihrig (2004) all use cross-country regressions to demonstrate that countries with lower inflation typically have lower rates of exchange rate pass-through. Bailliu and Fujii

<sup>19</sup> We can think of our economy as one in which there is an infinitesimally small cost associated with price changes by individual sellers. This will induce households to minimize the fraction of sellers who change prices but affect the overall price setting strategy only negligibly.

(2004) perform structural break tests for several countries and argue that those that experienced a reduction in inflation in the 1990's also experienced a fall in exchange rate pass-through. Devereux and Yetman (2002) also provide evidence that the relationship between inflation and pass-through is nonlinear and consistent with our finding that pass-through increases with inflation at a decreasing rate.

Panel A of Figure 3 also indicates a positive relationship between the average rate of inflation and the real price elasticity. This suggests that low inflation economies will be characterized by a greater degree of price stickiness than those in which average inflation is higher. Furthermore, as average inflation rises, the real price elasticity becomes positive, and the response of nominal prices to money growth shocks comes to resemble that exhibited by a standard cash-in-advance model with competitive markets (see footnote 15 above).

We also see in Panel B of Figure 3 that the economy is characterized by a negative relationship between average inflation and market power (measured by either the fraction of households who observe only one price or the average markup). This relationship is consistent with empirical evidence provided by several authors. Banerjee et al. (2001, 2007), Banerjee and Russell (2001), and Gali and Gertler (1999) all find a negative and significant long-run relationship between inflation and markups using time-series data for the United States, the United Kingdom, and Australia.

In order to understand these results, note that, as demonstrated by Head and Kumar (2005), when inflation is high, a relatively large fraction of buyers observe two prices. As discussed above, the lower this fraction, the more concentrated is the distribution of posted prices near its lower support. In this case, shocks lead to relatively small fluctuations in the return to search (since as described above shocks change mostly the relatively high prices, and with a small fraction of buyers observing a single price, these prices are posted with relatively low frequency). This in turn leads the search intensity response to either a cost or money growth shock to be relatively small and results in both greater cost pass-through and a stronger nominal (and thus smaller real) price response to money growth shocks. Indeed when inflation is sufficiently high so that nearly all buyers observe two prices, pass-through is nearly complete and the real price elasticity becomes positive.

The general result that both cost pass-through and the real price elasticity rise with average inflation implies that the magnitude of nominal price changes will be higher for economies with higher rates of inflation. This prediction of the model is consistent with the empirical findings of Gagnon (2009), who finds a positive relationship between inflation and the magnitude of price changes in highly disaggregated consumer prices in Mexico from 1994 to 2004. Note also that the result in the benchmark economy that cost pass-through is larger for cost increases than for cost reductions continues to hold as average inflation changes. In contrast, the finding that output responses are smaller for increases in money growth than for reductions does not hold if average inflation is high enough that the real price elasticity is positive.

Finally, Panel A of Figure 3 demonstrates that, although the markup is countercyclical in the benchmark economy and at low rates of inflation, it becomes pro-cyclical for economies with sufficiently high average inflation. Given that the effects of money growth shocks tend to dominate those of cost shocks in our benchmark economy, the markup becomes pro-cyclical at essentially that level of average inflation that makes the real price elasticity positive. As long as the markup exceeds 2% on average, it fluctuates counter-cyclically.

**4.4. Robustness.** We now briefly examine the robustness of our benchmark findings of incomplete cost pass-through, incomplete price adjustment to money growth shocks, and countercyclical markups to changes in preference parameters and search costs. First, since empirical estimates of the coefficient of relative risk aversion (i.e., the elasticity of intertemporal substitution) vary, we consider the effects of changing  $\alpha$ . For this exercise, we adjust the cost of search,  $\mu$ , as we change  $\alpha$  to maintain an average markup of 10%, holding all other parameters from the benchmark economy (including the stochastic process for costs and money growth) constant. Thus, the results of this exercise do not stem from changes in market power, which would occur as a result of changes in  $\alpha$ , ceteris paribus. Overall, our benchmark findings are all robust to

changes in  $\alpha$ , including the asymmetry results with regard to both cost pass-through and the real price elasticity.

Panel A of Figure 4 illustrates these results and shows that cost pass-through is decreasing in  $\alpha$ , approaches 1 as  $\alpha$  does, and becomes complete at  $\alpha = 1$  (log utility). The real price elasticity, in contrast, rises with  $\alpha$ , approaching zero from below. These results can be explained as follows. As  $\alpha$  rises, households prefer a smoother consumption profile and are increasingly willing to tolerate fluctuations in production costs to maintain smooth consumption. This accounts for reduced cost pass-through. Similarly, money-growth shocks induce smaller movements in real prices as  $\alpha$  rises, resulting in a closer connection between nominal price changes and money growth (and a real price elasticity that is closer to zero). Finally, although not shown in the diagram, the correlation between output and the markup is increasing in  $\alpha$ , but does not increase above  $-0.72$  in this range.

Next, we consider the relationship between price responses and market power. We vary the average markup by changing the search cost parameter,  $\mu$ , holding all other parameters constant at their benchmark levels. Panel B of Figure 4 depicts both cost pass-through and the real price elasticity for a range of search costs, which drives the fraction of buyers observing two prices from essentially one to very near zero. Now, as search costs increase, the measure of such buyers falls, resulting in greater market power and a higher average markup. This effect in turn reduces both cost pass-through and the real price elasticity.

For the range of search costs considered, cost pass-through ranges from near complete (when effectively all buyers observe two prices) to zero (when effectively none do). In all cases, however, cost pass-through is incomplete (i.e., strictly less than one). The real price elasticity rises as market power falls, and for low enough markups (less than approximately 2%), it becomes positive, indicating more than complete price adjustment to monetary shocks. So overall, the stronger is market power on average, the weaker is the adjustment of nominal prices to fluctuations in either production costs or the money growth rate. We also find that the correlation between output and the markup is decreasing in search costs and the average markup becomes pro-cyclical when the real price elasticity is positive.

Finally, we consider variations in the discount factor,  $\beta$ , holding the average markup constant by varying the search cost parameter,  $\mu$ . We can think of this exercise as similar to changing the period length, although we are unable to adjust the calibration given that our stochastic processes are estimated using quarterly data. Varying the discount factor is a useful exercise, because in our economy, period length pins down velocity. Thus, in our benchmark economy, annual velocity is four, whereas over the period we consider, M1 velocity is closer to six (we may think of this as being associated with  $\beta = 0.9932$ ).

Panel C of Figure 4 illustrates that as the discount factor rises, both cost pass-through and the real price elasticity fall. Over the range of  $\beta$ 's considered ( $0.05 \leq \beta \leq 0.995$ ), however, cost pass-through remains incomplete. Ceteris paribus, an increase in the discount factor works similarly to a reduction in the average inflation rate: it raises the real marginal value of a unit of currency and lowers all prices. By raising consumption, however, it lowers the reservation price by *more* than it lowers marginal costs, thereby compressing the price distribution. For a fixed search cost, this reduces households' incentive for search, reduces search intensity, and increases market power. Here we have, to some extent, compensated for the reduction in the returns to search by lowering search costs so as to maintain an average markup of 10%. This is not sufficient, however, to maintain constant "market power" in a broader sense. A higher discount factor effectively makes households more willing to hold onto money and wait until the next period for another chance to spend it. This reduces market power for any intensity of buyers' search. Thus, as  $\beta$  increases, a 10% average markup is associated with a larger fraction of buyers observing only one price, and, therefore, more market power for firms. This is associated with less cost pass-through and a lower real price elasticity.

We also report that the correlation between the markup and output is decreasing in  $\beta$  and becomes positive around  $\beta = 0.87$ , when the real price elasticity becomes positive. Finally, we note that cost pass-through is larger for cost increases than for cost reductions as we vary either

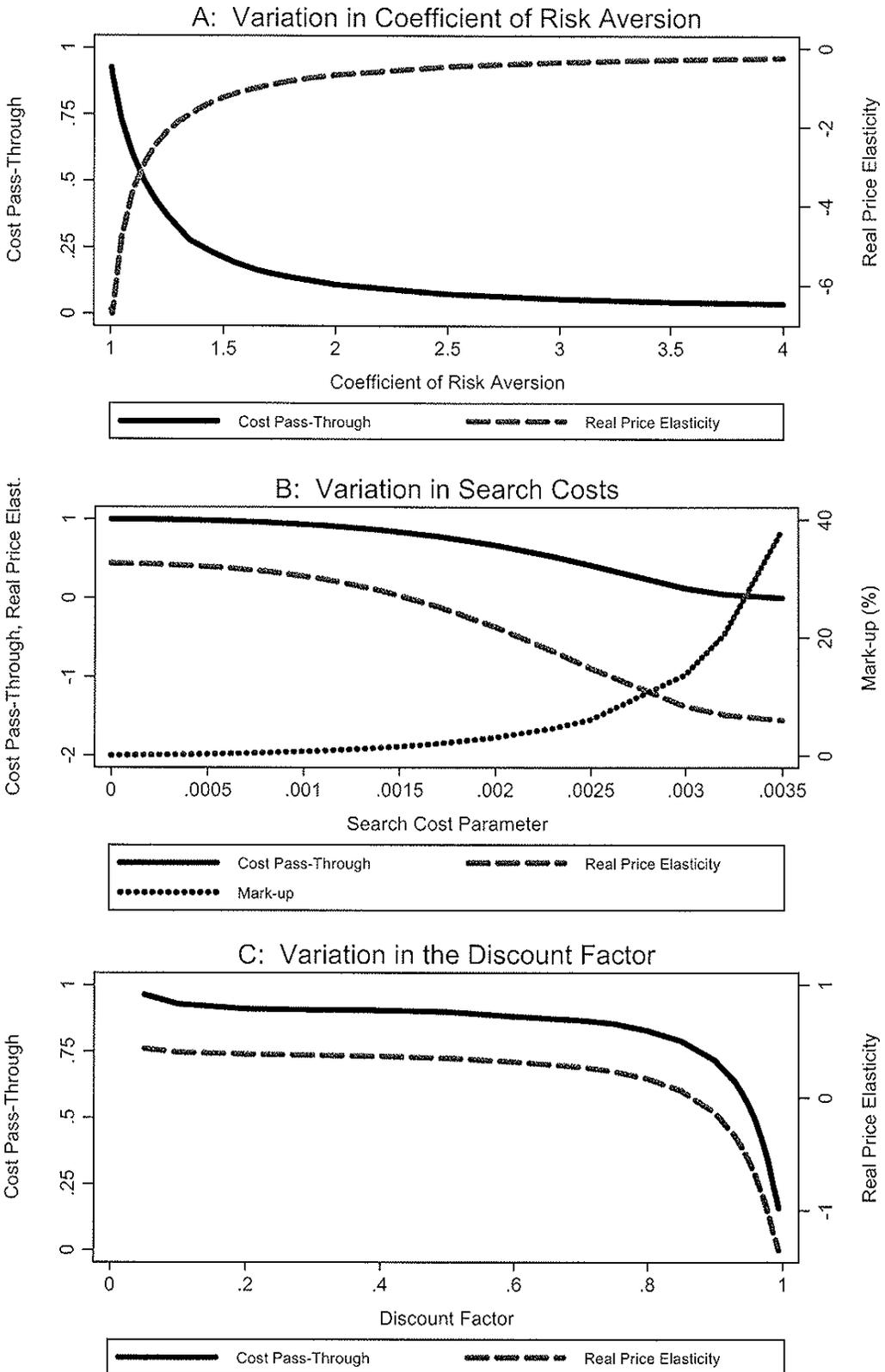


FIGURE 4

VARIATION IN ECONOMY PARAMETERS

$\mu$  or  $\beta$ . The benchmark result that output responses are smaller for increases in money growth than for reductions does not, however, continue to hold at levels of  $\mu$  or  $\beta$  when the real price elasticity becomes positive. The asymmetry results with respect to the magnitude of cost or money growth shocks continues to hold throughout these exercises.

In order to summarize, our finding of incomplete cost pass-through is robust to substantial changes to preference and cost parameters. Also, the response of the nominal price level to changes in the money growth rate is incomplete (i.e., nominal prices are sticky) when market power is sufficiently strong. Cost pass-through is larger for cost increases than decreases and output responses are smaller for money growth increases than for decreases when the economy exhibits incomplete nominal price adjustment. Finally, the economy generates countercyclical fluctuations of the average markup as long as there is sufficient market power that nominal price adjustment is incomplete.

## 5. CONCLUSION

This article has studied a stochastic monetary economy in which endogenous fluctuations in market power may cause both nominal and real prices to respond incompletely to stochastic fluctuations in productivity and the rate of money creation. Shocks of either type result in two potentially opposing effects. First, they induce individual sellers to change prices differentially. Second, the resulting changes in price dispersion induce households to adjust their search intensity. These two effects normally work in opposite directions, generating markup fluctuations that mitigate the response of prices to either type of shock. Limited adjustment of prices to shocks is thus driven by endogenous fluctuations in market power emanating from changes in consumers' search intensity.

A calibrated version of the economy can account for several empirical observations. First, the economy generates measures of cost pass-through that are incomplete and similar in magnitude to those estimated by several authors. Second, our results are consistent with several empirical studies suggesting that higher average inflation results in both a lower average markup and increasing sensitivity of prices to fluctuations in either productivity or money growth. Finally, our economy produces asymmetric price responses, with increases in either costs or money growth generating larger price responses than do reductions, as has been documented in other empirical studies.

Many of our findings are similar to the predictions of models with exogenous restrictions on price changes, in spite of the fact that in our economy prices are perfectly flexible. Our analysis and results do, however, differ in a number of ways. First, our approach has the advantage of not requiring the distribution of prices to be treated as a state variable, irrespective of whether individual prices change from period to period. Similarly, we are not required to impose assumptions regarding "indexation" in the presence of trend inflation. With regard to results, the nonlinearity associated with the marginal value of search naturally results in both asymmetric responses to positive and negative shocks and to a negative relationship between inflation and the average markup. Finally, although we do not dwell on the issue in this article (because it was dealt with in detail in Head and Kumar, 2005) our economy provides a rationale, the mitigation of market power, for an average inflation rate exceeding the (negative) rate of time preference.

Although our economy may be interpreted as predicting infrequent price changes by individual sellers, in this article we do not focus on aggregate inflation dynamics. For simplicity, we have abstracted from features of the economy that are normally associated with persistence in prices and inflation. For example, in our economy, inflation diverges from the money growth rate only because of fluctuations in the expected future value of money. In separate research, Head and Lapham (2006) consider persistence in international prices and inflation emanating from persistent heterogeneity across households in the spirit of Molico (2006), Molico and Zhang (2005), Berentsen et al. (2005), and Williamson (2008).

## APPENDIX

A.1. *Theoretical Results.*

*Extension of Proposition 1 of Head and Kumar (2005).* Let  $\hat{q}^+(\sigma)$  denote households' beliefs regarding the measure of buyers observing strictly more than one price. Then, given  $\hat{\mathbf{q}}(\sigma)$  and  $p_u(\sigma)$ , we have:

- (i) If  $\hat{q}^+(\sigma) = 0$ , then a household's optimal pricing strategy is to have sellers post  $p_u(\sigma)$  with probability one.
- (ii) If  $\hat{q}^+(\sigma) = 1$ , then a household's optimal pricing strategy is to have sellers post the marginal cost price,  $p^*(\sigma) = \phi/\bar{\Omega}(\sigma)$  with probability one.
- (iii) If  $\hat{q}^+(\sigma) \in (0, 1)$ , then the distribution of posted prices in that state is nondegenerate and continuous on a connected support.

PROOF. This follows directly from Lemmas 1 and 2 of Burdett and Judd (1983, pp. 959–61). In order to see this, note first that we may define a “firm equilibrium,” to use their terminology, as follows. Given beliefs regarding the search behavior of buyers,  $\hat{\mathbf{q}}(\sigma)$ , and a common reservation price,  $p_u(\sigma)$ , a *firm equilibrium* is a pair  $(F(p|\sigma), r)$  where  $F(\cdot|\sigma)$  is a distribution function with support  $\mathcal{F}(\sigma)$  and  $r = r(p)$  for all  $p \in \mathcal{F}$ . Next, note that  $\bar{p}$ ,  $p^*$ , and  $r$  in our notation correspond to  $\bar{p}$ ,  $r$ , and  $\Pi$  respectively, in theirs. Moreover, their probability of observing one price,  $q_1$ , is replaced here by  $1 - \hat{q}^+$ . (We exclude the possibility that  $\hat{q}_0(\sigma) = 1$  in any state as in this case any distribution of posted prices is consistent with household optimization.) ■

*Extension of Corollary 2 of Head and Kumar (2005).* If an SME exists, then  $\tilde{q}_1(\sigma) \in (0, 1)$  and  $\tilde{q}_2(\sigma) = 1 - \tilde{q}_1(\sigma)$ .

PROOF. We demonstrate the extension in steps, beginning with three preliminary results.

- (i) *Extension of Lemma 2 of Head and Kumar (2005):* Given  $\hat{F}(p|\sigma)$ ,  $\mathbf{q}(\sigma)$  has  $q_k(\sigma) > 0$  for at most two values,  $k^*$  and  $k^* + 1$ .

PROOF. Note first that if the distribution of posted prices is degenerate, then no household has incentive to have buyers observe more than one price. Thus, for price distributions that are degenerate, the claim is trivially true, since  $q_k > 0$  at most for  $k = 0$  and  $k = 1$ . For the case of a distribution that is nondegenerate and continuous on connected support, we first show that  $c^{k+1} - c^k$  is declining in  $k$ . It is clear that  $J^k(p|\sigma) \equiv 1 - [1 - \hat{F}(p|\sigma)]^k$  stochastically dominates in a first-order sense  $J^{k+1}(p|\sigma)$  so that the expected lowest price observed is declining in the number of price quotes observed,  $k$ , and thus that  $c^k$  is increasing in  $k$ . Moreover, it is straightforward to show that the expected lowest price observed declines at a decreasing rate, and thus that  $c^k$  increases at a decreasing rate. The remainder of the proof follows by directly applying the methods of Head and Kumar (2005) in the proof of their Lemma 2.

- (ii) *There can be no SME in which  $\tilde{q}^+(\sigma) = 1$  in any state.*

PROOF. Suppose that an SME exists with this property in some state. From above, we know that in this case the distribution of real posted prices in this state must be degenerate at the marginal cost price and there can be no gain to households from having a positive measure of its buyers observe a second price quote. Thus, all households will deviate from the conjectured equilibrium search strategy and set  $q^+(\sigma) = 0$ . This contradicts the claim that such an SME exists.

- (iii) *There can be no SME in which  $\tilde{q}^+(\sigma) = 0$  in any state.*

PROOF. Suppose that an SME with this property exists. From above, we have that in this case the distribution of real posted prices must be degenerate at the reservation price. In this case, however, since the household is indifferent between purchasing and holding money over until the next period, it must be the case that the return to the search

strategy  $q_1(\sigma) = 1$  is negative, as search costs are positive and, thus, no household will search. This is inconsistent with an SME.

From these last two results, we have that in any SME a positive measure of buyers must observe one or fewer prices *and* a positive measure must observe two or more prices. Combining this with the first result proves the extension. ■

*Note:* This finding contrasts with a result of Burdett and Judd (1983), who find that an equilibrium in which all buyers observe exactly one price and all sellers post the “monopoly” price always exists. They obtain this result by assuming that there is a monopoly price sufficiently below buyers’ reservation prices so that the surplus from exchanging at the monopoly price more than compensates the buyer for the cost of search. In our monetary economy, if the household expects to be a monopolist with probability one, it will price so as to extract all surplus from the trade. That is, the monopoly price is always equal to the buyers’ reservation price. Thus the result of Diamond (1971) obtains: buyers will not engage in costly search if it means trading at their reservation price with probability one. The result also corresponds to a result obtained in search-theoretic monetary models in which prices are determined by bargaining (see, e.g., Shi, 1995, or Trejos and Wright, 1995). In these models there can be no equilibrium with valued fiat money if sellers make take-it-or-leave-it offers to buyers.

A.2. *Data and Calibration.*

A.2.1. *Data sources.*

M1: M1, 1959–2007, Quarterly, Seasonally Adjusted; Federal Reserve Bank of St. Louis, FRED Database; <http://research.stlouisfed.org/fred2/series/M1SL?cid=25>.

Output per Hour: Nonfarm Business Output per Hour, Quarterly, 1959–2007, Seasonally Adjusted, Index with 1992=100; U.S. Department of Labor, Bureau of Labor Statistics; Series ID: PRS85006093; <http://data/bls/gov/data/home/htm>.

A.2.2. *Shock processes VAR.* We use the quarterly series for M1 to construct  $\gamma_t$  as  $\gamma_t \equiv \frac{M1_t}{M1_{t-1}}$  for  $t \in \{1959Q2, 2007Q4\}$ . We construct our measure of  $\phi_t$  by inverting the quarterly series for output per hour from 1959Q2 to 2007Q4, Hodrick-Prescott filtering it using a smoothing parameter of 1600 and normalizing the mean of the series to 0.1.

Letting  $\sigma_t \equiv (\gamma_t, \phi_t)$ , and  $\tilde{\sigma}_t \equiv \sigma_t - \bar{\sigma}$ , we run the following restricted VAR on the quarterly data series from 1959 to 2007:

$$(A.1) \quad \tilde{\sigma}_t = A\tilde{\sigma}_{t-1} + B\epsilon_t,$$

with  $\epsilon_t \sim N(0, I)$  and  $B$  restricted to be diagonal. The resulting estimates with standard errors in parentheses are

$$(A.2) \quad \hat{A} = \begin{bmatrix} 0.453 & 0.022 \\ (0.066) & (0.800) \\ -0.010 & 0.685 \\ (0.004) & (0.050) \end{bmatrix} \quad \hat{B} = \begin{bmatrix} 0.012 & 0 \\ (0.001) & \\ 0 & 0.001 \\ & (0.000) \end{bmatrix}.$$

A.2.3. *Shock process discrete approximation.* We use the methods of Tauchen (1986) to provide a finite state Markov-chain approximation of the VAR given by equation (A.1). In particular, we allow each stochastic variable to take on seven discrete values with the minimum and maximum values set to the series mean minus and plus three times the standard deviation of the stochastic variable, respectively. The discrete values for each variable in the approximation are as follows:

$$(A.3) \quad \hat{\gamma} \in \{0.9732, 0.9861, 0.9989, 1.0118, 1.0247, 1.0376, 1.0505\}$$

$$(A.4) \quad \hat{\phi} \in \{0.0962, 0.0975, 0.0987, 0.1000, 0.1013, 0.1025, 0.1038\}.$$

Let  $\hat{\sigma} \equiv [\hat{\sigma}_i^l]$  with  $\hat{\sigma}_i^l$  denoting the element in row  $i \in \{1, 2\}$  and column  $l \in \{1, 2, \dots, 7\}$  and let  $w_i \equiv \hat{\sigma}_i^{l+1} - \hat{\sigma}_i^l$  for  $i \in \{1, 2\}$  and  $\forall l \in \{1, 2, \dots, 6\}$ .

We also calculate the transition probabilities,  $\pi_{j,l,k,m}$ , for  $j, k, l, m \in \{1, 2, \dots, 7\}$  given in equation (3) in the discrete approximation as follows.

Let  $h_i((j, l), m) \equiv Pr(\sigma_{it} = \hat{\sigma}_i^m | \sigma_{it-1} = \hat{\sigma}_i^j \text{ and } \sigma_{2t-1} = \hat{\sigma}_i^l)$  for  $i \in \{1, 2\}$  and  $j, l, m \in \{1, 2, \dots, 7\}$ . Then

$$(A.5)$$

$$h_i((j, l), m) = \begin{cases} F_i \left( \frac{\hat{\sigma}_i^m - (\hat{A}_{k1}\hat{\sigma}_1^j + \hat{A}_{k2}\hat{\sigma}_2^l) + 0.5w_i}{\hat{B}_{ii}} \right) & \text{if } m = 1 \\ F_i \left( \frac{\hat{\sigma}_i^m - (\hat{A}_{k1}\hat{\sigma}_1^j + \hat{A}_{k2}\hat{\sigma}_2^l) + 0.5w_i}{\hat{B}_{ii}} \right) - F_i \left( \frac{\hat{\sigma}_i^m - (\hat{A}_{k1}\hat{\sigma}_1^j + \hat{A}_{k2}\hat{\sigma}_2^l) - 0.5w_i}{\hat{B}_{ii}} \right) & \text{if } 2 \leq m \leq 6 \\ 1 - F_i \left( \frac{\hat{\sigma}_i^m - (\hat{A}_{k1}\hat{\sigma}_1^j + \hat{A}_{k2}\hat{\sigma}_2^l) - 0.5w_i}{\hat{B}_{ii}} \right) & \text{if } m = 7. \end{cases}$$

Finally, given the independence of the  $\epsilon$ 's, the transition probabilities,  $\pi_{j,l,k,m}$ , are given by

$$(A.6) \quad \pi_{j,l,k,m} = h_1((j, l), k) \times h_2((j, l), m).$$

The discrete process that we construct in this manner provides a good approximation for the estimated continuous process (A.1), as is evident from a comparison of a set of data moments to moments from a simulation using the discrete approximation. In particular, the standard deviation and autocorrelation of  $\gamma$  is 0.0129 and 0.453 in the data and 0.0136 and 0.452 in the simulation. For  $\phi$ , the respective data moments are 0.0013 and 0.692, although simulated moments are 0.0011 and 0.696. Finally, the correlation between  $\gamma$  and  $\phi$  is  $-0.064$  in the data and  $-0.079$  in the simulation.

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