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INFLATION AND THE DISPERSION OF REAL WAGES*

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The article studies the effects of inflation on real wage dispersion in a search-monetary framework. The economy is characterized by frictions in both the goods and the labor markets. In the goods market, buyers and sellers bargain over prices, whereas in the labor market firms post wage offers. In equilibrium, a lower inflation rate increases the dispersion of real wages. This result is consistent with both the observed trends in wage dispersion and the inflation rate witnessed in the 1980s and the 1990s in the United States and the empirical literature linking reduced inflation to greater wage dispersion.

1. INTRODUCTION

In this article, I develop a search-theoretic general equilibrium monetary model in which lower anticipated inflation rate increases wage dispersion. It is motivated by the observed trends in the dispersion of wages and the inflation rate witnessed in the 1980s and the 1990s in the United States and the empirical literature linking reduced inflation to greater wage dispersion.

In the 1980s and 1990s, wage dispersion, including *residual* wage dispersion (i.e., differences in wages among workers with similar skills and job characteristics), increased dramatically in the United States (OECD, 1997; Katz and Autor, 1999). Over this period the inflation rate also declined significantly relative to what it had been in the 1970s. There is also an empirical literature that examines the relationship between the inflation rate and the dispersion of wages. Hammermesh (1986)

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analyzes the relationship between the inflation rate and the dispersion in the relative wage changes for the period 1955–81 in the United States using data from 20 two-digit manufacturing industries. He finds that higher inflation, especially unexpected inflation, reduced the dispersion in the relative wage changes. Erikson and Ichino (1995) examine the effect of inflation on wage earnings differentials over the period 1976–90 in Italy using the wage data taken from metal-manufacturing firms. They find that a higher inflation rate significantly reduced wage earnings differentials.[2]

These empirical findings are at variance with the predictions of the models that assume that firms face costs or other barriers to changing nominal wages. Inflation allows firms facing nominal wage rigidity to reduce real wages to offset negative demand shocks and bring real wages in line with productivity (Tobin, 1972; Akerlof et al., 1996). Many models with nominal rigidities imply that firms will follow a strategy of the [s,S] variety when setting wages (e.g., Sheshinski and Weiss, 1977; Benabou, 1988, 1992; Diamond, 1993).[3] In such models, a reduction in the trend inflation rate reduces the bound of real wages within which wages are not changed. That is, lower inflation reduces the range in which nominal wages do not respond to price level changes, resulting in less dispersion of real wages rather than more.

An additional problem with these models is that they lack a microfoundation for money. They cannot address questions such as why money circulates as a means of wage payment as well as a medium of exchange. It is also difficult to address questions such as what is the optimal level of inflation and whether changing the dispersion of real wages by inflation is good or bad for efficiency.

In this article, I address the issue of the effects of inflation on the dispersion of wages in a microfounded model of money. My model embeds a variant of the wage-posting model developed by Burdett and Mortensen (1998) in a search-theoretic monetary framework similar to Shi (1997, 1998, 1999).[4] It incorporates costly search in both the goods and the labor markets and on-the-job search by workers that are realistic features of economies.

In my model, the economy is populated by a large number of different types of households and firms. A household of particular type consumes a specific type of good. Similarly, a firm of particular type produces and sells a specific type of good. The goods are divisible and can be stored by their producers as inventory. Each household consists of a large number of identical buyers and identical workers. Each firm consists of a large number of identical sellers and identical recruiters.

[2] Lipsey and Swedenborg (1999) study the relationship between the price levels and wage dispersion for 15 OECD countries for the period 1979–90. They find that the price level is negatively related to wage dispersion.

[3] In these models, firms post prices and incur menu cost in changing them. The results of these models remain the same if we assume that firms post wages and face menu cost in changing them.

[4] Following the seminal work of Kiyotaki and Wright (1991, 1993), search-theory has become the dominant paradigm for the microfoundation of money. These models explicitly model the patterns of meetings, specialization in production and consumption, and the information structure that lead to the "double coincidence of wants problem" in the goods market, and intrinsically useless (fiat) money emerges as a medium of exchange endogenously (see also Lagos and Wright, 2005; Shi, 1995, 1997).

Buyers and sellers meet bilaterally and randomly in the goods market, where prices are determined through bargaining.

In the labor market, firms who want to hire workers open (or create) vacancies (or wage/job offers). Each vacancy is associated with a pre-committed real wage. Recruiters post vacancies (or wage/job offers). Both employed and unemployed workers search for suitable vacancies. Vacancies and workers are brought together by a constant returns to scale matching function. On-the-job search weakens the market power of wage-posting firms, and wage dispersion is an equilibrium outcome even though firms and workers are both ex ante homogeneous.[5]

In my model, a higher expected inflation by eroding the expected future value of money reduces the profitability of firms. Given other things, a lower profitability induces firms to post a smaller number of vacancies. A fall in the expected future value of money also raises the real reservation wage of unemployed workers (the real wage at which they are indifferent between working and not working). A higher real reservation wage lowers the return of firms from hiring workers at the margin, which further reduces the number of vacancies created.

A higher reservation wage and a fall in the level of vacancies reduce the dispersion of wages. A higher real reservation wage of unemployed workers increases the lower support of the real wage earnings distribution. The effect of inflation on the upper support, however, is mitigated by two factors. Firstly, firms posting the highest real wage do not face any competition from other firms to retain their workers, whereas firms posting lower real wages do. The result is that firms posting the highest real wage need not increase their real wage as much as the firms posting the lowest real wage. Secondly, the decline in the level of vacancies posted reduces the effectiveness of on-the-job search in weakening the market power of firms by lowering the matching rate of workers. Consequently, the support of the distribution of real wage earnings narrows and wage dispersion declines. In addition, a fall in the level of vacancies reduces employment and output. Finally, frictions in the labor market generate trading externalities. These externalities imply that the Friedman rule, which requires that the inflation rate be equal to the rate of discount, is not optimal.

My model is similar to Shi (1998), which first integrated the labor market search and matching model of Pissarides (1990) in a search-theoretic monetary framework.[6] The main difference is the way I model the wage-setting process. I use the wage-posting model rather than the ex post Nash bargaining. Mortensen (2003) and Manning (2003) provide evidence that wage posting is an appropriate characterization of wage-setting processes faced by nonunionized workers in industrialized countries. Variants of this model have been extensively used to explain

[5] By market power, I mean that firms pay wages less than the marginal product of labor. However, in this environment workers also possess market power in the sense that most of them are paid more than their reservation wage. This terminology is widely used (Mortensen, 2003; Manning, 2003).

[6] In Shi (1998) firms are part of the households. Households both own as well as operate firms. In this article, households own but do not directly operate firms. This separation is done at the suggestion of an anonymous referee. I show that for money to be valued, it is not necessary to assume that households produce one set of goods but desire to consume another set. The results of the article do not depend on whether there is separation between households or firms or not.

wage dispersion in industrialized countries (e.g., van den Berg and Ridder, 1998; Bontemps et al., 2000; Vurren et al., 2000). In addition, the basic mechanism of the Burdett–Mortensen model has been exploited by many other wage posting models that generate wage dispersion among similar workers (Coles, 2001; Postel-Vinay and Robin, 2002).

The rest of the article is organized as follows. Section 2 describes the model. In Section 3, the optimal strategies of households and firms are characterized. Section 4 derives conditions for the existence of a unique stationary monetary equilibrium with nondegenerate real wage earnings distribution. In Section 5, the effect of inflation rate on real wage dispersion is analyzed. Section 6 concludes. All proofs are contained in the Appendix.

2. THE ECONOMY

2.1. *Households, Firms, and Matches.* Time is discrete. Consider an economy comprised of many types of infinitely lived households and firms. Let the measure of types be unity. There is a large number of households and firms of each type. The measure of each type of households and firms is normalized to one. A household of type x derives utility only from consuming good x. Similarly, a firm of type x produces good x. All the households of particular type are identical. Similarly, all the firms of a given type are identical. Goods can be stored only by their producing firms as inventory.[7]

Households own but do not operate firms. Each household is endowed with a single share in a fully diversified portfolio of claims on the profit earnings by firms. For simplicity assume that these shares and claims on inventory cannot be used to buy goods in the goods market.[8] Firms maximize their profits and repatriate their profits to their owners (households) every period.

There are two markets: the goods market and the labor market.[9] Both markets are characterized by search frictions. Agents in both the markets are matched randomly and bilaterally through the matching processes described later. Since households of a given type are identical and so are the firms, random matching implies a particular household or a firm in the goods market cannot be located again in future. For the ease of exposition, in this article I assume that in the labor market a household of a particular type meets with a firm of a different type. This assumption coupled with the fact that an individual firm or household cannot be located again in future in the goods market rules out credit arrangements and exchanges must be *quid-pro-quo*. Thus, money is used as a medium of exchange as well as a means of wage payment.[10] Dividends are also paid in terms of money.

[7] The assumption that a good can be stored only by its producers rules out commodity money. The possibility of storage (or inventory) ensures that a firm's opportunity cost to selling goods in the goods market is strictly positive (see Section 3.4).

[8] In such models, this restriction is needed so that these claims do not replace money as a medium of exchange. One can assume that households can easily counterfeit such claims and sellers in the goods market cannot verify these claims (Aruba et al., 2006; Berentsen et al., 2006).

[9] The share market is suppressed as there will no trading in shares in equilibrium. The issues of dividend payments and trading in shares are further discussed in Section 3.1.

[10] Kumar (2008) analyzes conditions under which money emerges as a medium of exchange and a means of wage payment in detail.

Each household is endowed with $\hat{M}_0$ units of fiat money at time zero. At the beginning of each subsequent period, each household receives $(g-1)\hat{M}_{t-1}$ units of fiat money from the government as a lump-sum transfer, where $\hat{M}_t$ is the post-transfer per-household average holding of fiat money at time t in the economy ($\hat{M}_t = \hat{M}_{t-1} + (g-1)\hat{M}_{t-1} = g\hat{M}_{t-1}$). The government plays no role in the economy, other than making lump-sum transfers to the households. In what follows, as a convention, the variables that are taken as given by a particular household/firm are denoted with superscript "^".

Due to random matching, individual agents in these markets face uncertainty in their matching outcomes. This generates nondegenerate distributions of money holding, prices, employment, and inventories, which makes the model analytically intractable and numerically challenging. An additional complication is that my model generates dispersion in wage earnings. In order to make these distributions (except for wage earnings) degenerate and the analysis tractable, following Shi (1997, 1998, 1999), I use the construct of large households and firms.[11]

Each household of type x is assumed to comprise of unit measures of two types of members: buyers and workers. Workers are of two types—unemployed and employed. Each type of members in the household play a distinct role. Buyers buy the household's desired consumption goods in the goods market. Unemployed workers search for suitable jobs. Employed workers work and also search for better jobs. Let e_t^h and u_t denote the measures of employed workers and unemployed workers respectively in a household at time $t(e_t^h + u_t = 1)$.

Members of the household do not have independent preferences. Rather, the household prescribes the trading strategies for each type of members to maximize the overall household utility. The members of the household share equally in the utility generated by the household consumption. With this modeling device, the decisions of different household types are identical in a symmetric equilibrium, except for the types of goods they consume. Thus, I can analyze the behavior of a representative household.

Similarly, a firm consists of a large number of managers. Managers recruit workers and sell goods in the goods market. Assume that unit measure of managers (called recruiters) are engaged in recruiting activities and unit measure (called sellers) in selling activities. Just as in the case of households, these agents do not have independent preferences, but undertake activities in order to maximize firm's profit.[12] Large number of managers implies that idiosyncratic risks faced by individual managers are smoothed within the firm. In addition, it leads to identical terms of trade across matches in the goods market. With this construction of firms, the decisions of firms of different types are identical in a symmetric equilibrium, except for the types of goods they produce. Thus, I can analyze the behavior of a representative firm.

[11] An alternative framework that produces degenerate distributions of money holding and prices is examined by Lagos and Wright (2005).

[12] These managers need not be unpaid. One can assume that a *fixed number* of workers are required for managerial activities. These employees are chosen randomly at the beginning of every period from the existing pool of employees.

2.2. *Matching in the Goods and the Labor Markets.* The matching and trading process in the goods market is similar to Shi (1998). In the goods market, a buyer from a household of type x meets a seller of a firm of type x with probability ξ and vice versa. For analytical tractability, I assume that the meeting probability is fixed. Since, measures of buyers in a household and sellers in a firm are normalized to one, the measures of matched buyers in a household and matched sellers of a firm are equal to ξ in any time period.

In a match at time t, buyer j of type x exchanges $\hat{m}_t(j)$ units of money for $\hat{q}_t(j)$ units of good from the seller of type x, which implies a price of good x in this match of $\hat{p}_t(j) = \frac{\hat{m}_t(j)}{\hat{q}_t(j)}$. The average price across matches is denoted by $\hat{p}_t$. Here I am anticipating the result that any meeting between a buyer and a seller of the same type results in trade. This requires that each buyer j must have at least $\hat{m}_t(j)$ units of money. Similarly each seller j must have at least $\hat{q}_t(j)$ amount of goods. This will indeed be the case as both matched buyers and sellers receive positive surplus.

Note that as in Shi (1998), while making decisions at time t the household and the firm take terms of trade, $(\hat{m}_t(j), \hat{q}_t(j))$, or prices, $\hat{p}_t(j)$, as given. This assumption is similar to one made in the labor search models such as Pissarides (1990), where each firm takes the wage rate as given when it decides on the levels of capital and employment but has bargaining power over the wage rate with its matched workers. The terms of trade, $(\hat{m}_t(j), \hat{q}_t(j))$, or prices, $\hat{p}_t(j)$, are determined through Nash bargaining and are analyzed in Section 3.4.

In the labor market, workers and vacancies are matched through a matching function, which relates the flow of hiring to the average per-firm measure of vacancies, $\hat{v}_t$, and average per-household searching employed workers, $\hat{e}_t^h$, and unemployed workers, $\hat{u}_t(\hat{e}_t^h + \hat{u}_t = 1)$. The matching function $\mu(\hat{v}_t, \hat{e}_t^h, \hat{u}_t)$ is assumed to be concave, increasing, and subject to constant returns to scale. It is also assumed that the searching employed and unemployed workers are perfect substitutes, i.e., only the aggregate measure of the workers searching matters and not their composition. In addition, both employed and unemployed workers have equal search-intensity normalized to unity.[13] The aggregate flow of matches then is given by

$$(1) \qquad \mu\big(\hat{v}_t, \hat{e}_t^h + \hat{u}_t\big) = \mu(\hat{v}_t, 1) = \mu(\hat{v}_t), \text{ where } \mu'(\hat{v}_t) > 0, \lim_{\hat{v}_t \to 0} \mu(\hat{v}_t) \to 0,$$

where $\mu'(\hat{v}_t)$ denotes the first derivative of the matching function with respect to $\hat{v}_t$. Given the normalization with regard to the measure of households and their types, the measure of workers in the economy is unity; thus (1) also defines the matching rate of workers. Similarly, the matching rate of vacancies is given by

[13] With different search-intensities of employed and unemployed workers, the real reservation wage of unemployed workers depends not only on the disutility of work but also on the expected gain from search, which makes the model analytically complex. One can also endogenize the search-intensities of employed and unemployed workers. Endogenization of search-intensity leads to considerable analytical and computational complexity. The reason is that the model generates a nondegenerate wage earnings distribution, which induces nondegenerate distribution of search intensities, which is a high dimensional object on which the wage posting strategies of firms must depend.

$$(2) \qquad \frac{\mu(\hat{v}_t)}{\hat{v}_t}, \text{ where } \lim_{\hat{v}_t \to 0} \frac{\mu(\hat{v}_t)}{\hat{v}_t} = \infty, \lim_{\hat{v}_t \to \infty} \frac{\mu(\hat{v}_t)}{\hat{v}_t} = 0, \frac{d(\mu(\hat{v}_t)/\hat{v}_t)}{d\hat{v}_t} < 0.$$

Also assume that a newly formed match starts producing from the next period and continues until the match is dissolved.

A match can dissolve for two reasons: (i) idiosyncratic exogenous shocks, and (ii) the employee receives a better job offer. In both cases, a firm loses employees. However, only in the first case does an employed worker become unemployed. Assume that in each period fraction σ of a firm's existing matches are exogenously dissolved, with all such matches equally likely to fall in this group. This also implies that in any time period, fraction σ of employed workers in a household become unemployed.

2.3. *The Wage Determination Process.* The process of wage determination is modeled using a version of the wage-posting model developed by Burdett and Mortensen (1998). As in the Burdett–Mortensen model, a firm specifies wages to be paid to the prospective employees before recruiters search for workers. Searching for workers is costly for firms. The cost of search is discussed later. Assume that a firm fully specifies the nominal wages payable to the prospective employee for all time to come for each job/wage offer (or vacancy) as in Burdett and Mortensen (1998).

In the rest of the article, I use terms vacancies, job offers, and wage offers interchangeably. For simplicity, I abstract from nominal rigidities and assume that a wage offer promises to pay a constant real wage, w, to an employee defined as

$$(3) \qquad w = \frac{w_t}{\hat{p}_t},$$

where w_t and and $\hat{p}_t$ are the nominal wage payable and the average price level at time t, respectively.[14]

In order to preserve the construct of the representative firm, I assume that a firm chooses the measure of vacancies, v_t, and its distribution over real wages, $F_t(w)$ (which can be degenerate).[15] Recruiters belonging to the firm draw from the specified distribution and post vacancies. While choosing vacancies and its distribution, the firm takes the distribution of real wage offers in the economy at time t, $\hat{F}_t(w)$, with support, $\hat{\mathcal{F}}_t$, as given. Since the model has on-the-job search,

[14] One can assume that firms post infinite sequences of nominal wages to which they can credibly commit and then restrict attention to a stationary equilibrium, which supports constant real wage offers. However, the assumption that firms post constant real wage offers simplifies the exposition a great deal without affecting the results.

[15] In the Burdett–Mortensen model a particular firm offers a single wage to all its employees, which implies that firms differ in employment size. In the current environment, due to frictions in the goods market differing employment size leads to price dispersion, which makes the model analytically intractable. The current formulation not only makes the model analytically tractable, but also preserves the key aspect of the basic Burdett–Mortensen model that firms are identical not only in their productivity but also in the prices they receive. Wage dispersion arises solely due to the strategic interactions among recruiters and workers and not due to any heterogeneity with regard to prices and productivity.

the distribution of real wage earnings is different from the distribution of real wage offers. Let e_t^f and $G_t^f(w)$ denote the measure of employees and their distribution over real wages in the representative firm at time t. Similarly, let $G_t^h(w)$ be the distribution of real wage earnings of employed workers in the representative household at time t. In a symmetric equilibrium, $e_t^h = e_t^f$ and $G_t^h(w) = G_t^f(w)$.

In the next section, I analyze the optimal behavior of a representative household and a representative firm of an arbitrary type, x.

3. OPTIMAL DECISIONS OF THE HOUSEHOLD AND THE FIRM

3.1. *Timing.* The representative household at the beginning of period t enters with M_t units of post-transfer money, u_t, unemployed workers, e_t^h, employed workers, and their distribution over real wage earnings, $G_t^h(w)$. Similarly, the representative firm of type x enters with, e_t^f, employees, their distribution over real wage earnings, $G_t^f(w)$, and inventory of goods denoted by i_t.

At the beginning of period t, the household distributes available money balance, M_t, equally among buyers and chooses consumption for period t, c_t, a new money balance for the next period, M_{t+1}, and the job-acceptance strategies of workers and the trading strategies of buyers. Note that each buyer receives M_t units of money as the measure of buyers in the household is unity.

The firm produces goods using its existing employees, e_t^f. It distributes available goods, which includes current output, $f(e_t^f)$, plus the inventory, i_t, carried from the previous period, equally among sellers and prescribes their trading strategies. Thus each seller receives $f(e_t^f) + i_t$ units of goods. It also chooses next period employment level, e_{t+1}^f, and the inventory level, i_{t+1}, the measure of vacancies, v_t, and the associated distribution of real wage offers, $F_t(w)$, which are posted by the recruiters.

After these decisions, individual agents go to their respective markets and matches are formed. Matched buyers and sellers trade in the goods market. After trading in the goods market, buyers come back to the household with the purchased goods and any residual nominal money balances. Sellers come back with their nominal sales receipts and any unsold stock of goods. The firm pays wages to its employees and employees return to their respective households with their nominal wage receipts.

Similarly, in the labor market workers search among the posted real wage offers and accept or reject the offers received according to the prescribed job-acceptance strategies. Match dissolutions take place. Recall that the newly hired employees start working from the next period. Also only the existing matches receive the idiosyncratic exogenous shocks and not the new ones. Trading in the labor market and the exogenous dissolution of matches determine the next period's measure of employed workers, e_{t+1}^h, and their distribution over real wage earnings, $G_{t+1}^h(w)$, the measure of unemployed workers, u_{t+1}, and the measure of employees of firms, e_{t+1}^f, and their distribution over real wages, $G_{t+1}^f(w)$.

At the end of the labor market session workers go back to their respective households and consumption takes place. Since firms make strictly positive profit, there remains to determine how dividends are paid. Assume that at the end of the labor and the goods market sessions and after consumption, there is an asset market session, where households buy and sell shares and receive dividends. The asset market is competitive.[16] Since goods cannot be stored by the households and consumption has already taken place, households of a particular type do not have any incentive to buy only the shares of firms of their own type and dividends are paid only in terms of money. In equilibrium no trading in shares takes place.

After the end of the asset market session, the wage receipts of employed workers, the dividend received, and any residual money balances brought back by the buyers are added to the household nominal money balance for the next period. The unsold stock of goods are carried as inventory to the next period by the firm. Time moves to the next period $t + 1$.

3.2. *The Optimal Decisions of the Household.* Assume that the representative household maximizes the discounted sum of utilities from the sequence of consumption less the disutility incurred by employed workers from working.[17] The household's inter-temporal utility is represented by

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t}\left[U(c_t)-\phi e_t^h\right]; U'(c_t)>0,\, U''(c_t)<0,\, \lim_{c_t\to 0}U'(c_t)>0, \tag{4}$$

where $r, U(c_t)$, and ϕ, are the rate of time-preference, the utility derived from consumption c_t, and the disutility from working, respectively.

Recall that the measure of matched buyer in the household is ξ and a matched buyer j receives $\hat{q}_t(j)$ units of good in exchange for $\hat{m}_t(j)$ units of money. Since I focus on the symmetric equilibrium in which buyers in a household and sellers in a firm are assigned the same stocks and decision rules, I suppress index j from match-specific terms of trade from now on.

The money spent by an individual buyer in a match satisfies the following inequality:

$$\hat{m}_t \le M_t. \tag{5}$$

Consumption c_t satisfies the following inequality

$$c_t \le \xi\hat{q}_t. \tag{6}$$

[16] Berentsen et al. (2006), who integrate search and matching labor market model in the Lagos–Wright monetary framework, also assume that the profits earned by firms in the decentralized labor and goods markets are paid to the households in the centralized market. The shares cannot be used to buy goods in the decentralized goods market.

[17] The disutility from search in the goods and the labor markets is normalized to zero.

Denote the nominal dividends received by the household as $\hat{\pi}_{Mt}$ at time t. Then the budget constraint of the representative household is given by

$$M_{t+1} \leq M_t + (g-1)\hat{M}_t + \hat{\pi}_{Mt} + \hat{p}_t e_t^h \int_{\hat{\mathcal{F}}_t} w \, dG_t^h(w) - \xi \hat{m}_t. \tag{7}$$

The term on the left-hand side is the post-transfer money holding at the beginning of period, $t+1$. The first term on the right-hand side is the nominal money balances of the household at time t, the second term is the lump-sum monetary transfer at the beginning of period $t+1$, and the third term is the dividend received at time t. The fourth term is the total nominal wage payment received by the employed workers of the household. The final term is the money spent by the matched buyers at time t.

Before formally setting the optimization problems of the household, it is convenient to discuss the optimal job-acceptance strategies prescribed by the household to its workers. Let ω_{Mt} be the marginal value of money for the household, which is given by the Langrangian multiplier associated with the budget constraint (7). In the current environment, the household knows that in any period t fraction σ of the employed workers become unemployed and fraction $\mu(\hat{v}_t)$ receive new real wage offers, which they can accept or reject. Similarly, fraction $\mu(\hat{v}_t)$ of the unemployed workers realize real wage offers, which they can accept or reject. It can easily be shown that job acceptance strategies prescribed by the household have a reservation property.

The real reservation wage of an employed worker is the real wage he currently earns. Intuitively, the contribution of employed workers earning higher real wages to the current utility of the household is higher compared to employed workers earning lower real wages. At the same time, in the next period all employed workers face the same aggregate matching rate, $\mu(\hat{v}_{t+1})$, as well as the aggregate distribution of real wage offers, $\hat{F}_{t+1}(w)$. Therefore, it is optimal for the household to instruct employed workers to accept any new real wage offer higher than their current real wage.

The real reservation wage of an unemployed worker, $\underline{w}$, satisfies

$$\omega_{Mt+1}\hat{p}_{t+1}\underline{w} = \phi, \tag{8}$$

where ω_{Mt+1} is the marginal value of nominal money balances to the household at time $t+1$. Equation (8) equates the utility of working at the real reservation wage of unemployed workers $\underline{w}$ (the left-hand side) to the disutility of working (the right-hand side). Thus, it is optimal for the household to instruct unemployed workers to accept any real wage offer $w \geq \underline{w}$.

In a symmetric equilibrium, Equation (8) characterizes the common real reservation wage of unemployed workers, $\hat{\underline{w}}$. Since posting vacancy is costly, no firm would post a real wage offer below the common real reservation wage of unemployed workers as it would not be able to attract any worker. Thus, (8) also characterizes the lower support of the distribution of real wage offers, $\hat{F}_t(w)$, and an unemployed worker accepts any equilibrium real wage offer.

Next I set up the optimization problem of the household. Taking the aggregate distribution of real wage offers, $\hat{F}_t(w)$, the aggregate distribution of real wage earnings, $\hat{G}_t^h(w)$, the average level of vacancies, $\hat{v}_t$, the terms of trade in the goods market, $(\hat{m}_t, \hat{q}_t)$, the optimal choices of firms and other households, and the initial conditions $\{M_0, e_0^h, u_0, G_0^h(w)\}$ as given, the representative household of type x chooses the sequence $\{c_t, M_{t+1}\}$, $\forall t \geq 0$ to solve the following problem.

Household Problem (PH)

$$\max_{c_t, M_{t+1}} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[U(c_t) - \phi e_t^h\right] \tag{9}$$

subject to the constraints on money spent by an individual buyer in a match (5), the household's consumption (6), the budget constraint (7), and the law of motion of the employed workers, e_t^h,

$$e_{t+1}^h \leq (1-\sigma)e_t^h + \mu(\hat{v}_t)u_t. \tag{10}$$

The law of motion of the distribution of real wage earnings of the employed workers, $G_t^h(w)$, is given by

$$\begin{aligned} e_{t+1}^h G_{t+1}^h(w) \leq {} & [1-\sigma-\mu(\hat{v}_t)(1-\hat{F}_t(w))]e_t^h G_t^h(w) \\ & +\mu(\hat{v}_t)\hat{F}_t(w)u_t, \quad \forall w \in \hat{\mathcal{F}}_t. \end{aligned} \tag{11}$$

The constraints given in (10) and (11) incorporate the optimal job-acceptance strategies of workers as well as the condition that it is not optimal for the recruiters to post a real wage offer less than the common real reservation wage. The left-hand side of (10) is the measure of employed worker at time $t+1$. The first term on the right-hand side is the measure of employed workers at the beginning of the period t who remain in the same pool at the end of the period. An employed worker leaves this pool due to exogenous dissolution of the match. The second term is the measure of unemployed workers who receive wage offers.

The left-hand side of (11) is the measure of employed workers with the real wage earnings of w and less at the beginning of period $t+1$. The first term on the right-hand side is the measure of employed workers with real wage earnings of w and less at the beginning of the period t who remain in the same pool at the end of the period. An employed worker leaves this pool either due to the exogenous dissolution of match or because he receives a real wage offer higher than w. The second term is the measure of unemployed workers who receive the real wage offers of w and less. As discussed earlier, equilibrium real wage offers, w, are greater or equal to the reservation wage, $\underline{w}$, and thus constraints given in (10) and (11) will be binding.

Corresponding to the laws of motion for employed workers and their distribution over real wage earnings at the household level, there are laws of motion at the aggregate level, which are similar in form to (10) and (11) with e_t^h and $G_t^h(w)$ replaced by $\hat{e}_t^h$ and $\hat{G}_t^h(w)$, respectively for all t.

Turning to the optimal choices of c_t and M_{t+1}, consumption, c_t, is given by the equality constraint (6). Denote the Langrangian multiplier associated with the nominal money balance of an individual buyer (5) by λ_t; then the first order condition for the optimal choice of M_{t+1} is given by

$$\omega_{Mt} = \frac{1}{1+r}[\omega_{Mt+1} + \xi \lambda_{t+1}]. \tag{12}$$

This first order condition has the usual interpretation. The right-hand side of (12) is the discounted expected marginal benefit from carrying an additional unit of money next period. If the household carries one additional unit of money next period, then it relaxes the budget constraint (7) as well as the constraint on the nominal balances of matched buyers (5). Note that only fraction ξ of buyers are matched in any given period.

3.3. *The Optimal Decisions of the Firm.* Suppose that the representative firm has a production function given by

$$f\left(e_t^f\right) \text{ where } f'\left(e_t^f\right) > 0,\ f''\left(e_t^f\right) < 0,\ \lim_{e_t^f \to 0} f'\left(e_t^f\right) = \infty, \tag{13}$$

where e_t^f is the measure of employees at time t.

Recall that firms have to engage in costly recruitment activity. Suppose that in order to hire workers, firms have to advertise with advertising agencies and pay to them in terms of money. These advertising agencies are owned by the households and operate in a competitive market. A short-cut way to model this recruitment process is to assume that firms incur cost in terms of disutility of the representative household (or owner). This obviates the need to explicitly model the behavior of advertising agencies. Let k be the cost of creating and maintaining one unit of vacancy per period in terms of the disutility of the representative household (or owner).[18]

In the goods market at time t, the measure of matched sellers is ξ and each seller receives $\hat{m}_t$ units of money (under symmetry), and thus total sales in nominal terms is $\xi \hat{m}_t$. The profit of the firm in nominal terms at time t, π_{Mt}, is given by

$$\pi_{Mt} = \xi \hat{m}_t - e_t^f \hat{p}_t \int_{\hat{\mathcal{F}}_t} w \, dG_t^f(w), \tag{14}$$

[18] This assumption can be justified as follows. Suppose advertising agencies operate in a competitive market and charge price p_t^A per vacancy per unit of time in terms of money. Each advertising agency employs one person (different from production workers) and can at most handle one advertisement per unit of time. Suppose that a household incurs disutility k per nonproduction worker. Then if we allow free entry, the price charged per unit of vacancy advertised satisfies $p_t^A \hat{\omega}_{Mt} = k$. Thus, if a firm advertises v_t vacancies, the payment in terms of money is $p_t^A v_t$. Since the firm maximizes utility of its owners, in the utility terms the cost of advertising is $p_t^A v_t \hat{\omega}_{Mt} = k v_t$. By assuming that vacancy cost is incurred in terms of utility, I avoid explicitly modeling the behavior of advertising agencies and nonproduction workers. Alternatively, one can assume that a firm incurs vacancy cost in terms of lost production. In this case, number of vacancies directly affects the terms of trade in the goods market.

where the second term in the right-hand side is the total nominal wage payments made by the firm. Wage payments are made whether goods are sold or not.

Suppose that the firm creates v_t vacancies at time t. Then, the profit of the representative firm in utility terms at time t, π_t, is given by

$$\pi_t = \pi_{Mt}\hat{\omega}_{Mt} - kv_t. \tag{15}$$

The representative firm maximizes profit given in (15). Given the ownership structure of firms, this assumption is reasonable.

Let $\delta \in (0, 1)$ be the rate of depreciation for the inventory. The total units of goods sold by the matched sellers are $\xi\hat{q}_t$, and thus the evolution of inventory, i_t, over time is given by

$$i_{t+1} \leq (1-\delta)\big(f\big(e_t^f\big) + i_t - \xi\hat{q}_t\big), \tag{16}$$

where $f(e_t^f) + i_t - \xi\hat{q}_t$ is the total quantity of unsold goods at the end of the goods market session. Note that the units of goods sold by a matched seller, $\hat{q}_t$, satisfies the following inequality:

$$\hat{q}_t \leq f\big(e_t^f\big) + i_t. \tag{17}$$

Taking as given the aggregate distribution of real wage offers, $\hat{F}_t(w)$, the aggregate distribution of real wage earnings, $\hat{G}_t^h(w)$, the average level of vacancies, $\hat{v}_t$, the terms of trade, $(\hat{q}_t, \hat{m}_t)$, the job-acceptance strategies of workers, and the initial conditions $\{i_0, e_0^f, G_0^f(w)\}$, the firm's problem is to choose the sequence of $\{i_{t+1}, e_{t+1}^f, v_t, F_t(w)\}\ \forall t \geq 0$ to maximize

Firm Problem (PF)

$$\max_{i_{t+1}, e_{t+1}^f, v_t, F_t(w)} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \pi_t \tag{18}$$

subject to the law of motion of inventory (16), the quantity constraint on sellers (17), and the law of motion of the distribution of real wage earnings of employees, $G_t^f(w)$, given by

$$\begin{aligned} e_{t+1}^f G_{t+1}^f(w) \leq{}& [1-\sigma]e_t^f G_t^f(w) - \mu(\hat{v}_t)e_t^f \int_{\underline{\hat{w}}}^{w} (1-\hat{F}_t(z))\, dG_t^f(z) \\ &+ \frac{\mu(\hat{v}_t)}{\hat{v}_t}\hat{u}_t v_t F_t(w) + \frac{\mu(\hat{v}_t)}{\hat{v}_t}\hat{e}_t^h v_t \int_{\underline{\hat{w}}}^{w} \hat{G}_t^h(z)\, dF_t(z), \forall w \in \hat{\mathcal{F}}_t. \end{aligned} \tag{19}$$

The term on the left-hand side of (19) is the measure of employees earning real wage w and less at the beginning of the period $t+1$. The first term on the right-hand side is the measure of employees of the firm earning real wages w and less at the beginning of the period t who are not separated from their matches exogenously.

The second term is the measure of employees who leave their matches because they receive higher real wage offers. The third and fourth terms together give the measure of new matches formed on the vacancies posted with real wage offers of w and less.

The law of motion of the measure of employees, e_t^f, is given by

$$e_{t+1}^f \le \left[1 - \sigma - \mu(\hat{v}_t) \int_{\hat{\mathcal{F}}_t} (1 - \hat{F}_t(z))\, dG_t^f(z)\right] e_t^f + \frac{\mu(\hat{v}_t)}{\hat{v}_t}\left[\hat{u}_t + \hat{e}_t^h \int_{\hat{\mathcal{F}}_t} \hat{G}_t^h(z)\, dF_t(z)\right] v_t. \tag{20}$$

The term on the left-hand side of (20) is the measure of employees at the beginning of the period $t+1$. The right-hand side has two parts. The first part is the measure of employees of the firm at the beginning of the period t who do not leave the firm. An employee can leave the firm either due to the exogenous shock or because he receives a higher real wage offer. The second part gives the measure of new matches formed. It reflects the fact that $\hat{u}_t$ proportion of wage offers go to the unemployed workers and $\hat{e}_t^h$ proportion to the employed workers ($\hat{u}_t + \hat{e}_t^h = 1$). An unemployed worker accepts any equilibrium wage offer, whereas an employed worker accepts an offer only when it offers a real wage higher than what he currently earns.

Denote the Langrangian multiplier associated with the law of motion of inventory (16) by Ω_{it} and the Langrangian multiplier associated with the quantity constraint of the matched seller (17) by Ω_{qt}. The first-order condition for the choice of next period inventory, i_{t+1}, is

$$\Omega_{it} = \frac{1}{1+r}[(1-\delta)\Omega_{it+1} + \xi \Omega_{qt+1}]. \tag{21}$$

(21) equates the marginal cost of carrying an addition unit of inventory to its discounted expected marginal benefit. If the firm carries one extra unit of inventory next period, then it relaxes the next period constraints on the inventory and the quantity of goods that can be sold by matched sellers.

Now consider the optimal choice of the distribution of real wage offers, $F_t(w)$. Define the marginal value of an employee earning wage w, $\Omega_{et}^f(w)$, as

$$\Omega_{et}^f(w) = \frac{1}{1+r}\big[[\xi \hat{p}_{t+1}\hat{\omega}_{Mt+1} + (1-\xi)(1-\delta)\Omega_{it+1}] f'\big(e_{t+1}^f\big) - \hat{p}_{t+1}\hat{\omega}_{Mt+1} w + [1 - \sigma - \mu(\hat{v}_{t+1})(1 - \hat{F}_{t+1}(w))]\Omega_{et+1}^f(w)\big]. \tag{22}$$

Equation (22) can be interpreted as follows. An additional employee next period earning real wage w increases production by the marginal product, $f'(e_{t+1}^f)$. The marginal product can be sold with probability ξ, the value of which is $\hat{p}_{t+1}\hat{\omega}_{Mt+1} f'(e_{t+1}^f)$. With probability $1-\xi$, it remains unsold, the value of which is

$(1-\delta)\Omega_{it+1} f'(e^f_{t+1})$. The firm has to pay wages regardless of whether goods are sold or not, the value of which is $\hat{p}_{t+1}\hat{\omega}_{Mt+1} w$. The remaining term in the right-hand side gives the expected continuation value of the match.

As discussed earlier, the firm will not post any real wage offer less than $\underline{\hat{w}}$, since it will not be able to attract any worker. Conditional on $w \geq \underline{\hat{w}}$, the expected gross return on a real wage offer w posted at time t, $R_t(w)$, is

$$R_t(w) = \frac{\mu(\hat{v}_t)}{\hat{v}_t}\left[\hat{u}_t + \hat{e}^h_t \hat{G}^h_t(w)\right]\Omega^f_{et}(w), \quad \forall w. \tag{23}$$

This expected gross return is the product of the marginal value of the employee, $\Omega^f_{et}(w)$, and the expected measure of workers who will receive and accept the offer. In turn, the expected measure of workers who will receive and accept the offer is equal to the product of the aggregate matching rate of vacancies and the aggregate measure of workers with real reservation wage less than w. It is immediately clear from (23) that by posting a higher real wage offer, the firm can increase the expected measure of workers who will receive and accept the offer.

The firm will post real wage offer, w, such that

$$w \in \underset{w}{\operatorname{argmax}}\, R_t(w) \equiv R^*_t. \tag{24}$$

Let w^* be an optimal real wage offer. Then, the firm will post real wage offers other than w^* if and only if all the other posted real wage offers give return equal to R^*_t.

Next consider the optimal choice of vacancies. Utilizing (24), one can express the net expected return on a posted vacancy at time t, TR_t, as

$$TR_t \equiv -k + R^*_t. \tag{25}$$

The firm will create vacancies as long as the net expected return, $TR_t \geq 0$, and thus the optimal choice of the measure of vacancies, v_t, is given by

$$k = \frac{\mu(\hat{v}_t)}{\hat{v}_t}\left[\hat{u}_t + \hat{e}^h_t \hat{G}^h_t(w^*)\right]\Omega^f_{et}(w^*). \tag{26}$$

(26) equates the marginal cost of posting a vacancy to the expected marginal benefits.

3.4. *Goods Market and Terms of Trade.* Now I turn to the determination of the terms of trade in the goods market. I describe the process briefly as the details are given in Shi (1998, p. 326). In a match, a buyer gives up m_t units of money in exchange for q_t units of good from the seller. The associated price $p_t = \frac{m_t}{q_t}$. The trade at the terms (q_t, m_t) generates the following surpluses to the firm and the household

$$\text{Seller's surplus} : \hat{\omega}_{Mt} m_t - [(1-\delta)\Omega_{it} + \Omega_{qt}]q_t,$$
$$\text{Buyer's surplus} : U'(c_t)q_t - [\omega_{Mt} + \lambda_t]m_t.$$

As shown in Shi (1998), Nash bargaining with equal weights in a symmetric equilibrium ($\hat{\omega}_{Mt} = \omega_{Mt}$) implies that the terms of trade solves

$$p_t \omega_{Mt} = \Omega_{qt} + (1-\delta)\Omega_{it}, \tag{27}$$

$$U'(c_t) = p_t(\omega_{Mt} + \lambda_t). \tag{28}$$

The seller's surplus per sale is Ω_{qt} and the buyer's surplus per purchase is $p_t \lambda_t$.

4. SYMMETRIC STATIONARY MONETARY EQUILIBRIUM

This article restricts its attention to a symmetric and stationary monetary equilibrium with dispersed real wages. First, I require that all households have common marginal value of money, ω_{Mt}, consumption, c_t, employment, e_t^h, and the distribution of real wage earnings, $G_t^h(w)$, and choose identical job-acceptance strategies. Similarly, all firms have identical employment level, e_t^f, and the distribution of real wage earnings, $G_t^f(w)$, and choose identical levels of inventory, i_{t+1}, vacancies, v_t, and the nondegenerate distribution of real wage offers, $F_t(w)$. Secondly, the terms of trade in the goods market, $(\hat{m}_t, \hat{q}_t)$, are identical across matches. Thirdly, consumption, inventory, unemployment, employment, and vacancies are constant over time. Finally, the distribution of real wage offers, $F_t(w)$, and the distribution of real wage earnings of employees, $G_t^f(w)$, and employed workers, $G_t^h(w)$, are unchanging over time. Finally, money has value, i.e., the marginal value of real money balance, $\hat{p}_t \omega_{Mt}$, is strictly positive.

Denote the real money balance, $M \equiv \frac{M_t}{\hat{p}_t}$, the buyer's surplus per purchase, $\hat{p}_t \lambda_t = \lambda$, the real money balance with a buyer, $m \equiv \frac{m_t}{\hat{p}_t}$, and the marginal value of real money balances, $\Omega_M \equiv \hat{p}_t \omega_{Mt}$. From now on, I drop the subscript t from real variables.

DEFINITION 1. A symmetric stationary monetary equilibrium (SSME) with dispersed real wages is defined as a collection of the household's choice variables $X^h \equiv \{c, M, e^h, \underline{w}\}$, the firm's choice variables, $X^f \equiv \{i, e^f, v, F(w)\}$, the terms of trade in the goods market, $(\hat{q}, \hat{m})$, and the aggregate variables $\hat{X}^h$ and $\hat{X}^f$, such that

- given aggregate variables, $\hat{X}^h$ and $\hat{X}^f$, and the terms of trade in the goods market, $(\hat{q}, \hat{m})$, the household's choice variables X^h solve (PH);
- given aggregate variables, $\hat{X}^h$ and $\hat{X}^f$, the terms of trade in the goods market, $(\hat{q}, \hat{m})$, the firm's choice variables X^f solve (PF);
- the expected return on each posted real wage offer, defined in (23), $R(w) = R^* \forall w \in [\underline{w}, \bar{w}]$ where $\bar{w}$ is the highest real wage posted by the firm;
- the terms of trade in the goods market, $(\hat{q}, \hat{m})$, satisfy (27) and (28);

- aggregate variables are equal to the relevant household's and firm's variables, $\hat{X}^h = X^h$, $\hat{X}^f = X^f$;
- the marginal value of real money balances, Ω_M, is strictly positive and finite.

I focus on the stationary symmetric monetary equilibrium in which a matched buyer receives positive surplus, i.e., $\lambda > 0$. In the case $\lambda = 0$, one can easily show that there exists a continuum of monetary equilibria as in Shi (1997), in which the nominal price of goods, $\hat{p}_t = \frac{U'(\hat{c}_t)}{\hat{\omega}_{Mt}}$, falls over time at the rate $\frac{1}{1+r}$. Since such price behavior is not realistic, I ignore this case. Thus in the symmetric stationary monetary equilibrium considered, buyers' nominal cash balance expenditure constraint given in Equation (5) will be binding.

Given binding nominal cash balance expenditure constraint and the definition $\hat{p}_t \equiv \frac{\hat{m}_t}{\hat{q}_t}, \forall t$, in the stationary and symmetric equilibrium the average price level, $\hat{p}_t$, will grow at the rate equal to the money creation rate, i.e., the inflation rate

$$\frac{\hat{p}_{t+1}}{\hat{p}_t} = g \quad \forall t. \tag{29}$$

From now on I also suppress "^" from the aggregate variables. Since every employed worker is an employee, in the symmetric stationary equilibrium, $e^h = e^f$ and $G^h(w) = G^f(w)$. In the rest of the article, I also suppress superscripts h and f from these variables, and let e denote the employment level and $G(w)$, the distribution of real wage earnings. Equations (10) and (11) imply that employment, e, and the distribution of real wage earnings, $G(w)$, are given by

$$e = \frac{\mu(v)}{\sigma + \mu(v)} \tag{30}$$

and

$$G(w) = \frac{\sigma F(w)}{\sigma + (1 - F(w))\mu(v)}. \tag{31}$$

Equations (6), (16), and (17) imply that consumption c is given by

$$c = \frac{\xi}{\delta + \xi(1-\delta)} f(e). \tag{32}$$

Equations (12), (28), and (29) together give the expression for the marginal value of real money balance, Ω_M,

$$\Omega_M = \frac{\xi}{(1+r)g - 1 + \xi} U'(c). \tag{33}$$

Equations (21) and (27) imply that the marginal value of inventory, Ω_i is given by

$$\Omega_i = \frac{\xi}{r + \delta + \xi(1-\delta)} \Omega_M. \tag{34}$$

Using (27), (33), and (34), one can show that for any $\Omega_M > 0$, the surplus to the seller is strictly positive, i.e., $\Omega_q > 0$. For a stationary monetary equilibrium to exist the surplus to the seller must be strictly positive, i.e., $\Omega_q > 0$. If $\Omega_q = 0$, then (21) implies that the marginal value of inventory must grow at the rate $\frac{1+r}{1-\delta}$ (i.e., $\frac{\Omega_{it+1}}{\Omega_{it}} = \frac{1+r}{1-\delta}$). Thus in a symmetric stationary monetary equilibrium, sellers' quantity constraints (17) will be binding.

Equations (28), (29), and (33) imply that the growth rate of money supply, g, must exceed the rate of discount, $\frac{1}{1+r}$, in order to ensure that a matched buyer gets positive surplus, i.e., $\lambda > 0$. In the rest of the article, I assume that the growth rate of money supply exceeds the rate of discount, $g > \frac{1}{1+r}$.

Equation (8) implies that the real reservation wage of unemployed workers, $\underline{w}$, satisfies

$$\Omega_M \underline{w} = \phi. \tag{35}$$

Equation (35) is a key equation of the model. It implies that if the marginal value of real money balances, Ω_M, falls, then the real reservation wage of unemployed workers, $\underline{w}$, must rise in order to induce them to work. Similarly, if the marginal value of real money balances, Ω_M, rises, the real reservation wage of unemployed workers, $\underline{w}$, falls.

Equations (22) and (34) imply that the marginal value of an employee earning real wage $w, \Omega_e^f(w)$,

$$\Omega_e^f(w) = \frac{Af'(e) - w}{r + \sigma + \mu(v)(1 - F(w))} \Omega_M, \tag{36}$$

where A is a constant given by

$$A \equiv \frac{(1+r)\xi}{r + \delta + \xi(1-\delta)}.$$

Equations (26), (33), (35), and (36) together with the equilibrium condition that the expected gross return on all real wage offers posted be equal, $R(w) = R^*, \forall$ w, imply that the equilibrium measure of vacancies, v, implicitly solves

$$k = \frac{\mu(v)}{v} \frac{\sigma}{(\sigma + \mu(v))(r + \sigma + \mu(v))} \left[\frac{A\xi}{(1+r)g - 1 + \xi} U'(c) f'(e) - \phi \right]. \tag{37}$$

Equation (37) equates the marginal cost of posting vacancies to the discounted expected marginal benefit from the filled job at the real reservation wage of the unemployed workers. Using (30) and (32), Equation (37) can be reduced to an equation only in the level of vacancies, v. The existence of equilibrium depends crucially on whether (37) has a nontrivial solution or not.

LEMMA 1. *Equation (37) has a unique and finite solution,* $0 < v < \infty$.

Given the unique level of vacancy, v, the equilibrium condition that the expected gross return on each posted real wage offer is equal implies that the distribution of real wage offers, $F(w)$, is given by

$$(38)\quad F(w) = \frac{r + 2(\sigma + \mu(v))}{2\mu(v)} \left[1 - \sqrt{\frac{r^2 + 4(\sigma + \mu(v))(r + \sigma + \mu(v)) \frac{Af'(e) - w}{Af'(e) - \underline{w}}}{(r + 2(\sigma + \mu(v)))^2}} \right].$$

Putting $F(\bar{w}) = 1$ in (38), one can solve for the upper support of the real wage offer distribution, $\bar{w}$. $\bar{w}$ is given by

$$(39)\qquad \bar{w}/\underline{w} = 1 + \left[1 - \frac{\sigma(r + \sigma)}{(\sigma + \mu(v))(r + \sigma + \mu(v))} \right] \left(\frac{Af'(e)}{\underline{w}} - 1 \right).$$

The ratio given in (39) is my main object of interest. This ratio is increasing in vacancies, v, and the marginal product of labor, $f'(e)$. It is decreasing in the real reservation wage, $\underline{w}$.

For a unique positive and finite level of vacancies, $0 < v < \infty$, the marginal value of real money balances, Ω_M, is strictly positive and finite. Hence, I have following proposition.

PROPOSITION 1. *Under the assumption that $g > \frac{1}{1+r}$, there exists a unique SSME with dispersed real wages characterized by Equations (30)–(39).*

One can easily show that in the current environment the Friedman Rule, which requires that the growth rate of money supply, g, be equal to the rate of discount, $\frac{1}{1+r}$ (i.e., $g = \frac{1}{1+r}$), is not optimal. This result is similar to Shi (1998). As this result is commonly found in such an environment, the proof is omitted. Intuitively, there are two sources of inefficiency in the model—the binding buyer's nominal cash balance constraint and the trading externalities in the labor market. One more vacancy in the labor market makes searching workers better off, but it makes other recruiters worse off. Similarly, one more searching worker makes recruiters better off, but other workers worse off. The Friedman rule, in general, does not internalize these trading externalities.

One can easily derive the equilibrium effects of inflation on consumption, output, and employment in equilibrium, which are summarized in the following proposition.

PROPOSITION 2. *An increase in the inflation rate, g, in the SSME with dispersed real wages reduces vacancies, consumption, and output, and increases the unemployment rate.*

An increase in the inflation rate erodes the value of fiat money, which reduces the return of firms on vacancies for a given level of consumption. In addition, higher inflation rate reduces the expected benefit from working and increases the real reservation wage of unemployed workers for a given consumption level, which further lowers the return on vacancies. This induces firms to reduce the equilibrium level of vacancies posted, which leads to lower output and consumption and a higher unemployment rate.

In the next section, I analyze the effects of inflation rate on the dispersion of real wage offers and earnings.[19]

5. INFLATION AND REAL WAGE DISPERSION

A key issue determining the inflation's effect on real wage dispersion is its effect on the ratio of the marginal product of labor and the real reservation wage, $\frac{f'(e)}{\underline{w}}$ (see Equation (39)). A higher inflation rate, g, by reducing employment increases the marginal product of labor, whereas it may raise or lower the real reservation wage depending on whether the marginal value real money balance falls or rises (see Equation (35)). The following corollary follows from Proposition 2.

COROLLARY 1. *An increase in the inflation rate, g, reduces the marginal value of real money balances, Ω_M, and the ratio of marginal product of labor and the real reservation wage, $\frac{f'(e)}{\underline{w}}$, in the SSME with dispersed real wages.*

The intuition for this result is quite simple. Since a higher inflation rate induces firms to reduce the level of vacancies and thus employment, it must be that the rise in the marginal productivity of labor is more than offset by an increase in the real reservation wage. Thus, the ratio of marginal product of labor and the real reservation, $\frac{f'(e)}{\underline{w}}$, falls. Since real reservation wage, $\underline{w}$, rises, this also implies that the marginal value of real money balance, Ω_M, falls.

PROPOSITION 3. *In the SSME with dispersed real wages, an increase in the inflation rate, g, narrows the support of the distributions of wage offer, $F(w)$, and earnings, $G(w)$, in the sense that it reduces the ratio $\bar{w}/\underline{w}$.*

Inflation affects the support of the distributions of real wage offers and earnings through its effect on the real reservation wage of the unemployed workers, the level of vacancies, and the marginal product of labor. An increase in the inflation rate increases the real reservation wage of unemployed workers and raises the lower support of the distributions of both real wage offers and earnings. An increase in the inflation rate may also raise the upper support partly due to an increase in the marginal product of labor and partly due to the increase in the real reservation wage. But this effect is mitigated by two factors.

Firstly, recruiters posting the highest real wage do not face any competition from other firms to retain their workers, unlike firms posting lower real wages. Consequently, recruiters posting the highest real wage need not increase their wages as much as the recruiters posting the lowest real wage. Secondly, vacancies decline with an increase in the inflation rate, reducing the matching rate of workers and weakening the effectiveness of on-the-job search in reducing the market power

[19] Since the focus of the article is on the effect of inflation rate on the dispersion of real wages, the analysis of its effect on the average real wage earnings and offers is omitted. The analysis shows that a higher inflation rate may raise or lower the average real wage earnings and offers.

of recruiters. If the second factor is strong enough, then the upper support may in fact fall. These two factors imply that the support of the distributions of real wage offers and earnings narrows with an increase in the inflation rate.

Although, analytically, I am only able to show that higher inflation narrows the support of the distributions of real wage offers and earnings, through numerical computations it can be shown that a higher inflation rate reduces other measures of dispersions of real wages (e.g., 90th to 10th percentile ratio). Thus, my model is consistent with empirical observation that higher inflation is associated with reduced wage dispersion.

6. CONCLUSION

This article has embedded the wage posting model of Burdett and Mortensen (1998) in a search-theoretic monetary model to analyze the effects of inflation on the dispersion of real wages. The article shows that in equilibrium an increase in the inflation rate reduces real wage dispersion, a finding consistent with those of several empirical papers. An increase in the inflation rate also reduces the level of vacancies and output and increases the unemployment rate. The article proposes a mechanism through which higher inflation can reduce residual wage dispersion. In future work, I would like to extend the model to incorporate segmented labor market. This will allow me to examine the effects of inflation on the composition of vacancies (more or less skilled jobs) and inter-skill wage dispersion.

APPENDIX: PROOFS

PROOF OF LEMMA 1. The equilibrium level of vacancy, v, solves

$$k = \frac{\mu(v)}{v}\frac{\sigma}{\sigma+\mu(v)}\frac{[Af'(e)-\underline{w}]\Omega_M}{r+\sigma+\mu(v)} \tag{A.1}$$

which with appropriate substitution reduces to

$$k = \frac{\mu(v)}{v}\frac{\sigma}{(\sigma+\mu(v))(r+\sigma+\mu(v))} \times \left[\frac{A\xi}{(1+r)g-1+\xi}U'(c)f'(e)-\phi\right] \equiv T(v;g). \tag{A.2}$$

From (30) and (32) we have employment $e = \frac{\mu(v)}{\sigma+\mu(v)}$ and consumption $c = \frac{\xi}{1-(1-\xi)(1-\delta)}f(e)$. Equations (30) and (32) imply that $\frac{de}{dv}$ & $\frac{dc}{dv} > 0$. Given the assumptions that $\mu'(v) > 0$, $\frac{d(\mu(v)/v)}{dv} < 0$, $U''(c) < 0$, and $f''(e) < 0$, simple differentiation of the right-hand side of (A.2) w.r.t. vacancy, v, shows that $\frac{dT(v;g)}{dv} < 0$. In addition, under the assumptions that $\lim_{v\to 0}\frac{\mu(v)}{v} = \infty$, $\lim_{v\to 0}\mu(v) = 0$, $\lim_{c\to 0}U'(c) > 0$, and $\lim_{e\to 0}f'(e) = \infty$, $\lim_{v\to 0}T(v;g) = \infty$. Also $\lim_{v\to\infty}\frac{\mu(v)}{v} = 0$

implies that $\lim_{v\to\infty} T(v; g) \leq 0$. Since the left-hand side of (A.2) is constant, it implies that there exists a unique, $0 < v < \infty$, which solves (A.2). ■

PROOF OF PROPOSITION 2. Simple differentiation of the right-hand side of (A.2) *w.r.t.* inflation rate, g, shows that $\frac{dT(v;g)}{dg} < 0$ for a given v. Since the left-hand side is independent of g, an increase in the inflation rate, g, reduces the equilibrium level of vacancies and thus employment, consumption, and output. ■

PROOF OF COROLLARY 1. Recall that $\Omega_M \underline{w} = \phi$. Thus, (A.1) can be written as

$$k = \frac{\mu(v)}{v} \frac{\sigma}{\sigma + \mu(v)} \frac{\left[\frac{Af'(e)}{\underline{w}} - 1\right]\phi}{r + \sigma + \mu(v)}. \tag{A.3}$$

Since $\frac{\mu(v)}{v} \frac{\sigma}{\sigma + \mu(v)} \frac{1}{r + \sigma + \mu(v)}$ is a decreasing function of v, in order for the level of vacancies, v, to fall following an increase in the inflation rate, g, the ratio of the marginal product of labor to the reservation wage, $\frac{Af'(e)}{\underline{w}}$, must fall. Since the marginal product of labor, $f'(e)$, rises due to fall in employment, the real reservation wage, $\underline{w}$, must rise relatively more. As $\Omega_M \underline{w} = \phi$, the marginal value of real money balance, Ω_M, necessarily falls. ■

PROOF OF PROPOSITION 3. Follows from the simple differentiation of (39) and Corollary 1. ■

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