



PUBLIC DEBT AND WELFARE IN A QUANTITATIVE SCHUMPETERIAN GROWTH MODEL WITH INCOMPLETE MARKETS

Marco Cozzi

Department of Economics, University of Victoria
Victoria, B.C., Canada V8W 2Y2

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Abstract

This paper quantifies the welfare effects of counterfactual public debt policies using an endogenous growth model with incomplete markets. The economy features public debt, Schumpeterian growth, infinitely-lived agents, uninsurable income risk, and discount factor heterogeneity. Two versions of the model are specified, one allowing for households to hold equity in the group of innovating firms. The model is calibrated to the U.S. economy to match the degree of wealth inequality, the share of R&D expenditure in GDP, the firms exit rate, the average growth rate, and other standard long-run targets. When comparing balanced growth paths, I find large long-run welfare gains in equilibria characterized by governments accumulating public wealth. In some parameterizations, the equilibrium response of the growth rate is modest. However, welfare effects decompositions show that the growth component is still an important determinant of the welfare gains in the equilibria characterized by public wealth. The version of the model without equity is easier to solve computationally, allowing to consider transitional dynamics. Taking into account the dynamic adjustment to the new long-run equilibrium shows that the transitional welfare costs are not large enough to change the sign of the welfare effects stemming from a change in public debt. I find that eliminating public debt would lead to a 1.7% increase in welfare, while moving to a debt/GDP ratio of 100% would entail a welfare loss of 0.8%.

Keywords: Public debt, Heterogeneous Agents, Incomplete Markets, Endogenous Growth, Welfare.

JEL Classifications: D52, E21, E62, H63, O41.

Author Contact:

Marco Cozzi, Dept. of Economics, University of Victoria, P.O. Box 1700, STN CSC, Victoria, B.C., Canada V8W 2Y2; E-mail: mcozzi@uvic.ca; Tel: (250) 721-6535

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Contact details: *Department of Economics, University of Victoria, 3800 Finnerty Road, Victoria, BC V8P 5C2, Canada. Tel: 1-250-721-6535, E-mail: mcozzi@uvic.ca*

1 Introduction

The financial crisis and the global pandemic have represented two large macroeconomic shocks that compelled many governments to increase their public expenditures, with a concurrent decrease in tax revenues. In the near future, several countries will have to address whether the large accumulated public debts represent suboptimal outcomes, and whether considerable fiscal adjustments are justified on economic grounds.¹ [Aiyagari and McGrattan \(1998\)](#) is a seminal contribution showing that, in the presence of incomplete insurance markets, a sizable stock of public debt can be optimal. Their analysis focuses on steady-state comparisons, and assumes exogenous increases in productivity, implying that any change in public debt policy would leave the income growth rate unaffected. The assumption of exogenous technological progress is made in most papers studying the optimal quantity of public debt within an incomplete markets set-up, such as [Floden \(2001\)](#), [Desbonnet and Weitzenblum \(2012\)](#), [Rohrs and Winter \(2017\)](#), [Chatterjee, Gibson and Rioja \(2017\)](#), and [Chatterjee, Gibson and Rioja \(2018\)](#). Although the empirical literature has not reached a consensus on the magnitude of the causal relationship between public debt and growth, there is agreement that a big enough public debt will be detrimental for growth.² A popular view, stemming from the –controversial for some– results in [Reinhart and Rogoff \(2010\)](#), posits that there is a threshold value for the public debt/GDP ratio, estimated to be below the 100% mark. According to this perspective, public debt/GDP ratios above the threshold will slow down the income growth rate. Any econometric analysis tackling this issue is challenging because it has to deal with pervasive endogeneity problems. Finding valid instruments has proven difficult, and the thorough analysis in [Panizza and Presbitero \(2014\)](#) fails to find such a threshold. Using different data and methodologies, [Cecchetti, Mohanty and Zampolli \(2012\)](#), [Woo and Kumar \(2015\)](#), and [Salotti and Trecroci \(2016\)](#) all find a negative and significant point estimate of measures of public debt on the growth rate of productivity and income. These contributions support the notion that larger public debts are detrimental for growth, even though the estimated effect can be small. [Akcigit, Grigsby, Nicholas and Stantcheva \(2022\)](#) use four historical datasets to document that personal and corporate income taxes have significant negative effects on the quantity of innovation, which is a relevant margin for the mechanisms explored in this paper. Similarly, the growing body of empirical evidence on the effects of taxes on the quantity and quality of innovation, reviewed for example by [Stantcheva \(2021\)](#), finds that corporate and royalty taxes have a negative effect on firms' productivity growth.

The findings in [Desbonnet and Kankanamge \(2017\)](#) are another reason for endogenizing the growth process. They show that in the [Aiyagari and McGrattan \(1998\)](#) model the assumed growth rate has a large quantitative effect on the equilibrium welfare gains arising from a change in public debt policy. Motivated by these considerations, in this paper I quantitatively characterize the welfare effects of public debt in a Schumpeterian growth framework. I do so in a model featuring productivity-enhancing vertical innovations driven by investment in Research and Development (R&D), infinitely-lived agents facing income risk with incomplete insurance

¹In his recent survey, [Yared \(2019\)](#) argues that the observed rise in public debts is a phenomenon that has been unfolding over several decades. The more recent shocks have accelerated these trends, possibly exacerbating the distortions induced in the economy.

²For instance, think of the extreme case with such a high public debt that the interest costs are as large as aggregate GDP. In this situation, no resources would be allocated to investment.

opportunities, and a high degree of wealth concentration generated by discount factor heterogeneity.

In an endogenous growth framework, [Clemens and Heinemann \(2015\)](#) introduce incomplete markets in standard *AK* growth models with knowledge spillovers. They focus on the role of income shocks, but they do not consider creative destruction, the role of public debt, and cannot generate a realistic degree of wealth inequality, in part because they assume a simple stochastic income process.

The representative-agent version of the Schumpeterian growth model with capital accumulation that I extend has been used by [Howitt and Aghion \(1998\)](#) to argue that capital subsidies will raise the long-run growth rate, by [Nuno \(2011\)](#) to study the costs of business cycles, and by [Cozzi, Pataracchia, Pfeiffer and Ratto \(2021\)](#) to assess the relative importance of demand and supply factors in the recent slowdown of U.S. growth. [Chu and Cozzi \(2018\)](#) also study a Schumpeterian growth model with heterogeneous households, but they focus on the role of patents and R&D subsidies on income inequality, abstracting from uninsurable labor income risk.

A key element of the analysis is that R&D is capital intensive, and [Howitt and Aghion \(1998\)](#) provide some empirical evidence supporting this assumption. Any equilibrium effects on the capital costs are going to bring about endogenous responses in the rate of technological change. Changes in the interest rate affect multiple margins, two of which (the business stealing effect, and the rental cost of capital) have opposite effects on the equilibrium response of expected profits, hence on the growth rate. Moreover, the quantitative analysis is not straightforward, as in this class of models the growth rate could be inefficiently high in the decentralized equilibrium, with excessive investments in R&D displacing private consumption. If this were to be the case, public debt policies negatively affecting the growth rate could improve welfare.

The model is calibrated to the U.S. economy to match the degree of wealth inequality, the share of R&D expenditure in GDP, the firms' exit rate, the average growth rate, and other standard long-run targets. The calibrated model is used to quantify the welfare effects of counterfactual public debt policies, designating the long-run average of the U.S. public debt/GDP ratio as the status quo.

In one formulation of the model, the quantitative findings based on Balanced Growth Path (BGP) comparisons show that the equilibrium response of the endogenous growth rate is rather small. This result can potentially rationalize why some contributions in the empirical literature estimating the causal effect of increased public debts on the growth performance find negative effects, while others find no effect at all. One of the mechanisms that limits the effects on the growth rate is the behavior of one asset, the equity in the innovating firms held by the households. A reduction in the public debt/GDP ratio decreases the aggregate asset demand, triggering general equilibrium effects. Renting capital in the intermediates production becomes cheaper, profits increase, and so do dividends. The increase in the value of dividends causes the price of equity to rise. In this version of the model, the aggregate value of the asset market is relatively stable, as a decrease in the supply of assets stemming from a lower public debt is partially compensated by the opposite response of the value of equity in the innovating firms. This mechanism is what leads to the quantitatively small response of the growth rate, via a muted reaction of the interest rate. To allow for a stronger response of the growth rate, I consider another formulation of the model, without equity and with risk neutral entrepreneurs.

In both versions of the model, when comparing BGPs, I find large long-run welfare gains in equilibria characterized by governments accumulating public wealth. Welfare effects decompositions show that level effects

and growth effects reinforce each other. Moreover, the growth component is always an important determinant of the welfare gains in the equilibria characterized by public wealth.

The importance of transitional dynamics has been emphasized by [Desbonnet and Weitzenblum \(2012\)](#), and also in this paper I find that the long-run welfare analysis delivers either upper or lower bounds (depending on whether public debt is reduced or increased, respectively). The path toward a new BGP, triggered by an exogenous decrease in the long-run value of the public debt/GDP ratio, entails some welfare costs. Along the transition, public debt is reduced by increasing taxes, which has a quantitatively important effect. Taking into account the dynamic adjustment to the new long-run equilibrium, I find that the transitional welfare costs are not large enough to change the sign of the welfare effects stemming from a change in public debt. The results show that eliminating public debt would lead to a 1.7% increase in welfare, while moving to a debt/GDP ratio of 100% would entail a welfare loss of 0.8%.

In the literature, the closest papers are [Rohrs and Winter \(2017\)](#), [Chatterjee, Gibson and Rioja \(2017\)](#), [Chatterjee, Gibson and Rioja \(2018\)](#), and [Cozzi \(2019\)](#), none of which endogenizes the growth rate. Moreover, the arguments made in those contributions are not always applicable to my framework, as for instance the presence of monopolistic distortions in the Schumpeterian growth model complicates the comparison between the two set-ups. Another noteworthy difference is in the mechanism leading to a concentration of wealth that can match the U.S. data. Those contributions focus on the so-called superstar shock, first used by [Castaneda, Diaz-Gimenez and Rios-Rull \(2003\)](#). Although this is a tractable framework, it is not supported by the relevant empirical evidence on labor income dynamics and income shocks. At least since [Krusell and Smith \(1998\)](#), discount factor heterogeneity has been used extensively in quantitative models with incomplete markets, and [Epper et al. \(2020\)](#) provide evidence supporting this channel using Danish experimental data. This different modeling assumption can be important, because [Rohrs and Winter \(2017\)](#) identify inequality as the major driver of the welfare effects of public debt/GDP changes. [Chatterjee, Gibson and Rioja \(2017\)](#) and [Chatterjee, Gibson and Rioja \(2018\)](#) extend that framework to include public infrastructure, finding that this leads to a lower optimal debt level relative to the specification without infrastructure. Finally, [Cozzi \(2019\)](#) finds that both open-economy and life-cycle considerations are important factors accounting for the desirability of public wealth for economies with access to international financial markets.

The rest of the paper is organized as follows. Section 2 presents a model with endogenous Schumpeterian growth and incomplete markets. Section 3 is devoted to the model calibration. Section 4 discusses the main results. Section 5 concludes. Four appendices present the model and the numerical methods used to solve it in more detail.

2 A Model with Incomplete Markets and Schumpeterian Growth

The model extends the [Aiyagari and McGrattan \(1998\)](#) economy, allowing for endogenous growth, two sources of heterogeneity in labor income, and a high-degree of wealth concentration. Time is discrete, and denoted by t . In a first version of the model, there are two types of agents: workers and entrepreneurs. Workers earn a

stochastic labor income, while entrepreneurs earn stochastic profits. There are no state-contingent markets to insure against labor income risk, but workers can self-insure by saving in a risk-free asset.

Agents: Workers are infinitely-lived, and they differ both ex-ante, in their discount factors, and ex-post, due to idiosyncratic realizations of income shocks. Beside the workers, there is a measure one of risk-neutral and infinitely-lived entrepreneurs, whose consumption is denoted as C_e . Workers are subject to an exogenous borrowing limit, denoted as b .

Preferences: Workers' preferences are assumed to be time-separable and represented by the utility function $U(\cdot)$. Their utility is defined over stochastic consumption sequences $\{c_t\}_{t=0}^{\infty}$. They are utility-maximizers, and achieve this goal by choosing their consumption level (c_t), and asset holdings (a_{t+1}). These agents' problem can be defined as:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 U(c_0, c_1, \dots) = \max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where \mathbb{E}_0 represents the expectation operator over the idiosyncratic sequences of shocks, and $0 < \beta < 1$ is the heterogeneous subjective discount factor. The fraction μ^β of workers has discount factor $\underline{\beta}$, while the remaining fraction $1 - \mu^\beta$ has discount factor $\bar{\beta}$, with $\underline{\beta} < \bar{\beta}$. The discount factors are assigned randomly in the population, are uncorrelated with a worker's productivity, are time-invariant, and are perfectly observable already at time $t = 0$. As customary in the literature, I assume that $u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}$. The per-period utility function is strictly increasing and concave in consumption, satisfies the Inada conditions, and features a constant relative risk aversion (θ).

Endowments: All agents enter the economy with a zero asset endowment. Workers differ in their labor endowments $\epsilon_t^{\varepsilon, \phi}$, which are determined by two different channels. There is a stochastic component ε_t , whose natural logarithm follows a stationary $AR(1)$ process: $\ln \varepsilon_t = \zeta_y \ln \varepsilon_{t-1} + \xi_t$, with $\xi_t \sim N(0, \sigma_y^2)$. This is coupled with a fixed effect ϕ , whose natural logarithm is drawn from a normal distribution $\ln(\phi) \sim N(0, \sigma_\phi^2)$. At every point in time, the efficiency units a worker is endowed with are the product of these two components, which multiplied by the going wage (w_t) gives the stochastic labor earnings $y_t^w = w_t \epsilon_t^{\varepsilon, \phi} = w_t \times \varepsilon_t \times \phi$.³

Schumpeterian growth: The model considers a Schumpeterian growth mechanism that relies on capital as an essential factor of production, as in [Howitt and Aghion \(1998\)](#) and [Nuno \(2011\)](#), whose discrete time formulation I follow closely. Endogenous growth results from vertical innovations. Producers of final goods use labor and a continuum of intermediate goods M as inputs. The final goods sector is modeled as a constant returns to scale technology of the Cobb-Douglas form, which relies on aggregate labor L and the sum of all the intermediate goods M_i to produce the final output Y . The intermediate goods differ in their productivity $Z_{i,t}$ and each of them is produced by a monopolistic firm using capital as the only input. The amount of capital necessary to produce

³In the quantitative implementation, the shock ε is discretized with the Rouwenhorst method, using a 11-state Markov chain. For the fixed effect ϕ , I use five discrete types. Each type has a 20% mass, and the associated gridpoints ϕ_n are obtained by computing the conditional averages of the underlying continuous normal distribution. For more details, see Appendix A.

each intermediate good is proportional to its productivity, and more advanced products require increasingly capital-intensive techniques. In each period, there is a probability that the productivity of an intermediate good jumps to the technology frontier owing to the R&D activities of entrepreneurs. Entrepreneurs borrow resources from a perfectly competitive banking sector. They invest these funds in R&D activities, trying to increase their probability of making a discovery that is going to displace the current monopolist in the production of a specific intermediate good. If a discovery occurs, the successful entrepreneur introduces a new enhanced intermediate product in the relevant sector and becomes the new monopolist until replaced (stochastically) by another entrepreneur. The technology frontier (i.e., the productivity level of the most advanced sector) evolves endogenously as the result of positive spillovers from innovation activities.

Technology: The homogeneous final good is produced under perfect competition using labor and a continuum of intermediate products. Final output can be used interchangeably as a consumption good (C_t), a capital good (K_t), or an input to innovation (RD_t). The representative firm producing in the final good sector maximizes its profits having access to a Cobb-Douglas production function. The inputs are aggregate labor (L_t), the quantity of intermediate products of variety i ($M_{i,t}$), and the associated productivity index ($Z_{i,t}$).

$$Y_t = F(L_t, \{Z_{i,t}\}_i, \{M_{i,t}\}_i) = L_t^{1-\alpha} \int_0^1 Z_{i,t} M_{i,t}^\alpha di.$$

It follows that the profits π^Y in the final sector are given by:

$$\pi^Y = L_t^{1-\alpha} \int_0^1 Z_{i,t} M_{i,t}^\alpha di - \int_0^1 P_{i,t} M_{i,t} di - w_t L_t.$$

$M_{i,t}$ denotes the amount of intermediate product i , $P_{i,t}$ their monopolistic prices, while $Z_{i,t}$ is a productivity parameter embodied in the latest version of intermediate product i . The price of the final output is normalized to 1. Since the final good firm is a price taker, the first order conditions for profit maximization lead to a system of (inverse) demand equations $P_{i,t}(M_{i,t})$, one per intermediate good variety, given by:

$$P_{i,t}(M_{i,t}) = \alpha Z_{i,t} L_t^{1-\alpha} M_{i,t}^{\alpha-1}, \forall i. \quad (1)$$

Another first order condition delivers the (inverse) labor demand schedule:

$$w_t = (1 - \alpha) Y_t / L_t.$$

Intermediate goods: Each intermediate product i is produced by an incumbent monopolist using a capital-intensive technology (where $K_{i,t}$ is the capital used in sector i at time t):

$$M_{i,t} = K_{i,t} / Z_{i,t}. \quad (2)$$

Capital is divided by the productivity index $Z_{i,t}$ to capture the phenomenon that successive vintages of the intermediate product are produced by increasingly capital-intensive techniques. At the aggregate level, the model is deterministic, and the incumbent monopolist in each sector can correctly forecast the demand

for the intermediate good they are producing. Taking the demand function for their product as given, but understanding the consequences of setting different prices, each incumbent monopolist has the following (after-tax) profit function (where τ_f is the proportional tax rate on corporate profits π^f):

$$(1 - \tau_f)\pi_{i,t}^f = (1 - \tau_f)[P_{i,t}(M_{i,t})M_{i,t} - (r_t + \delta)K_{i,t}]. \quad (3)$$

Capital depreciates at the exogenous rate δ and r_t is the real rate of return to capital, so their sum $(r_t + \delta)$ is the user cost of capital. Substituting Eq. (1) and (2) into (3), the (after-tax) profit function can be rewritten as:

$$(1 - \tau_f)\pi_{i,t}^f = (1 - \tau_f)Z_{i,t} [\alpha L_t^{1-\alpha} M_{i,t}^\alpha - (r_t + \delta)M_{i,t}]. \quad (4)$$

Notice how the RHS of Eq. (4) is linear in $Z_{i,t}$. Each intermediate product is produced in the same amount M_t because the incumbents cannot innovate, their demand functions are symmetric, and the user cost of capital is the same for every firm. Aggregate quantities are the integral with respect to all intermediate products, namely the average productivity across all sectors is $Z_t = \int_0^1 Z_{i,t} di$, and aggregate capital is obtained as the product of average productivity times the average intermediate good:

$$K_t = \int_0^1 K_{i,t} di = Z_t M_t.$$

A convenient feature of this model is that in equilibrium the final good sector displays the familiar Cobb-Douglas aggregate production function in capital, labor and technological progress:

$$Y_t = F(L_t, Z_t, K_t/Z_t) = K_t^\alpha (Z_t L_t)^{1-\alpha}.$$

Compared to the standard neoclassical growth model, the equilibrium interest rate has a different formula. This is due to the monopolistic price distortions and the presence of profits in the intermediate good sector.

$$r_t = \alpha^2 \frac{Y_t}{K_t} - \delta. \quad (5)$$

Notice how in Eq. (5) the term α^2 replaces α in the corresponding version for the marginal product obtained in the neoclassical growth model with a Cobb-Douglas aggregate production function.

In terms of the functional distribution of income, the share $(1 - \alpha)$ of GDP belongs to labor earnings ($w_t L/Y_t = 1 - \alpha$), the share α^2 to capital earnings including depreciation ($(r_t + \delta)K_t/Y_t = \alpha^2$), and the share $\alpha(1 - \alpha)$ to profits ($\Pi_t/Y_t = \int \pi_{i,t}^f di/Y_t = \alpha(1 - \alpha)$). In every period, the flow of after-tax profits earned by the incumbent in the intermediate goods sector i is:

$$(1 - \tau_f)\pi_{i,t}^f = (1 - \tau_f)\alpha(1 - \alpha) \frac{Z_{i,t} Y_t}{Z_t}. \quad (6)$$

Innovation and Technological Change: Innovations result from entrepreneurship activities that advance technological knowledge. At any date there is a technology frontier that represents the most advanced technology across all sectors:

$$Z_t^{max} = \underset{i}{Max} Z_{i,t}.$$

The productivity $Z_{i,t}$ of an intermediate good in sector i can jump to the technology frontier with endogenous probability $p_{i,t}$. In order to achieve this goal, entrepreneurs in this sector have to undertake costly R&D activities ($RD_{i,t}$). Entrepreneurs invest resources $RD_{i,t}$ in an attempt to increase their probabilities of making a discovery and replace the current incumbents. If a discovery occurs, the successful entrepreneur introduces a new enhanced intermediate product in that sector and becomes the new monopolist, until another entrepreneur will create an even better version of that intermediate product. At the level of a single sector the evolution of the productivity index is stochastic. With probability $p_{i,t}$ the R&D activity is successful and the productivity jumps to $Z_{i,t+1} = Z_t^{max}$. With complement probability $1 - p_{i,t}$ the R&D activity is unsuccessful, the incumbent is not replaced by a new monopolist, and the productivity stays at its current value $Z_{i,t+1} = Z_{i,t}$.⁴ Entrepreneurs incur R&D costs and the probability of a successful innovation is assumed to be:

$$p_{i,t} = 1 - e^{-\left(\frac{RD_{i,t}}{\lambda Z_t^{max}}\right)^\psi}$$

where λ and ψ represent efficiency parameters of R&D. A higher value of λ results in a less effective expenditure in R&D, because a given investment $RD_{i,t}$ corresponds to a lower probability of a successful innovation. The amount spent on R&D is also adjusted by the technology frontier variable Z_t^{max} . This is meant to capture in a parsimonious way the increasing complexity of progress: as technology advances, the resource cost of further improvements increases proportionally. ψ controls the curvature of the probability of a successful innovation, and it helps matching the empirical share of R&D expenditures in GDP. Notice also how this formulation guarantees that the probability of an innovation is always well behaved (i.e., $0 \leq p_{i,t} < 1, \forall(i, t)$), and that R&D investments are essential for innovations to occur (i.e., $p_{i,t} = 0$ when $RD_{i,t} = 0$).⁵

Entrepreneurs: The presence of monopoly power gives the incumbent in each intermediate good variety the prospect of earning profits over the duration of their technological advantage. The value of being the incumbent in period t in sector i with current productivity level \widehat{Z} is denoted with $V_{i,t}^f(\widehat{Z})$, and satisfies the following Bellman equation:

$$V_{i,t}^f(\widehat{Z}) = \underset{M_{i,t}}{Max} \left\{ (1 - \tau_f)\pi_{i,t}^f + \left(\frac{1 - p_{i,t}}{1 + r_t} \right) V_{i,t+1}^f(\widehat{Z}) \right\} \quad (7)$$

$V_{i,t}^f(\widehat{Z})$ is the discounted value of after-tax profits that the incumbent is expected to obtain, given that the intermediate good variety that they previously developed has an associated (and fixed) productivity equal to \widehat{Z} . The Bellman equation displays the process of creative destruction, as it makes clear that the monopolistic position might be lost to a competitor with probability $p_{i,t}$ (hence, the complement probability $1 - p_{i,t}$ stands for the incumbent's chances of survival). Following the literature, I focus on the case where, in each period and sector, there is a single entrepreneur that tries to replace the incumbent by developing an improved version of

⁴After a successful innovation, the entrepreneur enters into Bertrand competition with the previous incumbent in that sector. Since the old intermediate good is of inferior quality, the incumbent exits. The appendix in [Howitt and Aghion \(1998\)](#) provides more details on how the strategic interaction unfolds.

⁵In a growing economy, rescaling $RD_{i,t}$ by Z_t^{max} prevents the probability of an innovation from becoming arbitrarily close to 1 in the long run. Absent this rescaling, the innovation process would eventually be deterministic. Notice also that, as long as $Z_0^{max} > 0$, the value $Z_t^{max} = 0$ can never be obtained.

the related intermediate good. Since entrepreneurs are assumed to be risk neutral, and can borrow the resources needed to finance the R&D expenditures from a competitive banking sector at the interest rate r_t , they choose $RD_{i,t}$ to maximize the expected discounted value of becoming an incumbent in their sector of operation i in the following period:

$$\underset{RD_{i,t}}{Max} \left\{ -RD_{i,t} + \left(\frac{p_{i,t}(RD_{i,t})}{1+r_t} \right) \mathbb{E}_t V_{i,t+1}^f(Z_t^{max}) \right\} \quad (8)$$

Notice how in Eq. (8) the optimal value function $V_{i,t+1}^f(\cdot)$, representing the discounted expected value of becoming an incumbent, is evaluated at $Z_t^{max} > \widehat{Z}$, as an entrepreneur's successful innovation brings the associated productivity to the technology frontier. This value function is deterministic, and the expectation operator can be dropped. Since the continuation value is the same in all sectors, and all entrepreneurs borrow at the same interest rate, R&D expenditures do not vary with i , $RD_{i,t} = RD_t$. Let's define $\rho_t \equiv \frac{RD_t}{\lambda Z_t^{max}}$. Taking the log of the first order conditions leads to a non-linear equation in ρ_t :

$$-\rho_t^\psi + (\psi - 1) \ln \rho_t + \ln \psi = \ln \left[\frac{(1+r_t)\lambda Z_t^{max}}{V_{t+1}^f(Z_t^{max})} \right] = \ln \left[\frac{(1+r_t)RD_t}{\rho_t V_{t+1}^f(Z_t^{max})} \right] \quad (9)$$

The advancement of the technology frontier Z_t^{max} is the mechanism that drives the aggregate economic growth. Innovations induce knowledge spillovers, because at any point in time the technology frontier is available to any successful innovator. This publicly available knowledge grows at a rate proportional to the aggregate rate of innovations, and each innovation moves the technology frontier by a factor $1 + \sigma > 1$. At any point in time, and across different varieties of intermediates, some entrepreneurs are going to be successful at innovating, while others are going to fail. Taking this into account, together with the assumption that the likelihood of a successful innovation is independent across intermediate varieties, a law of large numbers guarantees that at the aggregate level the average productivity will evolve according to the following equation:

$$Z_{t+1} = \int_0^1 \{p(\rho_t)(1+\sigma)Z_t + [1-p(\rho_t)]Z_{i,t}\} di = (1+\sigma)p(\rho_t)Z_t + [1-p(\rho_t)]Z_t = [1+p(\rho_t)\sigma]Z_t \quad (10)$$

The first term in the integral represents the probability that an innovation is going to occur for a given variety, multiplied by the implied increase in the related productivity index $Z_t^{max} = (1+\sigma)Z_t$.⁶ The second term in the integral represents the probability that an innovation is not going to occur, hence the productivity of all these intermediates is going to stay at their current level. The symmetry in the R&D investments across all varieties implies that also the probabilities of a successful innovation are symmetric.

Eq. (10) can be manipulated to express how the aggregate productivity is going to grow over time. Along a BGP, $\rho_t = \rho$ as both the R&D expenditures and the technology frontier will grow at the same rate. The (average) growth rate g is then determined as:

$$g = \frac{Z_{t+1}}{Z_t} - 1 = p(\rho)\sigma$$

⁶I assume that the formula $Z_t^{max} = (1+\sigma)Z_t$ applies along a BGP, and [Howitt and Aghion \(1998\)](#) prove that it holds asymptotically.

This equation shows that the growth rate is equal to the spillover effect weighted by the probability of an innovation. In this framework, the spillover effect σ is an exogenous parameter. Without a strictly positive knowledge spillover the economy cannot grow over time. The second element in the formula is $p(\rho)$, which is the endogenous business stealing effect, that is affected also by public policy. Different public debt policies will imply different interest rates and taxation schemes, which in turn are going to have an impact on the (rescaled) expenditure on R&D. This is going to affect the likelihood of an innovation, and ultimately the growth rate g .

Because of the positive growth rate, to use recursive methods, I need to de-trend the growing variables. Variables with a “tilde” are de-trended using the average productivity, e.g. $\tilde{Y}_t \equiv Y_t/Z_t$. Along a BGP, de-trended variables do not vary over time, and their time indexes can be dropped. Also, $\rho_t = \rho$ and $r_t = r$. Along a BGP, the de-trended average price of the intermediates is $\tilde{P} = \alpha(L/M)^{1-\alpha}$, which is stationary because M does not grow over time.

Since the incumbents’ optimal value functions are non-stochastic, and their productivity is fixed over the duration of their monopoly, they can be solved analytically:

$$V^f(Z_t^{max}) = \left(\frac{1+r}{r+p(\rho)} \right) (1-\tau_f)\pi_t^f \quad (11)$$

Substituting (11) into (9), evaluating (6) at an innovator’s productivity Z_t^{max} , and simplifying gets the non-linear equation that is used to compute ρ :

$$-\rho^\psi + (\psi - 1) \ln \rho + \ln \psi = \ln \left[\frac{(r+p(\rho))\lambda}{(1-\tau_f)\alpha(1-\alpha)\tilde{Y}} \right]$$

Taxes, Government Outlays, and Public Debt: Workers pay income taxes, and the proportional tax rate τ is set to satisfy the intertemporal budget constraint of the government. Every household is also entitled to a lump-sum public transfer tr_t .⁷ Successful entrepreneurs pay taxes on their profits, with proportional tax rate τ_f . The government finances wasteful and exogenous public expenditure G_t . Total tax revenues T_t are obtained from taxing labor income, asset income, and profits. Total government outlays are the sum of government purchases G_t and total public transfers TR_t .

The dynamics of public debt D_t are obtained by considering the intertemporal budget constraint of the government:

$$D_{t+1} = (1+r_t)D_t + G_t + TR_t - T_t$$

In a BGP, de-trended public debt is constant over time, and it is such that the related interest costs, adjusted for the growth rate, are equal to the difference between tax revenues and government outlays:

$$(r-g)\tilde{D} = \tilde{T} - \tilde{TR} - \tilde{G}$$

⁷Recall that a lump-sum transfer combined with a flat income tax is a parsimonious specification that delivers a progressive fiscal system.

In a BGP, de-trended aggregate output \tilde{Y} is constant, and the previous relationship can be considered relative to it:

$$(r - g) \frac{\tilde{D}}{\tilde{Y}} = \frac{\tilde{T} - \tilde{TR} - \tilde{G}}{\tilde{Y}} \quad (12)$$

The expression for total de-trended tax revenues is $\tilde{T} = \tau(\tilde{w}L + r\tilde{A}) + \tau_f\tilde{\Pi}$, where \tilde{A} are the total detrended assets held by the households, and $\tilde{\Pi}$ are detrended aggregate profits. Imposing the equilibrium condition in the asset market ($\tilde{K} + \tilde{D} = \tilde{A}$), substituting the formula for \tilde{T} into Eq. (12), defining $d \equiv \tilde{D}/\tilde{Y}$, $\gamma \equiv \tilde{G}/\tilde{Y}$, $\chi \equiv \tilde{TR}/\tilde{Y}$, $\kappa \equiv \tilde{K}/\tilde{Y} = \alpha^2/(r + \delta)$, and solving for τ gives:

$$\tau = \frac{\gamma + \chi + (r - g)d - \tau_f\alpha(1 - \alpha)}{1 - \alpha + \alpha^2 - \delta\kappa + rd}$$

For given policies on public debt, government expenditure, public transfers, and corporate taxes, this formula represents the equilibrium value of the tax rate that satisfies the intertemporal budget constraint of the government. Notice that τ depends on r and g , which are both equilibrium objects.

Equity in Innovating Firms: Many contributions in the Schumpeterian growth literature, such as [Howitt and Aghion \(1998\)](#) and [Nuno \(2011\)](#), are based on a representative agent framework. They typically assume that net profits are distributed back to the households in a lump-sum fashion. This setup is not suitable here, because the considerable size of profits would de facto undo the assumption of market incompleteness. Instead, in an alternative version of the model, I assume that households can hold equity in a mutual fund that owns all innovating firms, along the lines of [Krusell, Mukoyama and Sahin \(2010\)](#), among many others. This framework allows to re-interpret a risk neutral entrepreneur as a project that is alternative to the one currently in place.⁸ Such a project is set up when the previously developed blueprint has become operational, and can lead to a productivity-enhancing innovation. If the project fails, it is scrapped and substituted by a different one.⁹ The mutual fund finances all R&D activities in these alternative risky projects, and collects the stream of profits generated by the successful ones. The mutual fund's objective is to maximize the sum of expected discounted profits stemming from all the successful projects. For tractability, households are not allowed to hold equity on the venture of a specific project, which makes equity a risk-free asset. In this economy, there are three assets, one being the innovating projects, whose measure is equal to 1. Equity in the mutual fund represents a claim on the future profits of the successful projects. The equilibrium price of equity is the present discounted value of the stream of dividends, whose total value equals the aggregate profits net of the aggregate expenditure in R&D. Denoting the price of equity (after the dividends are paid) with P_t^q , and dividends with Δ_t , the pricing equation is:

⁸An advantage of this formulation is that it implies $C_e = 0$ in all equilibria. Although having wealthy entrepreneurs would be desirable, allowing for risk aversion is computationally intractable, because of the added complications in the price setting problem, which would depend on the wealth distribution.

⁹To fix ideas, think of CPU manufacturers. When a new lithography is developed, the old microprocessors stop being produced. The production of the new architecture typically requires brand new equipment and upgraded plants, and work on the next generation of chips starts immediately. Intel's infamous struggles to shrink their process node from 14nm to 10nm is an example of projects failing and getting replaced.

$$P_t^q = \frac{P_{t+1}^q + \Delta_{t+1}}{1 + r_t} \rightarrow \widetilde{P}^q = \left(\frac{1+g}{r-g} \right) \widetilde{\Delta}$$

where $1 + r_t$ represents the firms' discount rate, and \widetilde{P}^q and $\widetilde{\Delta}$ are the de-trended price of equity and dividends along a BGP. Since the total quantity of equity is normalized to one ($q = 1$), the equilibrium de-trended aggregate value of equity along a BGP (\widetilde{Q}) is:

$$\widetilde{Q} = \left(\widetilde{P}^q + \widetilde{\Delta} \right) * q = \left(\frac{1+r}{r-g} \right) \widetilde{\Delta}$$

The equilibrium condition in the asset market becomes:

$$\widetilde{K} + \widetilde{D} + \left(\frac{1+r}{r-g} \right) \widetilde{\Delta} = \widetilde{A}$$

In equilibrium, the tax rate that satisfies the intertemporal budget constraint of the government is obtained with the following equation:

$$\tau = \frac{\gamma + \chi + (r-g)d - \tau_f \alpha (1-\alpha)}{1 - \alpha + \alpha^2 - \delta \kappa + rd + \frac{r(1+r)}{r-g} [(1-\tau_f)\alpha(1-\alpha) - \eta]}$$

where η stands for the R&D/GDP ratio.

3 Calibration

Parameter values are assigned relying on a mix of (reduced-form) estimation and calibration (in equilibrium). The initial BGP is calibrated to match selected long-run features of the U.S. economy. Table 1 reports the full list of calibrated parameters with their values and empirical targets. The targets in the bottom part of the table are matched jointly, by searching over the related parameters' space.¹⁰

[Table 1 about here]

Preferences: The coefficient of relative risk aversion is set to $\theta = 2$. The two preference types are assumed to have the same mass, and $\mu^\beta = 0.5$. The two discount factors ($\underline{\beta}, \overline{\beta}$) are chosen to match a pre-tax equilibrium interest rate of 4.5% and a Gini wealth index of 0.8. In the model without equity, the calibrated values are $\underline{\beta} = 0.985$ and $\overline{\beta} = 0.986$. In the model with equity, the added supply of this additional asset leads to a larger aggregate wealth. A wider difference in the discount factors, which in this case are $\underline{\beta} = 0.981$ and $\overline{\beta} = 0.995$, is needed to match the concentration of wealth while hitting the same empirical target for the interest rate as above.

¹⁰The parameters in the two models, with or without equity, are calibrated separately. It turns out that the only two parameters that need to be re-calibrated are the two discount factors. Once these parameters are such that the same interest rate and wealth Gini index are matched, all remaining parameters can be kept at their values used in the other version of the model. In particular, since the aggregate labor supply is fixed and the interest rate is the same in the two economies, also the growth rate and the firms' exit rate are the same when using a mutual set of values for the R&D and the spillover effect parameters.

Technology: The curvature of the final good production function is set to $\alpha = 0.36$, to match a labor share of 64%. The capital depreciation rate is set to $\delta = 0.08$, a common value in the literature for a yearly model.

Stochastic Income process: The parameterization of the stochastic income process relies on the estimates obtained by [Guvenen \(2009\)](#) using Panel Study of Income Dynamics data. These estimates are $\zeta_y = 0.988$ and $\sigma_y^2 = 0.015$ for the AR(1) component, and $\sigma_\phi^2 = 0.058$ for the fixed effect.

Economic Growth: The efficiency of R&D parameters are set to $\lambda = 2425$ and $\psi = 0.191$. These values match a firms' exit rate of 10% and a R&D/output ratio of 2.28%. The former statistic is obtained from firm dynamics data over the 1995-2007 period, while the second one from NIPA data over 1947-2007 period. The growth rate matches its long run average (1.8%) from the Penn World Tables 9.0, so in equilibrium $g = 0.018$. Given that the probability of a successful innovation is 10%, this is obtained with a spillover effect equal to $\sigma = 0.18$.

Taxes and Government: I obtain the corporate tax rate using quarterly NIPA data on aggregate profits. I consider the percentage difference between the pre-tax and after-tax aggregate profits, and compute the long-run average of the implied quarterly tax rates. Over the 1947-2007 period, $\tau_f = 0.326$. Government consumption is set to match a public expenditure/GDP ratio of 21% ($\gamma = 0.21$), and the public transfer matches a transfers/GDP ratio of 8.2% ($\chi = 0.082$), which are the long-run averages for the U.S. economy. The public debt/GDP ratio is set to its long-run value of 60% ($d = 0.6$).

4 Quantitative Analysis

The calibrated model is used to quantify the welfare effects of counterfactual public debt policies, designating the long-run average of the U.S. public debt/GDP ratio as the status quo. Before discussing the quantitative results, I introduce more formally the criteria that will be used to assess the welfare effects.

4.1 Long-run Welfare Measures

As customary in this literature, I am going to posit a utilitarian social welfare function SW , which weights the agents' lifetime utilities along a BGP by their mass.¹¹

$$SW = \mu^\beta \int V(\tilde{a}, \varepsilon, \phi, \underline{\beta}) d\mu(\tilde{a}, \varepsilon, \phi, \underline{\beta}) + (1 - \mu^\beta) \int V(\tilde{a}, \varepsilon, \phi, \bar{\beta}) d\mu(\tilde{a}, \varepsilon, \phi, \bar{\beta}) \quad (13)$$

Let SW_i , $i = 0, 1$, denote the welfare attained with two different public debt policies. The index $i = 0$ stands for social welfare under the status-quo, while $i = 1$ under a counterfactual public debt policy.

¹¹The workers' optimal value functions $V(\cdot)$ and the stationary distributions $\mu(\cdot)$ appearing in Eq. (13) are carefully defined in Appendix A. Also, in order to make consistent comparisons between the welfare effects arising from the two versions of the model (with and without equity in innovating firms), I focus on the components of the social welfare function representing the workers' welfare. Namely, in the model without equity, I omit the entrepreneurs' welfare.

Welfare effects are computed using a consumption based equivalent variation (CEV). The welfare measure ϖ is the percentage increase in consumption in all states of the world that makes welfare in the baseline economy ($SW_0(\varpi)$) equal to welfare in the counterfactual one (SW_1). The overall welfare effect ϖ is given by:

$$SW_1 = SW_0(\varpi) \rightarrow \varpi = \left(\frac{SW_1}{SW_0} \right)^{\frac{1}{1-\theta}} - 1$$

Adapting the analysis in [Floden \(2001\)](#) and [Clemens and Heinemann \(2015\)](#), the overall welfare gain can be decomposed into four components. There are a level welfare effect (ϖ_{level}), a growth welfare effect (ϖ_{growth}), and two (one per BGP) risk welfare effects (ϖ_{risk_i}), such that:

$$1 + \varpi = \frac{(1 + \varpi_{level})(1 + \varpi_{growth})(1 + \varpi_{risk_1})}{(1 + \varpi_{risk_0})} \quad (14)$$

In particular, the four welfare effects are obtained from the following equation:

$$\left(\frac{SW_1}{SW_0} \right)^{\frac{1}{1-\theta}} = \frac{C_1}{C_0} \left(\frac{SW_1^g}{SW_0^g} \right)^{\frac{1}{1-\theta}} \left(\frac{SW_1}{\overline{SW}_1} \right)^{\frac{1}{1-\theta}} \left(\frac{\overline{SW}_0}{SW_0} \right)^{\frac{1}{1-\theta}} \quad (15)$$

In Eq. (15), \overline{SW}_i stands for the lifetime utility attained when all households consume the average consumption C_i , and SW_i^g stands for the lifetime utility that can be attributed to the equilibrium growth rate g_i . Note that, by construction, $\overline{SW}_i = C_i^{(1-\theta)} SW_i^g$ and $SW_i^g = \frac{\mu^\beta}{1-\beta(1+g_i)^{(1-\theta)}} + \frac{1-\mu^\beta}{1-\beta(1+g_i)^{(1-\theta)}}$.

4.2 Long-run Results

This subsection presents the quantitative findings for the long-run analysis. I will first report the results of the model with equity, and [Figure 1](#) displays eight plots representing the equilibrium response of a set of aggregate outcomes for this version of the model. The results of the model without equity are discussed next, and [Figure 2](#) presents the corresponding plots.

[[Figure 1](#) about here]

Results for the model with equity: The first panel plots the long-run welfare effects, including their decomposition in level and growth components. A stark finding is that large falls in public debt lead to large welfare gains, and vice versa. For instance, eliminating public debt entails a welfare gain of 0.5%, while moving to a debt/GDP ratio of 150% entails a welfare loss of 0.9%. The decompositions of the overall welfare gains show that both the level and growth effects are important, with the former accounting for approximately 2/3 of the overall welfare measure, and the latter for the remaining 1/3. The risk welfare effect in the counterfactual economy, ϖ_{risk_1} , changes monotonically in d , but its response is quantitatively small. Unless the size of the change in public debt is especially big, the ratio between the two gross risk welfare effects, $(1 + \varpi_{risk_1})/(1 + \varpi_{risk_0})$, is found to stay almost constant, and they essentially cancel out in Eq. (14). At most, the change

in the risk effect accounts for 1.0% of the overall welfare effect, which happens when the counterfactual public debt policy is at its lower bound.

The second panel shows the behavior of (detrended) aggregate output and consumption, relative to their values in the initial BGP. Both variables respond monotonically in the long-run value of public debt, with quantitatively large responses, up to 3.3% for output and 2.0% for consumption. Changes in public debt policy can lead to an improved “static” allocation of resources. In particular, drops in the interest rate (plotted in panel 4) play several roles, one of which being the increased use of capital in the intermediates production, leading to an increase in the final good sector’s output. Finally, the percentage response of aggregate consumption is smaller than the percentage response of output, suggesting that a sizable part of the gap might be accounted for by the response of R&D expenditures (with the remainder being allocated to finance the costs of the new level of investment in physical capital).

The third panel shows that changes in the growth rate are limited in size, but they are still important enough to cause a sizable change in welfare. For instance, considering the two extreme values in the range of counterfactual debt policies, moving from the highest to the lowest value of d , the growth rate changes from 1.815% to 1.850%. As for the interest rate, when considering the same change in d , it drops by almost one percentage point, moving from 4.7% to 3.8%. In turn, this leads to a lower discounting of prospective profits of R&D activities, with the end result of a faster growth rate. From the households’ point of view, the fall in r could affect their welfare negatively, as it makes saving a less effective instrument to self-insure against the labor income risk. However, the general equilibrium response of the tax rate (plotted in panel 5) tames this channel, because the fall in the after-tax interest rate is much less pronounced, as it changes from 3.5% to 3.1%.

In this version of the model, the behavior of the tax rate is clear-cut, as it responds monotonically. Considering a change in d from its highest to its lowest value, τ changes from 26.4% to 18.2%. This substantial drop in the proportional tax rate is possible because the government moves from a situation where interest payments are a cost on the accumulated stock of public debt, to a situation where they are an income originating from the accumulated stock of public wealth that helps financing the government outlays.¹²

The sixth panel confirms that the response of R&D expenditure is more than proportional. When public debt decreases, not only the R&D expenditure increases in levels, as output becomes larger, but it also increases monotonically as a share of GDP. However, when d changes from its highest to its lowest value, the share changes by less than 0.2 percentage points.

The changes in the wealth Gini index are depicted in panel 7. Decreases in public debt exacerbate wealth inequality, which increases at a linear rate. The fall in the tax rate is partially responsible for this outcome, as capital income is taxed less heavily, leading to a decrease in the (implicit) redistribution via this channel.

The last panel displays the behavior of (detrended) profits and dividends, relative to their values in the initial BGP. Both variables respond monotonically in the long-run value of public debt. When the long-run value of public debt is lower compared to the status quo, the percentage increase of dividends is lower than that of output, because proportionally higher costs of R&D need to be subtracted from the firms’ profits.

¹²It is worth pointing out that, in all counterfactual experiments, the corporate tax rate is kept fixed at its calibrated value. If fiscal adjustments were to involve changes in τ_f , the welfare results would be even stronger.

In terms of the mechanism at play, a reduction in the public debt/GDP ratio decreases the aggregate asset demand, triggering general equilibrium effects. Renting capital in the production of the intermediates becomes cheaper, profits increase, and so do dividends. This puts an upward pressure on the price of equity. In the asset market, part of the decrease in public debt is compensated by the increase in the demand for capital, and by the market value of equity in the innovating firms. At the same time, the higher profits stimulate innovation, with a rise in R&D expenditure that is found to be quantitatively modest. In this version of the model, the aggregate value of the asset market is relatively stable, which is what leads to the minor response of the growth rate, via a muted reaction of the interest rate.

The model shows that, in the empirically relevant range of actual public debt changes, the effects on the growth rate are monotonic, but they are minimal. Therefore, given the limited size of the response, any econometric analysis on market data, contaminated by the short-run effects of the business cycle and measurement issues, would not be able to detect them. This result can rationalize why a number of empirical studies do not find a systematic effect of changes in public debt on the growth rate.

The alternative version of the model, considered next, eliminates the possibility for households to hold equity in the group of innovating firms. In this environment, the successful entrepreneurs receive the net profits and use them to finance their consumption expenditures. This eliminates a source of demand for household savings, leading to a more pronounced response of the interest rate, and of the growth rate, in turn.

[**Figure 2** about here]

Results for the model without equity: The first panel in Figure 2 shows that, in terms of the welfare gains of reducing public debt, the qualitative implications are for the most part unaffected. Economies with a lower public debt/GDP ratio do enjoy a higher welfare. Notably, the long-run welfare effects are convex rather than concave in d , which is due to the more pronounced response of the interest rate. In this version of the model, I consider a smaller range of values for the counterfactual long-run public debt. One reason to limit it is that, for low enough values of d , the equilibrium interest rate falls below the equilibrium growth rate, and the economy becomes dynamically inefficient. Contrasting panels 3 and 4 reveals that this occurs for values of the debt/GDP ratio lower than -100% .

Overall, the interest rate response in this model is much stronger, leading to more conspicuous effects on the growth rate. Moreover, the increase in the supply of intermediates reduces the monopolistic distortion per unit produced, as their de-trended price falls.

Quantitatively, compared to the model with equity, the welfare effects are considerably larger. The level and growth decompositions retain the same properties of the overall welfare gains. In terms of their relative importance, the level component accounts for between 54% and 58% of the total welfare effect, while the growth component accounts for between 22% and 26%. It follows that, for this version of the model, the change in risk is substantial, and it accounts for between 24% and 16% of the overall welfare effect. This is made apparent by the much larger changes in wealth inequality depicted in panel 7. In this version of the model, compared to the

model with equity, the percentage change in wealth inequality caused by the same counterfactual public debt policy is up to ten times larger.

Panel 5 highlights another notable difference in the results of the two versions of the model. In the model without equity, the change in the tax rate is non-monotonic, as it starts increasing quite steeply for $d < -0.5$. The reason behind this result is that the fall in the interest rate is much stronger. This leads to two consequences: a substantial decrease in the tax revenues obtained from taxing capital income, and a large decrease in the asset income obtained on the stock of public wealth. Since both government expenditure and public transfers are a fixed share of income, total government outlays are larger in counterfactual economies with a lower debt/GDP ratio. Eventually, public expenditure needs to be financed with a larger tax rate, which is unavoidable if the interest rate becomes negative.

To conclude with, in the model without equity, the average change in the growth rate arising from a 10% increase in public debt is approximately -0.01 . This represents a tenfold increase compared to the other version of the model, but still falls short of some empirical estimates, which are discussed below.

Discussion: To better understand the mechanisms behind the equilibrium outcomes, it is informative to focus on the behavior of profits, and the associated expected value of being an incumbent. Although the profit share of income is constant across different BGPs, the response of expected profits is theoretically ambiguous. There are several effects, some going in opposite directions. As for the entrepreneurs, a decrease in the interest rate makes borrowing to invest in R&D cheaper. If in equilibrium the value of being an incumbent does not fall excessively, this leads to an increase in ρ . Therefore, the business stealing effect intensifies as p increases. Ultimately, this is what leads to a faster growth rate. As for the incumbents, a decrease in the interest rate affects multiple margins. First, there is a static effect. Producing intermediates becomes less expensive, because the cost of renting capital falls. The incumbents increase their production, irrespective of the fall in the price of the intermediate goods, as this choice maximizes their per-period profits. Second, there is a dynamic effect, linked to the continuation value in Eq. (7). Inspecting $(1-p)/(1+r)$, the adjusted discount factor of the intermediates firms, indicates that in general there is an ambiguous effect because both the numerator and the denominator fall. If the business stealing effect were to dominate, the expected future profits could decrease, and also the value of being an incumbent could be negatively affected. If this were to be the case, the asserted increase in ρ would not materialize, and the growth rate would not speed up. Quantitatively, in all the counterfactual experiments considered here, I find that the relative response of the interest rate vis-a-vis the business stealing effect is monotonic, with the former effect always dominating the latter. It follows that, in either version of model, the adjusted discount factor of the intermediate firms is always decreasing in d , just like the expected value of being an incumbent.

Robustness analysis: Some empirical contributions, e.g. [Woo and Kumar \(2015\)](#), find a sizable negative effect of public debt on the growth rate. Therefore, I modify both versions of the model to obtain larger responses. The decreasing returns in the probability of a successful innovation are shut down, by setting the

parameter $\psi = 1$.¹³ This modification allows a given change in R&D expenditure to have a larger effect on the likelihood of a successful innovation, and ultimately on the growth rate.¹⁴ In this version of the two models, the average change in the growth rate arising from a 10% increase in public debt is around -0.04 . This figure is inside the 90% confidence intervals derived from the estimation results reported in Table 5 of [Woo and Kumar \(2015\)](#), which refers to the sample of advanced economies. The results in both the model with and without equity are qualitatively similar, but quantitatively stronger. An important drawback of these cases is that the R&D expenditure share of GDP in the initial BGP is 12%, which is well above its actual value.

4.3 Results with Transitional Dynamics

In this subsection, I present the results based on solving the model without equity along the transition towards the new BGP.¹⁵ The equilibrium dynamics are triggered by a change in public debt policy. At time $t = 0$, the economy is in the initial BGP, characterized by a public debt/GDP ratio equal to $d_0 = 0.6$. The government then announces the new long-run value of d , together with the commitment to reach this target in 15 periods, at a linear rate. This makes the whole sequence $\{d_t\}_{t=1}^{t=\bar{t}}$ known to the private sector, causing a change in their optimal choices, which shape the equilibrium dynamics of both prices and aggregate variables.

[**Figure 3** about here]

The six panels in [Figure 3](#) plot selected equilibrium outcomes. The first panel shows the welfare effects arising from a number of different public debt policies. All the other panels depict the time paths of a given variable under two specific policies. One policy ($d = 0$, plotted with the black solid line) consists of eliminating public debt, while the other ($d = 1$, plotted with the blue dashed line) consists of reaching a public debt/GDP ratio of 100%.¹⁶

The curve shown in the first panel displays the welfare effects, which include the benefits and costs of the short-run dynamics. The main consideration is that big reductions in public debt are still welfare improving, unlike public debt increases that affect welfare negatively. Allowing for transitional dynamics has important implications. In the considered range, the welfare effects of all public debt counterfactuals are approximately half as much as the corresponding long-run effects. Nevertheless, these adjustments are not strong enough to

¹³The model parameters are recalibrated to match the same targets as in the baseline specification (but one, the R&D expenditure share of GDP). In particular, setting $\lambda = 0.97$ matches the firms' turnover rate of 10%.

¹⁴The results are plotted in [Figure 4](#) in Appendix D. In the model with equity there is an additional complication, as in some counterfactuals the economy becomes dynamically inefficient ($r < g$). In those instances, which happen for public wealth values in excess of 100% of GDP, the value of equity would become negative because of its price. To avoid this outcome, I impose a lower bound on the equity price, which prevents it from going below zero.

¹⁵The economy is assumed to reach the new BGP in a finite number of periods, at time \bar{t} , and the solution algorithm is outlined in Appendix C. Solving this model is computationally demanding, leading to consider a limited range for public debt, with 11 evenly spaced gridpoints. Also, solving the transitional dynamics of the model with equity poses a number of computational challenges, and the related analysis is left for future work.

¹⁶Notice that, since the two cases are not symmetric around the initial debt value, the response of the endogenous variables should not be expected to be the same in absolute value.

lead to a reversal in the sign of the welfare effects. The welfare gains of eliminating public debt are still found to be large, being 1.7%. Instead, moving to a debt/GDP ratio of 100% is associated with a welfare loss of 0.8%, which is also sizable. This result differs from the typical finding in the literature that the short-run welfare costs more than offset the long-run gains. This is the argument made, for example, by [Desbonnet and Weitzenblum \(2012\)](#), [Rohrs and Winter \(2017\)](#) and [Chatterjee, Gibson and Rioja \(2017\)](#).

The second panel plots the public debt/GDP ratio dynamics, namely the exogenous paths in $\{d_t\}_{t=1}^{t=\bar{t}}$ that the private sector takes as given. After 15 periods, the ratio settles to its new long-run value, while it takes about 100 periods for the other variables to complete their transitions.

The third panel displays the evolution of productivity, showing that the economy transitioning to a higher public debt/GDP ratio is lagging behind. In general, economies transitioning to higher values of d would endure a productivity gap. In the two cases discussed here, after 100 periods, this gap is 3.8%. This is due to the response of the growth rate, portrayed in the fourth panel. The economy with a high long-run d experiences a decrease in its growth rate, which settles relatively quickly around a value close to its new BGP. The no long-run debt economy goes through more complicated dynamics in its growth rate, which increases non-monotonically to its permanently higher value. A noteworthy behavior related to the growth rates is that they do not jump considerably when the new public debt policy is implemented. Entrepreneurs anticipate the long-run benefits of a lower public debt, which increases their expected profits and the incentives to invest in R&D. The R&D expenditure dynamics track the behavior of the growth rate, as this expenditure is the only endogenous variable affecting it. The value of R&D does not necessarily increase on impact. Since capital is given when the sequence of public debts is announced, R&D could increase only if consumption were to react with a discrete downward adjustment, but such a response would be very costly in terms of welfare. At the same time, taxes increase and the slow initial response of R&D can be attributed to the higher burden of taxation. The need to finance higher taxes leads to a drop in consumption, but the households' desire to smooth it prevents it from falling even further, which does not allow to leave some additional resources to be allocated to R&D. As taxes fall, R&D expenditure rises at an increasing rate, to reap the expected benefits as quickly as possible, until taxes reach their new long-run value.

The fifth panel shows the paths of the proportional tax rate $\{\tau_t\}_{t=1}^{t=\bar{t}}$. These are fairly straightforward, as tax revenues need to increase sharply to eliminate public debt, or decrease similarly quickly for d_t to rise. The debt elimination experiment features a “cold turkey” behavior. In the first year of the reform, the tax rate jumps by 6 percentage points, with a subsequent (slow but persistent) decline due to the faster increase in income. When the dynamics in the debt/GDP ratio are completed, the tax rate falls immediately to a value in the neighborhood of the new BGP equilibrium. The second case is essentially the mirror image of the first one, as a long-run increase in d triggers the opposite adjustments. In this instance, the change in the tax rate after the first period is more pronounced, because the growth rate falls quickly (and almost monotonically) to its lower long-run value.

The last panel plots the dynamics of de-trended consumption relative to the initial BGP (\widetilde{C}_0 is then equal to 1). In the debt elimination counterfactual, Households react to the increased taxes by marginally reducing their consumption, while the decrease in savings is more pronounced. The overall effect on the interest rate is

almost negligible, as the decrease in savings is accompanied by the fall in public debt. When the public debt adjustment is completed, households devote the additional disposable income to increasing their consumption, and this adjustment is very sharp. De-trended consumption overshoots its new BGP value, and the final part of the transition unfolds with a monotonic decrease. In the $d = 1$ counterfactual, consumption does not increase when taxes fall, as households forecast the reduced income growth, and increase their savings to smooth the effects of this unfavorable outcome.

5 Conclusions

I addressed the question of whether governments should accumulate public debt in the long-run, using an endogenous growth model with rich workers' heterogeneity and technological progress that responds to different public debt policies. I extended the Schumpeterian growth model proposed by [Howitt and Aghion \(1998\)](#), embedding the elements that have been found to rationalize large public debts, namely uninsurable income risk, a tight borrowing constraint, and a high degree of wealth inequality. I considered two different assumptions regarding the structure of the asset market, which make the quantitative implications of the model consistent with the wide range of estimates of the growth rate response caused by changes in public debt. Both versions lead to the same qualitative answer, namely that decreasing the long-run public debt/GDP ratio enhances welfare.

Quantitatively, I find that the foregone economic growth more than offsets the improved consumption smoothing (or the lack thereof, due to the absence of complete markets) that larger public debts bring about. Although the equilibrium effect on the growth rate can be quantitatively modest, it still has a first order consequence on welfare.

The version of the model with the simpler asset market structure is easier to solve computationally, allowing to consider transitional dynamics. Taking into account the dynamic adjustment to the new long-run equilibrium shows that the transitional welfare costs are not large enough to change the sign of the welfare effects stemming from a change in public debt. I find that eliminating public debt would lead to a 1.7% increase in welfare, while moving to a debt/GDP ratio of 100% would entail a welfare loss of 0.8%.

The model ignored the possibility for government expenditure to increase output or affect the utility of the household: I leave these extensions for future research.

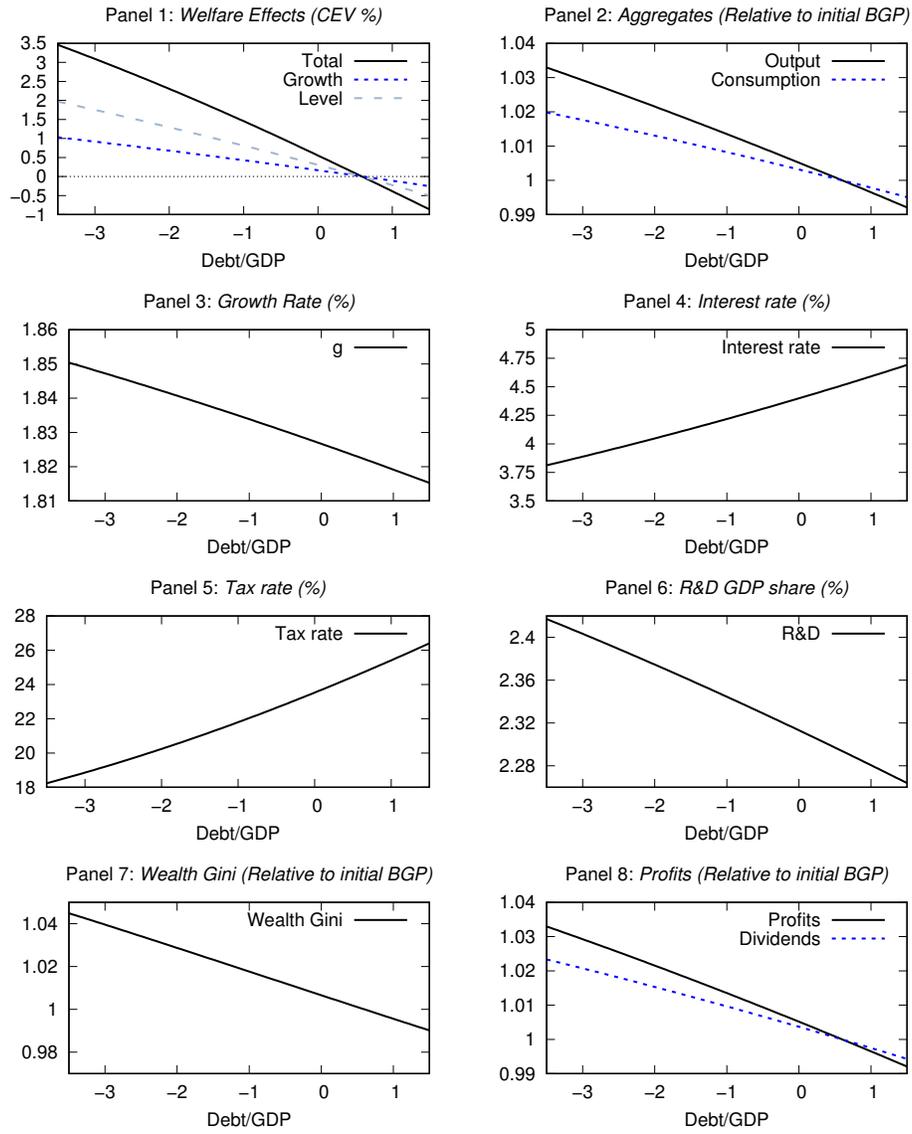


Figure 1: Long-run Welfare Effects Profile and Equilibrium Outcomes in the Model with Equity in Innovating Firms.

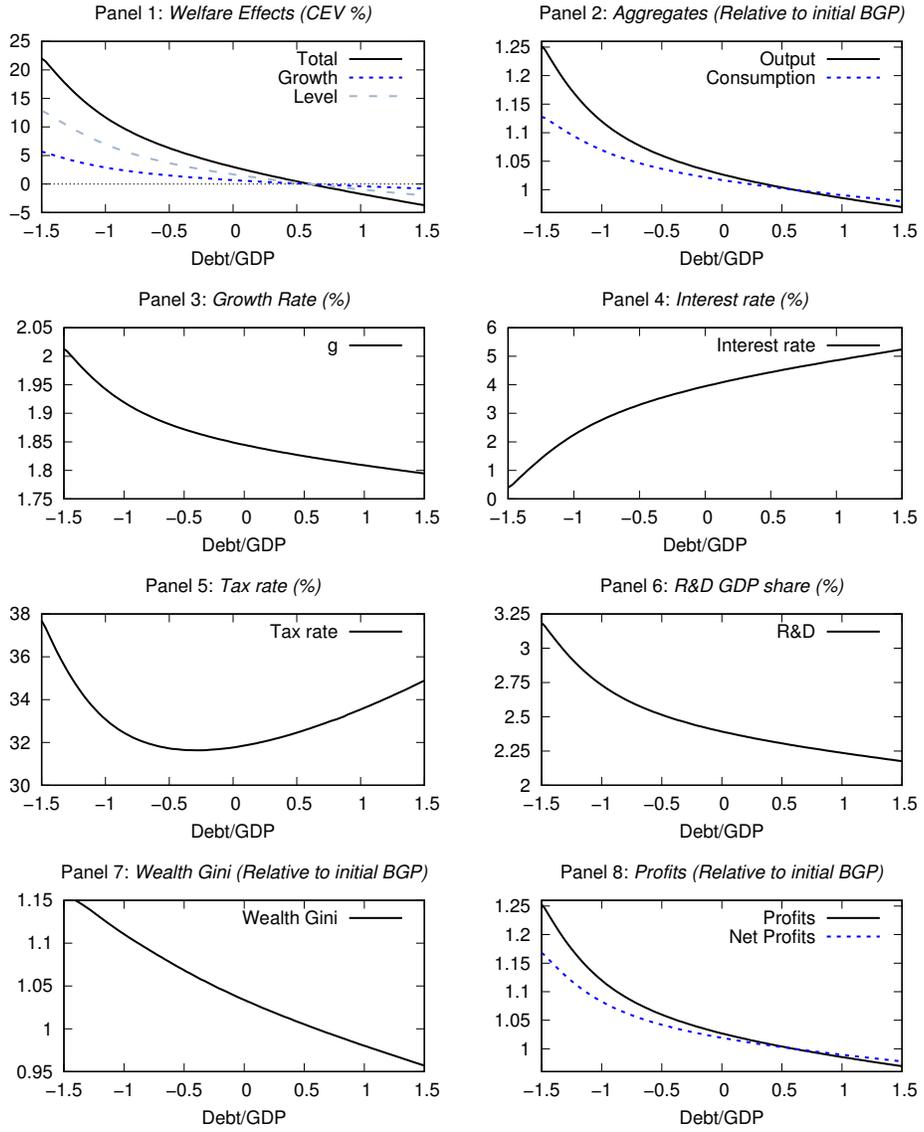


Figure 2: Long-run Welfare Effects Profile and Equilibrium Outcomes in the Model without Equity in Innovating Firms.

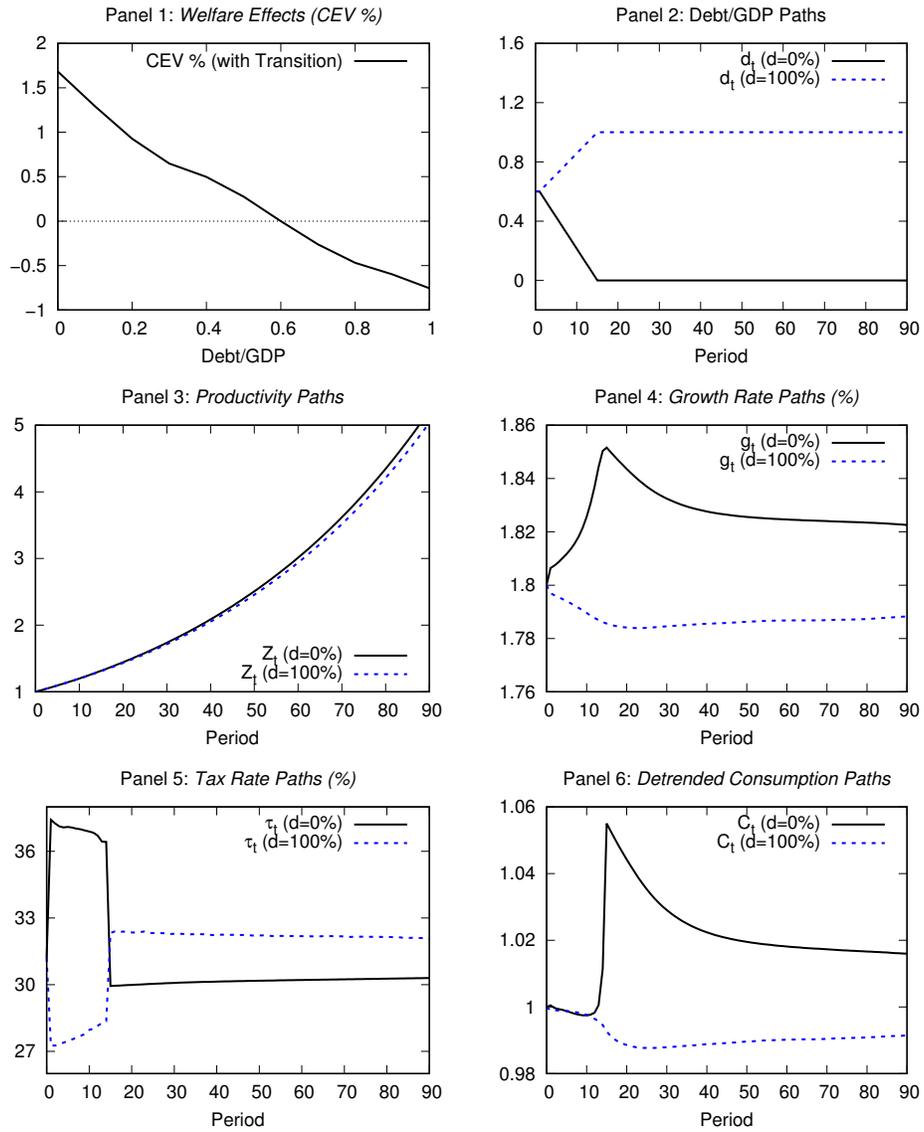


Figure 3: Welfare Effects Profile and Transitional Dynamics in the Model without Equity in Innovating Firms.

<i>Parameters Set Externally</i>	<i>Value</i>	<i>Target</i>
θ - <i>Relative Risk Aversion</i>	2.0	<i>Elasticity of Intertemporal Substitution = 0.5</i>
δ - <i>Capital depreciation rate</i>	0.08	<i>Capital depreciation estimates</i>
α - <i>1-Labor share</i>	0.36	<i>Labor share= 64%</i>
σ_y^2 - <i>Variance of the income shocks</i>	0.015	<i>Guvenen (2009) estimates on PSID data</i>
ζ_y - <i>Persistence of the income shocks</i>	0.988	<i>Guvenen (2009) estimates on PSID data</i>
σ_ϕ^2 - <i>Variance of the fixed effect</i>	0.058	<i>Guvenen (2009) estimates on PSID data</i>
γ - <i>Government Consumption</i>	0.21	<i>G/GDP = 21%</i>
χ - <i>Lump-sum transfer</i>	0.082	<i>Transfers/GDP = 8.2%</i>
τ_f - <i>Profits Tax Rate</i>	0.326	<i>Average Corporate Tax Rate = 32.6%</i>
d - <i>Public Debt/GDP</i>	0.60	<i>D/GDP = 60%</i>
b - <i>Borrowing limit</i>	0	<i>No borrowing allowed</i>
<i>Parameters Set in Equilibrium</i>	<i>Value</i>	<i>Joint Targets</i>
$\underline{\beta}$ - <i>Rate of time preference, low type</i>	0.981	<i>Interest rate = 4.5%</i>
$\bar{\beta}$ - <i>Rate of time preference, high type</i>	0.995	<i>Wealth Gini index = 0.8</i>
λ - <i>Weight in prob. of successful innovation</i>	2425	<i>Firms exit rate = 10%</i>
ψ - <i>Curvature in prob. of successful innovation</i>	0.191	<i>R&D/GDP = 2.28%</i>
σ - <i>Spillover Effect</i>	0.18	<i>Growth rate = 1.8%</i>

Table 1: Model Calibration. The model period is one year. The parameters set externally are the same in both versions of the model. The parameters set in equilibrium refer to the model with equity.

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Appendix A - The Model and its Recursive Representation

Stationary Equilibrium

In this Section, I first define the problem of the agents in their recursive representation. Then I provide a formal definition of the recursive competitive equilibrium. The individual state variables are: the preference type $\beta \in \mathcal{B} = [\underline{\beta}, \bar{\beta}]$, the fixed effect $\phi \in \mathcal{F} = \{\phi_{\min}, \dots, \bar{\phi}, \dots, \phi_{\max}\}$, the persistent shock component of the labor endowment $\varepsilon \in \mathcal{E} = \{\varepsilon_{\min}, \dots, \bar{\varepsilon}, \dots, \varepsilon_{\max}\}$ and asset holdings $a \in \mathcal{A} = [-b, \bar{a}]$. The grid for ϕ is obtained discretizing the underlying normal distribution, using $N_\phi = 5$ points and partitioning the p.d.f. with equal areas of size $1/N_\phi$. Each fixed effect type is then assigned the conditional average implied by each partition, computed with the formulas for the truncated normal, and a share in the workforce equal to $1/N_\phi$. The shock ε is discretized with the Rouwenhorst method, using a 11-state Markov chain. The transition function of the labor endowment shocks is represented by the matrix $\Pi(\varepsilon', \varepsilon) = [\pi(v, z)]$, where each element $\pi(v, z)$ is defined as $\pi(v, z) = \Pr\{\varepsilon_{j+1} = z | \varepsilon_j = v\}$, $v, z \in \mathcal{E}$. In every period the exogenous labor endowments are given by $\varepsilon_{\varepsilon, \phi} = \varepsilon\phi$. The stationary distribution of the workers is denoted by $\mu(a, \varepsilon, \phi, \beta)$.

A.1 - Problem of the Workers

The value function of a worker whose current asset holdings are equal to a , whose current efficiency units shock is ε , and whose fixed effect is ϕ , and whose discount factor is β is denoted with $V(a, \varepsilon, \phi, \beta)$. The problem of these agents can be represented as follows:

$$V(\tilde{a}, \varepsilon, \phi, \beta) = \max_{\tilde{c}, \tilde{a}'} \left\{ \frac{\tilde{c}_j^{1-\theta}}{1-\theta} + \beta(1+g)^{(1-\theta)} \sum_{\varepsilon'} \pi(\varepsilon', \varepsilon) V(\tilde{a}, \varepsilon', \phi, \beta) \right\} \quad (16)$$

s.t.

$$\tilde{c} + (1+g)\tilde{a}' = (1 + (1-\tau)r)\tilde{a} + (1-\tau)\tilde{w}\varepsilon_{\varepsilon, \phi} + \tilde{tr}$$

$$\tilde{a}_0 = 0, \quad \tilde{c} \geq 0, \quad \tilde{a}' \geq 0$$

The workers have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they transit from their current efficiency units ε to the value ε' . These agents pay labor and capital income taxes, both with a proportional tax rate τ , but receive a lump-sum public transfer tr . Finally, they are born with no wealth, and are subject to an exogenous borrowing constraint.

A.2 - Recursive Stationary Equilibrium

Since in equilibrium the economy is growing along a BGP, the dynamic programming problem is non-stationary. Every non stationary variable needs to be transformed into their stationary counterpart. This is achieved dividing a generic variable X_t at time t by the average technological index Z_t , $\tilde{X}_t \equiv X_t/Z_t$, where the tilde denotes the transformed variable.

Definition 1 For given public policies $\{\tau_f, \gamma = \widetilde{G}/\widetilde{Y}, \chi = \widetilde{TR}/\widetilde{Y}, d = \widetilde{D}/\widetilde{Y}\}$ a recursive stationary equilibrium is a set of (transformed) decision rules, $c(\widetilde{a}, \varepsilon, \phi, \beta)$, value functions, $V(\widetilde{a}, \varepsilon, \phi, \beta)$, prices $\{P, r, \widetilde{w}\}$, normalized $R\&D$ expenditure ρ , endogenous growth rate $g = (1 - e^{-\rho^\psi})\sigma$, and a set of stationary distributions, $\mu(\widetilde{a}, \varepsilon, \phi, \beta)$ such that:

- Given relative prices $\{r, \widetilde{w}\}$, taxes and transfers $\{\tau, \chi\}$, the individual policy functions $c(\widetilde{a}, \varepsilon, \phi, \beta)$, $a'(\widetilde{a}, \varepsilon, \phi, \beta)$ solve the household problem (16), and $V(\widetilde{a}, \varepsilon, \phi, \beta)$ are the associated value functions.
- Given relative prices $\{P, r, \widetilde{w}\}$ and public policies, $\widetilde{K}/L = M/L$ solves the final good sector firm's problem.
- Given relative prices $\{P, r, \widetilde{w}\}$ and public policies, ρ solves the entrepreneur's problem (8).
- The labor market is in equilibrium, and the labor input L corresponds to the total supply of labor efficiency units

$$L = \int_{\mathcal{A} \times \mathcal{E} \times \mathcal{F} \times \mathcal{B}} \epsilon^{\varepsilon, \phi} d\mu(\widetilde{a}, \varepsilon, \phi, \beta)$$

- The asset market clears (with equity $\widetilde{Q} = 0$ in the entrepreneurs version of the model)

$$\widetilde{K} + \widetilde{D} + \widetilde{Q} = \int_{\mathcal{A} \times \mathcal{E} \times \mathcal{F} \times \mathcal{B}} a'(\widetilde{a}, \varepsilon, \phi, \beta) d\mu(\widetilde{a}, \varepsilon, \phi, \beta)$$

- The goods market clears (with $\widetilde{C}_e = 0$ in the equity version of the model)

$$\widetilde{Y} = \widetilde{C} + \widetilde{I} + \widetilde{G} + \widetilde{RD} = \int_{\mathcal{A} \times \mathcal{E} \times \mathcal{F} \times \mathcal{B}} c(\widetilde{a}, \varepsilon, \phi, \beta) d\mu(\widetilde{a}, \varepsilon, \phi, \beta) + \widetilde{C}_e + \delta \widetilde{K} + \widetilde{G} + \widetilde{RD}$$

- The tax τ satisfies the government's intertemporal budget constraint

$$(r - g) \frac{\widetilde{D}}{\widetilde{Y}} = \frac{\widetilde{T} - \widetilde{TR} - \widetilde{G}}{\widetilde{Y}} \rightarrow (r - g)d = \tau(1 - \alpha + \alpha^2 - \delta\kappa + rd) + \tau_f \alpha(1 - \alpha) - \chi - \gamma$$

- The stationary distribution $\mu(\widetilde{a}, \varepsilon, \phi, \beta)$ satisfies

$$\mu(\widetilde{a}', \varepsilon', \phi, \beta) = \sum_{\varepsilon} \pi(\varepsilon', \varepsilon) \int \nu(\widetilde{a}, \varepsilon, \phi, \beta, \widetilde{a}', \varepsilon') d\mu(\widetilde{a}, \varepsilon, \phi, \beta) \quad (17)$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the above equation (17), where $\nu(\cdot)$ is the transition function.

Appendix B - Computation

- All codes solving the model economies and simulating samples of agents were written in the FORTRAN 95 language, relying on the Intel Fortran Compiler, build 17.1 (with the IMSL library). They were compiled selecting the O2 option (maximize speed), and without automatic parallelization. They were run on a 64-bit PC platforms, running Windows 10 Professional, with an Intel *i7 – 8700k* Quad Core processor clocked at 4.7 Ghz.
- I use 101 evenly spaced points for the public debt/GDP grid d_j . For either version of the model, the computations of the sequence of counterfactual public debt policies takes up to 8 hours to complete. Typically between 30 and 100 iterations on the endogenous variables are needed to find each equilibrium, as a slow updating factor is needed to guarantee convergence.
- In the actual solution of the models I need to discretize the continuous state variable a . I rely on an unevenly spaced grid, with the distance between two consecutive points increasing geometrically. This is done to allow for a high precision of the policy rules at low values of a , where the change in curvature is more pronounced. I use 1001 points, as increasing the number of points does not affect the results considerably. The lowest value is the borrowing constraint b and the highest one being a value a_{\max} high enough not to be binding ($a_{\max} = 500$).
- In the model, ε is discretized with the Rouwenhorst method, using a 11-state Markov chain. This method has several desirable properties, especially when working with highly persistent processes.
- The optimal decision rules are obtained using the endogenous grid method. I start from an initial guess on the policy functions, $a'_0(\tilde{a}, \varepsilon, f, \beta)$, and keep on iterating until convergence. A slow updating factor is needed to guarantee convergence.
- The stationary distributions are computed relying on their definitions (17), without performing a simulation.
- The asset market is in equilibrium when the current guess for the interest rate r_0 achieves a capital excess demand which is less than 0.1% of the market size. In turn, this implies that the excess demand in the final good market is always less than 0.1% of the market size.
- The welfare measures SW_i are based on (13), which is the weighted sum of the two numerical integrals of the discount factor dependent value functions (integrated with respect to the stationary distributions).

Appendix C - Solution Algorithms

This algorithm represents the computational procedure used to solve the BGP version of the model:

1. Generate a discrete grid over the detrended asset space $\{0, \dots, \tilde{a}_{\max}\}$.
2. Generate a discrete grid over the income shocks with the Rouwenhorst method $\{\varepsilon_{\min}, \dots, \varepsilon_{\max}\}$.
3. Generate a discrete grid over the fixed effect $\{\phi_{\min}, \dots, \phi_{\max}\}$.
4. Generate a discrete grid over the discount factors $\{\underline{\beta}, \bar{\beta}\}$.
5. Set the value of public debt/GDP at its long-run value d .
6. Guess the interest rate r_0 .
7. Guess the tax rate τ_0 .
8. Get the capital demand \tilde{K}_0 and wages \tilde{w}_0 .
9. Find the monopolists' profits $\tilde{\pi}_0$.
10. Find the optimal rescaled R&D expenditure ρ_0 .
11. Get the equilibrium growth rate g_0 .
12. Transform the HH's problem into its stationary formulation.
13. Get the saving functions $a'(\tilde{a}, \varepsilon, \phi, \beta)$ and the value functions $V(\tilde{a}, \varepsilon, \phi, \beta)$.
14. Get the stationary distributions $\mu(\tilde{a}, \varepsilon, \phi, \beta)$.
15. Get the equilibrium income tax rate τ_1 .
16. Get the aggregate capital supply and check the asset market clearing; Get r_1 .
17. Update r'_0, τ'_0 (with a relaxation method).
18. Iterate until asset market clearing and until the government intertemporal budget constraint is satisfied.
19. Get the consumption functions $c(\tilde{a}, \varepsilon, \phi, \beta), \tilde{C}_e$ and check the final good market clearing.
20. Compute social welfare SW , the associated welfare effects, and their decompositions.
21. Set the value of public debt/GDP to a counterfactual value on a grid d_j , repeating steps 6-20 for all the points in this grid.

This algorithm represents the additional steps used to solve the model with transitional dynamics:

1. Assume that at time $t = 0$ the economy is in the initial BGP with $d_0 = 0.6$, and compute it using the algorithm above.
2. Set the length of time needed to complete the whole transition to the final BGP to $\bar{t} = 100$.
3. Assume that at time $t = \bar{t}$ the economy is in the final BGP, with $d_{\bar{t}} = d$, and compute it using the algorithm above.
4. Set the length of time needed to complete the public debt dynamics to $\bar{t}_d = 15$.
5. Assume that the public debt dynamics evolve linearly, which allows to postulate the whole sequence for the public debt/GDP ratio $\{d_t\}_{t=1}^{t=\bar{t}}$.
6. Guess sequences for all the transitional equilibrium objects $\{r_t, \tau_t, g_t\}_{t=1}^{t=\bar{t}}$.
7. Get the capital demand and wages $\{\widetilde{K}_t, \widetilde{w}_t\}_{t=1}^{t=\bar{t}}$.
8. Find the monopolists' profits $\{\widetilde{\pi}_t\}_{t=1}^{t=\bar{t}}$.
9. Find the optimal rescaled R&D expenditure $\{\rho_t\}_{t=1}^{t=\bar{t}}$.
10. Get the equilibrium growth rate $\{g_t\}_{t=1}^{t=\bar{t}}$.
11. Transform the HH's problem into its stationary formulation.
12. Solve the household problems backward, relying on the fact that at time $t = \bar{t}$ the decision rules are the ones for the final steady-state.
13. Solve the transitional distributions forward, relying on the fact that at time $t = 0$ the distributions are the ones for the initial steady-state.
14. Aggregate the decision rules and compute the new guesses for the transitional equilibrium objects.
15. Iterate until the asset market clears in every period of the transition, and the sequences $\{r_t, \tau_t, g_t\}_{t=1}^{t=\bar{t}}$ converge.
16. Compute social welfare SW at time $t = 0$, and obtain the associated welfare effects.
17. Set the value of the final public debt/GDP to another counterfactual value on a grid d_j , repeating steps 3-16 for all the points in this grid.

Appendix D - Additional Plots

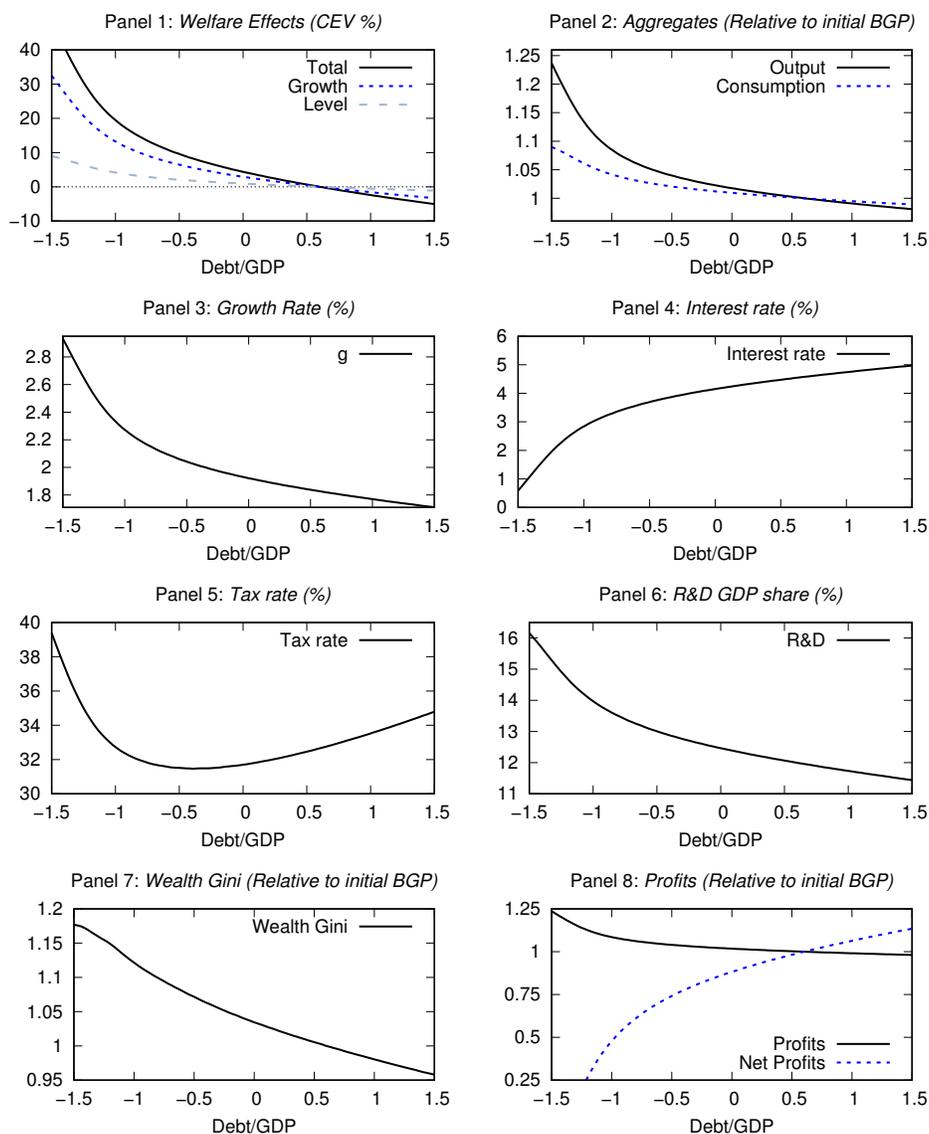


Figure 4: Long-run Welfare Effects Profile and Equilibrium Outcomes in the Model without Equity in Innovating Firms, with $\psi = 1$.