University of Victoria | Economics

Department Discussion Paper DDP1903

ISSN 1914-2838

EARNING RISKS, PARENTAL SCHOOLING INVESTMENT, AND OLD-AGE INCOME SUPPORT FROM CHILDREN

Alok Kumar
Department of Economics, University of Victoria
Victoria, B.C., Canada V8W 2Y2

April, 2019

Abstract

Old-age income support is an important motive for parents to invest in schooling of their children in developing countries. At the time parents choose schooling investment for their children, both parental future income and return from schooling are uncertain. This paper analyzes effects of parental income risk and human capital investment risk on parental schooling investment using alternative models (altruism and educational loan model) of determination of old-age income support in a model with intergenerational transfers. It finds that effects of these risks on schooling investment depend on whether old-age income support is state-contingent. When income support is state-contingent, increasing parental income risk (human capital investment risk) has a positive (negative) effect on schooling investment. However, when income support is not state-contingent, effects of these two types of risks may get reversed. Numerical analysis using Indonesian data suggests that risks have significant negative effect on schooling investment.

Keywords: schooling, parental income risk, human capital investment risk, old-age income support, risk-sharing.

JEL Classifications: O15, D60, I20, I25, J22

Author Contact:
Alok Kumar, Dept. of Economics, University of Victoria, P.O. Box 1700, STN CSC, Victoria, B.C., Canada V8W 2Y2; E-mail: kumara@uvic.ca; Voice: (250) 721-8543.

Earning Risks, Parental Schooling Investment, And Old-Age Income Support From Children

Alok Kumar*

April 2019

Abstract

Old-age income support is an important motive for parents to invest in schooling of their children in developing countries. At the time parents choose schooling investment for their children, both parental future income and return from schooling are uncertain. This paper analyzes effects of parental income risk and human capital investment risk on parental schooling investment using alternative models (altruism and educational loan model) of determination of old-age income support in a model with intergenerational transfers. It finds that effects of these risks on schooling investment depend on whether old-age income support is *state-contingent*. When income support is state-contingent, increasing parental income risk (human capital investment risk) has a positive (negative) effect on schooling investment. However, when income support is *not state-contingent*, effects of these two types of risks may get reversed. Numerical analysis using Indonesian data suggests that risks have significant negative effect on schooling investment.

Keywords: schooling, parental income risk, human capital investment risk, old-age income support, risk-sharing

JEL Code: O15, D60, I20, I25, J22

Acknowledgement: The author thanks seminar participants at the Delhi School of Economics, the Indian Statistical Institute, the UNU-WIDER and the HECER for their insightful comments and suggestions. The responsibility for any error in the paper is entirely mine.

Address: * Department of Economics, University of Victoria, Victoria, British Columbia, Canada, V8W 2Y2, Email: kumara@uvic.ca, Fax: 1-250-721-6214.

1 Introduction

Many elderly parents rely on income-support from their children in developing countries. Empirical evidence from developing countries shows that there is a significant positive relationship between educational level of children and transfers made by them to their older parents (Lee et. al. 1994, Lillard and Willis 1997).

There is a growing empirical literature which suggests that risks and volatility have significant effect on schooling in developing countries. Flug et. al. (1998) and Checchi and Garcia-Penalosa (2004) using cross-country evidence find that macro-economic volatility has a negative effect on schooling. Additionally, evidence suggests that different types of risks have different effects on schooling. Kaufmann (2014) using household data from Mexico finds that the uncertainty about returns from schooling (human capital investment risk) has a negative effect on college attendance of youths.[1] However, evidence regarding effects of household income risk on schooling is mixed. Portner (2009) using household data from rural Guatemala finds that the hurricane risk has a significant positive effect on schooling. But, Kazianga (2012) and Foster and Gehrke (2017) find that household income risk has a significant negative effect on schooling in rural Burkina Faso and India respectively.

There is a large theoretical literature which has analyzed the relationship between old-age income support provided by children and parental schooling investment (Raut 1990, Rangazas 1991, Baland and Robinson 2000, Raut and Tran 2005). However, these analyses have largely ignored effects of various risks, which parents face while making schooling decisions for their children. Parents may have imperfect information about the future rate of return from schooling. In addition, the future income of parents may be risky. Given inadequate development and coverage of financial markets in most of the developing countries, parents have limited opportunity to (formally) insure against such risks (Dercon 2005).

The main aim of this paper is to theoretically examine interactions among earnings risks, old-age income support by children, and parental investment in schooling of their children. It analyzes effects of parental income risk

[1] Most studies on effects of human capital investment risk have been done in the context of developed countries (see Hartog and Diaz-Serrano 2013 for a review). These studies provide mixed evidence with regard to effects of human capital investment risk on schooling with some finding positive effect and others negative effect.

and human capital investment risk on parental schooling investment in the unitary household framework. It studies effects of these risks under two alternative models of determination of old-age income support: altruism and repayment on the past investment incurred by parents for children. Empirical literature suggests that one motive does not explain inter-generational transfers and their relative importance may vary across regions and income groups (Rapoport and Docquier 2006 and Cox and Fafchamps 2008).

In the model, there are two periods. A family consists of a parent and a child. Both parent and child are altruistic. Parental utility depends not only on its own consumption, but also on consumption of its child. Similarly, utility of child depends not only on its own consumption, but also on consumption of its parent.

Parent chooses its saving and schooling investment for child in the first period. Schooling investment in the first period increases human capital (earnings) of child in the second period. While making first-period decisions, parent faces two kinds of uninsurable risks. Its own second period income and return from schooling are uncertain.

Adult child provides income-support to its parent in the second period. I consider alternative models of determination of old-age income support. First I analyze the case in which old-age income support is motivated by the altruism of child towards its parent (pure altruism model) similar to Rangazas (1991), Baland and Robinson (2000), Bommier and Dubois (2004), and Raut and Tran (2005). Altruistic child chooses old-age income support voluntarily.

In the second model, schooling investment by parent is modelled as a loan to child and old-age income support by child as a repayment on the loan (educational loan model) as suggested by the work of Lillard and Willis (1997). Parent chooses both schooling investment and old-age income support (terms of loan). I consider two cases. Parent can choose state-contingent terms of loan (repayment contingent on realizations of incomes of parent and child) or fixed terms of loan (repayment independent of realizations of incomes of parent and child). In both the pure altruism model and educational loan model with state-contingent terms, old-age income support plays a role of risk-sharing instrument, but not in the case of fixed-contract model.[2]

[2]Empirical evidence regarding the response of old-age income support to realized incomes of parents and children is mixed. Some studies find that old-age income support is responsive to incomes of parents and children, but others do not (Rapoport and Docquier 2006, Cox and Fafchamps 2008). Whether the old-age income support is state-contingent or not has been used to distinguish among various motives of old-age income support.

I find that effects of increasing risks (mean preserving spread) crucially depend on whether old-age income support is state-contingent and whether parent faces binding borrowing constraint. When old-age income support is state-contingent, an increase in parental income risk has a positive effect on schooling investment. When it is *not* state-contingent, an increase in parental income risk has a negative effect on schooling investment.

In contrast, an increase in the human capital investment risk reduces schooling investment of an unconstrained parent, when old-age income support is state-contingent. However, either when support is not state-contingent or parent is borrowing-constrained, an increase in human capital investment risk may increase schooling investment, particularly when the degree of relative risk-prudence is high.

Numerical results show that the human capital investment risk has relatively larger effect on the schooling investment of unconstrained parents. But the parental income risk has relatively larger effect on the schooling investment of borrowing-constrained parents.

In terms of policy, I find that income assistance to parents in the first period has a positive effect on schooling. However, effect of income assistance to parent in the second period may have a positive or a negative effect on schooling investment. Public education combined with old-age income pension system cannot achieve socially efficient allocations, unless the government provides insurance against earnings risks. This result is different from Boldrin and Montes (2005) who in a model without uncertainty show that public education combined with pension system can achieve socially efficient allocations.

This paper relates to various strands of theoretical literature on effects of risks on schooling. Applebaum and Katz (1991) analyze effects of income uncertainty of parents and (adult) children on schooling investment and fertility decisions of parents. In their model, only fertility decisions are made under uncertainty. Income risks have no effect on schooling investment, when fertility is exogenous. In the case of endogenous fertility, an increase in parental income risk has a negative effect on schooling investment. However, an increase in income risk of (adult) children may have a positive or a negative effect on schooling investment.

Pouliot (2006) examines effects of human capital investment risk on schooling investment of children, when there is bequest from *altruistic parents to children*. He finds that human capital risk leads to inefficiently low level of schooling investment. This paper does not analyze effects of increasing risks on schooling and its interaction with old-age income support.

There is a theoretical literature which analyzes effects of human capital investment risk in which an individual undertakes schooling investment to increase its own future income. In a seminal work Levhari and Weiss (1974) show that increasing human capital investment risk has a negative effect on schooling investment. Subsequent studies show that the effect of human capital investment risk on schooling depends on the riskiness of alternative investment opportunities (Paroush 1976 and Williams 1979), type (linear or multiplicative) of human capital risk (Kodde 1986 and Singh 2010) and possibility of delaying entry to or exit from college (Hogan and Walker 2007 and Jacobs 2007).

Qualitatively, these models are equivalent to models in which altruistic parents who do not face binding borrowing constraint make schooling investment and there is a bequest from parents to children in all states. These studies do not analyze interaction among parental income and human capital risks, schooling investment, borrowing constraint and types and motives of old-age income support by children which is the focus of the paper.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 analyzes effects of parental income and human capital investment risks on schooling, when old-age income support is motivated by altruism. Section 4 analyzes effects of these risks on schooling, when old-age income support is a repayment on educational loan. Section 5 quantitatively analyzes the interaction among risks, schooling, and old-age income support using Indonesian Life and Family Survey data 2014-15. This is followed by conclusion. All proofs are contained in the appendix.

2 Model

There are two periods. Consider a household consisting of a parent (p) and a child (k). Parent and child live in both periods, $t = 1, 2$. Denote earnings of parent in period $t = 1, 2$ by y_t. Let R be the risk-free rate of return on saving.

In the first-period, parent chooses schooling investment for its child. Earnings of adult child (second period) depends on schooling investment made by the parent in the first period. The human capital function of child is given by, $\phi h(e)$, where ϕ is the productivity parameter. Assume that $\phi h(e)$ is an increasing and concave function of e and $\phi h(0) > 0$.

Since the focus of the paper is on analyzing effects of parental income risk and human capital investment risk, suppose that the second period parental income, y_2, and the human capital function productivity parameter, ϕ, are random variables with strictly positive and finite support.

$$y_2 \sim (\overline{y}_2, \sigma^2_{y_2}) \ \& \ \phi \sim (1, \sigma^2_{\phi}) \tag{2.1}$$

where mean of ϕ is normalized to 1 for notational simplicity. Let $\sigma_{y_2,\phi}$ denote the co-variance between y_2 and ϕ.

Parent makes saving and schooling investment decisions before values of these random variables are known. Assume that insurance markets do not exist to insure against these risks.

Both parent and child are altruistic. Parental utility depends not only on it own consumption but also on consumption of child. Similarly, utility of child depends not only on its own consumption but also on consumption of its parent. For simplicity, I assume that child consumes only in the second period.[3] The parental expected utility is given by

$$W^p = U(c^p_1) + \beta E[U(c^p_2) + \delta U(c^k)] \text{ with}$$

$$U_c() > 0, \ U_{cc}() < 0, \ U_{ccc}() > 0, \ \& \ U_{cccc}() < 0 \tag{2.2}$$

where function $U()$ is the period utility function, E is the expectation operator, $0 < \beta < 1$ is the discount rate and $0 < \delta < 1$ is the degree of parental altruism towards its child.[4] c^p_t is parental consumption in period $t = 1, 2$ and c^k is consumption of child in period 2.

The expected utility of child is given by:

$$W^k = E[V(c^k) + \lambda V(c^p_2)] \text{ with}$$

$$V_c() > 0 \text{ and } V_{cc}() < 0 \tag{2.3}$$

where function $V()$ is the period utility function and $0 < \lambda < 1$ is the degree of child's altruism towards its parent.

[3] One can easily introduce consumption by child in the first period without affecting results.

[4] For any function $z(x)$, $z_x()$, $z_{xx}()$, $z_{xxx}()$ & $z_{xxxx}()$ refer to first, second, third, and fourth derivative respectively.

Parent chooses schooling investment, e, and its saving, s, in the first period and its consumption in both periods, c_t^p for $t = 1, 2$. Child chooses its consumption, c^k, in the second period. Finally, parent receives transfers from its child (old-age income support), $\tau \geq 0$, in the second period. Alternative ways in which these transfers are determined are discussed below. Since the focus of the paper is on the interaction among risks, old-age income support, and schooling investment, I only consider cases in which there is transfer from child to parent in the second period. The budget constraints faced by parent and child can be written as

$$c_1^p + s + e = y_1; \tag{2.4}$$

$$c_2^p = y_2 + Rs + \tau \text{ and} \tag{2.5}$$

$$c^k = \phi h(e) - \tau. \tag{2.6}$$

In terms of timing, first period decisions are made before values of random variables are known. Second period decisions are made after values of random variables are known.

Efficient Allocations

For future use, I derive allocations chosen by the social planner. Suppose that there is a large number of ex-ante identical households (unit measure). There is a social planner which puts weight $\mu \geq 0$ on the utility of children. It can use lump-sum transfers/taxes to smooth consumption across time, individuals, and households. Since there is no aggregate uncertainty, the social planner problem is to

$$\max_{c_1^p, s, e, c_2^p, c^k} U(c_1^p) + \beta[U(c_2^p) + \delta U(c^k)] + \beta\mu[V(c^k) + \lambda V(c_2^p)]$$

subject to resource constraints

$$c_1^p + s + e = y_1 \text{ and} \tag{2.7}$$

$$c_2^p + c^k = \overline{y}_2 + Rs + h(e). \tag{2.8}$$

Using first-order conditions, it is straightforward to show that efficient allocations (denoted by *) are characterized by

$$e^* : \ h_e(e^*) = R; \tag{2.9}$$

$$s^* : \ U_c(c_1^{p*}) = \beta R[U_c(c_2^{p*}) + \lambda\mu V_c(c_2^{p*})]; \tag{2.10}$$

$$c_2^{p*} : [U_c(c_2^{p*}) - \delta U_c(c^{k*})] = \mu[V_c(c^{k*}) - \lambda V_c(c_2^{p*})]; \tag{2.11}$$

and resource constraints (2.7) and (2.8).

(2.9) shows that the social planner equates the rate of return on schooling to the rate of interest. Since, the social planner can increase second-period resources both by increasing schooling investment and saving, at margin it is indifferent between the two.

(2.10) shows that due to two-sided altruism and $\mu > 0$, the social marginal benefit of saving (RHS of 2.10) has two parts. An increase in saving increases the marginal utility of both parents and children. Either when $\lambda = 0$ (one-sided altruism) or $\mu = 0$, it reduces to

$$s^* : \ U_c(c_1^{p*}) = \beta R U_c(c_2^{p*}). \tag{2.12}$$

(2.11) characterizes the socially optimal inter-generational distribution of consumption in the second period. When $\mu = 0$, the social planner values utility of children only as much parents. In this case, (2.11) reduces to $U_c(c_2^{p*}) - \delta U_c(c^{k*}) = 0$, which is similar in form to the case where parents optimally choose distribution of consumption in the second period (see 4.6 below). For any $\mu > 0$, the social planner values utility of children more than parents. In the limiting case $\mu \to \infty$, the socially optimal distribution of consumption in the second period is given by $V_c(c^{k*}) - \lambda V_c(c_2^{p*}) = 0$. This is similar in form to the case where children optimally choose distribution of consumption in the second period (see 3.1 below).

3 Pure Altruism Model and Schooling

First, I analyze the case in which τ is chosen by child in the second period and assume that parameter values are such that there is no net transfer from parent to child for all realizations of random variables. The necessary condition is that $U_c(c_2^p) \geq \delta U_c(c^k)$ for all realizations of random variables.[5] This may arise if the income of child is relatively high compared to its parent in the second period and/or the degree of parental altruism towards its child is low.

The interaction between parent and child can be modeled as a two-period transfer game between them. For time-consistency one needs to solve this problem recursively starting in the second period.

The second period problem of child is to

$$\max_{\tau} V(c^k) + \lambda V(c_2^p)$$

subject to the budget constraints (2.5)-(2.6). The first order condition is:

$$W_\tau^k = -V_c(\phi h(e) - \tau) + \lambda V_c(y_2 + Rs + \tau) \leq 0;\ \forall\ \tau \geq 0\ \&\ \tau W_\tau^k = 0. \quad (3.1)$$

(3.1) shows that for the optimal choice of τ, child compares its marginal cost to its marginal benefit. If the marginal cost exceeds the marginal benefit then $\tau = 0$. This can occur if child's realized income or its degree of altruism towards parent is low and parental realized income and savings are high.

Differentiating (3.1) with respect to e, s, y_2, and ϕ, I have

$$\frac{d\tau}{de} = \frac{V_{cc}(c^k)\phi h_e(e)}{V_{cc}(c^k) + \lambda V_{cc}(c_2^p)} > 0; \quad (3.2)$$

$$\frac{d\tau}{ds} = -\frac{\lambda V_{cc}(c_2^p)R}{V_{cc}(c^k) + \lambda V_{cc}(c_2^p)} < 0; \quad (3.3)$$

$$\frac{d\tau}{d\phi} = \frac{V_{cc}(c^k)h(e)}{V_{cc}(c^k) + \lambda V_{cc}(c_2^p)} > 0; \quad (3.4)$$

$$\frac{d\tau}{dy_2} = -\frac{\lambda V_{cc}(c_2^p)}{V_{cc}(c^k) + \lambda V_{cc}(c_2^p)} < 0; \quad (3.5)$$

(3.2) and (3.4) show that a higher schooling investment and a higher realized value of ϕ increase transfers by child to parent. In contrast, (3.3) and (3.5) show that a higher parental saving and a higher realized value of y_2 reduce transfers by child. (3.4) and (3.5) also imply that transfers will be

[5] The sufficient condition is that $U_c(y_2^{max} + Ry_1) \geq \delta U_c(\phi^{min} h(0))$, where y_2^{max} and ϕ^{min} are the upper support of y_2 and lower support of ϕ respectively.

state-contingent. A higher schooling investment and a higher realized value of ϕ increase the income of child in the second period and thus reduce the marginal cost of transfer. A rise in parental saving and a higher realized value of y_2, in contrast, reduce the marginal benefit from transfer.

Parent takes into account effects of its choices on transfers by child. The parental problem is to

$$\max_{e,s} W^p = U(c_1^p) + \beta E[U(c_2^p) + \delta U(c^k)]$$

subject to (2.4)-(2.6) and (3.1). Since the focus of the paper is on effects of risks on schooling investment, I assume that parameters values are such that $e > 0$.

The first order conditions are

$$W_e^p = -\ U_c(c_1^p) + \beta E\Big[\delta U_c(c^k)\phi h_e(e) + \{U_c(c_2^p) - \delta U_c(c^k)\}\frac{d\tau}{de}]\Big] = 0 \ \& \quad (3.6)$$

$$W_s^p = -\ U_c(c_1^p) + \beta E\Big[U_c(c_2^p)R + \{U_c(c_2^p) - \delta U_c(c^k)\}\frac{d\tau}{ds}\Big] \leq 0;\ s \geq 0 \ \& \ sW_s^p = 0. \quad (3.7)$$

These first order conditions can be interpreted in a standard fashion, with parent equating the (perceived) marginal benefit of its choices to its (perceived) marginal cost. In the case of schooling investment, the marginal cost of schooling investment is loss in the utility of parent from consumption forgone in the first-period. The marginal benefit of schooling investment has two parts. Firstly, it increases earnings of child. Secondly, it increases old-age income support from child. (3.7) shows that if the marginal cost of saving is higher than the marginal benefit, parent will not save. This can happen if the first period income of parent is relatively low or it discounts future more heavily.

I first consider the case in which the borrowing constraint does not bind, $s > 0$. Then combining (3.6) and (3.7) I have

$$EU_c(c_2^p)[\frac{d\tau}{de} - R - \frac{d\tau}{ds}] + \delta EU_c(c^k)[\phi h_e(e) - \frac{d\tau}{de} + \frac{d\tau}{ds}] = 0. \quad (3.8)$$

For the rest of the analysis, I assume that the period utility function of child, $V()$, is such that consumptions of child, c^k, and parent, c_2^p, in the second period are linear and increasing functions of *the second period joint family income* $(y_2 + \phi h(e) + Rs)$ when $\tau > 0$.

$$c^k = A^k + D^k(y_2 + \phi h(e) + Rs) \text{ and } \tag{3.9}$$

$$c_2^p = A^p + D^p(y_2 + \phi h(e) + Rs) \tag{3.10}$$

where A^i & D^i for $i = p, k$ are constants.[6]

(3.8) -(3.10) imply that the optimal level of schooling investment, e, is characterized by

$$E[BU_c(c_2^p) + \delta U_c(c^k)]\phi h_e(e) = RE[BU_c(c_2^p) + \delta U_c(c^k)] \tag{3.11}$$

where B is a constant given by

$$B \equiv \frac{V_{cc}(c^k)}{\lambda V_{cc}(c_2^p)} = \frac{D^p}{D^k}. \tag{3.12}$$

It is convenient to define the second period indirect utility function of parent as

$$\tilde{U}(I) \equiv U(c_2^p(I)) + \delta U(c^k(I)) \tag{3.13}$$

where $I \equiv y_2 + \phi h(e) + Rs$ is the second period joint family income. Then using (3.14) and the co-variance decomposition, (3.11) can be written as

$$[h_e(e) - R]E\tilde{U}_I(I) = -h_e(e)Cov(\tilde{U}_I(I),\ \phi). \tag{3.14}$$

Similarly, (3.7) can be written as

$$W_s^p = -U_c(c_1^p) + \frac{D^k \beta R}{1+B} E\tilde{U}_I(I) \leq 0; s \geq 0 \ \&\ sW_s^P = 0. \tag{3.15}$$

(3.14) shows that schooling investment depends on the sign and size of the covariance of the (indirect) marginal utility of income of parent in the second

[6] This requires that the marginal rate of substitution between c^k and c_2^p is a function of ratio between c^k and c_2^p. For example, if $V(c) = -\exp^{-\alpha c}$ then $A^k = -A^p = -\frac{\ln \lambda}{2\alpha}$ and $D^k = D^p = 1/2$. If $V(c) = \frac{c^{1-\alpha}}{1-\alpha}$, then $A^k = A^p = 0$ and $D^k = 1 - D^p = \frac{\lambda^{-1/\alpha}}{1+\lambda^{-1/\alpha}}$ and for $V(c) = c^\alpha$, $A^k = A^p = 0$ and $D^k = 1 - D^p = \frac{\lambda^{\frac{1}{\alpha-1}}}{1+\lambda^{\frac{1}{\alpha-1}}}$.

period with ϕ. If the covariance term is negative, then schooling investment is inefficiently low.

To develop the intuition, I take second-order Taylor series approximation of the RHS of (3.14) in the neighborhood of mean values of y_2 and ϕ, $(\overline{y}_2, 1)$. This implies that

$$Cov(\tilde{U}_I(I),\ \phi) \approx \tilde{U}_{II}(I)\{\sigma_{y_2,\phi} + h(e)\sigma_\phi^2\} \tag{3.16}$$

Proposition 1: Suppose that parent receives old-age income support, $\tau > 0$, in all states and parental saving is strictly positive, $s > 0$. Schooling investment is inefficiently low $e < e^*$, if *the second period joint family income* $I \equiv (y_2 + \phi h(e) + Rs)$ is *positively correlated* with the productivity parameter of the human capital investment function, ϕ. The sufficient condition is that

$$\sigma_{y_2,\phi} + \sigma_\phi^2 h(e^*) > 0. \tag{3.17}$$

(3.17) ensures that there is a negative covariance between second period marginal utility of income of parent and ϕ. The negative covariance between the two reduces the expected marginal benefit of schooling investment relative to saving. This induces parent to save more. The reason is that the marginal rate of return on schooling investment is high when the second period marginal utility of consumption of parent (and child) is low. Thus, parent chooses an inefficiently low level of schooling investment.

(3.17) also shows that if there is only human capital investment risk, then schooling investment will be inefficiently low. This result is similar to Lehvari and Weiss (1974) and Pouliot (2006). However, in the presence of parental earnings risks, schooling investment can be inefficiently high or low depending on whether (3.17) is satisfied. This result relates to Paroush (1976) who finds that if the rate of return on saving is risky, schooling investment can be inefficiently high or low.

Increasing Risks (Mean Preserving Spread)

Now consider effects of increasing human capital investment risk, σ_ϕ^2, and parental income risk, $\sigma_{y_2}^2$. Taking second order Taylor approximation of (3.14) and (3.15) in the neighborhood of $(\overline{y}_2,\ 1)$, I have

$$W_e^p \approx (h_e(e) - R)[\tilde{U}_I(I) + \frac{\tilde{U}_{III}(I)}{2}\{\sigma_{y_2}^2 + 2h(e)\sigma_{y_2,\phi} + h^2(e)\sigma_{\phi^2}\}]$$

$$+h_e(e)\tilde{U}_{II}(I)\{\sigma_{y_2,\phi} + h(e)\sigma_{\phi^2}\} = 0 \text{ and} \tag{3.18}$$

$$W_s^p \approx -U_c(c_1^p) + \frac{D^k\beta R}{1+B}[\tilde{U}_I(I) + \frac{\tilde{U}_{III}(I)}{2}\{\sigma_{y_2}^2 + 2h(e)\sigma_{y_2,\phi} + h^2(e)\sigma_{\phi^2}\}] = 0. \tag{3.19}$$

Using the Cramer's rule, effect of a change in an exogenous variable, z, on schooling investment can be derived as follows

$$\frac{de}{dz} = \frac{W_{es}^p W_{sz}^p - W_{ss}^p W_{ez}^p}{H} \tag{3.20}$$

where $H \equiv W_{ee}^p W_{ss}^p - W_{es}^{p\,2}$. For the maximum to exist, it must be the case that $H > 0$.

From (3.18)-(3.20), it is clear that the effect of an increase in risk depends on the sign and size of the co-variance term and how increasing risk affects them. Since the major problem faced by developing countries is low level of schooling, I only consider the case in which (3.17) holds and the optimal choice of $e < e^*$.

Assumption 1: The variance and covariance of random variables, $(y_2,\ \phi)$ are such that $\sigma_{y_2,\phi} + \sigma_\phi^2 h(e^*) > 0$.

To develop intuition, I analyze three cases. In the first case, I assume that y_2 and ϕ are independent of each other, $\sigma_{y_2,\phi} = 0$. With this assumption, one can analyze effects of increasing one risk independent of another risk. In the second case, I assume that they are perfectly positively correlated, $\sigma_{y_2}^2 = \sigma_\phi^2 = \sigma_{y_2,\phi}$. This can arise if both parent and child are expected to work in a similar occupation and region. Finally, I consider the case in which they are perfectly negatively correlated, $\sigma_{y_2}^2 = \sigma_\phi^2 = -\sigma_{y_2,\phi}$. This case provides the maximum opportunity to share risks.

In the case of perfectly correlated risks, an increase in risk increases earnings risk for both parent and child symmetrically i.e. at the joint family level. In the rest of the paper, I refer to this case as *the joint family earnings risk.*

Assumption 2: The second period indirect utility function of parent, $\tilde{U}(I)$, is such that the *absolute risk-prudence* of parent, $\tilde{\rho}(I) \equiv -\frac{\tilde{U}_{III}(I)}{\tilde{U}_{II}(I)}$ is decreasing $(\rho_I(I) \leq 0)$ in the second period joint family income, I.

Proposition 2:

(I) Suppose that y_2 and ϕ are independent of each other, $\sigma_{y_2,\phi} = 0$. An increase in parental income risk, $\sigma^2_{y_2}$, has a positive effect on schooling investment. However, an increase in human capital investment risk, σ^2_ϕ, has a negative effect on schooling investment.

(II) Suppose that y_2 and ϕ are perfectly correlated. An increase in joint family earnings risk has a negative effect on schooling investment.

The intuition for these results are as follows. Parental income risk does not distort the relative rate of return between saving and schooling investment. As shown above (3.16), $\sigma^2_{y_2}$ does not affect the *Cov* term and thus the attractiveness of schooling relative to saving directly. Thus as it rises, the precautionary saving motive induces parent to increase schooling investment.

Given Assumption 1, a higher σ^2_ϕ increases the negative covariance between the parental marginal utility of consumption in the second period and ϕ making schooling investment less attractive. The risk-prudent parent reduces schooling investment in order to reduce its exposure to this risk. Similarly, an increase in the joint family earnings risk reduces schooling investment.

Borrowing Constraint

Now consider the case when $s = 0$.[7] The main difference with the previous case is that in response to risks parent can only adjust schooling investment.

Proposition 3: Suppose that the borrowing constraint binds, $s = 0$. Define the degree of *relative risk-prudence* of parent as $\tilde{r}\rho(I) \equiv \tilde{\rho}(I)I \equiv -\frac{\tilde{U}_{III}(I)}{\tilde{U}_{II}(I)}I$.

(I) If parental income risk and human capital investment risk are independently distributed, $\sigma_{y_2,\phi} = 0$, then an increase in $\sigma^2_{y_2}$ increases schooling investment. An increase in σ^2_ϕ increases (reduces) schooling investment if

[7] The sufficient condition is that $y_1 < y_2^{min} + \phi^{min}h(0)$ where y_2^{min} and ϕ^{min} are lower support of y_2 and ϕ respectively.

$$\tilde{r}\rho(I)\frac{h(e)}{I} > (<)2. \tag{3.21}$$

(II) If parental income risk and human capital investment risk are perfectly positively correlated, $\sigma_{y_2}^2 = \sigma_\phi^2 = \sigma_{y_2,\phi}$, then an increase in joint family earnings risk increases (reduces) schooling investment if

$$\tilde{r}\rho(I)\frac{h(e)+1}{I} > (<)2. \tag{3.22}$$

(III) If parental income risk and human capital investment risk are perfectly negatively correlated, $\sigma_{y_2}^2 = \sigma_\phi^2 = -\sigma_{y_2,\phi}$, then an increase in joint family earnings risk increases (reduces) schooling investment if

$$\tilde{r}\rho(I)\frac{h(e)-1}{I} > (<)2. \tag{3.23}$$

As before, a higher $\sigma_{y_2}^2$ increases schooling investment due to precautionary saving motive. The effects of a higher σ_ϕ^2 is more complicated. An increase in σ_ϕ^2 implies that the rate of return from schooling investment becomes more risky. It affects parental decision in two ways. Firstly, the precautionary saving motive induces higher schooling investment. Secondly, schooling investment itself becomes risky as one extra unit of investment does not increase earnings of child for sure, which has a negative effect on schooling. These two countervailing effects on schooling investment are similar to the (positive) precautionary saving motive effect and the (negative) substitution effect of the capital income risk on saving (Leland 1968, Sandmo 1970).

(3.21)-(3.23) show conditions under which the precautionary saving motive dominates the substitution effect and a higher σ_ϕ^2 has a positive effect on schooling investment. These conditions show that when the degree of *relative risk-prudence* and the share of earnings of child in the second period total family income are high, the precautionary saving motive is likely to dominate the substitution effect.[8] Intuitively, the more risk-prudent parent and child are, more parent would like to insure against the occurrence of

[8]For an example, suppose that the period utility function is of CRRA form, $U(c) = \frac{c^{1-\alpha}}{1-\alpha}$. Then, (3.21) implies that $\tilde{r}\rho(I)\frac{h(e)}{I} = (\alpha+1)\frac{h(e)}{I}$.

low family income in the second period. In addition, larger is the share of earnings of child in the second period total family income, it is more likely that the low family income in the second period will occur when the human capital investment is risky for a given schooling investment. Both of these induce parent to invest more in schooling.

(3.22)-(3.23) also show that an increase in joint family earnings risk is more likely to have positive effect on schooling investment if ϕ and y_2 are positively correlated. The reason is that the low family earnings are more likely to occur for a given schooling investment.

Panels A and B of Table 1 summarize main results. These results show that unconstrained parent responds differently to human capital investment and joint family earnings risk. It can adjust its saving in response to increasing human capital investment and joint family earnings risk. For it there is no precautionary motive for schooling investment and thus its schooling investment falls with increasing human capital/joint family earnings risk.

Since market allocations are inefficient and in particular schooling investment is inefficiently low, it raises the question whether providing income transfer/subsidies to parents by the government can raise schooling investment. The other important issue is whether provision of public schooling will lead to efficient allocations. Boldrin and Montes (2005) in an overlapping generations model with no uncertainty where young individuals face binding borrowing constraint show that mere provision of public education does not lead to efficient allocations. However, if public education is combined with public pension for old, it can lead to efficient allocations. Below I address these set of issues.

Income Transfers

Proposition 4:

(I.) *Lump-Sum/Unconditional Income Transfer*: Suppose that parents receive g_1 and g_2 as lump-sum transfers from the government in the first and second period respectively. An increase in income transfer to parent in the first-period, g_1, has a positive effect on schooling investment. Similarly, an increase in income transfer to parent in the second period, g_2, has a positive effect on schooling investment of an unconstrained parent. However, an increase in g_2 to the borrowing-constrained parent can increase or decrease schooling investment.

(II.) *Conditional Income Transfer*: Let $\xi < 1$ be the transfer per-unit of schooling investment to parents in the first period.[9] An increase in income transfer per-unit of schooling investment, ξ, has a positive effect on schooling investment.

The proposition shows that an increase in g_1 and ξ raises schooling investment of both types of parents. However, an increase in g_2 has a differential effect. The intuition for these results are as follows. An increase in g_1 and ξ reduces the marginal cost of schooling investment for parents inducing them to raise schooling investment.

An increase in g_2 increases future income of parents, which induces an unconstrained parent to reduce its saving and increase schooling investment. When a parent is borrowing-constrained, it cannot adjust its saving in response to an increase in g_2. The schooling investment is affected in two opposing ways. Firstly, an increase in g_2 reduces the marginal benefit of schooling investment, inducing it to reduce schooling investment. However, an increase in g_2 also reduces the negative covariance of the marginal utilities of consumption of parent and child with ϕ, which encourages schooling investment. The overall effect depends on which effect dominates.

Public Education and Pension

As discussed earlier, there can be many efficient allocations depending on the value of μ. The efficient level of schooling investment does not depend on the value of μ, but saving would differ. I just consider the case where $\mu = 0$, i.e. the social planner values utility of children same as parents. This is closest to the case considered in Boldrin and Montes (2005).

Suppose that there is a government which can save and borrow required amount at the risk-free rate R. It imposes income tax (transfer) in the second period on old parents, $g_2^p(y_2) = \overline{y}_2 - y_2$ and on adult children, $g_2^k(\phi) = (1-\phi)h(e)$. In addition, old parents receive lump-sum pension from the government, τ_2^p, which is financed by equal amount of lump-sum tax on adult children.

Suppose that the government undertakes public investment in schooling equal to e^*. It finances schooling expenditure by imposing lump-sum income

[9]The budget constraints are $c_1^p + s + (1-\xi)e = y_1 + g_1$, $c_2^p = y_2 + g_2 + Rs + \tau$, and $c^k = \phi h(e) - \tau$. Also assumption that $\xi < 1$ ensures that there is interior solution to schooling investment.

tax $\tau_1^e = e^*$ in the first period on unconstrained parents (before the introduction of public schemes). On the other hand, it imposes lump-sum income tax $\tau_2^e = Re^*$ in the second period on borrowing-constrained parents (before the introduction of public schemes).

Parents while taking decisions take these taxes (transfers) and public investment in schooling as given. Assume that for initially borrowing-constrained parents, $U_c(y_1) < \beta R U_c(y_2 + g_2^p(y_2) - \tau_2^e + \tau_2^p)$.[10] Then, if the government chooses τ_2^p which satisfies equation (2.12), it is straight-forward to show that market allocations will be efficient.

To achieve efficient allocations, it is crucial to have taxes such as $g_2^p(y_2)$ and $g_2^k(y_2)$. These taxes ensure that in the second-period each generation has identical net earnings i.e. they fill the role of missing insurance markets. In the absence of $g_2^p(y_2)$, there will be inefficiently high level of saving due to precautionary saving motive. In the absence of $g_2^k(\phi)$, consumption of children will be variable and the government can increase their (ex-ante) utility by providing certain consumption. Due to uninsurable risks, public education and public pension are not enough to ensure efficient allocations as in Boldrin and Montes (2005).

4 Educational Loan Model

4.1 State-Contingent Contracts

In this case, parent chooses both schooling investment, e, and old-age income support from child, τ, in the first-period before realizations of random variables. (e & τ) can be interpreted as an implicit loan contract, whose terms are set by parent. In this section, I assume that parent chooses τ contingent on realizations of random variables. Next section analyzes the case in which parent chooses fixed repayment term.[11]

The contract should be incentive-compatible i.e. child should be better-off by accepting the contract and not reneging on it. It raises two questions. Firstly, how is the contract enforced? One enforcement mechanism can be using judicial procedures. However, many developing countries do not have

[10] This condition ensures that it will be optimal for these parents to do saving.

[11] A parent would choose the contract which provides it with higher utility. It will compare the indirect utility functions under these two cases. With the fixed contract the probability of reneging by child will increase and the risk-sharing role of τ will be reduced.

well-functioning judicial system. In addition, if such contract involves minor children, it may not be legally recognized. Instead as is standard in the literature (Becker 1991, Cox 1987, Raut and Tran 2005), I assume that the contract is enforced through social norms. Reneging on repayment by child imposes social cost, $F > 0$, on child.

The other issue is that if child does renege, whether it does not provide any support to parent or it provides partial support (less than the contractual obligation). Given that child is altruistic, it is reasonable to assume that in the case of reneging, child provides support to parent equal to what it would voluntarily choose. Then the minimum level of support by child is given by (3.1).

Let τ^a be the solution of (3.1). Then the no-reneging constraint is

$$V(c^k) + \lambda V(c_2^p) \geq V(c^k(e, \tau^a)) + \lambda V(c_2^p(e, \tau^a)) - F, \ \forall \ y_2 \ \& \ \phi. \tag{4.1}$$

where $c(e, \tau^a)$ denotes consumption when old-age income support is τ^a for a given e. Assume that

$$V(c^k(e, \tau^a)) + \lambda V(c_2^p(e, \tau^a)) - F \geq 0, \ \forall \ y_2 \ \& \ \phi. \tag{4.2}$$

(4.2) ensures that in the case child reneges, it is still optimal for it to provide transfer equal to τ^a.

The participation constraint is given by

$$E[V(c^k) + \lambda V(c_2^p)] \geq E[V(c^k(0, \tau^a)) + \lambda V(c_2^p(0, \tau^a))] \tag{4.3}$$

(4.3) ensures that child is better-off with the loan contract, (e, τ), rather than without it. Note that the no-reneging constraint comes in play only when child accepts the contract. For the rest of the analysis, I assume that the parameter values are such that the participation constraint is slack and child is better-off accepting the loan contract.

The parental problem is to

$$\max_{e,s,\tau} W^p = U(c_1^p) - \delta M(l) + \beta E[U(c_2^p) + \delta U(c^k)]$$

subject to (2.4)-(2.6), and (4.1). Let γ^i be the Langarangian multiplier associated with the no-reneging constraint for the *ith* realization of random variables, (y_2, ϕ).

The first-order conditions for $e,\ s$ & τ are

$$W_e^p = -\,U_c(c_1^p)) + \beta E[\delta U_c(c^k) + \gamma^i(-V_c(c^k) + V_c(c^k(e,\tau^a)))]\phi h_e(e) = 0; \quad (4.4)$$

$$W_s^p = -\,U_c(c_1^p) + \beta RE[U_c(c_2^p) + \gamma^i\lambda(-V_c(c_2^p) + V_c(c_2^p(s,\tau^a))] \leq 0;\ s \geq 0 \,\&\, sW_s^p = 0; \quad (4.5)$$

$$W_\tau^p = U_c(c_2^p) - \delta U_c(c^k) - \gamma^i[-V_c(c^k) + \lambda V_c(c_2^p)] \leq 0 \ \&\ \tau W_\tau^p = 0 \ \& \quad (4.6)$$

$$W_{\gamma^i}^p = V(c^k(e,\tau^a)) + \lambda V(c_2^p(e,\tau^a)) - F - V(c^k) - \lambda V(c_2^p) \leq 0 \ \&\ \gamma^i W_{\gamma^i}^p = 0. \quad (4.7)$$

By comparing (3.1) and (4.6) it is easy to show that for a given e and s, parent would choose a higher τ compared to τ^a. Old-age income support will be higher under the educational loan model compared to the pure altruism model. It implies that child will have incentive to renegotiate τ in the second period and $F > 0$ is required to ensure that child does not renege on the contract.

In the rest of the section, I analytically analyze the case in which the no-reneging constraint does not bind for all $i \in (y_2, \phi)$, $\gamma^i = 0$. This can happen if the social cost of reneging and the degree of altruism of parent and child towards each other are high. Throughout I assume that the period utility function of the parent $U()$ is such that the second period consumption of parent and child are linear and increasing functions of the second-period joint family income (i.e. similar in form to 3.9 and 3.10).[12]

The analysis of the case in which the no-reneging constraint binds for some realizations of random variables is conducted numerically in section 5. The main difference is that in the case no-reneging constraint binds, parent will be able to transfer less from child compared to unconstrained case, potentially reducing the attractiveness of schooling investment.

[12]For example, if $U(c) = -\exp^{-\alpha c}$ then $A^k = -A^p = \frac{\ln\delta}{2\alpha}$ and $D^p = D^k = 1/2$. If $U(c) = \frac{c^{1-\alpha}}{1-\alpha}$, then $A^k = A^p = 0$ and $D^p = 1 - D^k = \frac{\delta^{-1/\alpha}}{1+\delta^{-1/\alpha}}$ and for $U(c) = c^\alpha$, $A^k = A^p = 0$ and $D^p = 1 - D^k = \frac{\delta^{\frac{1}{\alpha-1}}}{1+\delta^{\frac{1}{\alpha-1}}}$.

Consider the case when the borrowing constraint does not bind, $s > 0$. In this case, it straight-forward to show that the optimal level of schooling investment is characterized by

$$EU_c(c_2^p)[\phi h_e(e) - R] = 0. \tag{4.8}$$

(4.8) is similar to (3.14) in form except that $E[BU_c(c_2^p) + \delta U_c(c^k)]$ is replaced by $EU_c(c_2^p)$. The optimal choices of e and s are characterized by equations similar in form to (3.14) and (3.15) with $\tilde{U}(I)$ replaced by $U_c(c_2^p)$. Similar is the case when the borrowing constraint binds, $s = 0$.

Define the absolute and relative risk-prudence of the parent as $\rho(c_2^p(I)) = -\frac{U_{III}(c_2^p(I))}{U_{II}(c_2^p(I))}$ and $r\rho(c_2^p(I)) = -\frac{U_{III}(c_2^p(I))}{U_{II}(c_2^p(I))} I$ respectively. Assume that $\rho(c_2^p(I))$ and $r\rho(c_2^p(I))$ are declining in I. Since equations characterizing optimal choices are similar in form to the previous case, all the results summarized in Propositions 1 to 4 continue to apply with $\tilde{r}\rho(I)$ replaced by $r\rho(c_2^p(I))$ in equations (3.21)-(3.23). As before, public schooling investment combined with public pension system and income stabilization taxes (transfers) can achieve efficient allocations.

4.2 Fixed Contract

Now consider the case in which parent chooses fixed repayment contract. τ is chosen in the first-period, but the actual repayment occurs in the second period. In this case, the old-age income support no longer plays the role of risk-sharing instrument. Throughout this section, I assume that the no-reneging constraint does not bind, $\gamma_i = 0$. The analysis of the case in which the no-reneging constraint binds is conducted numerically in section 5.

The optimal choices of e, and s continue to given by (4.4), and (4.5) respectively (with $\gamma_i = 0$). The optimal choice of τ is given by

$$W_\tau^p \equiv EU_c(c_2^p) - \delta EU_c(c^k) = 0. \tag{4.9}$$

Parent chooses τ which equates the average marginal utilities of consumption of parent and child (as perceived by parent) in the second period.

Proposition 5:

(I) Unconstrained Parent ($s > 0$): An increase in $\sigma_{y_2}^2$ reduces schooling investment. However, an increase in σ_ϕ^2 may increase or reduce schooling investment

(II) Borrowing-Constrained Parent ($s = 0$): An increase in $\sigma^2_{y_2}$ increases schooling investment. However, an increase in σ^2_ϕ increases (decreases) schooling investment if

$$r\rho(c^k)\frac{h(e)}{c^k} > (<) 2 \tag{4.10}$$

where $r\rho(c^k) \equiv -\frac{U_{ccc}(c^k)}{U_{cc}(c^k)}c^k$ is the relative risk-aversion of child as perceived by parent.

5(I) shows that an increase in $\sigma^2_{y_2}$ reduces schooling investment of an unconstrained parent, which is in contrast to the result in Proposition 2. Since τ does not allow for risk-sharing, the parent cannot rely on increased transfer from child when its realized income is low. Thus, the precautionary saving motive induces parent to reduce schooling investment and increase saving in response to increase in $\sigma^2_{y_2}$.

It also shows that an increase in σ^2_ϕ may have a negative or a positive effect on schooling investment. As discussed earlier, the effect of increasing σ^2_ϕ on schooling investment depends on the (positive) precautionary motive and the (negative) substitution effect. Since there is no risk-sharing between child and parent, essentially parent has one (risky) asset to deal with human capital investment risk. Thus, schooling investment may increase if the precautionary motive dominates.

5(II) shows that qualitatively, effects of increasing risks on schooling investment for a borrowing-constrained parent remain as before (Proposition 3). An increase in $\sigma^2_{y_2}$ induces parent to increase τ. But since it reduces consumption of child, it also induces parent to increase schooling investment. Regarding the effects of σ^2_ϕ, (4.10) provides condition under which the precautionary motive dominates the substitution effect. This result is in contrast to the results in Proposition 2. Table 1 Panel C summarizes main results under fixed contract.

Regarding transfer from the government, as shown in the proof of Proposition 5, an increase in g_1 and ξ has a positive effect on schooling investment. An increase in g_2 has a positive effect on schooling investment of unconstrained parents. But it has a negative effect on schooling investment of borrowing-constrained parents. Public schooling investment combined with public pension system and income stabilization taxes (transfers) can achieve efficient allocations.[13]

[13] In the analysis, I have considered the cases where old-age income support is either

5 Quantitative Analysis

For analytical tractability, I have analyzed effects of small risks and assumed that parents receive transfer in all states and/or no-reneging constraint does not bind. However, parents may not receive transfers in all states and no-reneging constraint can bind for some realizations of random variables. While making schooling investment and setting repayment terms parents would take into account these possibilities. To examine these cases, I simulate responses of two types of parents: unconstrained and borrowing constrained parents to increasing risks under alternative models of old-age income support.

Assume that the period utility function is of logarithmic form

$$U(x) = \ln x. \tag{5.1}$$

Similar to Becker (1991) and Rustuccio and Urrutia (2004), the human capital investment function is specialized to

$$\phi h(e) = a \exp^{\zeta}(e + m)^{\alpha}. \tag{5.2}$$

where a is the productivity parameter, $\exp^{\zeta}$ is the wage of (adult) child per-unit of human capital, and m is the government expenditure on schooling per child. Wages are assumed to be *log-normally distributed* with mean zero and variance σ_{ζ}^2.

The parental endowment income in the second period is assumed to be

$$y_2 = \exp^{\eta} \tilde{y}_2 \tag{5.3}$$

where $\exp^{\eta}$ is the wage of parent per-unit of its human capital. Assume that parental wage is *log-normally distributed* with mean zero and variance σ_{η}^2.

For the quantitative analysis, I take pure altruism model as baseline case. Then I choose parameter values to match some salient features of household earnings and schooling in Indonesia where old-age income support by adult children is widespread (Park 2003, Raut and Tran 2005). The Indonesian Human Development Index report 2013 (HDI 2013) estimates that a child

chosen by parent or child. Literature suggests that such transfers may also be determined by social norms (or exogenous factors) and not chosen by either parents or children (Cox and Fafchamps 2006). One can model this situation as parents receiving fixed amount $\tau \geq 0$ from children for all realizations of y_2 and ϕ. A special case is when $\tau = 0$. One can show that effects are similar to that of fixed contract highlighting the role of risk-sharing. Proofs are available on request.

born in Indonesia in 2011-12 is expected to have 13 years of schooling. Accordingly I set time-period to be 13 years.

I use fifth wave of the Indonesian Family Life Survey (ILFS 5) 2014-15 undertaken by the Rand Corporation to generate estimates of a sub-set of parameter values related to household income and earnings. This survey covered 16,931 households in both rural and urban areas.

To generate an estimate of earnings risk, I estimate a model of household earnings function of a following form:

$$\ln hin = \alpha X + \kappa \tag{5.4}$$

where $\ln hin$ is the log of household earnings, X is the matrix of household characteristics, κ is the residual. Household characteristics include family size, number of self-employed workers, government workers, private salaried wage workers, and casual wage workers, number of children below 5, number of individuals 60 years and above, place of residence whether rural or urban and in which sub-districts, and schooling level of household-head and its age and gender. I proxy earnings risks of both child and parent by the variance of residual of estimated household earnings function.

The estimated model suggests that the conditional mean of log of household earnings is equal to $\ln 16.99$ ($\approx$ 23.9 million Rupiah $\approx US\$$ 1730) and its variance to be 0.34. I set $\sigma_\eta^2 = \sigma_\zeta^2 = 0.34$. I assume that both risks are independent, $\sigma_{\zeta,\eta} = 0$.

Assume that the first period endowment income of an unconstrained parent, y_1, is 23.9 and of (borrowing) constrained parent 11.96 which is 50% less.[14] I choose second period endowment income, $\tilde{y}_2$, such that the present value of expected endowment income of both types of parents remain (approximately) the same as in the baseline case.

Taking log of equation 5.2, I have

$$\ln \phi h(e) = \ln a + \alpha \ln(e + m) + \zeta. \tag{5.5}$$

To derive estimates of a and α, I use deterministic version of equation 5.5. Assume that equation 5.5 holds for both current adults and children. I proxy schooling expenditure incurred by years of schooling.

In 2013 mean year of schooling for adults (25 years and above) is estimated to be approximately 6 years (HDI 2013). As discussed earlier the expected

[14] All income and expenditure items are in million rupiah.

mean year of schooling for children born in 2011-12 is estimated to be 13 years. Thus these children will have 7 years of additional schooling. The average rate of return on schooling in Asian countries is estimated to be 4.5% per year of schooling (Peet et.al. 2015).

The average income of households with 6 years of schooling for adults is 23.9 million Rupiah (log 16.99). Assuming 7 years of additional schooling and with the return of 4.5% per year of schooling, the expected earnings of child is estimated to be 32.5 million Rupiah (log 17.30). Using these two conditions, I get estimate of $a = 11.7$ and $\alpha = 0.40$.

The government expenditure on education as a percentage of GDP in Indonesia has averaged 3.5% over the 2005-14 (World Bank 2016). In the same period, the government expenditure as a percentage of GDP in Indonesia has averaged 15.5% (World Bank 2016). This implies that the government expenditure on education as a percentage of private GDP has averaged 4.4%. Accordingly, I set $m = 0.79$ which is 4.4% of the weighted average of first-period endowment incomes of parents of both types.

There is no estimate available for the degree of altruism of parents and children towards each other. I set $\delta = \lambda$. Then I choose δ, $\tilde{y}_2$, R & β such that: (i) The second period expected income for unconstrained parent, $Ey_2 + Rs$, is equal to average annual income of adults between 60-75 years. The ILFS 5 data suggests that average annual income of adults between 60-75 years was 7.98 million rupees. (ii) Unconstrained parent receives transfers from child for most of the realizations of random variables (over 95%).[15] (iii) Parental saving rate is equal to the household saving rate in Indonesia. The IFC (2008) reports that the household saving rate in 2003-04 was 16.7%. (iv) The rate of interest $R > \frac{1}{\beta}$. Given incomplete markets and uninsurable risks, it is realistic to assume that the rate of interest is higher than the inverse of discount rate. The implied values are $\delta = \lambda = 0.95$, R =1.53, $\beta = 0.68$, and $\tilde{y}_2 = 1.6$ for the unconstrained parent and $\tilde{y}_2 = 19.84$ for the constrained parent. The parameter values are reported in Table 2.

For comparison purpose, Table 3 reports results for the deterministic environment. I also report welfare level of parents, $Welp$ and children, $Welk$, calculated as follows:

$$Welp = \ln c_1^p + \beta E[\ln c_2^p + \delta \ln c^k] \text{ and} \tag{5.6}$$

[15]The constrained parent receives transfers from child for half of the realizations of random variables, partly due to its relatively high endowment income in the second period.

$$Welk = \beta E[\ln c^k + \lambda \ln c_2^p]. \tag{5.7}$$

Table 3 shows that schooling investment by unconstrained parent (7.58) is almost four times higher than of constrained parent (1.94). Welfare level of unconstrained parent is higher than of constrained parent. But welfare level of child of unconstrained parent is lower than of child of constrained parent. This is due to the fact that the child of constrained parent does not provide income support to its parent.

5.1 Increasing Risks

Pure Altruism

Table 4 reports results for the baseline model. With the chosen parameter values, saving is 3.9985. The schooling investment by unconstrained and borrowing-constrained parents are 5.0152 and 3.0073 respectively. Thus adding risks reduces schooling investment of unconstrained parent by 34% and increases its saving by 550% relative to the deterministic case. However, schooling investment of constrained parent is higher. For the constrained parent precautionary motive dominates negative substitution effect resulting in its schooling investment being higher than the deterministic case. These results are consistent with analytical results derived earlier. Results also show that risks reduce welfare levels of both types of parents and their children.

To examine effects of risks, I increase one risk at a time by 50% keeping the other risk at the baseline value (0.34). Panel A shows that a higher human capital investment risk, σ_ζ^2, increases saving. It has a negative effect on schooling investment and welfare of unconstrained parent and its child. Results show that a higher σ_ζ^2 has a marginal effect on schooling investment of constrained parent. Welfare of unconstrained parent and its child falls.

Panel B shows that a higher parental income risk, σ_η^2, has a marginal effect on saving and schooling investment and welfare of unconstrained parent and its child. On the other hand, it has a significant positive effect on the schooling investment of borrowing-constrained parent. Though the schooling investment of borrowing-constrained parent rises, its own welfare and of its child falls. The reason for relatively small effect of parental income risk on saving, schooling investment, and welfare of unconstrained parent and its child is that it has relatively low endowment income in the second period.

These results show that the human capital investment risk has relatively larger effect on the schooling investment of unconstrained parent. But, the parental income risk has relatively larger effect on the schooling investment of borrowing-constrained parent.

State Contingent Contract

For this model the value of one additional parameter, the psychic cost of reneging, F, needs to be pinned down. Note that in the case of reneging, children still make transfers to parents equal to what they will voluntarily choose.

I choose value of F such that the unconstrained parent receives transfers in 95% of realizations of random variables similar to the pure altruism model. Rest of the parameter values remain the same. Table 5 reports the results. Panel A shows that in the baseline case ($\sigma_\zeta^2 = \sigma_\eta^2 = 0.34$) schooling investment by unconstrained parent is higher and saving is lower compared to the pure altruism model. Additionally, welfare levels of unconstrained parent and its child are lower. The reason that unconstrained parent invests more in schooling and save less is that the expected transfer from its child per-unit of schooling investment is higher compared to the pure altruism model. Schooling investment becomes more attractive relative to saving.

On the other hand, schooling investment of constrained parent is less compared to the pure altruism model. The reason is that it has relatively high endowment income in the second period and thus values old age income support relatively less. Overall results show that the effect of changes in risk on schooling investment and saving remains as in the pure altruism model.

Fixed Contract

The fixed-contract economy differs from state-dependent contract in the sense that parents receive fixed amount of old-age income support for realizations of random variables for which children do not renege. For other realizations of random variables, parents receive what children would voluntarily transfer. Old-age income transfer still plays risk-sharing role, though less than the previous models.

For simulation purpose, I change values of two parameters. I increase the value of F to 0.25 and reduce the value of λ (the degree of altruism of children) to 0.50. With these changes, fixed contract is used by unconstrained parents in about 50% of the realizations of random variables.[16]

Table 6 reports the results. Panel A shows that a higher human capital investment risk, σ_ζ^2, increases schooling investment of both constrained and unconstrained parents, though marginally. This suggests that the precautionary motive dominates the negative substitution effect. It reduces welfare levels of both types of parents and their children. Saving falls marginally.

Panel B shows that a higher parental income risk, σ_η^2, has a marginal negative effect on schooling investment of unconstrained parents and their savings. However, it increases schooling investment of constrained parents significantly. Results also show that a higher parental income risk reduces welfare levels of both types parents and their children. These results are consistent with the analytical results derived earlier.

6 Conclusion

This paper theoretically analyzed effects of parental future income risk and human capital investment risk on parental schooling investment when children provide old-age income support to parents in a two-period model. It considered alternative models of determination of old-age income support: pure altruism model and educational loan repayment model with state-dependent and fixed contracts. It finds that the effects of risks crucially depend on type of risks, whether there is risk-sharing and whether parents are borrowing-constrained.

When there is risk-sharing, an increase in parental income risk has a positive effect on schooling investment. In the absence of risk-sharing, an increase in parental income risk has a negative effect on schooling investment. In the case of human capital investment risk, increasing risk has a negative effect on schooling investment if there is risk-sharing and parent is not borrowing-constrained. However, if parent is borrowing constrained or there is no risk-sharing, an increase in the human capital investment risk may have a positive effect on schooling investment.

[16] If one uses parameter values of state-dependent contract model, then the child of unconstrained parent reneges for 95% of realizations of random variable. The fixed contract is used in less than 5% of the realizations of random variables. I also simulate the state-dependent model with new values of F and λ. The effects of risks remain similar to ones discussed earlier.

Numerical analysis shows that the human capital investment risk has relatively larger effect on the schooling investment of unconstrained parents. But, the parental income risk has relatively larger effect on the schooling investment of borrowing-constrained parents.

In terms of policy, providing income-assistance to young parents can increase schooling investment. However, income stabilization measures may have a positive or a negative effect on schooling investment depending on whether there is risk-sharing and whether borrowing constraint binds. I find that the provision of public education along with properly designed old-age pension scheme and income insurance can lead to socially efficient allocations.

For analytical tractability, I have analyzed partial equilibrium effects of risks. In future work, I would like to examine their general equilibrium effects. With changes in risks, as parents rebalance their portfolios it will affect interest rate in the economy. Any change in human capital investment will also affect wages and thus return from schooling. Such general equilibrium effects may strengthen or moderate effects of increasing risks on schooling investment.

Table 1

Effects of Increasing Risks on Schooling Investment

Risk-Sharing

Panel A: Independent Risks ($\sigma_{y_2,\phi} = 0$)

	$\sigma^2_{y_2}$	σ^2_{ϕ}
τ & $s > 0$	+	-
$\tau > 0$ & $s = 0$	+	+(-) if $\tilde{r}\rho(I)\frac{h(e)}{I} > (<)2$

Panel B: Joint Family Earnings Risk

($\sigma_{y_2,\phi} = \sigma^2_{\phi}$ or $\sigma_{y_2,\phi} = -\sigma^2_{\phi}$)

τ & $s > 0$	-
$\tau > 0$ & $s = 0$	+(-) if $\tilde{r}\rho(I)\frac{h(e)+1}{I} > (<)2$ & $\sigma_{y_2,\phi} = \sigma^2_{\phi}$
$\tau > 0$ & $s = 0$	+(-) $\tilde{r}\rho(I)\frac{h(e)-1}{I} > (<)2$ & $\sigma_{y_2,\phi} = -\sigma^2_{\phi}$

Panel C: Fixed Contract (No Risk-Sharing)

	$\sigma^2_{y_2}$	σ^2_{ϕ}
τ & $s > 0$	-	?
$\tau > 0$ & $s = 0$	+	+(-) if $r\rho(c^k)\frac{h(e)}{c^k} > (<)2$

Table 2
Parameter Values

Parameter	Values
β	0.68
R	1.53
$\delta = \lambda$	0.95
σ_η^2	0.34
σ_ζ^2	0.34
a	11.7
α	0.40
g	0.79
Unconstrained Parents	
y_1	23.91
$\tilde{y}_2$	1.6
Constrained Parents	
y_1	11.96
$\tilde{y}_2$	19.84
State-Dependent Contract	
F	0.02
Fixed-Contract	
F	0.25
λ	0.50

Table 3
Deterministic Case

	Variables			
	e	s	$Welp$	$Welk$
Unconstrained Parents	7.5799	0.7240	6.5590	3.8131
Borrowing Constrained Parents	1.9364	0	6.3876	4.0771

Table 4
Effect of Increasing Risks: Pure Altruism Model

	Variance	Variables			
Panel A: Human Capital Risks		e	s	$Welp$	$Welk$
Unconstrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	5.0142	3.9985	6.3962	3.6965
	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .51$	4.5028	4.8268	6.3367	3.6578
Constrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	3.0083	0	6.2086	4.0065
	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .51$	2.9450	0	6.1577	3.9446
Panel B: Parental Income Risks		e	s	$Welp$	$Welk$
Unconstrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	5.0142	3.9985	6.3962	3.6965
	$\sigma_\eta^2 = .51,\ \sigma_\zeta^2 = .34$	5.0223	4.0035	6.3979	3.6989
Constrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	3.0083	0	6.2086	4.0065
	$\sigma_\eta^2 = .51,\ \sigma_\zeta^2 = .34$	3.1519	0 6.1733	3.9867	

Table 5
Effect of Increasing Risks: State-Dependent Model

	Variance	Variables			
Panel A: Human Capital Risks		e	s	$Welp$	$Welk$
Unconstrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	6.5652	2.0801	6.3823	3.6481
	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .51$	5.6421	3.4618	6.3295	3.6255
Constrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	2.2429	0	6.2215	3.9313
	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .51$	2.2061	0	6.1405	3.8444
Panel B: Parental Income Risks		e	s	$Welp$	$Welk$
Unconstrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	6.5552	2.0801	6.3823	3.6481
	$\sigma_\eta^2 = .51,\ \sigma_\zeta^2 = .34$	6.5436	2.1016	6.3772	3.6437
Constrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	2.2429	0	6.2215	3.9313
	$\sigma_\eta^2 = .51,\ \sigma_\zeta^2 = .34$	2.3249	0	6.1795	3.8977

Table 6
Effect of Increasing Risks: Fixed Contract Model

	Variance	Variables			
Panel A: Human Capital Risks		e	s	$Welp$	$Welk$
Unconstrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	2.8634	7.3908	6.3138	2.8999
	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .51$	2.8917	7.3175	6.2671	2.8442
Constrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	2.3544	0	6.1932	2.9972
	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .51$	2.3623	0	6.1387	2.9386
Panel B: Parental Income Risks		e	s	$Welp$	$Welk$
Unconstrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	2.8634	7.3908	6.3138	2.8999
	$\sigma_\eta^2 = .51,\ \sigma_\zeta^2 = .34$	2.8540	7.4051	6.3037	2.8922
Constrained Parents	$\sigma_\eta^2 = .34,\ \sigma_\zeta^2 = .34$	2.3544	0	6.1932	2.9972
	$\sigma_\eta^2 = .51,\ \sigma_\zeta^2 = .34$	2.5357	0	6.1459	2.9861

Appendix: Proofs

Taking second order Taylor series approximation in the neighborhood of $(\overline{y}_2, 1)$, I have

$$EU_c(c_2^p)\phi \approx U_c(c_2^p) + \frac{D^{p2}}{2}U_{ccc}(c_2^p)[\sigma_{y_2}^2 + h^2(e)\sigma_\phi^2 + 2h(e)\sigma_{y_2,\phi}]$$

$$+D^p U_{cc}(c_2^p)[\sigma_{y_2,\phi} + h(e)\sigma_\phi^2]. \quad (A1)$$

$$EU_c(c_2^p) \approx U_c(c_2^p) + \frac{D^{p2}}{2}U_{ccc}(c_2^p)[\sigma_{y_2}^2 + h^2(e)\sigma_\phi^2 + 2h(e)\sigma_{y_2,\phi}]. \quad (A2)$$

$$Cov(U_c(c_2^p), \phi) \approx D^p U_{cc}(c_2^p)[\sigma_{y_2,\phi} + h(e)\sigma_\phi^2]. \quad (A3)$$

$$EU_c(c^k)\phi \approx U_c(c^k) + \frac{D^{k^2}}{2}U_{ccc}(c^k)[\sigma_{y_2}^2 + h^2(e)\sigma_\phi^2 + 2h(e)\sigma_{y_2,\phi}]$$

$$+D^k U_{cc}(c^k)[\sigma_{y_2,\phi} + h(e)\sigma_\phi^2]. \quad (A4)$$

$$EU_c(c^k) \approx U_c(c^k) + \frac{D^{k^2}}{2}U_{ccc}(c^k)[\sigma_{y_2}^2 + h^2(e)\sigma_\phi^2 + 2h(e)\sigma_{y_2,\phi}]. \quad (A5)$$

$$Cov(U_c(c^k), \phi) \approx D^k U_{cc}(c^k)[\sigma_{y_2,\phi} + h(e)\sigma_\phi^2]. \quad (A6)$$

$$D^k E\tilde{U}_I(I)\phi = [BU_c(c_2^p)+\delta U_c(c^k)]+\frac{1}{2}[BD^{p2}U_{ccc}(c_2^p)+\delta D^{k^2}U_{ccc}(c^k)][\sigma_{y_2}^2+h^2(e)\sigma_\phi^2+2h(e)\sigma_{y_2,\phi}]$$

$$+[BD^p U_{cc}(c_2^p) + \delta D^k U_{cc}(c^k)][\sigma_{y_2,\phi} + h(e)\sigma_\phi^2]. \quad (A7)$$

$$D^k E\tilde{U}_I(I) \approx [BU_c(c_2^p)+\delta U_c(c^k)]+\frac{1}{2}[BD^{p2}U_{ccc}(c_2^p)+\delta D^{k^2}U_{ccc}(c^k)][\sigma_{y_2}^2+h^2(e)\sigma_\phi^2+2h(e)\sigma_{y_2,\phi}]. \quad (A8)$$

$$D^k Cov(\tilde{U}_I(I), \phi) \approx [BD^{p2}U_{cc}(c_2^p) + \delta D^{k2}U_{cc}(c^k)][\sigma_{y_2,\phi} + h(e)\sigma_\phi^2]. \qquad (A9)$$

Proposition 1

$$Cov(I, \phi) \equiv Cov(y_2 + \phi h(e) + Rs, \phi) = \sigma_{y_2,\phi} + h(e)\sigma_\phi^2. \qquad (A10)$$

Suppose that (3.17) holds. Then for any $e \geq e^*$ (3.17) will hold and $Cov(\tilde{U}(I), \phi) < 0$ given risk-aversion. This implies that for any $e \geq e^*$ the LHS of (3.14)≤ 0, but the RHS of (3.14) > 0. Thus, any $e \geq e^*$ is not a solution of (3.14). If there is a solution to (3.14), it must be the case that $e < e^*$.

Note that at the optimal e, it must be the case that

$$\sigma_{y_2,\phi} + h(e)\sigma_\phi^2 > 0 \qquad (A11)$$

to ensure that $Cov(\tilde{U}_I(I), \phi) < 0$ and $e < e^*$.

Proof of Propositions 2, $\tau, s > 0$

Differentiating (3.18) and (3.19) and evaluating the expressions at $h_e(e) = R$, I have

$$W_{es}^p = D^k R\tilde{U}_{III}(I)[\sigma_{y_2,\phi} + h(e)\sigma_\phi^2]h_e(e) > 0; \qquad (A12)$$

$$W_{ss}^p = U_{cc}(c_1^p) + \frac{D^k\beta R^2}{1+B}[\tilde{U}_{II}(I) + \frac{\tilde{U}_{IIII}(I)}{2}\{2\sigma_{y_2,\phi} + h^2(e)\sigma_\phi^2 + \sigma_{y_2}^2\}] < 0 \quad (A13)$$

and

$$W_{ee}^p = h_{ee}(e)[\tilde{U}_I(I) + \frac{\tilde{U}_{III}(I)}{2}\{2\sigma_{y_2,\phi} + h^2(e)\sigma_\phi^2 + \sigma_{y_2}^2\} + D^k\tilde{U}_{II}(I)\{\sigma_{y_2,\phi} + h(e)\sigma_\phi^2\}]$$

$$+ D^k h_e(e)[\tilde{U}_{III}(I)\{\sigma_{y_2,\phi} + h(e)\sigma_\phi^2\} + \tilde{U}_{II}(I)h_e(e)\sigma_\phi^2]. \qquad (A14)$$

Since $W^p_{es} > 0$ and $W^p_{ss} < 0$, it must be the case that $W^p_{ee} < 0$ for maximization.

Suppose now that $\sigma_{y_2,\phi} = 0$. Then, Differentiating (3.18) and (3.19) w.r.t. $\sigma^2_{y_2}$ and σ^2_{ϕ}, I have

$$W^p_{e,\sigma^2_{y_2}} = 0 \;\&\; W^p_{s,\sigma^2_{y_2}} = \frac{1}{2}\frac{D^k \beta R}{1+B}\tilde{U}_{III}(I) > 0. \tag{A15}$$

$$W^p_{e,\sigma^2_{\phi}} = D^k \tilde{U}_{II}(I) h(e) h_e(e) < 0 \;\&$$

$$W^p_{s,\sigma^2_{\phi}} = \frac{D^k \beta R}{1+B}\frac{\tilde{U}_{III}(I)}{2} h^2(e) > 0. \tag{A16}$$

The effect of $\sigma^2_{y_2}$ follows from $(A15)$ and the Cramer's rule.

The sign of $\frac{de}{d\sigma^2_{\phi}}$ depends on the sign of the expression of $W^p_{e,s} W^p_{s,\sigma^2_{\phi}} - W^p_{s,s} W^p_{e,\sigma^2_{\phi}}$. Now,

$$W^p_{e,s} W^p_{s,\sigma^2_{\phi}} - W^p_{s,s} W^p_{e,\sigma^2_{\phi}} = -[U_{cc}(c^p_1) + \frac{D^k \beta R^2}{1+B}\{\tilde{U}_{II}(I) + \frac{1}{2}\tilde{U}_{IIII}(I)\sigma^2_{y_2}\}] W^p_{e,\sigma^2_{\phi}} +$$

$$\frac{1}{2}\frac{D^{k^2} \beta R^2}{1+B} h^3(e) h_e(e) [\tilde{U}^2_{III}(I) - \tilde{U}_{II}(I) * \tilde{U}_{IIII}(I)] \sigma^2_{\phi}. \tag{A17}$$

Since the first expression in the RHS of $A17$ is negative, for $\frac{de}{d\sigma^2_{\phi}} < 0$, it is sufficient to show that the last expression in the RHS of $A17$ is also negative i.e.

$$[\tilde{U}^2_{III}(I) - \tilde{U}_{II}(I) * \tilde{U}_{IIII}(I)] < 0. \tag{A18}$$

The assumption that the absolute risk-prudence for the parent, $\tilde{\rho}(I)$, is declining in I is sufficient to ensure that $(A18)$ holds. The effect of σ^2_{ϕ} follows from $(A18)$.

When y_2 and ϕ are perfectly positively correlated, $\sigma_{y_2,\phi} = \sigma^2_{\phi}$.

$$W^p_{e,\sigma^2_{\phi}} = D^k \tilde{U}_{II}(I)(h(e)+1) h_e(e) < 0 \;\&$$

$$W^p_{s,\sigma^2_{\phi}} = \frac{D^k \beta R}{1+B}\frac{\tilde{U}_{III}(I)}{2}(h(e)+1)^2 > 0. \tag{A19}$$

When y_2 and ϕ are perfectly negatively correlated, $\sigma_{y_2,\phi} = -\sigma^2_\phi$. Note that in this case, at equilibrium, e, $h(e) > 1$.

$$W^p_{e,\sigma^2_\phi} = D^k\tilde{U}_{II}(I)(h(e)-1)h_e(e) < 0 \;\&$$

$$W^p_{s,\sigma^2_\phi} = \frac{D^k\beta R}{1+B}\frac{\tilde{U}_{III}(I)}{2}(h(e)-1)^2 > 0. \tag{A20}$$

Using the steps discussed above one can show that $\frac{de}{d\sigma^2_\phi} < 0$ if $\tilde{\rho}(I)$ is declining in I. This proves **Part II**.

Proof of Proposition 3 (Borrowing Constraint $s = 0$):

Taking second order Taylor approximation of (3.6) around $(\overline{y}_2,\ 1)$, I have

$$W^p_e \approx -U_c(c^p_1)+$$

$$\frac{D^k\beta h_e(e)}{1+B}[\tilde{U}_I(I)+\frac{\tilde{U}_{III}(I)}{2}\{\sigma^2_{y_2}+2h(e)\sigma_{y_2,\phi}+h^2(e)\sigma_{\phi^2}\}+\tilde{U}_{II}(I)\{\sigma_{y_2,\phi}+h(e)\sigma_{\phi^2}\}] = 0. \tag{A21}$$

First note that for the maximum to exist, it must be the case that $W^p_{e,e} < 0$. Differentiating (A21) *w.r.t.* $\sigma^2_{y_2}$ and σ^2_ϕ, I have

$$\frac{de}{d\sigma^2_{y_2}} = -\frac{1}{2}\frac{D^k\beta h_e(e)}{1+B}\frac{\tilde{U}_{III}(I)}{W^p_{e,e}} > 0; \tag{A22}$$

$$\frac{de}{d\sigma^2_\phi} =$$

$$-\frac{D^k\beta h(e)h_e(e)}{(1+B)W^p_{e,e}}[\frac{1}{2}\tilde{U}_{III}(I)h(e)+\tilde{U}_{II}(I)];\ \text{if } \sigma_{y_2,\phi} = 0; \tag{A23}$$

$$-\frac{\beta(h(e)+1)h_e(e)}{(1+B)W^p_{e,e}}[\frac{1}{2}\tilde{U}_{III}(I)(h(e)+1)+\tilde{U}_{II}(I)(h(e)+1)];\ \text{if } \sigma_{y_2,\phi} = \sigma^2_\phi; \tag{A24}$$

$$-\frac{\beta(h(e)-1)h_e(e)}{(1+B)W^p_{e,e}}[\frac{1}{2}\tilde{U}_{III}(I)(h(e)-1)+\tilde{U}_{II}(I)(h(e)-1)];\ \text{if } \sigma_{y_2,\phi} = -\sigma^2_\phi. \tag{A25}$$

Proposition 3 follows from $(A22 - A25)$.

Proof of Proposition 4

First suppose that $s > 0$. Then differentiating (3.18) and (3.19) $w.r.t.$ g_1 and g_2 and evaluating the expression at $h_e(e) = R$, I have

$$W^p_{e,g_1} = 0 \ \& \ W^p_{s,g_1} = -U_{cc}(c^p_1) > 0. \tag{$A26$}$$

$$W^p_{e,g_2} = \frac{W^p_{e,s}}{R} > 0 \ \& \ W^p_{s,g_2} = \frac{W^p_{s,s} - U_{cc}(c^p_1)}{R}. \tag{$A27$}$$

Using Crammer's rule and the expressions above

$$\frac{de}{dg_1} = -U_{cc}(c^p_1)\frac{W^p_{e,s}}{H} > 0 \ \& \ \frac{de}{dg_2} = -U_{cc}(c^p_1)\frac{W^p_{e,s}}{HR} > 0. \tag{$A28$}$$

Suppose now that the parent is borrowing-constrained, $s = 0$. Then differentiating (3.6), I have

$$W^p_{e,g_1} = -U_{cc}(c^p_1) > 0 \ \& \tag{$A29$}$$

$$W^p_{e,g_2} = \frac{D^k\beta h_e(e)}{1+B}[\tilde{U}_{III}(I)\{\sigma_{y_2,\phi}+h(e)\sigma^2_\phi\}+\tilde{U}_{II}(I)+\frac{\tilde{U}_{IIII}(I)}{2}\{2h(e)\sigma_{y_2,\phi}+h^2(e)\sigma^2_\phi+\sigma^2_{y_2}\}]. \tag{$A30$}$$

As is clear that the first term in $(A30)$ is positive, but the last two terms are negative. Thus, W^p_{e,g_2} can be positive or negative.

$$\frac{de}{dg_1} = \frac{de}{dy_1} = \frac{U_{cc}(c^p_1)}{W^p_{e,e}} > 0. \tag{$A31$}$$

$$\frac{de}{dg_2} = -\frac{W^p_{e,g_2}}{W^p_{e,e}}. \tag{$A32$}$$

Proposition 4(I) follows from $(A28)$, $(A31)$, $(A32)$ and Cramer's rule.

First suppose that $s > 0$. The optimal choice of e is given by

$$W^p_e = (\xi - 1)U_c(c^p_1) + \frac{D^k\beta h_e(e)}{1+B}E\tilde{U}_I(I)\phi = 0. \tag{$A33$}$$

The optimal choice of s continues to be given by (3.15). Combining $(A33)$ and (3.15), I have

$$[h_e(e) + (\xi - 1)R]E\tilde{U}_I(I) = -h_e(e)Cov(\tilde{U}_I(I),\ \phi). \qquad (A34)$$

Then taking second-order Taylor approximation of the RHS (A35)and differentiating (A34) and (3.19) *w.r.t.* ξ and evaluating expression at $[h_e(e) + (\xi - 1)R] = 0$, I have

$$W^p_{s,\xi} = -U_{cc}(c^p_1)e > 0. \qquad (A35)$$

$$W^p_{e,\xi} = R[\tilde{U}_I(I) + \frac{\tilde{U}_{III}(I)}{2}\{\sigma^2_{y_2} + 2h(e)\sigma_{y_2,\phi} + h^2(e)\sigma_{\phi^2}\}] > 0. \qquad (A36)$$

Now suppose that $s = 0$. Then in this case, one can show that

$$\frac{de}{d\xi} = -\frac{U_c(c^p_1) + U_{cc}(c^p_1)(\xi - 1)e}{W^p_{e,e}} > 0. \qquad (A37)$$

Part II follows from $(A36)$ and $(A37)$ and Cramer's rule.

Proof of Proposition 5

First consider the case in which $s > 0$. The optimal choices are characterized by

$$W^p_e \approx (h_e(e) - R)[U_c(c^p_2) + \frac{1}{2}U_{ccc}(c^p_2)\sigma^2_{y_2}] + \delta U_{cc}(c^k)h(e)h_e(e)\sigma^2_\phi = 0; \quad (A38)$$

$$W^p_\tau \approx [U_c(c^p_2) + \frac{1}{2}U_{ccc}(c^p_2)\sigma^2_{y_2}] - \delta h^2(e)[U_c(c^k) + \frac{1}{2}U_{ccc}(c^k)\sigma^2_\phi] = 0 \ \& \quad (A39)$$

$$W^p_s \approx -U_c(c^p_1) + \beta R[U_c(c^p_2) + \frac{1}{2}U_{ccc}(c^p_2)\sigma^2_{y_2}] = 0. \qquad (A40)$$

Differentiating (A38-A40), I have

$$W^p_{e,e} = h_{ee}(e)[U_c(c^p_2) + \frac{1}{2}U_{ccc}(c^p_2)\sigma^2_{y_2}] +$$

$$\delta[U_{ccc}(c^k)h(e) + U_{cc}(c^k)]h_e(e)\sigma_\phi^2; \quad (A41)$$

$$W^p_{e,\tau} = -\delta U_{ccc}(c^k)h(e)h_e(e)\sigma_\phi^2 < 0; \quad (A42)$$

$$W^p_{e,s} = 0; \quad (A43)$$

$$W^p_{s,\tau} = \beta R[U_{cc}(c_2^p) + \frac{1}{2}U_{cccc}(c_2^p)\sigma_{y_2}^2] < 0; \quad (A44)$$

$$W^p_{s,s} = U_{cc}(c_1^p) + RW^p_{s\tau} < 0; \quad (A45)$$

$$W^p_{\tau,\tau} = \frac{W^p_{s\tau}}{\beta R} + \delta[U_{cc}(c^k) + \frac{1}{2}U_{cccc}(c^k)\sigma_\phi^2]h^2(e) < 0; \quad (A46)$$

$$W^p_{e,\sigma^2_{y_2}} = 0;\ W^p_{\tau,\sigma^2_{y_2}} = \frac{1}{2}U_{ccc}(c_2^p) > 0 \ \&\ W^p_{s,\sigma^2_{y_2}} = \frac{\beta R}{2}U_{ccc}(c_2^p) > 0 \ \& \quad (A47)$$

$$W^p_{e,\sigma^2_\phi} = \delta U_{cc}(c^k)h(e) < 0\ ;\ W^p_{\tau,\sigma^2_\phi} = -\frac{\delta}{2}U_{ccc}(c^k)h^2(e) < 0 \ \&\ W^p_{s,\sigma^2_\phi} = 0. \quad (A48)$$

For maximization, $|H| > 0$. Since $W^p_{e,s} = 0$, $|H| = W^p_{e,e}(W^p_{\tau,\tau}W^p_{s,s} - W^{p2}_{e,\tau}) - W^p_{s,s}W^{p2}_{e,\tau}$. Note that $W^p_{\tau,\tau}W^p_{s,s} - W^{p2}_{e,\tau} > 0$. Then, for maximization it is sufficient that $W^p_{ee} < 0$.

Now,

$$|H|\frac{de}{\sigma^2_{y_2}} = -W^p_{\tau,\sigma^2_{y_2}}W^p_{e,\tau}W^p_{s,s} < 0 \ \& \quad (A49)$$

$$|H|\frac{de}{\sigma^2_\phi} = W^p_{e,\sigma^2_\phi}(W^p_{\tau,\tau}W^p_{s,s} - W^{p2}_{e,\tau}) - W^p_{\tau,\sigma^2_\phi}W^p_{e,\tau}W^p_{s,s}. \quad (A50)$$

Proposition 5(I) follows from (A41)-(A50).

In the case $s = 0$. The optimal choices are characterized by (A39) and

$$W^p_e \approx -U_c(c_1^p) + \beta\delta h_e(e)[U_c(c^k) + \frac{1}{2}U_{ccc}(c^k)\sigma_\phi^2 + U_{cc}(c^k)h(e)\sigma_\phi^2] = 0. \quad (A51)$$

$$W^p_{e,e} = -U_{cc}(c^p_1)\frac{dl}{de} + \beta\delta h_{ee}(e)[U_c(c^k) + \frac{1}{2}U_{ccc}(c^k)\sigma^2_\phi + U_{cc}(c^k)h(e)\sigma^2_\phi]$$

$$+\beta\delta h^2_e(e)[(1+\sigma^2_\phi)U_{cc}(c^k) + \{\frac{1}{2}U_{cccc}(c^k) + U_{ccc}(c^k)h(e)\}\sigma^2_\phi]. \qquad (A52)$$

$$W^p_{e,\tau} = -\delta[U_{cc}(c^k) + \frac{1}{2}U_{cccc}(c^p_2)\sigma^2_\phi]h_e(e) > 0; \qquad (A53)$$

$$W^p_{\tau,\tau} = [U_{cc}(c^p_2) + \frac{1}{2}U_{cccc}(c^p_2)h(e)\sigma^2_{y_2}] + \delta[U_{cc}(c^k) + \frac{1}{2}U_{cccc}(c^k)\sigma^2_\phi] < 0; \quad (A54)$$

Since $W^p_{\tau,\tau} < 0$, for maximization it must be the case that $W^p_{e,e} < 0$.

$$W^p_{e,\sigma^2_{y_2}} = 0; \; W^p_{\tau,\sigma^2_{y_2}} = \frac{1}{2}U_{ccc}(c^p_2) > 0 \; \& \qquad (A55)$$

$$W^p_{e,\sigma^2_\phi} = \beta\delta h_e(e)[\frac{1}{2}U_{ccc}(c^k) + U_{cc}(c^k)h(e)] \; \& \; W^p_{\tau,\sigma^2_\phi} = -\frac{\delta}{2}U_{ccc}(c^k) < 0. \quad (A56)$$

Proposition 5(II) follows from (A52)-A(56) and Cramer's rule.

Income-Assistance

Suppose that $s > 0$. Then,

$$w_{eg_1} = w_{\tau g_1} = 0, \; w_{sg_1} = -U_{cc}(c^p_1) > 0. \qquad (A57)$$

$$W^P_{eg_2} = 0, \; W^P_{\tau g_2} = U_{cc}(c^p_2) + \frac{1}{2}U_{cccc}(c^p_2)\sigma^2_{y_2} < 0, \; W^P_{sg_2} = \beta R w_{\tau g_2} < 0. \quad (A58)$$

The effects of g_1 and g_2 follow from (A57), (A58) and Cramer's rule.

In the case of conditional transfer, if $s > 0$, the optimal choice of e is given by

$$W^p_e \approx (h_e(e) + (\xi - 1)R)[U_c(c^p_2) + \frac{1}{2}U_{ccc}(c^p_2)\sigma^2_{y_2}] + \delta U_{cc}(c^k)h(e)h_e(e)\sigma^2_\phi = 0; \qquad (A59)$$

Then,

$$W^P_{e\xi} = R[U_c(c^p_2) + \frac{1}{2}U_{ccc}(c^p_2)\sigma^2_{y_2}] > 0$$

$$W^P_{\tau\xi} = 0, \; W^P_{sg^l_1} = -U_{cc}(c^p_1)e > 0. \qquad (A60)$$

In the case $s = 0$, the optimal choice of e is given by

$$W^p_e \approx (\xi - 1)U_c(c^p_1) + \beta\delta h_e(e)[U_c(c^k) + \frac{1}{2}U_{ccc}(c^k)\sigma^2_\phi + U_{cc}(c^k)h(e)\sigma^2_\phi] = 0. \qquad (A61)$$

$$W^P_{e\xi} = U_c(c^p_1) + (\xi - 1)U_{cc}(c^p_2)e > 0 \;\&\; W^P_{\tau\xi} = 0. \qquad (A62)$$

Using (A60), (A62), and Cramer's rule one can derive effect of ξ.

References

[1] Applebaum, E. and E. Katz (1991), "The Demand for Children in the Absence of Capital and Risk Markets: A Portfolio Approach", *Oxford Economic Papers*, 43, 292-304.

[2] Baland, J. and J. Robinson (2000), "Is Child Labor Inefficient?", *Journal of Political Economy*, 108, 663-79.

[3] Becker, G. (1991), "A Treatise on the Family", *Harvard University Press, Enlarged Edition*, London.

[4] Boldrin, M. and Montes, A. (2005), "The Intergeneratinal State Education and Pensions", *Review of Economic Studies*, 72, 651-664.

[5] Checchia, D. and Cecilia Garca-Penalosa (2004) "Risk and the Distribution of Human Capital", *Economic Letters*, 82(1), 53-61.

[6] Cox, D. (1987), "Motives for Private Income Transfers", *Journal of Political Economy*, 95(3), 508-546.

[7] Cox, D. and M. Fafchamps (2008), "Chapter 58: Extended Family and Kinship Networks: Economic Insights and Evolutionary Directions", in T.P. Schultz and A. S. John (ed.) *Handbook of Develoment Economics, Volume 4*, Elsvier, 3711-3784.

[8] Dercon, S. (2005), *Insurance Against Poverty*, Oxford University Press, Oxford.

[9] Flug, K., A. Spilimbergo and W. Wachtenheim (1998), "Investment in Education: Do Economic Volatility and Credit Constraints Matter?", *Journal of Development Economics*, 55, 465-481.

[10] Foster, A. and Gehrke, E. (2017), "Consumption Risk and Human Capital Accumulation in India", NBER Working Paper No. 24041.

[11] Hartog, J. and L. Diaz-Serrano (2013), "Schooling as a Risky Investment: A Survey of Theory and Evidence", *Foundations and Trends in Microeconomics*, 9, 159-331.

[12] HDI (2013), "Human Development Report 2013 : Indonesia", available at *http://hdr.undp.org/sites/default/files/Country-Profiles/IDN.pdf* .

[13] Hogan, V. and I. Walker (2007), "Education Choice under Uncertainty: Implications for Public Policy", *Labour Economics*, 14, 894-912.

[14] IFC (2008), "The Household Survey and Monetary Policy in Indonesia", IFC Bulletin No. 26313, International Finance Corporation.

[15] Jacobs, B. (2007), "Real Opions and Human Capital Investment", *Labour Economics*, 14, 913-925.

[16] Kaufmann, K. (2014), "Understanding the Income Gradient in College Attendance in Mexico: The Role of Heterogeneity in Expected Returns, *Quantitative Economics*, 5, 583630.

[17] Kazianga, H. (2012), "Income Risk and Household Schooling Decisions in Burkina Faso", *World Development*, 40, 1647-62.

[18] Kodde, D. (1986), "Uncertainty and the Demand for Education", *The Review of Economics and Statistics*, 68, 460-467.

[19] Lee, Y., W. Parish, and R. Willis (1994), "Sons, Daughters, and Intergenerational Support in Taiwan", *American Journal of Sociology*, 99, 1010-1041.

[20] Leland, H. (1968), "Saving and Uncertainty: The Precuationary Demand for Money", *Quarterly Journal of Economics*, 2, 465-473.

[21] Levhari, D. and Y. Weiss (1974), "The Effect of Risk on the Investment in Human Capital", *American Economic Review*, 64, 950-963.

[22] Lillard, L. and R. Willis (1997), "Motives for Intergenerational Transfers: Evidence from Malaysia", *Demography*, 34(1), 115-134.

[23] Park, C. (2003), "Intra-Household Transfers between Relatives in Indonesia: Determinants and Motives", *Economic Development and Cultural Change* , 51, 929-944.

[24] Paroush, J. (1976), "The Risk Effect and Investment in Human Capital", *European Economic Review*, 8, 339-347.

[25] Peet, E., G. Fink, and W. Fawazi (2015), "Returns to Education in Developing Countries: Evidence from the Living Standards and Measurement Study Surveys", *Economics of Education Review* , 49, 69-90.

[26] Portner, C. (2009), "Gone With the Wind? Hurricane Risk, Fertility and Education", Mimeo University of Washington available at http://papers.economics.ubc.ca/legacypapers/portner.pdf.

[27] Pouliot, W. (2006), "Introducing Uncertainty into Baland and Robinsons Model of Child Labor", *Journal of Development Economics*, 79, 264-272.

[28] Rangazas, P. (1991), "Human Capital Investment in Wealth-Constrained Families with Two-Sided Altruism", *Economics Letters*, 35, 137-141.

[29] Rapoport, H. and F. Docquier (2006), "The Economics of Migrant's Remittances", in S. Kolm and J. Mercier (eds) *Handbook of the Economics of Giving, Altruism, and Reciprocity*, 1138-1195, North Holland.

[30] Raut, L. (1990),"Capital Accumulation, Income distribution and Endogenous Fertility in an Overlapping Generations General Equilibrium Model", *Journal of Development Economics*, 34, 123-150.

[31] Raut, L. and L. Tran (2005), "Parental Human Capital Investment and Old Age Transfers from Children: Is it a Loan Contract or Reciprocity for Indonesian Families?", *Journal of Development Economics*, 77, 389-414.

[32] Rustuccia, D. and C. Urrutia (2004), "Intergenerational Persistence of Earnings: The Role of Early and College Education", *American Economic Review*, 94, 1354-1378.

[33] Sandmo, A. (1970), "The Effect of Uncertainty on Saving Decisions", *Review of Economic Studies*, 37, 352-360.

[34] Singh, A. (2010), "Human Capital Risk in Life-Cycle Economies", *Journal of Monetary Economics*, 57, 729-738.

[35] Williams, J. (1979), "Uncertainty and the Accumulation of Human Capital over The Life-Cycle", *The Journal of Business* , 52(4), 521-548.

[36] World Bank (2016), "World Development Indicators", available at *http://data.worldbank.org/indicator.*