OLD-AGE INCOME SUPPORT, HUMAN CAPITAL INVESTMENT, AND EFFICIENCY: ROTTEN-KID THEOREM MEETS SAMARITAN’S DILEMMA

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Abstract

This paper studies the interaction between investment by parents in the human capital of their children and income support received by parents from them in their old age (reverse transfers). I find that reverse transfers lead to an inefficiently high level of human capital investment, when children choose reverse transfers non-cooperatively. The choice of human capital investment imposes a positive externality on parents. An increase in the human capital investment of a child not only increases transfers from him/her to parents, but also increases transfers from the other child. When children cooperate, this externality is internalized leading to efficient level of human capital investment. Old-age pension scheme financed by taxes on children can lead to efficient level of human capital investment by parents.

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1 Introduction

In recent decades, due to increasing longevity of individuals and rising old-age dependency ratio, providing a decent standard of living to elderly population has become an important policy issue in both developing and developed countries (ILO 2012). Traditionally, poorer elderly individuals have relied on support from their children, particularly in developing countries. For example, evidence shows that majority of elderly parents receive support from their children in many Asian and Latin American countries (Table 3, Becker et. al. 2012). Even in developed countries, nearly one-third of elderly parents receive support from their children.

There is a large literature which has examined motives of old-age income support provided by children (e.g. Lillard and Willis 1997, Raut and Tran 2005, Cox and Fafchamps 2008). This literature suggests that old-age income support provided by children is one of the most important motives for investment made by parents in the human capital of their children in developing countries. Empirical evidence also finds that there is a significant positive relationship between educational level of children and transfers made by them to their older parents (e.g. Lee et. al. 1994, Lillard and Willis 1997, Frankenberg et. al. 2002, Kim 2010, Lei et. al. 2012).

In this paper, I develop a model to analyze the interaction between old-age income support and human capital investment of children in a unitary household framework (Becker 1974), where parents have multiple (heterogeneous) children. The analysis shows that when parents have multiple children, old-age income support provided by children significantly alters the nature of interaction between parents and children. The equilibrium outcomes depend on whether (adult) children choose their support cooperatively or non-cooperatively.

In the model, there are two periods. A family consists of parents and two children. Both parents and children are altruistic. Parental utility depends not only on their own consumption, but also on the utility enjoyed by their
children. Similarly, the utility of children depends not only on their own consumption, but also on the utility enjoyed by their parents. Parents choose level of human capital investment for their children in the first period, which increases human capital (earnings) of children in the next period. Children can differ in terms of their human capital/earnings functions. They give transfers to parents (reverse transfer) in the second period.\textsuperscript{1} While choosing transfers children can either cooperate with each other or can choose their transfers non-cooperatively. In the main part of the paper, I assume that there is no uncertainty about their degree of altruism towards their parents. In the later part, I consider the case in which there is uncertainty about the altruism of children. In particular, they can default on the old-age income support.

In the certainty case, I find that when parents receive transfers from both children in the second period and children act non-cooperatively, reverse transfers lead to an inefficiently high level of human capital investment. The choice of human capital imposes a positive externality on parents. An increase in the human capital of one child not only increases transfers from him/her to parents in the second period, but also increases transfers from the other child. This positive externality induces parents to choose inefficiently high level of human capital investment. When children cooperate, the positive externality inherent in the choice of human capital investment is internalized. In this case, reverse transfers lead to efficient human capital investment by parents.

The result that old-age income support provided by children can lead to inefficiently high level of human capital investment (certainty case) is in contrast to the implications of the models with one child (e.g. Raut 1990, Rangazas 1991, Becker et. al. 2012).\textsuperscript{2} In these models, old-age income support

\textsuperscript{1}Altruism is considered to be one of the most important motives for inter-generational transfers in developing countries (e.g. Lee et. al. 1994, Lillard and Willis 1997, Franken-berg et. al. 2002, Raut and Tran 2005). See Cox and Fafchamps (2008) for a thorough review of various motives for inter-generational transfers and evidence.

\textsuperscript{2}These models allow for multiple children, but they behave as if there is one child or choose their support co-operatively.
support by child leads to efficient level of capital investment (as in the cooperative case). This result is also in contrast to models without old-age income support (e.g. Davis and Zhang 1995, Kumar 2013, Becker et. al. 2012). In these models, parents choose inefficiently low level of human capital investment, when they cannot give bequests to their children.

In the model, the positive externality associated with the human capital investment arises due to the family structure and the bilateral altruism between parents and children. When children act non-cooperatively, reverse transfers create the problem of common pool. Since, a child knows that the other child will increase the transfer if the parental utility falls, he/she does not fully compensates parents for the loss in the parental utility from increased human capital investment. On the other hand, parents recognizing that they are going to receive transfers from both children, choose inefficiently high level of human capital investment.

These results relate to both the Rotten-Kid theorem (Becker 1974) and the Samaritan’s dilemma (Buchanan 1975, Lindbeck and Weibull 1988). When children do not cooperate, the resulting allocations suffer from both the Rotten-Kid type of inefficiency and the Samaritan’s dilemma type of inefficiency. As discussed earlier, parents in order to increase transfers from children in the second period choose human capital investment which reduces the net family income (the Rotten-Kid type of inefficiency). In addition, transfers from children incentivize parents to under-save (or over-consume) in the first period relative to the efficient level for a given income (the Samaritan’s dilemma type of inefficiency). Essentially, parents exploit the concerns of altruistic children in order to increase their utility. Cooperation among children solves the Rotten-Kid type of inefficiency, but the Samaritan’s dilemma type of inefficiency still remains.

The analysis also shows that when older parents receive income support

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3 In the model, we have a case of rotten parents who reduce net family income in order to maximize their welfare.

4 See Bruce and Waldman (1990) for the analysis of these two types of inefficiencies.
from their children, human capital investment (though inefficient) becomes independent of parental income. The reason is that a change in parental income affects human capital investment only to the extent it affects the relative consumption of parents and children. However, when parental income changes, parents adjust their savings and children adjust their transfers leaving the relative consumption and thus human capital investment unaffected.

In the case, where children can default on old-age income support, I find that the human capital investment is inefficient in both cooperative and non-cooperative case. In particular, in the cooperative case, the human capital investment can be inefficiently low. The reason is that the risk of default reduces the expected return from the human capital investment. This result is similar to Chakrabarti et. al (1993), who examine the effect of uncertainty about reverse altruism on the human capital investment in a model with one child.

The analysis raises important issue about when children are likely to cooperate. Children will cooperate only when each child is better-off compared to non-cooperation. In general, for a given level of human capital investment and savings of parents, cooperation leads to larger transfer from children to parents compared to non-cooperation. Thus, the second period consumption of parents will be higher and the total consumption of children will be lower under cooperation relative to non-cooperation, for a given level of human capital investment and savings. Interestingly, the analysis finds a higher degree of reverse altruism and the greater degree of inequality aversion by children may not result in cooperation. The reason is that a greater degree of reverse altruism and inequality aversion increases transfers from children both when they cooperate and when they do not. Thus, the relative gap between parental consumption and the consumption of children will be lower in the non-cooperative case. A further redistribution of consumption between parents and children which cooperation entails may not make children better-off compared to non-cooperation.
The analysis shows that there can be inefficiency in both human capital investment and savings. In this context, I analyze the effects of old-age public pension scheme financed by taxes on (adult) children. The analysis shows that a public pension scheme can induce parents to choose efficient levels of both human capital investment and savings, when the government is a Stackleberg leader and it optimally chooses taxes and transfers. Such pension scheme can internalize both types of inefficiencies.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 analyzes the case in which children receive transfers (bequests) from parents in the second period. Section 4 analyzes the reverse case in which parents receive transfers from their children in the second period. Section 5 analyzes the case in which there is uncertainty about the altruism of children. Section 6 discusses the policy implications. Section 7 concludes the paper.

2 Model

Consider a two-period overlapping generations model. There are large number of households. A household consists of parents and two children: \( m \) and \( n \). Parents and children live in both periods, \( t = 1, 2 \). Parents discount future at the rate normalized to one.

Let \( A_t \) be the earnings of parents in period \( t = 1, 2 \). The earnings of (adult) children in the second period depends on the human capital investment, \( e^i \), \( i = m, n \), made by parents in the first period. Assume that the human capital/earnings function of a child, \( h_i(e^i) \) for \( i = m, n \), is a strictly increasing and concave function of the human capital investment, \( e^i \). Assume that the earnings functions are heterogeneous \( h^m(e^m) \neq h^n(e^n) \). Heterogeneity in the earnings function can arise due to differing abilities or other factors such as gender bias. For concreteness, assume that child \( m \) has a superior
earnings function. For any $e^m = e^n$, $h^m(e^m) > h^n(e^n)$ and $h^m(e^m) > h^n(e^n)$.\(^5\) Thus, child $m$ has higher total as well as marginal return on the human capital investment.

Both parents and children are altruistic. The parental utility depends not only on their own consumption but also on the consumption of children. Similarly, utility of children depends not only on their own consumption but also consumption of parents. For notational simplicity, I assume that children consume only in the second period.\(^6\) The parental utility function is given by

$$W^p = U(c^p_1) + U(c^p_2) + \delta(U(c^m) + U(c^n)) \quad (2.1)$$

where function $U()$ is the period utility function and $c^m$ and $c^n$ are consumption of child $m$ and $n$. $U()$ is a twice continuously differentiable, strictly increasing, and concave function of consumption. $c^p_t$ is the consumption by parents in period $t = 1, 2$. Parameter $0 < \delta < 1$ measures the degree of parental altruism.

The utility function of child $i = m, n$ is given by:

$$W^i = U(c^i) + \lambda U(c^p_2) \forall i = m, n. \quad (2.2)$$

Parameter $0 < \lambda < 1$ captures the degree of altruism by children towards their parents.

Parents choose their savings, $k$, and the human capital investment, $e^i \geq 0$, for children $i = m, n$ in the first period and their own consumption in both periods. I normalize the rate of return on savings to one. Parents can give bequests, $b^i \geq 0$ for $i = m, n$, to their children in the second period. Children can also give transfers, $\tau^i \geq 0$; $i = m, n$, to their parents in the second

\(^5\)Throughout the paper, for any function $F(x)$, $F_x(x)$ and $F_{xx}(x)$ denote first and second derivatives respectively.

\(^6\)One can easily introduce consumption by children in the first period without affecting any of the results.
period. The non-negativity restriction on bequests and transfers imply that parents or children cannot leave debt to each other or coerce the other into transferring resources.

The budget constraints faced by parents and children are

\[ c_1^p + k + e^m + e^n = A_1; \] (2.3)

\[ c_2^p + b^m + b^m = A_2 + k + \tau^m + \tau^n \] \& (2.4)

\[ c^i = b^i - \tau^i + h^i(e^i) \forall i = m, n. \] (2.5)

Since \( \tau^i \) is similar to negative transfers from parents to children, what is important for the analysis and the decision making by parents and children are the net transfers between parents and children. If one interprets \( b^i \) \& \( \tau^i \) as net transfers, then when \( b^i > 0, \tau^i = 0 \). Similarly, when \( \tau^i > 0, b^i = 0 \).

3 Parents to Children Transfer

As a benchmark and for future reference, I briefly analyze the case in which there is a transfer from parents to children, \( b^i > 0, \tau^i = 0 \), for \( i = m, n \) (see Davis and Zhang 1995, Kumar 2013 for a detailed analysis). The parental optimization problem is

\[
\max_{c_1^p, c_2^p, e^m, e^n, b^m, \tau^n, k} \sum_{t=1}^{2} U(c_t^p) + \sum_{i=m, n} \delta^i U(c^i)
\]

subject to the budget constraints 2.3-2.5. As the focus of the paper is on the effects of transfers between parents and children on the human capital investment, to begin with I assume that parents can save or borrow the desired amount and the parameters of the model are such that the optimal human capital investment, \( e^m \) \& \( e^n > 0 \). The implications of corner solutions
for the human capital investment and savings are discussed in section 6.2.

The first order conditions associated with the optimal choices are

\[ e^i : U_c(c'_p) = \delta U_c(c^i)h_e(e^i), \text{ for } i = m, n; \]  

(3.1)

\[ b^i : U_c(c'_2) = \delta U_c(c^i), \text{ if } b^i > 0, \text{ for } i = m, n; \]  

(3.2)

\[ b^i : U_c(c'_2) \geq \delta U_c(c^i), \text{ if } b^i = 0, \text{ for } i = m, n \& \]  

(3.3)

\[ k : U_c(c'_1) = U_c(c'_2). \]  

(3.4)

(3.1) equates the marginal cost of human capital investment for child \( i \) with its marginal benefit. An additional unit of human capital investment in child \( i \) reduces the utility of parents by \( U_c(c'_p) \) in the first period. At the same time, it increases the utility of parents by \( \delta U_c(c^i)h_e(e^i) \) in the second period.

Other first-order conditions can be interpreted in a similar fashion. (3.2) equates the marginal cost of giving bequest to the \( i \)th child with its marginal benefit. An additional unit of bequest reduces the utility of parents by \( U_c(c'_2) \) in the second period. At the same time, it increases the utility of parents by \( \delta U_c(c^i)h_e(e^i) \). If the marginal cost of bequest to the \( i \)th child exceeds the marginal benefit, then parents will not give any bequest to the \( i \)th child. (3.3) characterizes this condition. This case can arise when parental income is low, income of children is high, and/or parents are less altruistic towards their children. Finally, (3.4) equates the marginal cost of savings to its marginal benefit.

Using the optimal strategies of parents, one can easily show that (see Davis and Zhang 1995, Kumar 2013):

**Proposition 1:**
(i) The human capital investment is at efficient level for a child who receives positive bequest \((b^i > 0)\), \(h^e_i(e^m) = 1\) for all \(i = m, n\). The human capital investment is inefficiently low for a child who does not receive bequest \((b^i = 0)\), \(h^e_i(e^i) > 1\) for all \(i = m, n\).

(ii) If the bequest constraint binds for only one child, it must bind for the child with a superior earnings function \((m)\), \(b^m = 0 \& b^n > 0\).

(iii) When parents cannot give bequest to both children, the human capital investment for child with a superior earnings function \((m)\) can be higher or lower than the other child \((n)\), \(e^m \gtrless e^n\).

The first part of proposition shows that the efficient level of human capital investment is characterized by equality between the marginal rate of return from human capital and the rate of return on financial savings. In addition, bequest plays a crucial role in achieving the efficient level of human capital investment. The reason is that parents have two instruments to increase consumption of children in the second period: human capital investment and giving bequest which is financed by savings. When parents are using both instruments, they must be indifferent between the two. On the other hand, when they are not giving bequest, it must be the case that the return from human capital investment is higher. Also, the proposition shows that when both children receive bequests \((b^m, b^n > 0)\), then \(h^e_m(e^m) = h^e_n(e^n)\). Parents equate the marginal rate of return from human capital investment across two children.

The second and the third parts of the proposition follow from the fact that parents would like to provide same level of utility to both children. Since, child \(m\) has higher earnings for a given level of human capital investment, parents provide bequest to child \(n\), when they can give bequest to only one child. When they cannot give bequest to both children, they may invest more or less in the human capital of child \(n\) compared to child \(m\).
4 Children to Parents Transfer

Now I consider the case in which parents may not leave bequests to their children, but receive transfers from them in the second period. As discussed above, when \( i \)th child does not receive bequest \((b^i = 0)\), \( U_c(c^i_{p2}) \geq \delta U_c(c^i) \).

4.1 Transfer to Parents From Both Children: Non-Cooperative Case

Consider the case in which parents do not leave bequest to either of the children \((b^i = 0 \text{ for } i = m, n)\), but may receive transfers from both children \((\tau^i \geq 0 \text{ for } i = m, n)\) in the second period. This may arise if the incomes of children are relatively higher than of parents in the second period and/or the degree of altruism by children towards their parents is high.

To begin with, assume that children choose their transfers \textit{non-cooperatively}. This problem can be modeled as a two-period transfer game between parents and children. For time-consistency one needs to solve this problem recursively starting in the second period.

The second period problem of \( i \)th child is to

\[
\max_{\tau^i} U(c^i) + \lambda U(c^i_{p2})
\]

subject to budget constraints (2.4)-(2.5), given the transfers from the \( j \)th child, \( \tau^j \). The first order condition are:

\[
U_c(h^i(c^i) - \tau^i) = \lambda U_c(A_2 + k + \tau^i + \tau^j), \text{ if } \tau^i > 0, \forall i = m, n. \quad (4.1)
\]

\[
U_c(h^i(c^i) - \tau^i) \geq \lambda U_c(A_2 + k + \tau^i + \tau^j) \text{ if } \tau^i = 0, \forall i = m, n. \quad (4.2)
\]
(4.1) equates the marginal cost of giving transfer to parents by the \(i\)th child to its marginal benefit. If the marginal cost exceeds the marginal benefit then \(\tau^i = 0\). (4.2) characterizes this condition. This can occur if the \(i\)th child’s income or its degree of altruism towards parents is low and parental income and savings and the transfer from the \(j\)th child are high.

(4.1) and (4.2) show that parents are more likely to receive transfer from the child with superior earnings function (\(m\)) than the other child for a given level of human capital investment and their own income. Reason is that the earnings of child \(m\) is higher than child \(n\) for a given level of human capital investment. This reduces the perceived marginal cost of transfers to parents from child \(m\) relative to transfers from child \(n\).

From (4.1) it follows that when parents receive transfers from both children, \(\tau^m, \tau^n > 0\), then

\[
U_c(c^m) = U_c(c^n). \tag{4.3}
\]

(4.3) shows that when both children give transfers, the marginal utilities of their consumption as perceived by them are equalized. This also implies that children choose transfers such that both of them have same level of consumption, \(c^m = c^n\).

Differentiating, (4.1) with respect to \(e^i\) and \(k\) for \(i = m, n\), I have

\[
\frac{d\tau^i}{de^i} = \frac{U_{cc}(e^i)}{U_{cc}(e^i) + \lambda U_{cc}(c_p^2)} > 0, \quad \forall i \neq j = m, n; \tag{4.4}
\]

\[
\frac{d\tau^i}{de^j} = \frac{U_{cc}(c_j)h^i_j(c_j)}{U_{cc}(c_j) + \lambda U_{cc}(c_p^2)} > 0, \quad \forall i \neq j = m, n \& \tag{4.5}
\]

\[
\frac{d\tau^i}{dk} = -\frac{\lambda U_{cc}(c_p^2)}{U_{cc}(e^i) + \lambda U_{cc}(c_p^2)} < 0, \quad \forall i \neq j = m, n. \tag{4.6}
\]

(4.4) and (4.6) show that a higher level of human capital investment increases transfers by children to parents, but a higher parental saving re-
duces the transfers by children. A higher amount of human capital investment increases the income and the consumption of children in the second period, which induces children to increase their transfers to parents. A rise in parental savings, on the other hand, increases the consumption of parents in the second period, which reduces the need for transfers from both children.

These predicted relationships among transfers to parents, human capital investment and incomes of parents and children are supported by empirical evidence. There is a large empirical literature which shows a significant positive relationship between education level of children and transfers by them to parents in many developing countries (e.g. Lee et. al. 1994 for Taiwan, Lillard and Willis 1997 for Malaysia, Frankenberg et. al. 2002 for Indonesia, Kim 2010 for South Korea, Lei et. al. 2012 for China). Empirical evidence also suggests a significant positive association between income of children and transfers to parents and a significant negative relationship between income of parents and transfers to parents (Lee et. al. 1994, Lillard and Willis 1997, Frankenberg et. al. 2002, Kim 2010, Lei et. al. 2012).

(4.5) shows that an increase in human capital investment of one child increases transfers by the other child as well i.e. a child subsidizes human capital investment of its sibling. The reason for this cross-effect is the family structure and the assumption that children care about parents. For a given transfer from the $i$th child, $\tau_i$, an increase in the human capital investment of $i$th child by parents reduces parental utility. Since, $j$th type of child cares about parents, it induces him/her to increase transfer to parents, $\tau_j$. As shown below, this cross-effect generates a common pool problem between the two types of children and imposes a positive externality on the choice of human capital investment by parents.

There is substantial qualitative and quantitative evidence that older children help to subsidize their siblings’ education. Frankenberg et. al. (2002) find that adult children increase their transfers to parents when their siblings are in school in Indonesia. There is also evidence of older children working.
in order to support their younger siblings and finance their schooling (Edmonds 2006, Manacorda 2006; see Parish and Willis 1993 for discussion of this literature related to South and East Asia). Qualitative research documents this phenomenon for many regions (e.g. Java (Wolf 1991) and Inner Mongolia (Ping and Shaohua 2009)), where young women have left home to work and send money to help their younger siblings stay in school. Studies done on effects of remittances from migrant workers also find a significant positive effect of remittances on education and health of children in recipient households (see Rapoport and Ramirez 2006 for a thorough review of evidence).

Turning to the parental problem, they take into account the effects of their choices on transfers by children to them. The parental problem in the first period is to

$$\max_{e^m, e^n, k} \sum_{t=1}^{2} U(c^p_t) + \sum_{i=m,n} \delta U(c^i)$$

subject to (2.5)-(2.7) and (4.1).

The first order conditions are

$$e^i : U_c(c^1_i) = \delta U_c(c^j) h^1_i(e^i) +$$

$$\left( U_c(c^2) - \delta U_c(c^i) \right) \frac{d\tau^i}{de^i} + \left( U_c(c^2) - \delta U_c(c^j) \right) \frac{d\tau^j}{de^j}; \forall i \neq j = m, n \quad (4.7)$$

$$k : U_c(c^m) = U_c(c^2) + (U_c(c^2) - \delta U_c(c^m)) \frac{d\tau^m}{dk} + (U_c(c^2) - \delta U_c(c^n)) \frac{d\tau^n}{dk} \quad (4.8)$$

(4.7) and (4.8) show that reverse transfer increases the marginal benefit of human capital investment and reduces the marginal benefit from savings
relative to the case when parents give bequests to their children.

**Proposition 2:** (*Reverse transfers from both children to parents: Non-Cooperative Case*) The human capital investment by parents is inefficiently high, $h_i^*(e^i) < 1$ for both $i = m, n$.

Proof: See Appendix

The reason for the inefficiently high levels of human capital investment is that its choice imposes a positive externality on parents. A higher level of human capital investment in a child not only increases transfers from that child but also from the other child. In order to increase their second period consumption and transfers from children, parents choose inefficiently high levels of human capital investment in the first period.

Essentially, the family structure and the bilateral altruism create the problem of common pool. Since, one type of child knows that other type of child will increase the transfer if parental utility falls, he/she does not fully compensates parents for the loss in parental utility from increased human capital investment. On the other hand, parents recognizing that they are going to receive transfers from both types of children choose inefficiently high level of human capital investment.

Transfers from children also lead to inefficient choice of savings by parents. Parents in order to increase transfers from children under-save (or over-consume) in the first period relative to the efficient level for a given income. The efficient level of savings equates the marginal utility of consumption across two-periods. As is evident from (4.8), $U_c(x_1^p) < U_c(x_2^p)$. For a given income, by rearranging the consumption pattern in favor of second period consumption, parents can be made better-off. This result relates to the Samaritan dilemma (Buchanan 1975, Lindbeck and Weibull 1988) which pertains to strategic interactions among an altruistic donor and a recipient. The donor cares about the welfare of the recipient. The recipient exploits the
concern of the altruistic donor resulting in inefficiently high level of transfer from the donor to the recipient.

Next, I summarize the pattern of human capital investment and reverse transfers.

**Proposition 3:** *(Reverse transfers from both children to parents: Non-Cooperative Case)* Parents choose human capital investment for children such that the marginal rates of return from human capital investment across children are equalized, \( h^m_e(e^m) = h^n_e(e^n) \). In addition, the child with superior earnings function has a larger human capital investment and transfers more to parents, \( e^m > e^n \) and \( \tau^m > \tau^n \).

Proof: See appendix.

As discussed earlier, when both children give transfers, the marginal utilities of their consumption as perceived by them are equalized. Since \( e^m = e^n \), transfers from children also equalize the marginal utilities of consumption of children as perceived by parents (\( \delta U_c(c^i) \)). This induces parents to choose human capital investment such that return from human capital investment including transfers is equalized across children. This occurs when the marginal return from human capital investment is same across children.

The analysis shows that the outcomes are very different when children provide old-age income support compared to the situation in which they do not provide support to parents. Firstly, old-age income support can lead to inefficiently high rather than inefficiently low level of human capital investment. Secondly, when children do not provide old-age income support and human capital investment is at inefficient level, a marginal increase in parental income or marginal redistribution of wealth from children to parents raises the human capital investment. However, as shown below, when parents receive old-age income support an autonomous (marginal) increase in parental income or redistribution of wealth from children to parents does not
affect the human capital investment by parents for a wide variety of utility functions.

**Proposition 4:** (*Reverse transfers from both children to parents: Non-Cooperative Case*) If the period utility function of parents and children is either a quadratic function or a power function, or an exponential function $U(c_i) = \alpha c_i - \beta c_i^2$ or $\alpha c_i^\beta$ or $-\exp(-\alpha c_i)$, then the human capital investment by parents is independent of parental income, $A_t$ for $t = 1, 2$.

Proof: See Appendix.

In the model, changes in parental income affect human capital investment only to the extent that it affects the relative consumption of parents and children. However, when the income of parents changes, parents adjust their savings and children adjust their transfers leaving the relative consumption unaffected. The result is that a change in parental income does not affect the human capital investment. This also implies that any anticipated change in the autonomous income of children (independent of changes in human capital function) does not affect the parental choice of human capital investment. In particular, an increase in the autonomous income of $ith$ child will have a positive effect on its transfer, but will have a negative effect on the transfer from the other child. The analysis suggests that public policies involving (lump-sum) marginal redistribution of wealth between parents and children may not affect the human capital investment.

So far, I have analyzed the case in which parents receive transfer from both children. Given the heterogeneity in the earnings function, however they may not receive transfer from both children. It is possible that parents receive transfers only from the child with a superior earnings function ($m$). At the same time, if the income of the child with inferior earnings function is

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7 Let $c^i = h^i(c^i) - \tau^i + I^i$, where $I^i$ for $i = m, n$ be the autonomous income of $ith$ child. Then, any change in $I^i$ will not affect human capital investment.
very low, then parents may not receive transfers from him/her or may even leave bequest to that child.

**Proposition 5:** In the case parents leave bequest to the child with inferior earnings function, $b^n > 0$, but receive transfers from the child with superior earnings function in the second period, $\tau^m > 0$, the human capital investment for both children is characterized by $h^m_e(e^m) = h^n_e(e^n) = 1$ and the human capital investment is at efficient level.

Proof: See Appendix

This case is likely to emerge, when there is relatively large earnings differential between siblings. In this case, parents serve as a mechanism by which there is transfer between siblings.

The reason that the reverse transfer from child $m$ leads to efficient level of human capital investment in this case is very simple. Since, parents receive transfers from only one child, the choice of human capital investment does not imposes a positive externality on parents. In addition, savings by parents only affect transfers by child $m$. Note that parents still under-save relative to the efficient level.

Finally, in the case that parents receive transfer from child $m$, but do not leave bequest to child $n$, it is straightforward to show that $h^m_e(e^m) = 1$, but $h^n_e(e^n) > 1$. The human capital investment is at efficient level for the child from whom parents receive transfer, but inefficiently low for the child from whom it does not receive transfer. This implies that the human capital investment for the child with inferior earnings function will be inefficiently low. This suggests that low income level of parents may exacerbate the inequality in human capital investment across siblings. This can be particularly relevant in explaining gender differences in human capital investment observed in many developing countries.

Many Asian and Middle Eastern societies have strong patrilineal tradition. In such societies, it is expected that sons would provide support to their
older parents and take care of them. In these societies, girls are expected to leave their parental home after marriage (patrilocality) and take care of family members of husband. Traditionally, women have less control over their own earnings as well as family’s income and less say in their allocations.

These traditions and cultural practices are likely to reduce the likelihood of older parents receiving transfers from their female children. Indeed, it is a common saying (e.g. in South Asia) that raising girls are like watering the garden of neighbors. Evidence from many Asian countries show that married female children are less likely to provide support to older parents and the amount of transfers provided them is smaller than sons (e.g. Lee et. al. 1994, Lillard and Willis 1997, Frankenberg et. al. 2002, Kim 2010, Lei et. al. 2012).

Since, these traditions and cultural practices reduce the likelihood and amount of old-age income support provided by female children to their parents, from the parental point of view, the effects of these traditions and cultural practices are similar to the earnings function bias against women. These traditions and cultural practices are likely to induce parents to invest less in the human capital of female children compared to male children, particularly in rural areas where these traditions and practices are likely to be more wide-spread and more rigidly followed.

4.2 Cooperation Among Children

Now, I consider the case when both children can cooperate and choose their transfers to parents to maximize their joint utility. Let $\mu^i \in (0, 1)$ be the weight on the utility of child $i = m, n$ with $\mu^m + \mu^n = 1$ in the joint utility function. Then the optimization problem for children is to

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8Suppose that the earnings function of female child is $h_f(e^f)$. But she controls $\sigma \in (0, 1)$ fraction of her earnings. From the point of view of parents, her effective earnings function is $\sigma h_f(e^f)$. Also, these cultural practices impose restrictions on the labor force participation of adult women. One can also interpret $\sigma$ as the probability that female child will participate in the labor force.
\[
\max_{c^m,c^n,\tau^m,\tau^n} \sum_{i=m,n} \mu^i U(c^i) + \lambda U(c^p) \\
\text{subject to} \\
c^m + c^n + \tau^m + \tau^n = h^m(e^m) + h^n(e^n).
\] (4.9)

The first order conditions are

\[
\tau_i : \mu^i U_c(c^i) = \lambda U_{c^p}(c^p), \ \forall \ i = m, n \& (4.10)
\]

\[
c^i : \mu^i U_c(c^i) = \mu^j U_c(c^j), \ \forall \ i \neq j = m, n. \quad (4.11)
\]

(4.10) equates the joint marginal cost of transfer from children to parents to its marginal benefit. The comparison of (4.1) and (4.12) shows that when children cooperate, they take into account the indirect effect of transfer to parents from one child on other child. When children do not cooperate they ignore this indirect effect. (4.13) equalizes the joint marginal utility of consumption across children.

From (4.10) and (4.11) it follows that

\[
\frac{d\tau^i}{dk} + \frac{d\tau^j}{dk} = -\frac{\lambda U_{cc}(c^p)}{\mu^i U_{cc}(c^i) + \lambda U_{cc}(c^p)} < 0, \ \forall \ i \neq j = m, n. \quad (4.13)
\]

(4.12) and (4.13) imply that

\[
\frac{d\tau^m}{de^m} + \frac{d\tau^n}{de^n} - \frac{d\tau^m}{dk} - \frac{d\tau^n}{dk} = 1, \ \text{if} \ h^m(e^m) = h^n(e^n). \quad (4.14)
\]
The parental problem is to

$$\max_{e^m, e^n, k} \sum_{t=1}^{2} U(e^p_t) + \sum_{i=m,n} \delta U(e^i)$$

subject to (2.7)-(2.10), (4.10), and (4.11). The first order conditions are

$$e^i : U_c(c^p_1) = U_c(c^p_2)[\frac{d\tau^i}{de^i} + \frac{d\tau^j}{de^i} + \delta U_c(c^i)[h^i_c(e^i) - \frac{d\tau^i}{de^i} - \frac{d\tau^j}{de^i}], \forall i \neq j = m, n \& (4.15)$$

$$k : U_c(c^p_1) = U_c(c^p_2)[1+\frac{d\tau^i}{dk} + \frac{d\tau^j}{dk}] - \delta U_c(c^i)[\frac{d\tau^i}{dk} + \frac{d\tau^j}{dk}], \forall i \neq j = m, n. \ (4.16)$$

Then, using (4.12-4.16), one can easily show that

$$h^m_e(e^m) = h^n_e(e^n) = 1 \ (4.17)$$

solves (4.15) and (4.16). Thus, when children co-operate, reverse-transfer leads to efficient level of human capital investment. The reason is that due to cooperation among children, the positive externality inherent in the choice of human capital investment is internalized.

Note that, parents still under-save relative to the efficient level for a given income. (4.17) also implies that any marginal change in (autonomous) income of parents and children will only affect transfers and savings and not the human capital investment similar to the case when children do not co-operate.

4.3 Cooperative or Non-Cooperative Equilibrium

The analysis raises a very important question regarding the process of deciding about old-age income support. In particular, when are children likely to cooperate? The existing empirical literature which has examined the effects
of old-age income support (cited above) has ignored this issue. Though cooperation among children leads to efficient level of human capital investment by parents, children will cooperate only when each child is better-off compared to non-cooperation.

In general, *for a given level* of human capital investment and savings of parents, cooperation leads to larger transfer from children to parents compared to non-cooperation. Thus, the second period consumption of parents will be higher and the total consumption of children will be lower under cooperation relative to non-cooperation, *for a given level* of human capital investment and savings. Overall, whether a child is better-off under cooperation will depend on factors such as the weight of the utility of a child in the joint utility function, \( \mu_i \), the degree of reverse altruism, \( \lambda \), the inequality aversion of children, composition of family and social norms and institutions.

As is clear from (4.3), in the non-cooperative case, both children will have same utility. However, in the cooperative case a child with a lower \( \mu_i \) will have lower consumption and utility compared to the other child (4.11) and thus will have less incentive to cooperate. Therefore, whether cooperation occurs or not will depend crucially on the relative weight of a child in the joint utility function. This in turn is likely to depend on the relative bargaining power of children, which may depend on their earning capacity, education, and social norms and institutions.\(^9\)

The other important factor can be the extent to which children care about the inequality of consumption between themselves and parents, which will depend on the degree of reverse altruism, \( \lambda \), and their inequality aversion. Interestingly, a higher \( \lambda \) and/or the greater degree of inequality aversion

\(^9\)In recent years, following the work of Chiappori (1992), Blundell et. al. (2007) etc. a large empirical literature has developed which have examined the determinants of the relative decision making power of household members. However, this literature has focussed on the determinants of the relative decision making power of spouses. This literature suggests that human capital, earnings capacity of members, government policies and social and cultural norms are important determinants of spousal relative decision making power.
may not result in cooperative equilibrium. The reason is that a higher \( \lambda \) or the greater degree of inequality aversion increases transfers from children both under cooperation and non-cooperation. Thus, the relative gap between parental consumption and the consumption of children will be lower in the non-cooperative case. A further redistribution of consumption between parents and children which cooperation entails may not make children better-off compared to non-cooperation. Thus, whether cooperation will occur may depend crucially on the interaction between degrees of reverse altruism and inequality aversion.

To see this assume that \( \mu^m = \mu^n \) in the cooperative case and the period utility function is of the CRRA form \( U(c) = \frac{c^{1-\alpha}}{1-\alpha} \). \( \forall \alpha \neq 1 \) & \( \ln c \) if \( \alpha = 1 \), where \( \alpha = -c \frac{U_{cc}(c)}{U_c(c)} \) is the elasticity of marginal utility. \( \alpha \) is also a measure of the degree to which children are concerned with the distribution of consumption between themselves and parents. The higher \( \alpha \) is, the stronger is the inclination to smooth consumption levels, other things remaining the same. As shown in the appendix, for a given level of savings, \( k \), and the human capital investment, \( e^i \), children will cooperate only if the following condition is satisfied:

\[
\alpha \ln(2 + 2 \lambda B) + (1 - \alpha) \ln(2 + B) \geq \ln 2 + \ln(1 + B) \quad \forall \alpha \neq 1 \& \quad (4.18)
\]

\[
(1 + \lambda)[\ln(2 + \lambda) - \ln(1 + \lambda)] \geq \ln 2 \quad \text{if} \quad \alpha = 1 \quad (4.19)
\]

where \( B \equiv \lambda^{\frac{1}{\alpha}} \). Numerical simulation (see the appendix) shows that for a given \( \lambda \) this condition is satisfied when \( \alpha \) is low. This happens because with low \( \alpha \) the relative gap in the consumption of children and parents is relatively high under non-cooperation for a given \( \lambda \). Also, the loss of utility to children from the marginal redistribution of consumption towards parents is relatively low. In this case, cooperation makes children better-off. Opposite happens
when $\alpha$ is high.

The composition of family can be another important factor. For example, in societies where joint family system (e.g. South Asia) is prevalent, cooperation among children is more likely to occur. On the other hand, in societies (e.g. Western countries) where nuclear family system is a norm, non-cooperation is more likely to occur. Also, there can be important differences between the rural and the urban areas. For example in South Asian countries, joint family system is more prevalent in the rural areas than in the urban areas. In that case, cooperation is more likely to occur in the rural areas than in the urban areas.

The gender composition of children can be another important factor. As discussed earlier, in patrilineal societies married daughters leave parental home and usually they have less say in the decision making. In that case, cooperation is more likely to occur if parents have two sons rather than one son and daughter or two daughters.

The other important factor can be mobility. In societies with high mobility or migration, children are more likely to live in different locations, making cooperation among them more difficult. In such societies non-cooperative equilibrium is more likely to emerge. Interestingly, this may incentivize parents, who expect non-cooperative equilibrium to occur, to undertake additional human capital investment, which may also then increase the likelihood of children migrating and living apart.

5 Uncertain Reverse Altruism

So far I have examined equilibrium outcomes under certainty. In this section, I extend the model to incorporate uncertainty about the degree of reverse altruism, which has received considerable attention in the literature (e.g. Chakrabarti et. al. 1993, Leroux and Pestieau 2014). Assume that the parents in the first period are uncertain about the degree of reverse altruism
and believe that with positive probability the realized value of the degree of reverse altruism can be such that children will not provide old-age income support regardless of the size of human capital investment.\textsuperscript{10} This uncertainty is resolved at the beginning of the second period.

For analytical simplicity, assume that the degree of reverse altruism takes two values 0 and $\lambda$ with probability $p$ and $1-p$ respectively. I focus on the case, where the value of $\lambda$ is high enough such that (given the values of other parameters) both children provide old-age income support to parents.\textsuperscript{11}

Thus, parents receive income support from both children or from none.

With the introduction of the possibility of default by children, the human capital investment by parents becomes risky. The budget constraints for parents and children are modified as follows. For parents,

\[ c^p_2 = A_2 + k + \tau^m + \tau^n, \] \hspace{1cm} (5.1)

For children,

\[ c^i = b^i - \tau^i + h^i(e^i) \forall i = m, n, \] \hspace{1cm} (5.2)

in the non-cooperative case and

\[ c^m + c^n + \tau^m + \tau^n = h^m(e^m) + h^n(e^n) \] \hspace{1cm} (5.3)

\textsuperscript{10} Such uncertainty can arise due to variety of reasons such as imperfect information about innate characteristics of children, family disputes, changing social norms, increased mobility etc.

\textsuperscript{11} Let $e^{i0}$ be the equilibrium human capital investment, when parents cannot give bequest ($b^i = 0$ for $i = m, n$) and there is no reverse altruism. Now let $\lambda$ be the degree of reverse altruism, such that $U_c(c^i) \leq \lambda U_c(e_{i2}^i)$ in the non-cooperative case and $\mu U_c(c^i) \leq \lambda U_c(e_{i2}^i)$ in the cooperative case for $e^{i0}$ for $i = m, n$. Now suppose that $\lambda > \lambda$.
in the cooperative case.

Now the parents maximize their expected utility

\[
E \left[ \sum_{t=1}^{2} U(c_{t}^{p}) + \sum_{i=m,n} \delta^{i} U(c_{t}^{i}) \right]
\]

where \( E \) is the expectation operator.

**Proposition 6: Uncertain Reverse Altruism**

(i) In the non-cooperative case, parents do not equalize the marginal rate return from human capital across children, \( h_{e}^{m}(e^{m}) \neq h_{e}^{n}(e^{n}) \). In addition, the human capital investment for children can be inefficiently high or low, \( h_{e}^{i}(e^{i}) \gtrless 1 \) for \( i = m, n \).

(ii) In the cooperative case, when children have equal weight, \( \mu^{i} = \mu^{j} \), the parents equalize the marginal rate of return from human capital across children, \( h_{e}^{m}(e^{m}) = h_{e}^{n}(e^{n}) \). However, in this case the human capital investment for both children is inefficiently low \( h_{e}^{i}(e^{i}) > 1 \) for \( i = m, n \).

Proof: See Appendix.

In the non-cooperative case, the introduction of the possibility of default by children alters results in two ways. Firstly, the marginal utility of consumption of children is no longer equalized and \( c^{m} \neq c^{n} \), in the case parents do not receive transfers from children. Thus, the parents who care equally about both children are no longer able to equalize the marginal rate return from human capital across children. Secondly, the possibility of default reduces the expected return from human capital investment for parents and thus parental investment can be lower than the efficient level. Note that it is possible that the parents choose efficient level of human capital investment for a child. However, the human capital investment can be efficient for at most one child and not both.
In the cooperative case with equal weight, the marginal utility of consumption of children is equalized whether they provide transfers to parents or not. Thus, the parents chose human capital investment such that the marginal rate of return from human capital investment is equalized across children. However, since the expected return from the human capital is lower, the parents reduce human capital investment below the efficient level. The result that with the possibility of default, the human capital investment will be inefficiently low in the co-operative case, generalizes the result of Chakrabarti et al. (1993), who only consider a family with one child (or identical children).

In general, with unequal weights ($\mu^i \neq \mu^j$), the marginal rate of return from human capital investment will not be equalized, $h^m_e(e^m) \neq h^n_e(e^n)$. In addition, if children do not choose their consumption co-operatively when the degree of reverse altruism is low ($c^i = h^i(e^i)$), then again in this case $h^m_e(e^m) \neq h^n_e(e^n)$ even with equal weights ($\mu^i = \mu^j$).

6 Policy Implications

In this section, I analyze the policy implications of the model. I first begin with the analysis of the effects of a public pension scheme on the human capital investment, a issue which has received considerable attention in the literature (e.g. Chakrabarti et. al. 1993, Leroux and Pestieau 2014). As shown above, the reliance of elderly parents for income support on their children may lead to inefficient level of human capital investment either because of non-cooperation among children or uncertainty about the degree of reverse altruism. Public pension scheme may potentially lead to efficient level of human capital investment.
6.1 Public Pension Scheme

I analyze a pension scheme where parents receive old-age pension from the government financed by taxes imposed on (adult) children. Suppose that the government can commit to tax and transfer scheme and it acts as a Stackleberg leader. The government chooses taxes and transfers to maximize the utility of parents. As shown below, such pension scheme replicates the allocations achieved when parents can leave debt to their children, $b^i < 0$.

Timing is as follows: The government announces transfers to parents, $G$, and taxes on children, $\tau^m$ & $\tau^n$, at the beginning of the first period. Taking transfers as given, parents choose human capital investment, $e^i$, for $i = m, n$, and savings, $k$, in the first-period. The government makes transfers to parents, $G$, in the second period, financed by imposing tax, $\tau^i$, for $i = m, n$ on children of type $i = m, n$ in the second period.

The parental problem is

$$\text{max} \sum_{t=1}^{2} U(c^p_t) + \sum_{i=m,n} \delta U(c^i)$$

subject to (2.7)-(2.10), given $G$, $\tau^m$, & $\tau^n$. The optimal choices of parents will be characterized by (3.1) and (3.4) respectively.

The government optimization problem is to

$$\text{max} \sum_{t=1}^{2} U(c^p_t) + \sum_{i=m,n} \delta U(c^i)$$

subject to (3.1), (3.4), and $G = \tau^m + \tau^n$. The first order condition is

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12It is easy to show that in a pension system in which parents contribute in the first period and receive pension in the second period based on their contribution will have no effect on the human capital investment. Such pension scheme will just affect amount of savings, $k$, and transfers from children, $\tau^m$ & $\tau^n$.

13The government optimization problem remains the same, whether there is uncertainty or not.
\[ \tau^i : [-U_c(c^p_1) + U_c(c^p_2)] \frac{dk}{d\tau_i} + [-U_c(c^e_i) + \delta U_c(c^e_i) h^i(e^i)] \frac{de^i}{d\tau_i} \]

\[ [-U_c(c^p_1) + \delta U_c(c^e_i) h^i(e^i)] \frac{de^j}{d\tau_i} + U_c(c^p_2) - \delta U_c(c^e_i) = 0, \forall i \neq j = m, n. \tag{6.1} \]

The government while choosing taxes/transfers take into account their effects on the parental choices of human capital investment and savings. From (3.1), (3.4), and (6.1), it follows that

\[ \tau^i : U_c(c^e_2) = \delta U_c(c^e_i) \forall i = m, n & \tag{6.2} \]

\[ h^m(e^m) = h^n(e^n) = 1. \tag{6.3} \]

Such pension scheme leads to efficient level of human capital investment. In addition, it leads to efficient level of savings. Thus, such pension scheme can resolve both the Rotten-Kid Theorem and the Samaritan’s dilemma types of inefficiencies.

It is interesting to note that this pension scheme achieves the same allocations which would occur when parents can leave debt to their children, \( b^i < 0 \). In the model, the inefficiency of human capital investment arises because parents would like to leave debt to their children, but are not able to do so.\(^{14}\) The pension scheme provides a mechanism through which parents can leave debt to their children.

The analysis shows that the introduction of pension scheme may lead to higher or lower human capital investment of children compared to pre-pension levels, depending on whether children cooperate or not and whether there is uncertainty about the degree of reverse altruism. Chakrabarti et. al. (1993) in their model (with one child) and uncertain reverse altruism

\(^{14}\)In almost all countries, children are not legally responsible for debts of their parents.
find that the introduction of public pension scheme leads to reduction in the human capital investment of children. The reason is that the provision of public pension increases the future wealth of parents, which induces them to reduce their human capital investment.

However, in my model public pension scheme would lead to higher human capital investment when children cooperate and there is uncertainty about the degree of reverse altruism. Only in the case, children do not cooperate, it is possible that the public pension reduces the human capital investment. The reason is that in my model the government acts as a Stackleberg leader. On the other hand, Chakrabarti et. al. (1993) only consider a marginal lump-sum intergenerational redistribution.

The effect of public pension on private savings has been a matter of considerable debate among researchers (see Feldstein and Liebman 2005 for a thorough review). In particular, whether public pension crowds-out private savings. The model provides an environment in which public pension scheme can lead to higher financial savings, as it mitigates the problem of under-saving by parents arising from the Samaritan’s dilemma type of inefficiency.

### 6.2 Other Policy Implications

Our analysis suggests that the low quality of schooling witnessed in most of the developing countries adversely affects human capital investment by parents not only by reducing its return, but also by reducing the likelihood and the amount of old-age income support provided by their children. From policy perspective, it implies that improving the quality of schooling will increase human capital investment not only because the rate of return on the human capital investment will rise, but also because the likelihood and the amount of old-age income support provided to parents by their children will increase.

Our analysis also suggests that when parents do not face borrowing constraint and they receive old-age income support from their children, the neg-
ative effect of low parental income on the human capital investment gets mitigated. However, when financial markets are imperfect and poor parents are unable to borrow \((k = 0)\), then it is easy to show that parents may under-invest in the human capital of their children even when they receive transfers from both children. In addition, human capital investment is no longer independent of income. In this case, providing income assistance to parents in the first period will increase human capital investment.

As discussed earlier, old-age income support from children incentivizes parents to under-save in order to elicit more transfers from their children. This may induce some of the low income parents, who would have saved in the absence of old-age income support from children, to reduce their savings to zero and also to cut back on the human capital investment of children. This will occur if for these parents transfers from children are more responsive to parental savings than to human capital investment. Interestingly, in such a case cooperation among children will lead to inefficiently low level of human capital investment by parents. The analysis suggests that the Samaritan’s dilemma type of inefficiency interacting with imperfections in the financial market may reduce human capital investment by low income parents, who are unable to borrow.

Another group which is likely to benefit from income-assistance programs are parents who are so poor that they are not able to send their children to schools \((e^i = 0)\). Our results suggest continuing need to provide financial support to poor households who face borrowing constraint and/or cannot afford to send their children to school and to increase their access to credit market. Micro-finance programs, interest-free educational loans, and scholarships targeted towards poorer households will increase investment in education.

The analysis also shows that the relationship between human capital investment and parental income/wealth can be highly non-monotonic. The effect of parental income on human capital investment of their children depends on (apart from other things) number of children and parental expec-
tation about receiving old-age income support from their children. Parents with low income, but expecting old-age income support from their children may invest more in their human capital than parents with high income who expect to leave bequest to their children and also parents with low income who do not expect to receive old-age income support from their children. In addition, the effect of (unconditional) income transfers to poor households will depend on whether they expect old-age income support from children and the degree of imperfections in the financial market. These predictions of the model are consistent with a large number of studies which find mixed evidence with regard to income elasticities of schooling demand in developing countries. Some empirical studies find income elasticities to be large, but others find income elasticities to be positive but quite small (see Orazem and King 2007 for a review and references). The analysis also suggests that the relationship between quality and number of children can be non-monotonic, which is in contrast to Becker and Lewis (1973) model with one-sided altruism which posits a negative relationship between number and quality of children.\textsuperscript{15}

7 Conclusion

In this paper, I developed a model to study the interaction between investment by parents in the human capital of their children and old-age income support provided by them to their parents in a model in which both parents and children are altruistic. The analysis shows that when parents have multiple children, old-age income support provided by children significantly alters the nature of interaction between parents and children. The equilibrium outcomes depend on whether (adult) children choose their income support cooperatively or non-cooperatively.

\textsuperscript{15}The empirical evidence on relationship between number and quality of children is mixed. Becker and Lewis (1973) find a significant negative relationship between the two. But Raut and Tran (2005) find a significant positive relationship between the two.
When children choose their income support *non-cooperatively*, then old-age income support leads to an inefficiently *high* level of human capital investment by parents. When children cooperate, such support leads to efficient level of human capital investment.

The reason for the inefficiently high levels of human capital investment by parents when children act non-cooperatively is that its choice imposes a positive externality on parents. A higher level of human capital investment for one type of child not only increases transfers from that type of child but also from the other type of child. In order to increase their second period consumption and transfers from children, parents choose inefficiently high levels of human capital investment in the first period. When children cooperate, this positive externality is internalized.

The analysis also shows that when there is uncertainty about the degree of altruism of children and they can default on old-age income support, the human capital investment is inefficient in both cooperative and non-cooperative cases. In particular, in the *cooperative case*, the human capital investment can be inefficiently low. The reason is that the risk of default reduces the expected return from the human capital investment.

The analysis also finds that the old-age income support by children leads to under-saving by parents. Old-age income support incentivizes parents to under-save relative to the efficient level.

The analysis suggests that there is scope for public provision of old-age pension. A public pension scheme financed by taxes on (adult) children can induce parents to choose efficient levels of both human capital investment and savings, when the government is a Stackleberg leader and it optimally chooses taxes and transfers. Such pension scheme can internalize both types of inefficiencies.
Appendix

Proof of Proposition 2

These results can be shown as follow. From (4.7) and (4.8), I get

\[
[U_c(c_2^p) - \delta U_c(c^i)h^i_c(e^i)] + [U_c(c_2^p) - \delta U_c(c^i)] \left[ \frac{d\tau^i}{dk} - \frac{d\tau^i}{de^i} \right] = \\
-[U_c(c_2^p) - \delta U_c(c^i)] \left[ \frac{d\tau^j}{dk} - \frac{d\tau^j}{de^i} \right]; \quad \forall \ i \neq j = m, n. \quad (1)
\]

Now note that (4.4) and (4.6) imply that

\[
\frac{d\tau^i}{dk} - \frac{d\tau^i}{de^i} = -\lambda U_{cc}(c_2^p) + U_{cc}(c^i)h^i_c(e^i) - \frac{U_{cc}(c^i)}{U_{cc}(c^2) + \lambda U_{cc}(c_2^p)}, \quad \forall i \neq j = m, n. \quad (2)
\]

(2) implies that

\[
\left| \frac{d\tau^i}{dk} - \frac{d\tau^i}{de^i} \right| \geq 1 \text{ for } h^i_c(e^i) \leq 1. \quad (3)
\]

Suppose that \( h^i_c(e^i) = 1 \), then (3) implies that (1) can hold either when

\[
U_c(c_2^p) = \delta U_c(c^m) = \delta U_c(c^n) \text{ or } \frac{d\tau^j}{dk} - \frac{d\tau^j}{de^i} = 0. \quad (4)
\]

But then (4) implies that either \( b^i > 0 \) for \( i = m, n \), which is a contradiction or \( \frac{d\tau^j}{dk} - \frac{d\tau^j}{de^i} = 0 \) (i.e. the choice of human capital investment in one type of child and savings do not affect the transfers from the child of other type).

Now, I show that the human capital investment is not only at an inefficient level, but it is more than the efficient level. Note that the right hand side of (1) is strictly positive. Thus, for any \( e^i \) for \( i = m, n \), which solves (1), the left hand side of (1) should also be strictly positive i.e.

\[
[U_c(c_2^p) - \delta U_c(c^i)h^i_c(e^i)] > -[U_c(c_2^p) - \delta U_c(c^i)] \left[ \frac{d\tau^i}{dk} - \frac{d\tau^i}{de^i} \right]. \quad (5)
\]

Then (3) implies that (5) can hold only when \( h^i_c(e^i) < 1 \), for \( i = m, n. \).
Thus, in the case of reverse transfers from both children, the human capital investment is inefficiently high.

**Proof of Proposition 3**

(4.7) can be written as

$$U_c(c^p)\left[\frac{d\tau^m}{de^m} + \frac{d\tau^n}{de^n} - \frac{d\tau^m}{de^m} + \frac{d\tau^n}{de^n}\right] + \delta U_c(c^m)\left[h^m_c(e^m) - \frac{d\tau^m}{de^m} + \frac{d\tau^m}{de^n}\right]$$

$$-\delta U_c(c^n)\left[h^n_c(e^n) - \frac{d\tau^n}{de^n} + \frac{d\tau^n}{de^m}\right] = 0. \quad (6)$$

(4.3) implies that $$c^m = c^n$$. Then using (4.4) and (4.5), (6) can be written as

$$(U_c(c^p) - \delta U_c(c^m))(1 - \delta \lambda)U_{cc}(c^m)[h^m_c(e^m) - h^n_c(e^n)]\left[\frac{1}{U_{cc}(c^m) + \lambda U_{cc}(c^p)} + \frac{1}{\lambda U_{cc}(c^p)}\right]$$

$$= \delta U_c(c^m)[h^m_c(e^m) - h^n_c(e^n)]. \quad (7)$$

(7) is satisfied when $$h^m_c(e^m) = h^n_c(e^n)$$. However, as discussed earlier despite this equality, human capital investment remains inefficiently high. From (2.5), it also follows that in the reverse transfer case since $$e^m > e^n$$ and $$c^m = c^n$$, $$\tau^m > \tau^n$$. Child $$m$$ transfers more to parents than child $$n$$.

**Proof of Proposition 4**

Using (4.1), (4.7) and (4.8) can be written as,

$$e^i : \frac{U_c(c^p)}{U_c(c^p)} = \delta \lambda h^i_c(e^i) + \frac{1}{1 - \delta \lambda de^i} \frac{d\tau^i}{de^i} - \frac{1}{1 - \delta \lambda de^i} \frac{d\tau^i}{de^i}, \forall i = m, n & \quad (8)$$
\[ k : \frac{U_c(c^*_1)}{U_c(c^*_2)} = 1 + \frac{1}{1 - \delta\lambda} \frac{d\tau^m}{dk} + \frac{1}{1 - \delta\lambda} \frac{d\tau^n}{dk}. \tag{9} \]

(4.1) and (4.4)-(4.6) imply that in the case \( U(c^i) = \alpha c^i - \beta c^{2i} \) or \( \alpha c^{2i} \) or \( - \exp^{-\alpha c^i} \) for \( i = p, m, n \), \( \frac{d\tau^i}{dk} \) and \( \frac{d\tau^i}{db^i} \) are independent of income of parents and children and only depend on \( h^*_i(e^i) \). Then, (8) and (9) imply that human capital investment is independent of parental income.

**Proposition 5:**

Now, the second period budget constraint for parents is given by: \( c^p_2 = A_2 + k + \tau^j - b^i \). The first order condition for the transfer from child \( m \) is modified to

\[ U_c(h^j(e^j) - \tau^j) = \lambda U_c(A_2 + k - b^i + \tau^j). \tag{10} \]

Using (10), one can derive

\[ \frac{d\tau^j}{de^j} = \frac{U_{cc}(c^j) h^j_e(e^j)}{U_{cc}(c^j) + \lambda U_{cc}(c^2_j)} > 0 \& \tag{11} \]

\[ \frac{d\tau^j}{dk} = -\frac{\lambda U_{cc}(c^p_2)}{U_{cc}(c^j) + \lambda U_{cc}(c^p_2)} = -\frac{d\tau^j}{db^i} < 0. \tag{12} \]

(12) shows that an increase in \( k \) and a reduction in \( b^i \) reduce transfers from the \( j \)th child to parents.

The parental problem in the first period is to choose \( e^m, e^n, b^m, \& k \) to maximize their utility subject to (2.7)-(2.10), and (10). The optimal choice of \( e^i \) continue to be given by (3.1). The other first order conditions are

\[ b^i : U_c(c^p_2) \left[ 1 - \frac{d\tau^j}{db^i} \right] = \delta U_c(c^i) - \delta U_c(c^j) \frac{d\tau^j}{db^i}; \tag{13} \]

\[ e^j : U_c(c^p_1) = \delta U_c(c^j) \left[ \frac{d\tau^j}{de^j} - h^*_j(e^j) \right] - U_c(c^p_2) \left[ \frac{d\tau^j}{de^j} \right]; \tag{14} \]
\[ k : U_c(c_1^p) = U_c(c_2^p) \left[ 1 + \frac{d\tau^j}{dk} \right] - \delta U_c(c^j) \frac{d\tau^j}{dk}. \]  

(15)

Using (3.1), (12), (13) and (15), one can derive

\[ \delta U_c(c^j) = U_c(c_1^p). \]  

(16)

Then, (3.1) and (3.2) imply that \( h^1_c(e^i) = 1 \). Thus, the human capital investment for the \( i \)th child continues to remain at the efficient level. Turning to the human capital investment for the \( j \)th child, combining (14) and (15) I have,

\[ U_c(c_2^p) \left[ 1 + \frac{d\tau^j}{dk} - \frac{d\tau^j}{de^j} \right] = \delta U_c(c^j) \left[ h^1_c(e^j) + \frac{d\tau^j}{dk} - \frac{d\tau^j}{de^j} \right]. \]  

(17)

Now, note that (11) and (12) imply that \( \frac{d\tau^j}{dk} - \frac{d\tau^j}{de^j} = -1 \), when \( h^1_c(e^j) = 1 \). Then, for \( U_c(c_2^p) \geq \delta U_c(c^j) \) (17) implies that \( h^1_c(e^j) = 1 \). Thus, reverse transfer from the \( j \)th child leads to the efficient choice of human capital investment for the \( j \)th child by parents.

**Proposition 6:**

Let us denote the second period consumption of parents and children by \( c_2^p \) and \( c_i^0 \) respectively for \( i = m, n \), when children do not provide income support to parents. Note that when children provide support to parents (with probability \( 1 - p \)), the optimal strategy of children continues to be characterized by (4.1) in the non-cooperative case and by (4.10) in the cooperative case.

The Non-Cooperative Case:

Combining the first order-conditions for the optimal choices of human capital investment of children, I have
\[(1-p) \left[ (U_c(c^m_p) - \delta U_c(c^m)) (1 - \delta \lambda) U_{cc}(c^m) [h^m_c(e^m) - h^n_c(e^n)] \right] \frac{1}{U_{cc}(c^m) + \lambda U_{cc}(c^m_p)} + \frac{1}{\lambda U_{cc}(c^m_p)} \]

\[-(1-p) \left[ \delta U_c(c^m) [h^m_c(e^m) - h^n_c(e^n)] \right] + p \left[ \delta U_c(c^m) h^m_c(e^m) - \delta U_c(c^m_0) h^n_c(e^n) \right] = 0. \tag{18} \]

Now suppose that \( h^m_c(e^m) = h^n_c(e^n) \). Then given that \( c^m = c^n \) when both children make transfers, terms involving \( 1 - p \) will be equal to zero. However, \( c^m_0 > c^n_0 \) and thus equation (18) will not be satisfied. Thus it must be the case that \( h^m_c(e^m) \neq h^n_c(e^n) \), in equilibrium.

Combining the first order condition for the optimal choices of the human capital investment and savings, I have

\[(1-p) \left[ [U_c(c^m_p) - \delta U_c(c^i)] h^i_c(e^i) \right] + [U_c(c^j_p) - \delta U_c(c^j)] \left[ \frac{d\tau}{dk} - \frac{d\tau^i}{de^i} + \frac{d\tau^j}{de^j} \right] \]

\[+ p[U_c(c^m_0) - \delta U_c(c^m_0) h^m_c(e^m)] = 0 \forall i \neq j = m, n. \tag{19} \]

Now suppose that \( h^i_c(e^i) = 1 \). Given that \( \frac{d\tau^i}{dk} - \frac{d\tau^i}{de^i} = 1 \) when \( h^i_c(e^i) = 1 \), then for (19) to be satisfied, it must be the case that

\[-(1-p)[U_c(c^m_p) - \delta U_c(c^i)] \left[ \frac{d\tau^j}{dk} - \frac{d\tau^j}{de^j} \right] = p[U_c(c^m_0) - \delta U_c(c^m_0)] \forall i \neq j = m, n. \tag{20} \]

As is clear from (4.5), (4.6), and (20), for a particular set of parameter values, it is possible that the parents choose efficient level of human capital investment for a child, \( h^i_c(e^i) = 1 \). However, in general, that will not be the case. In addition, the human capital investment can be efficient for at most one child and not both.

Also, it is possible that \( h^i_c(e^i) > 1 \) is a solution for (19). Note that for
For the expression involving \((1-p)\) in (19) to hold, the expression involving \(p\) must be strictly positive. Thus, for \(h^i_\epsilon(e^i) > 1\) to be the solution in equilibrium, it must be the case that \(\frac{U^c_i(c^i_0) - U^c_i(c^i_0)h^i_\epsilon(e^i)}{\mu^c_i(h^i_\epsilon(e^i))} > 1\).

**The Cooperative Case:**

Combining the first order conditions for the human capital investment, I have

\[
(1-p) \left[ U^c_i(c^i_2) \left[ 1 + \frac{d\tau^i}{de^i} + \frac{d\tau^j}{de^j} - \frac{d\tau^i}{de^j} \right] \right] + (1-p) \left[ \delta U^c_i(e^i) \left[ \frac{d\tau^i}{de^i} - \frac{d\tau^j}{de^j} \right] \right] \\
-(1-p) \left[ \delta U^c_j(e^j) \left[ h^j_\epsilon(e^j) \right] \right] - p \left[ \delta U^c_i(e^i) h^i_\epsilon(e^i) - \delta U^c_j(e^j) h^j_\epsilon(e^j) \right], \quad \forall i \neq j = m, n. \tag{21}
\]

Now note that when both children make transfers, we have \(\mu_i U^c_i(e^i) = \mu_j U^c_j(e^j)\). Then in the case, \(\mu_i = \mu_j\), it is straightforward to show that \(h^i_\epsilon(e^i) = h^j_\epsilon(e^j)\) solves (21). However, when \(\mu_i \neq \mu_j\), \(h^i_\epsilon(e^i) = h^j_\epsilon(e^j)\) will not be the solution.

Now combining the first order conditions for the human capital investment and savings, I have

\[
(1-p) \left[ U^c_i(c^i_2) \left[ 1 + \frac{d\tau^i}{dk} + \frac{d\tau^j}{dk} - \frac{d\tau^i}{dk} \right] \right] + (1-p) \left[ \delta U^c_i(e^i) \left[ \frac{d\tau^i}{dk} + \frac{d\tau^j}{dk} - \frac{d\tau^i}{dk} \right] \right] \\
+p \left[ U^c_i(c^i_0) - \delta U^c_i(c^i_0)h^i_\epsilon(e^i) \right], \quad \forall i \neq j = m, n. \tag{22}
\]

Now note that \(\left| \frac{d\tau^i}{dk} + \frac{d\tau^j}{dk} - \frac{d\tau^i}{dk} \right| = 1\) when \(h^i_\epsilon(e^i) = h^j_\epsilon(e^j)\). This implies that when \(h^i_\epsilon(e^i) = h^j_\epsilon(e^j) = 1\), terms involving \((1-p)\) will be zero. However, the term involving \(p\) will be strictly positive. Thus, \(h^i_\epsilon(e^i) = h^j_\epsilon(e^j) = 1\)
is not a solution of (21). Note also that for any \( h_x^i(e^i) = h_x^j(e^j) < 1 \), the LHS of (22) is strictly positive. Thus, only possible solution for (22) is \( h_x^i(e^i) = h_x^j(e^j) > 1 \).

**The Co-operative or Non Co-operative Solution**

**Non-Cooperation:**

First I derive the transfers and the indirect utility functions of children when they do not cooperate for a given \( e^i \) and \( k \). Let \( B \equiv \lambda \frac{1}{\alpha} \). With the CRRA utility function, (4.1) implies that

\[
e^p = Bc^m = Bc^n. \tag{23}
\]

Then using the budget constraints, one can show that

\[
\tau^m = \frac{1}{2 + B}[(1 + B)h^m(e^m) - h^n(e^n) - A - k]; \tag{24}
\]

\[
\tau^n = \frac{1}{2 + B}[(1 + B)h^n(e^n) - h^m(e^m) - A - k]; \tag{25}
\]

\[
c^m = c^n = \frac{1}{2 + B}[h^m(e^m) + h^n(e^n) + A + k] \tag{26}\]

\[
e^p = \frac{B}{2 + B}[h^m(e^m) + h^n(e^n) + A + k]. \tag{27}
\]

Using (26) and (27), one can derive the indirect utility functions of children under non-cooperation

\[
V_{NC}^m = V_{NC}^n = \frac{1 + B}{(1 - \alpha)(2 + B)^{1-\alpha}}[h^m(e^m) + h^n(e^n) + A + k]^{1-\alpha} \forall \alpha \neq 1 \tag{28}\]
\[ = \lambda \ln \lambda + \lambda \ln 2 + (1 + \lambda) \left[ \ln h^m(e^m) + h^n(e^n) + A + k \right], \text{ if } \alpha = 1. \quad (29) \]

**Cooperation:**

Now I derive the transfers and the indirect utility of children when they cooperate for a given \( e^i \) and \( k \). Suppose that \( \mu^m = \mu^n \). Then (4.10) and (4.11) imply that

\[ c^p = 2^{\frac{1}{2}} B c^m = 2^{\frac{1}{2}} B c^n. \quad (30) \]

Then using the budget constraints one can show that

\[ \tau = \tau^m + \tau^n = \frac{1}{2 + 2^{\frac{1}{2}} B} \left[ 2^{\frac{1}{2}} B (h^m(e^m) + h^n(e^n)) - 2(A + k) \right]; \quad (31) \]

\[ c^m = c^n = \frac{1}{2 + 2^{\frac{1}{2}} B} \left[ h^m(e^m) + h^n(e^n) + A + k \right] \& \quad (32) \]

\[ c^p = \frac{2^{\frac{1}{2}} B}{2 + 2^{\frac{1}{2}} B} \left[ h^m(e^m) + h^n(e^n) + A + k \right]. \quad (33) \]

Using (32) and (33), one can derive the indirect utility functions of children under cooperation

\[ V^m_C = V^n_C = \frac{(2 + 2^{\frac{1}{2}} B)^\alpha}{2(1 - \alpha)} [h^m(e^m) + h^n(e^n) + A + k]^{1-\alpha}, \forall \alpha \neq 1 \& \quad (34) \]

\[ = \lambda \ln \lambda - \ln 2 + (1 + \lambda) \left[ \ln [h^m(e^m) + h^n(e^n) + A + k] - \ln (1 + \lambda) \right], \text{ if } \alpha = 1. \quad (35) \]
Children will cooperate if $V_C^i \geq V_{NC}^i$ for $i = m, n$. Then from (28), (29), (34), and (35), it follows that children will cooperate if

$$\frac{(2 + 2\alpha B)^{\alpha}}{2} \geq \frac{1 + B}{(2 + B)^{1-\alpha}} \forall \alpha \neq 1 \&$$ (36)

$$(1 + \lambda)[\ln(2 + \lambda) - \ln(1 + \lambda)] \geq \ln 2, \text{ if } \alpha = 1.$$ (37)

Taking logs of (36), the condition for co-operation can be written as

$$\alpha \ln(2 + 2\alpha B) + (1 - \alpha) \ln(2 + B) \geq \ln 2 + \ln(1 + B) \forall \alpha \neq 1.$$ (38)

### Table

Cooperation (C) or Non-Cooperation (N)

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References


