HAS THE CANADIAN PUBLIC DEBT BEEN TOO HIGH?
A QUANTITATIVE ASSESSMENT

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March, 2019

Abstract

This paper provides a quantitative analysis on whether the historically sizable public debt that the Canadian governments have accumulated might be close to its welfare maximizing level. As the public provision of liquidity to borrowing constrained individuals coupled with an increased supply of safe assets can be welfare improving, I consider a two-region model with an integrated asset market and incomplete insurance markets. The home country features a rich life-cycle setup, where the income dynamics rely on state of the art estimates obtained from previous studies using income tax returns. The main features are ex-ante labor earnings heterogeneity, both in levels and in growth rates, together with persistent and permanent shocks. When the public expenditure is assumed to be wasteful, I find that the optimal quantity of public debt for Canada is negative, meaning that the government should be a net saver. When the government, with a portion of its expenditure and consistent with the Canadian experience, finances valuable public goods the long-run public debt is still found to be inefficiently large, but closer to the welfare maximizing level.

Keywords: Public debt, Incomplete markets, Welfare.

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1 Introduction

A rationale for governments to accumulate public debt is the provision of liquidity to borrowing constrained individuals. In an influential paper, Aiyagari and McGrattan (1998) found that the increase in the asset demand brought about by larger values of public debt has important general equilibrium effects, and that the long-run average of the U.S. public debt is close to the optimal one. In this paper, I undertake an equilibrium analysis tailored to the Canadian economy. The Canadian experience represents an interesting case, because the long-run average of the public debt has been sizable, and the pre-tax labor income dynamics have been similar to the U.S. ones. However, the resulting labor income inequality has been dealt with a more generous level of redistribution and a more extensive provision of public goods. Finally, the size of these two economies is drastically different, with plausibly very different implications for the magnitude of general equilibrium effects.

Some of the main findings in Aiyagari and McGrattan (1998) rest on three important assumptions: a closed economy framework, infinitely-lived households, and wasteful public expenditures.

In a closed economy setup, an increase in public debt is associated with an increase in the aggregate demand for assets, leading to a large response of the equilibrium interest rate. In steady-state comparisons, this can be beneficial for social welfare, as saving in a risk-free asset becomes a more effective tool to smooth income fluctuations. It goes without saying that such an effect would not be present in a basic small open economy framework. With interest rates determined in international financial markets, and domestic public debt held by foreign savers, welfare improvements induced by higher interest rates would not be present. However, the OECD data plotted in Figure 1 show that the Public Debt/GDP ratio is large and positive also for small economies with access to international financial markets. Clearly, the public provision of liquidity channel, relaxing the household borrowing constraint with public transfers partially financed by public debt, would still be operational. These arguments call for an open economy analysis, to better measure the importance of the liquidity provision explanation. In order to address these aspects, I consider a model with two regions, with the home economy being calibrated to Canadian microeconomic and macroeconomic evidence, and the other economy representing the Rest-of-the-World (RoW, hereafter). A priori, the two-region model does not rule out the possibility of large public debts to be welfare improving via the improved self-insurance attained by saving in a safe asset. One might expect considerably smaller GE effects, which is what I actually find. However, these could still be important in terms of the welfare effects, especially in an environment where savings have multiple goals, and the demand for redistribution is driven by several concurrent causes (namely, the degree of market incompleteness, and the nature of labor market shocks).

[Figure 1 about here]

A model with infinitely-lived households prevents some key life-cycle dimensions to play a role in how detrimental incomplete insurance possibilities might be. In particular, young and asset poor individuals typically prefer more public liquidity, unless this positive effect on their welfare is offset by lower wages (due to a crowding out of investment in physical capital) and more expensive borrowing (due to an increase in the asset demand). Recently, also Peterman and Sager (2018) have included life-cycle motives in their analysis of optimal public
debt, finding that this has important implications for their welfare measure, the expected utility of the newborns. However, their contribution differs from mine, as I consider two open economies, a richer specification for the income process, a calibration targeting the Canadian economy and an extension with more general preferences that leads to a different explanation for large public debts. In terms of the income dynamics, I rely on state of the art estimates obtained from income tax returns by Baker and Solon (2003). Their econometric specification nests the standard persistent/permanent decomposition of income shocks into a framework with heterogeneity in the growth rate of labor earnings during the life-cycle. This formulation leads to substantial heterogeneity in income dynamics during the active years in the labor market, together with different pensions upon retirement. To the best of my knowledge, there are no other contributions in the literature of optimal public debt considering such a general specification for the income dynamics. The considerable amount of redistribution present in Canada via the fiscal system might be effective in counteracting the considerable earnings risk that the Canadian households are facing. This begs the question of whether the liquidity provision attained with public debt is warranted, as it might inefficiently provide excessive insurance against labor market shocks. A two-region life-cycle model allows to quantify more accurately the importance of the GE effects on wages and the interest rate. This is a critical aspect, as these influence the effectiveness of self-insurance via the accumulation of risk-free assets, versus the lost income and lower pensions arising from a lower level of aggregate capital. In this setup I find that, under several modeling assumptions, the optimal quantity of debt for Canada is large and negative. This means that the Federal and Provincial governments should be net savers, rather than being net borrowers. To facilitate the comparison with calibration studies done on the U.S., I then conduct some quantitative experiments assuming that the Canadian economy is closed. In these counterfactual scenarios, I do find that sizable public debts can be optimal, at least according to some welfare measures. I also find a generational conflict, as in this case the welfare effects of very young individuals are opposite in sign to those of older ones, an outcome that is not observed in the open economy versions of the model.

A model with a wasteful public expenditure has some appealing features, as a positive welfare effect of public debt could then be interpreted as a lower bound. However, in a scenario where public debt is found to be unequivocally detrimental for welfare, the notion of an entirely wasteful public expenditure can be problematic. Governments do provide valuable services, either in terms of public infrastructure, a hypothesis considered in the literature by Chatterjee, Gibson, and Rioja (2017) and Chatterjee, Gibson, and Rioja (2018), or in terms of publicly provided goods that directly affect the welfare of the households. This instance is ruled out by most papers in the literature of public debt with incomplete markets, as typically public spending is interpreted as consumption of the public sector, and it is wasteful by assumption (differently from public transfers, which can be beneficial as stressed by Floden (2001)). The Canadian experience is noteworthy in this dimension, as key goods and services, such as health and education, are publicly provided. I extend the model to capture this feature, by including a public good that affects the utility of the households. Since there is limited direct information on the utility weight that individuals might place on private vs. public goods, and the results can be heavily affected by this element, I postulate a stylized political economy mechanism to identify this key parameter. The status quo public policy is determined according to a voting maximizing mechanism. This can be considered as a model with office motivated politicians, that select the level of public debt in order to
maximize the political support for this policy. In this light, I then provide an alternative explanation for large public debts. Lowering the public debt can be achieved by partially cutting the public expenditure. If the wasteful consumption of the public sector cannot easily be targeted as a source of budget savings, the welfare of the households will eventually be negatively affected, when the provision of public goods falls below the related optimality principle. The results show that the long-run Canadian public debt has been inefficiently high, and that a positive –yet moderate– public debt is optimal.

I perform an extensive robust analysis. The main results found in the two-region model without valuable public goods go through also when considering Canada as a small open economy, when financial markets demand a risk premium for larger Debt/GDP ratios, and when the government finances pure public goods also relying on public deficits, but without adjusting the public expenditure in the same direction.

In the next subsection I briefly present some evidence related to public debt and public expenditure in the Canadian context.

1.1 The Canadian Public Debt and Public Expenditure in the Long Run

Figure 2 includes three panels, which plot the time series of Canadian Public Debt and Public Expenditure. The top panel displays the longest available series for the gross Canadian Public Debt, expressed as a fraction of GDP. The Canadian Public Debt has been quite volatile, with the Public Debt/GDP ratio being anywhere between 30% and 150%. As observed in several other countries, public debt spiked during WWII, and quickly fell to less extreme values shortly thereafter. In this dataset, the long-run average of the Public Debt/GDP ratio in the post WWII era (1952-2012) is 68.5%, while the average for the whole sample period is 61.8%. The middle panel relies on Cansim data, which measure the net Public Debt/GDP ratio. These data cover a considerably shorter period, but are quarterly and disaggregated by level of government. It is interesting to notice that in the last decade the Federal Public debt as a share of GDP has been declining, but it has been replaced by Public debt accumulated by other levels of government. The quantitative model will be constrained by data limitations in terms of the reference time period, especially with respect to the income dynamics. It is worth reporting that also the Cansim data in the 1990-2007 period, namely closer to the income data and pre-dating the great recession, is 68.5%. For these reasons, I am going to consider this figure as the long-run value of the Canadian Public Debt/GDP ratio.

The bottom panel displays the time series of Canadian Public Expenditure/GDP ratio. Abstracting from the short-run fluctuations, a noteworthy feature of this series is the emergence of two long-run levels of public expenditure. The first regime covers the years from the early 1960’s until the early 1990’s, where Public Expenditure/GDP ratios around 25% were the norm. Starting in the 2000’s, this ratio has settled around a much lower value of approximately 21%. This is a further indication that the Public Debt dynamics are

1For a more sophisticated treatment of the political-economy equilibrium in models with incomplete markets see Corbae, D’Erasmo and Kuruscu (2009). Early computational models of voting, for example Krusell, Quadrini and Rios-Rull (1997), circumvented some of the numerical challenges by limiting the amount of heterogeneity.

2This series was compiled by Abbas, Belhocine, El-Ganainy and Horton (2011), and it is included in a database maintained by the IMF.
positively correlated with the dynamics of Public Expenditure.

[Figure 2 about here]

1.2 Related Literature

The literature on public debt has been very active of late, mainly because of the large public debts accumulated by several countries in the aftermath of the great recession, and the recent public debt crises that have been observed in Europe. Alesina and Passalacqua (2016) and Yared (2018) are two comprehensive surveys reviewing both the classical and the more recent explanations for public debt. Both contributions point out the need for methodical quantitative work, especially in models with a political economy explanation for public debt.

There is limited work on the role of aggregate shocks and public debt in models with household heterogeneity. Desbonnet and Kankanamge (2017) consider a simple version of the RBC model with incomplete markets, finding that the optimal amount of public debt is negative. In their paper, they also perform a comprehensive decomposition analysis for the steady-state version of the model. However, they do not allow for an open economy analysis or for a life-cycle component, which is found by Peterman and Sager (2018) to be the most important element in their decomposition of the welfare effects. Since the income processes these two contributions are working with are quite parsimonious, it is not straightforward that similar quantitative effects would be obtained in models with more general and empirically grounded income dynamics. Rohrs and Winter (2015) and Rohrs and Winter (2017) emphasize the distinction between the private vs. public provision of liquidity, also accounting for the transitional dynamics. The computational burden of my framework does not make feasible the study of the transition between steady states. However, given the open economy set-up, I speculate that the transition could be completed relatively quickly, making the steady-state comparison possibly quite accurate. As mentioned above, Chatterjee, Gibson, and Rioja (2017) argue that the role of public investment is key in the determination of the optimal debt/GDP ratio. Instead, in my most general setup, I will focus on a different role of the public sector, namely the provision of public goods.

Although understanding how the Canadian public debt has been accumulated to what I will find to be an inefficiently high level is beyond the scope of this paper, Desbonnet and Weitzenblum (2012) argue that the short-run gains can create a temptation to deviate toward higher levels of debt. Another possible argument is that short lived governments have used generous public transfers for electoral purposes, an instance that is discussed extensively in Yared (2018). In this paper I am going to propose a model with public goods provision that is closer in spirit to the latter channel. For tractability, I do not consider a model with endogenous default, an element included in D’Erasmo and Mendoza (2016).

The rest of the paper is organized as follows. Section 2 presents an open economy model for Canada. Section 3 discusses the model calibration. Section 4 presents the results and Section 5 a model extension with public goods. Section 6 concludes, and a set of Appendices present an in depth discussion of the theoretical model, more information on the data used for the quantitative analysis, and additional results.

5
2 A Two-Region Model

I start by considering a model with two asymmetric economies that share an integrated asset market. There are no restrictions to capital movements, but both economies have incomplete markets, whereby the workers cannot buy insurance for the labor market risks they are facing. For tractability, there is no international mobility of workers. Since the home economy is representing Canada, its size is considerably smaller compared to the RoW economy.\(^3\)

One might wonder why the two-region framework is to be favored compared to the simpler Small Open Economy (SOE) one. One reason is that Canada’s shares in global GDP and in capital markets are not trivially small, being 2.16% and 2.26%, respectively. Although it is plausible to expect a limited response of the interest rate arising from different public debt policies, under incomplete markets and with rich income dynamics this could still give rise to positive welfare effects of public debt. The SOE framework eliminates the possible positive role of public debt through improved self insurance by assumption. In a SOE an increase in public debt is absorbed by the world financial market without any effect on the foreign and domestic interest rates. On the one hand, this precludes the crowding out of investment, which would lead to a lower domestic capital stock, output and wages. At the same time, in the SOE scenario, public debt is partially wasteful as the additional asset income is enjoyed by foreigners, leaving the domestic residents to pay higher taxes that translate into a worsening of the current account. Solving the model with the SOE setup, I found that the optimal steady-state public debt is unambiguously negative and large, because public wealth would provide income subsidies rather than taxes to the domestic households via the asset income accrued on the accumulated public wealth. This outcome is extremely far from the available empirical evidence on public debt, questioning the suitability of the SOE assumption.\(^4\)

The two-region model is an extension of the Huggett (1996) economy, appropriately modified to allow for several sources of heterogeneity in labor income, public debt, and two open economies.\(^5\) The home economy has an Overlapping Generations (OLG) structure. One of the reasons of working with an OLG model is to accommodate the Canadian empirical evidence on the labor income dynamics, that emphasize the importance of the life-cycle elements. Newborn agents are ex-ante identical, but the idiosyncratic realizations of a series of shocks induce substantial ex-post heterogeneity. In particular, upon entering the labor market, the workers observe two characteristics that affect their earnings potential: the immutable individual effects for both the entry level \((f_0)\) and the growth rate of labor income \((f_\beta)\).

Given the rich dynamics displayed by the Canadian labor income data, the model allows for rich labor

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\(^3\)Because of the NAFTA agreement, considering the U.S. and Mexico as the relevant group of foreign economies would seem appealing. However, the zero long-run average of the Canadian net lending position is inconsistent with the sizable net borrowing position of the U.S., invalidating the two-region set-up.

\(^4\)I also solved the SOE version of the model closing the model assuming an ad-hoc domestic interest rate determination rule, adapting the ideas discussed by Schmitt-Grohe and Uribe (2003). I estimated the interest rate elasticity to public debt on a sample of OECD countries, such that the interest rate is decreasing in the home country’s public wealth. Also in this case the optimal public debt/GDP ratio was found to be negative and large, in excess of 200%. For more details see Appendix D.

\(^5\)For more details on the model, see Appendix A, Rios-Rull (1994), Rios-Rull (1996), Huggett (1996), and Guvenen (2007), among many others.
earnings dynamics, in order to capture the demand for redistribution and liquidity provision, and the need for self insurance.\footnote{Since the T-4 tax returns that form the basis for the estimation of the stochastic income process do not include the number of hours worked, it is not possible to estimate a process for wages or allow for a flexible labour supply margin.}

Regarding the mortality risk, I consider two different formulations. In one case, individuals can buy an actuarially fair life insurance, where there is pooling of the mortality risk across all ages. This formulation has the advantage of leaving the newborns with just their labor income, the government transfers and no additional resources, as they are born with no assets and receive no accidental bequests.\footnote{I also tried a more conventional formulation with annuities that affect the ex-post interest rate of the surviving agents on an age-to-age basis. Since agents can borrow, this formulation had the drawback that the surviving agents had to take care of the outstanding debts of the agents dying with negative asset holdings.} In another case, the accidental bequests are redistributed uniformly to all alive agents.

Production in both the home economy and in the foreign one relies on aggregate labor and aggregate capital to produce a homogeneous good that can be sold internationally. The production functions are Cobb-Douglas, with region-specific inputs and parameters. The input markets are perfectly competitive, and the firms maximization problem leads to the usual equilibrium conditions equating marginal products to the related input price.

The RoW has incomplete insurance markets, and the workers are subject to idiosyncratic income shocks. For tractability, I assume that the RoW region is populated by infinitely lived agents, that are subject to an exogenous borrowing constraint, and that their income dynamics are represented by a simple AR(1) stochastic process.

The interpretation of the RoW region is that there is a measure $\mu^{RoW}$ of foreign economies, each sharing the same preferences and the same technology, a simple Cobb-Douglas production function with aggregate labor ($L^{RoW}$) and capital ($K^{RoW}$) as inputs, and with a Total Factor Productivity parameter ($Z^{RoW}$).\footnote{I chose this formulation, instead of an alternative one with the RoW using an aggregate production function with RoW labor and capital as inputs, because it is possible to obtain all the relevant parameters by computing cross-country averages.} Also in these countries there are incomplete insurance markets and the workers are facing labor income risk, captured by an AR(1) income process. Given the absence of aggregate shocks, at the aggregate level each foreign country displays the same outcomes, and they can be easily considered as a unitary region.

Figure 4 depicts the determination of the equilibrium in the two-region economy. In the plot, the label CA stands for Canada (the Home economy), and RoW for the Rest of the World. As in the textbook two-large-economies model, the equilibrium interest rate is found when the current accounts of the two regions are consistent with each other. Given that the Canadian data on the long-run average of its net financial position show a value that is virtually zero, in the initial steady-state (namely, under the status quo Canadian public debt), the current accounts of both regions are forced by the calibration to be equal to zero. Notice that the home economy has two variables determining the total demand for assets, which are the demand for capital by firms ($K^{d,CA}$) coupled with the demand for savings to hold the public debt ($D^{CA}$). In both economies there is a supply of assets coming from the aggregation of the saving decisions of the households in Canada ($A^{s,CA}$) and in the other region ($A^{s,RoW}$). Any increase in the domestic public debt/GDP ratio brings about an outward shift in the Canadian asset demand. This causes an imbalance in the current accounts, which triggers an interest
rate adjustment, until the general equilibrium is restored.\(^9\)

[Figure 4 about here]

3 Calibration

In order to assign values to the parameters of the Two-Region model, I rely on a mix of (reduced-form) estimation and calibration (in equilibrium) methods. The initial steady-state is calibrated to reproduce selected long-run features of both the Canadian economy and the RoW. I begin by presenting the calibration for the home economy, then I move to present the one for the RoW region.

3.1 Home Economy Parameters

Table 1 reports the list of the calibrated parameters with their values and associated empirical targets. These parameters are the ones that apply to Canada, and that can be set externally, without solving the model.

[Table 1 about here]

**Demographics:** The following parameters are related to the demographic aspects: \(J_R, J, \pi^d_j\). Agents become economically active at age 25, retire at age \(J_R = 66\), and they can live up to \(J = 101\) years. These are conventional and fairly innocuous assumptions, also made in other studies calibrating OLG models. The survival probabilities \(\pi^d_j\) are obtained from the Canadian Life Tables. In particular, I averaged the values of each age-dependent survival rate reported for every year of the 1990-2007 period.\(^{10}\)

**Technology:** The labor share is computed from the Penn World Tables 9.0 (PWT9.0), and its long-run average implies a capital share \(\alpha = 34.5\%\). Also the capital depreciation rate is obtained from the PWT9.0, and its long-run average is equal to \(\delta = 0.0422\).

**Preferences:** The risk aversion is set to \(\theta = 1.50\), which matches an elasticity of intertemporal substitution of 0.75, a common value in the literature. In the closed economy version of the model, that eventually will be considered, the discount factor \(\beta\) is chosen to match an equilibrium capital/income ratio of 3.1. This value represents the long-run average of this statistic computed in the PWT9.0. The corresponding value is \(\beta = 1.017\).\(^{11}\)

\(^9\)It is worth mentioning that the welfare effects that will be reported do not stem from the OLG economy being dynamically inefficient, as in all equilibria (status-quo and counterfactuals) the interest rate (net of depreciation) is always positive. This guarantees that the economy is never in the dynamically inefficient region.

\(^{10}\)The two top panels in Figure 11 in Appendix D plot the survival probabilities over the life cycle together with the resulting age distribution.

\(^{11}\)In a previous version of the paper, also in the closed economy version of the model the discount factor \(\beta\) was chosen to match an equilibrium interest rate of 2.79\%. This value corresponds to the long-run average of the real interest rate of Canadian treasury
**Income process and pensions:** Thanks to the availability of both longitudinal surveys and administrative data, the literature on earnings dynamics has been proposing several econometric models to capture their evolution over the life-cycle. Even though the jury is still out on what is the most accurate statistical process capturing them, yet parsimonious enough to be used for quantitative equilibrium analysis, there is extensive evidence supporting labor income growth heterogeneity. Baker and Solon (2003) is one of the few systematic studies on Canadian administrative data, while Baker (1997), Guvenen (2009), Haider (2001) and Hause (1980) are other influential contributions in this field.

The baseline parameterization of the exogenous stochastic income process mainly relies on the values reported by Baker and Solon (2003). Their estimates, based on the T-4 tax return data, are $\sigma_s = 0.0837$ for the standard deviation (s.d., hereafter) of the permanent –Random Walk– income shocks (RW, hereafter), while $\rho_{\epsilon} = 0.54$ and $\sigma_{\epsilon} = 0.3579$ are the estimates of the autocorrelation and s.d. of the innovations for the persistent (AR1) component. The heterogeneity at labor market entry is captured by the dispersion of the two individual effects: $\sigma_{f_{\alpha}} = 0.058$ is the s.d. of the level fixed effect, while $\sigma_{f_{\beta}} = 0.0095$ is the s.d. of the slope fixed effect. Finally, the covariance between the level and slope fixed effects is $\sigma_{f_{\alpha}, f_{\beta}} = -0.0031$. Notice that the resulting correlation index between the two fixed effects is negative and very high, being $-0.89$. This implies that individuals with large labor earnings at labor market entry tend to have low wage growth over the life-cycle, while workers with low entry earnings enjoy a faster income grow. The top panel in Figure 3 shows these trajectories for three worker types.

![Figure 3 about here](image-url)

The average growth type included in the Figure represents 25.4% of the working age population, while both the low and high growth types each represent 11.4% of the working age population. Recall, that the workers have two correlated fixed effects: a level fixed effect and a growth one. Given the strong negative correlation between the two, the high growth types start their career with a low level of earnings and vice versa. An interesting feature of the earnings dynamics attributable to the fixed effects is that, absent the other shocks, close to the retirement age these three worker types would enjoy virtually the same level of earnings, of about $57,500. There is a type of labor income convergence taking place, as upon retirement (abstracting from other shocks) these worker types command virtually the same labor earnings. A possible interpretation of these dynamics sees on one side of the spectrum individuals with high educational achievements, who often begin their careers with low paying internships and co-op experiences. On the other side of the spectrum there could

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12Figure 10 in Appendix D plots the joint bi-normal density implied by these estimates.

13I discretize the fixed effects with six types in each dimension, for a total of thirty six possible pairs of fixed effects values (i.e., worker types). Notice that, not to clutter the figure, the average type plotted in the graph corresponds to the four middle types in the quantitative model. In order to maximize the accuracy of the numerical computations, I undertook a form of pruning of the discrete points in the support of the bi-normal density (taking into account that the two random variables are negatively correlated), and the symmetry of the shares at the extreme points of the grid is obtained by construction. By way of comparison, the other two extreme categories, high level-high growth and low level-low growth types, represent less than 0.001% of the labor force.
be athletes hired by professional teams, young entertainers, and other individuals involved in the show-business or other artistic careers, whose pay can be relatively large from early on, and possibly decline over time. In terms of the consumption smoothing behavior, and the latent demand for public liquidity, the low growth types are in a favorable position. Their high entry earnings allow them to start saving early on, providing them with a buffer stock of wealth that can be used against negative income shocks. For these agents, self insurance is extremely effective and not very costly, as they are also saving for retirement and to smooth their high labor earnings over their life-cycle. Differently, the high growth types are constrained for two reasons: they would like to borrow both against their substantial future incomes and to partially offset the negative income shocks.

Baker and Solon (2003) do not provide estimates for two parameters that are needed in the model for the computation of the labor efficiency profiles: the average growth rate of labor earnings during the life-cycle and the average intercept. Since the tax return data are confidential, I computed these two parameters estimating with OLS a simple log-wage regression on Labour Force Survey (LFS) data. The LFS is a representative sample of the Canadian workforce, and the estimates obtained in the two samples should be similar. The average of the level fixed effect is $m_{fα} = 10.10$ and the average of the slope fixed effect is $m_{fβ} = 0.013$.\(^{14}\)

In terms of dollar values, the deterministic component of earnings might appear low (2010 is the base year). However, one has to take into consideration that total labor earnings are on average higher compared to the values depicted in this plot, because the average of the other stochastic components of the income process is different from one. By construction, the Rowenhurst method used to discretize the AR1 component ensures that its corresponding average in levels is equal to one. Differently, the average of the RW component in levels is greater than one, as the shocks in logs are symmetric around zero.

The bottom panel of Figure 3 plots the earnings dynamics over the life cycle attributable to the Random Walk component. Two patterns are worth pointing out. First, since the variance of the shocks increases linearly, the dispersion of these permanent shocks grows over the life cycle. Second, since these shocks in logs are symmetric around zero, their average is zero, while the average of the RW component in levels is greater than one, because of Jensen’s inequality.

The pension replacement rate is taken from OECD (2015), and its value is $φ_R = 0.479$, while the pension contributions are financed with a proportional tax on the current labor earnings. The equilibrium value of the tax guarantees a balanced budget. There is dispersion in the pensions, as the pension benefits can be conditioned on the individual fixed effects. For computational reasons, I cannot keep track of the actual individual contributions to the system, as it would require an additional continuous state variable. The simpler approach used here assigns pensions on the basis of the average labor market histories, namely it considers the role of both the level and

\(^{14}\)I relied on the monthly LFS from January 1997 to December 2007. Notice that before 1997 wage data were not included in the LFS, so they cannot be used. Because of measurement error concerns, in each wave of the dataset and for every age group, I trimmed both the top and bottom 5% of the hourly wages. The wage data are adjusted for inflation by using the monthly CPI index. The model is yearly and the Baker and Solon (2003) T-4 data refer to yearly earnings, hence to make the data consistent with each other I multiplied the hourly wages in the LFS by the usual number of weekly hours worked. I then multiplied the weekly earnings by an estimate of the number of weeks worked in a year consistent with the data on aggregate number of hours worked available for the Canadian Economy. In the log-wage regression, I included a full set of time dummies, and I added to the intercept the time average of their estimated coefficients. I also performed the estimation on the whole sample, without trimming, and the values were quite similar. More details are reported in Appendix B.
growth fixed effect, while it collapses the effect of the other shocks to their unconditional averages.

**Taxes and Government:** The income taxes are proportional and are set to satisfy the intertemporal budget constraint of the government. The consumption tax rate is $\tau_c = 0.124$, which is the average of the estimates reported by Mendoza, Razin and Tesar (1994), which include both the PST and the GST.

The wasteful government consumption is set to match the long-run average over the 1990-2007 period of the public expenditure/GDP ratio, which is $\gamma = 0.214$. Over the same time horizon, and as described in Section 1.1, the public debt/GDP ratio is set at $D/Y = 0.685$.\(^{15}\)

**Asset Markets:** The borrowing limit is set to $a = 0.31$, such that households can borrow up to 20% of the average labor earnings. I conducted some robustness checks, setting $a = 0$ and the results are not heavily affected by this parameter.

### 3.2 Rest of the World Parameters

A number of parameters for the RoW economy can be directly estimated using the PWT9.0. Table 2 reports the full list of the calibrated RoW parameters with their values and associated empirical targets.

[Table 2 about here]

**Demographics:** The agents in the RoW region are assumed to be infinitely lived. This assumption circumvents the challenging step of obtaining life-cycle profiles for productivity and survival rates.

**Technology:** The labor share and the capital depreciation rate are computed from the PWT9.0. Since the data quality for some developing Countries can be problematic, I exclude the effect of potential outliers by considering the median in each annual cross-section. I then take the time average of this statistic’s time series. Computed this way, the RoW labor share is 55.9%, which implies $\alpha_{\text{RoW}} = 44.1\%$. A similar procedure gives a capital depreciation rate equal to $\delta_{\text{RoW}} = 0.0432$. In order to obtain the GDP for the RoW, the number of employed workers is needed. I compute this figure from the PWT9.0.

**Preferences:** Since there is no evidence that Canadian and foreign households differ systematically in their attitudes towards risk, I set the risk aversion parameter equal in the two economies $\bar{\theta} = \theta_{\text{RoW}} = 1.50$.

\(^{15}\)Even though the series are longer, I chose this time period because the data on public expenditure show two clear regimes, an earlier one that lasted until the early 90’s with the ratio hovering around the 25% value, and a more recent one with the ratio consistently close to the 20% figure.
Income process and Labor input: Since many Countries are lacking good-quality panel data with household-level information on labor income, for the RoW I postulate a simple AR(1) process. The persistence and the s.d. of the autocorrelated income shocks are obtained averaging the available estimates in the literature on income dynamics.\(^{16}\) This procedure gives \(\rho_{\text{RoW}} = 0.949\) and \(\sigma_{\text{RoW}} = 0.276\). Each country’s labor input averages to 1. The measure of the countries is estimated dividing the total number of workers in the PWT9.0 by the average number of workers, which gives \(\mu_{\text{RoW}} = 126.8\).

Government: Since I focus on the welfare effects for the home economy, stemming from policies implemented by the domestic government, I assume that the countries in the RoW region have an inactive public sector.

### 3.3 Parameters Set in Equilibrium

A number of parameters do not have a direct empirical counterpart, and need to be set in equilibrium. These parameters are reported in the middle section of Table 2.

The Canadian data on the national net lending/net borrowing position (Cansim series v62305900) show that the long-run average is virtually zero. This figure implies that the long-run average of the Canadian current account is approximately equal to zero. I then calibrate the Canadian discount factor to match this target in the initial steady state, obtaining \(\beta = 1.0123\). The discount factor of the RoW is set to match an equilibrium interest rate \(r^* = 2.79\%\). This figure corresponds to the average real interest rate for the period 1990-2007 paid by the Canadian Treasury bills. Since capital and public debt are both risk-free, arbitrage forces their rate of return to be equal. In a perfectly integrated financial market the interest rate for the RoW and home economies must be the same. Given that this paper focuses on the welfare effects of Canadian public debt, I chose to target the rate of return of this asset, and \(\beta_{\text{RoW}} = 0.958\) achieves this goal. The TFP for the home economy is normalized to 1, while I set the TFP for the RoW region to match the long-run average of Canada’s share in global asset markets. More precisely, I match the ratio between Canada’s and the RoW’s capital stocks, both taken from the PWT9.0. This figure is equal to \(K_{\text{CA}}/K_{\text{RoW}} = 2.26\%\), and \(Z_{\text{RoW}} = 0.31\) matches this target.

The transfers of the Canadian government \(\xi\) are also calibrated in equilibrium. This variable affects the degree of inequality in the economy, which is reduced by these unconditional (lump-sum) transfers. \(\xi = 0.19\) matches an income inequality pre-post government ratio of \(\text{Var}[\log(y^{\text{post}})]/\text{Var}[\log(y^{\text{pre}})] = 46\%\), which is the average value of this statistic reported by Brzozowski, Gervais, Klein and Suzuki (2010).

### 4 Results

In this section, I discuss the quantitative results related to the two-region model. The analysis is based on implementing a series of counterfactual public debt policies, considering the steady state implications for welfare, prices, aggregate variables and taxes.

\(^{16}\)In particular, I consider the estimates for the U.S. reported by Guvenen (2009), for Sweden reported by Domeij and Floden (2010), for Russia reported by Gorodnichenko, Peter and Stolyarov (2010), for France, Germany, Italy, Spain, and the U.K. reported by Le Blanc and Georgarakos (2013), and for Japan reported by Lise et al. (2014).
Figure 5 presents the equilibrium outcomes for the home economy. The first panel conveys a strong message. According to this model, in Canada the long-run public debt has been inefficiently large. The welfare of both the newborns and the households taken as a whole could be drastically improved by moving from a policy of public debt to a policy of public savings. The behavior of both welfare effect measures is very much alike and they also are quantitatively similar. A striking result is the size of the welfare gains of moving from the long-run public debt/GDP ratio of 68.5% to public savings. For example, a public debt/GDP ratio of −200% would entail welfare gains in excess of 20%.

The second panel displays the response of the interest rate. With an overall change of four times the size of Canadian GDP, the interest rate moves from 2.77% to 2.81%. As expected, this is quantitatively small, yet different from a completely flat behavior.

The third panel is devoted to representing the response of GDP and consumption. Given the marginal change in the interest rate, aggregate output only declines mildly, showing an insignificant effect of crowding out of investment in physical capital. Differently, the change is aggregate consumption is drastic. This is due to the rising costs of servicing the public debt. This moves from being an additional source of income on top of the public transfers, to being a disbursement mostly paid to foreigners.

This behavior is mirrored by the increase in the proportional tax rate on income, presented in the fourth panel. This variable changes from 25% to 42%, the status quo being characterized by a tax rate of about 35%.

By way of comparison, I conduct some quantitative experiments assuming that the Canadian economy is closed. Figure 6 plots the related results. In this counterfactual scenario, I do find that public debt could be optimal, at least according to some welfare measures, reconciling my results with those of Aiyagari and McGrattan (1998). The first panel in the figure shows that social welfare (the most natural measure to base the comparison on, as they have an infinitely lived households setup, while this version of the model has annuities) peaks at a positive and large value of public debt of 105%. Just like in Aiyagari and McGrattan (1998), this welfare measure is exceptionally flat for a large range of public debt policies. Interestingly, the newborns would prefer to have large public savings of 175%, a result that mirrors the findings in Peterman and Sager (2018), albeit in quite a different environment. This is due to the compounded effect of larger public transfers (partially financed by the public asset income), and larger wages per efficiency units, that ensure a better consumption smoothing in the early stages of the life-cycle. The related lower interest rates allow for more capital used in production, leading to larger output and larger aggregate consumption. The response of the tax rate is non-monotonic. Compared to the newborns’ welfare, Social welfare responds differently, as most individuals in the economy hold some wealth and are partially damaged by lower equilibrium interest rates, especially the retirees.

The discrepancy in the welfare measures is made clear by the age profiles of the welfare effects depicted in Figure 7. Unlike in the two-region economy, where the welfare effects have the same sign for virtually all ages,
in a closed economy there is a generational conflict, as the younger individuals have welfare effects that are opposite in sign compared to most of the older ones. Moreover, this conflict can be acute as the heterogeneity in the welfare effects can be extreme, the range being $[-12\%, +1\%]$, abstracting from the very few individuals older than 90.

4.1 Discussion

The quantitative results are in line with the intuition. A small economy with access to integrated financial markets can only have a second order effect in the determination of the interest rates. As a consequence, the liquidity provision effect of public debt is diluted, limiting its effectiveness in improving the self-insurance of borrowing constrained individuals. The results show that the liquidity effect channel is unlikely to be an important explanation for why also small economies have accumulated large public debts. Although the data show a positive correlation between the size of GDP and the Debt/GDP ratio, in the subsample including the smaller economies there is substantial variation in the latter variable. The results in this section plausibly make the liquidity provision rationale for public debt a quantitatively irrelevant explanation for several small and open Countries. In the next section I extend the model to allow for valuable public services, and to quantify whether a stylized political channel can help explaining the large Canadian public debt.

5 A Model with Public Goods

In this extension, the per-period utility function is defined over consumption $c$ and a pure public good $g$. I assume that there is a Cobb-Douglas aggregator between the private and public good, with $\eta$ denoting the weight of private consumption in the utility:

$$u(c, g) = \left( \frac{c^\eta g^{1-\eta}}{1-\eta} \right)^{\frac{1}{1-\theta}}.$$ 

Given the previous quantitative findings, in this section I am going to consider Canada as a SOE. The government is still in charge of distributing public transfers $TR$, and collecting taxes. However, now a given fraction of total government expenditure $G$ takes the form of a pure public good $g$. In my definition of publicly provided services I include several categories. The first one is the fraction of health expenditure that is covered by the public sector, which for Canada is approximately 70\%.

The other items are public spending on education, parks and defense. According to this notion of public services, their total value accounts for 12.8\% of GDP. In per-capita terms, this corresponds to 2010 CAD$5,919. It goes without saying that a number of these items do not qualify as pure public goods. However, since the demographic structure is exogenous, the model would imply the same results if I were to model them as publicly provided private goods. Incidentally, this formulation captures the fact that fiscal reforms cutting public expenditures might be only imperfectly substituted with their private counterpart.

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17 The difference between publicly provided goods and total government expenditure can be interpreted as the inefficiency of the public sector, namely a rents extraction mechanism, which is however assumed to be exogenous and not affected by different public debt policies.

18 In the last two decades, this share has been remarkably stable. The remaining 30\% is split between private insurance (12\%), out of pocket (15\%), and other sources (3\%).

19 Notice that I do not include police expenditures, as they are typically financed with property taxes set at the municipal level.
5.1 An Exploratory Fiscal Rigor Mechanism

A plausible candidate explanation for large public debts is that government spending also provides services that are valuable in terms of people’s well-being. In order to reduce public debt, typically “austerity” policies have to be implemented, with cuts in public expenditure generating the resources for the desired adjustment. Symmetrically, large public debts can be incurred when public expenditure rises quickly.

Figure 8 displays the mechanism at play in this extension and hints at a critical feature of this approach. The first two panels show the welfare effects of changing the value of public debt, which partially maps into a decrease of the quantity of public goods provided (on the basis of a correlation that will be estimated below). The first panel focuses on the welfare of the newborns, while the second one on social welfare. Different curves refer to the welfare effects for different values of the weight $\eta$. The plots show that both welfare effects behave similarly, qualitatively and quantitatively. It goes without saying that if the weight were to be equal to one, and if we were to consider a negative correlation between public debt policy and public goods provision, we would go back to the results of the previous section. The solid lines refer to a case where the individuals are characterized by an extremely high weight on private consumption, $\eta = 0.95$. As in the previous section, the welfare effects respond almost linearly to changes in public debt. It is apparent that larger weights bring about a non-linearity, as the gains in terms of lower taxes are offset by a decrease in the provision of public goods. Not only for a large enough $\eta$ the curves are no longer monotonic, but also the slope can now become positive for low values of public debt, turning into negative for higher values of public debt. This argument should make clear that, absent a reliable estimate for $\eta$, almost anything goes. Different weights of private consumption in the utility function imply different preferences for the quantity of public goods that should be provided. As the plot shows, with low values of $\eta$, individuals prefer larger public debts that are partially used to finance public goods.\(^\text{20}\)

The welfare analysis depends crucially on the rate of substitution between private and public goods. A reliable estimate of the curvature parameter in the utility function is needed. Figure 8 shows how the two welfare measures that I consider are affected by different values of the weight $\eta$. The lower the weight of private consumption in the utility function, the more concave the welfare measures become, making a positive optimal public debt more likely. I then use a revealed political preference approach: I calibrate this parameter in order to maximize the political support of the steady-state public policy.\(^\text{21}\)

From the third panel in the plot it is immediate to see what is the source of identification for the parameter $\eta$. Different values of $\eta$ imply different votes in favor of policies different from the status quo. In particular, I

\(^{20}\)Since I assume that there is no congestion, the analysis is providing an upper bound.

\(^{21}\)Notice that I cannot rely on the median voter theorem, as the policy is multidimensional. Furthermore, individuals differ in several dimensions: keeping track of the indirect utility of all agent types is computationally intractable, as for any possible public policy there is a continuous asset distribution for $$(6 \times 6 \times 5 \times 5 \times 41) + (6 \times 6 \times 35) = 38,160$$ types. Although there is no guarantee that the indirect utilities are single-peaked, I find that the number of votes are well-behaved.
consider two perturbations around the long-run value of public debt, and I select the value of $\eta$ that minimizes the (average) probability of the long-run public debt policy to be defeated in a proportional election.\footnote{In the quantitative implementation, the proportional votes respond smoothly in the parameter and there is no evidence of multiple political equilibria (e.g., if the curves were to display multiple maxima in the neighborhood of the steady-state value). Finally, I also impose the constraint that no alternative policy achieves a majority of votes.}

Many model parameters are calibrated following the same steps used in the previous Section. However, some of them need to be calibrated in equilibrium. Table 3 reports the list of the calibrated parameters with their values and associated empirical targets.

[Tables 3 and 4 about here]

The interest rate is set again to $r = 2.79\%$, but now it is exogenous and fixed. The discount factor is chosen to set the long-run current account to zero, and $\beta = 0.9882$ matches this target. The utility weight of consumption is set to the value that maximizes the political support for the observed long-run public debt, giving $\eta = 0.717$. For a given utility weight of consumption, the curvature of the utility function needs to be adjusted, to obtain the target relative risk aversion of 1.5. In the benchmark case, $\theta = 1.685$ matches this value.

Another, perhaps less trivial, aspect is the relationship between public debt and government expenditure. A mechanical implication of steady state analysis is that, for a fixed level of taxes, there is a long-run negative relationship between public debt and government expenditure. The reason is straightforward, because the government’s intertemporal budget constraint has to be satisfied. For constant taxes, lower public debts determine lower interest payments, freeing up resources that can be used to increase government expenditure. However, empirically there is no such negative relationship. In fact, I ran regressions on OECD data that show a positive correlation, and are reported in Table 4.\footnote{It is worth stressing that these regressions are not designed to identify a causal effect. The goal is to obtain an estimate of the conditional partial correlation between public debt and public expenditure.} An interpretation of this finding is that, in order to obtain changes in the level of public debt, considerable fiscal adjustments have to be undertaken. Governments worried about the size of their public debts typically implement cuts in their public expenditures, entering a period of fiscal rigor. Symmetrically, public debts increase in periods of fiscal recklessness. Imposing this correlation in the counterfactual analysis allows to mimic the implications of either cuts in public spending implemented to reduce the public debt, or increases in public spending that imply a rising public debt. Unfortunately, the size of the estimate varies considerably depending on the method used. I used both a Fixed Effects (FE) estimator on the OECD panel data, and a simple OLS estimator on the cross sectional data obtained from computing each country’s time averages. Since there are advantages and drawbacks for both estimators, I consider the quantitative implications of both sets of estimates.\footnote{This argument tends to suggest that the welfare analysis should consider transitional dynamics. However, the steady-state model is already computationally demanding (the calibration takes more than 150 hours), making the computation of the transition not feasible. Moreover, a transitional analysis is not necessarily desirable as it would require arbitrary assumptions on the length of the adjustment period and the timing of tax changes. Since there is virtually no guidance in the choice of these variable, there is too large a number of potential combinations that prevent it from making it operational.}
Figure 9 displays the welfare effects together with the responses of both aggregate consumption and public goods. The two top panels display the results based on the partial correlation between public debts and public expenditure estimated with OLS on the time-averages by country. The bottom panels refer to the FE estimates on the panel data. Compared to the wasteful public expenditure case the results are strikingly different. On the one hand, unlike in the closed version of the model, the behavior of the two welfare measures is extremely similar. As discussed above, the welfare response is no longer monotonic. Moreover, the calibrated weight implies that welfare peaks at positive values of the public debt ratio, which are not too far from the status-quo. Because of the sharp negative fall in the public good, the FE case shows that the status quo, supported by a majority of the voters, is a relatively robust equilibrium. This is because the change in welfare afforded by moving to the optimum is relatively small. For a debt ratio below 60% the welfare effects become negative, meaning that a reduction in public debt coupled with a reduction in public goods (and taxes) can be detrimental. The results are similar, but less drastic for the OLS case. Not only public debt reductions can determine a welfare loss, but also these losses can be extremely large. In the OLS (FE) case, individuals are indifferent between the status quo and a public debt/GDP ratio of 33% (60%). The social welfare maximizing debt ratios of 47% (63%) would improve social welfare by 1.83% (1.61%), while the newborns’ maximizing debt ratios of 49% (63%) would improve the newborns’ welfare by 1.44% (1.24%).

These results have a clear interpretation: Canada should not increase its long-run public debt, even if this were to be incurred to finance a proportional increase of publicly provided goods and services. The long-run public debt is already inefficiently high, but because of the asymmetry of the welfare effects response, public debt could be reduced with an associated welfare gain. However, if the cuts in publicly provided goods were to be too extreme, this adjustment could easily lead to large welfare losses.

6 Conclusions

Given the prevalence of public debt also in small economies, the results of the benchmark model cast doubts on the liquidity provision rationale for public debt. Quantitatively, in a model with two open economies, the negligible response of the interest rate dampens one of the positive effects of public debt. Political economy mechanisms and the associated distortions, advocated for example by Alesina and Passalacqua (2016), Battaglini (2011) and Yared (2018), seem to be quantitatively relevant, a more plausible explanation and a promising avenue for future research.

To conclude, the results show that the long-run value of Canadian Public Debt has been too high. In a two-region model with incomplete markets with wasteful government consumption there is a stark message: the Canadian government should have accumulated public wealth. The amount of redistribution already in place and the minimal equilibrium effects on the interest rate make public debt an ineffective tool for the provision of resources to liquidity constrained individuals. Taking into consideration valuable government services, together with public debt changes positively correlated with changes in public expenditure, I still find that Canadian public debt has been excessive. However, quantitatively the welfare gains of moving to the optimal quantity of debt are substantially smaller, and imply a positive and sizable stock of public debt. For an empirically plausible
parameterization of the model, the political channel implies an over-accumulation of public debt, which can be reduced only at the expenses of cutting public expenditure. The results show that the welfare gains of reducing the long-run level of public debt are fairly small. However, the welfare losses of increasing it can be substantial. The quantitative framework shows social welfare losses of 2.5% and 5.5% when moving to a 80% or 90% Public Debt/GDP ratio, respectively, so Public Debt should not be increased. The political economy framework is very stylized: future work could consider alternative, and perhaps more realistic, voting protocols, as done in Azzimonti, Battaglini and Coate (2016) or Corbae, D’Erasmo and Kuruscu (2009).
Figure 1: The Liquidity Channel Vs. the OECD Evidence. Public Debt/GDP ratio and GDP relative to the USA, OECD data 1995-2015.
Figure 2: Public Debt/GDP, IMF and Cansim data, Table 0378-0125; Public Expenditure/GDP. The Gross Public Debt/GDP
Panel 1: Deterministic Component ($1,000)

Panel 2: Random Walk Component

Figure 3: Pre-tax Earnings Dynamics over the Life-Cycle, 2010 CAD$.
Figure 4: Equilibrium determination in the two-region economy. CA stands for Canada (the Home economy) and RoW for the Rest of the World. In the initial steady-state, under the status quo public debt, the current accounts of both regions are equal to zero.
Figure 5: Equilibrium outcomes for the two-region economy.
Figure 6: Equilibrium outcomes for the closed economy.
Figure 7: Age-Dependent Welfare Effects.
Figure 8: Welfare effects and proportional votes responses to different values of the parameter $\eta$. 
Figure 9: Equilibrium outcomes for the Small Open Economy. The top panels refer to the analysis done on the basis of the partial correlation between estimated with OLS on the time-averages by country. The bottom panels refer to the Fixed Effects estimates on the Panel data.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model Period</strong></td>
<td>Year</td>
<td>Frequency of T-4 Tax Return Data</td>
</tr>
<tr>
<td>J - Maximum Age</td>
<td>76</td>
<td>Certain death at age 101</td>
</tr>
<tr>
<td>( J_R ) - Maximum Working Age</td>
<td>41</td>
<td>Retirement at age 66</td>
</tr>
<tr>
<td>( \pi^d_j ) - Death probabilities</td>
<td>-</td>
<td><strong>Canadian Life Tables, average for 1990-2007, see Appendix D</strong></td>
</tr>
<tr>
<td>( \delta ) - Capital depreciation rate</td>
<td>0.0422</td>
<td><strong>Capital depreciation estimates, PWT9.0</strong></td>
</tr>
<tr>
<td>( \alpha ) - 1-Labor share</td>
<td>0.345</td>
<td>Labor share of output = 65.5%, PWT9.0</td>
</tr>
<tr>
<td>( \theta ) - Risk Aversion</td>
<td>1.5</td>
<td><strong>Elasticity of Intertemporal Substitution = 0.75</strong></td>
</tr>
<tr>
<td>( \rho_e ) - Persistence of the AR(1) income shocks</td>
<td>0.540</td>
<td>Baker and Solon (2003)</td>
</tr>
<tr>
<td>( \sigma_e ) - S.d. of the AR(1) income shocks</td>
<td>0.358</td>
<td>Baker and Solon (2003)</td>
</tr>
<tr>
<td>( \sigma_s ) - S.d. of the Random Walk income shocks</td>
<td>0.0837</td>
<td>Baker and Solon (2003)</td>
</tr>
<tr>
<td>( \sigma_{\text{fs}} ) - S.d. of the level fixed effect</td>
<td>0.366</td>
<td>Baker and Solon (2003)</td>
</tr>
<tr>
<td>( \sigma_{\text{fs}} ) - S.d. of the slope fixed effect</td>
<td>0.0095</td>
<td>Baker and Solon (2003)</td>
</tr>
<tr>
<td>( \sigma_{\text{fs}} ) - Covariance between fixed effects</td>
<td>−0.0031</td>
<td>Baker and Solon (2003)</td>
</tr>
<tr>
<td>( m_{\text{fs}} ) - Mean of the level fixed effect</td>
<td>10.102</td>
<td>** Intercept of a Log-earnings regression on LFS data**</td>
</tr>
<tr>
<td>( m_{\text{fs}} ) - Mean of the slope fixed effect</td>
<td>0.0131</td>
<td>** Gradient of a Log-earnings regression on LFS data**</td>
</tr>
<tr>
<td>( \gamma ) - Government Consumption</td>
<td>0.214</td>
<td><strong>G/GDP = 21.4%, average for 1990-2007</strong></td>
</tr>
<tr>
<td>( \tau_c ) - Consumption Tax Rate</td>
<td>0.124</td>
<td><strong>Average GST and PST, Mendoza, Razin and Tesar (1994)</strong></td>
</tr>
<tr>
<td>( \phi_R ) - Pension Replacement Rate</td>
<td>0.479</td>
<td><strong>Average pension/average earnings ratio, OECD (2015)</strong></td>
</tr>
<tr>
<td>( d ) - Government Long-Run Debt</td>
<td>0.685</td>
<td><strong>Debt/GDP = 68.5%, average for 1990-2007</strong></td>
</tr>
<tr>
<td>( a ) - Borrowing limit</td>
<td>0.31</td>
<td><strong>Households can borrow up to 20% of average labor earnings</strong></td>
</tr>
</tbody>
</table>

Table 1: Calibration of the Two-Region Model, Parameters Set Externally for Canada
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta^{RoW}$</td>
<td>0.0432</td>
<td>Capital depreciation estimates, PWT9.0</td>
</tr>
<tr>
<td>$\alpha^{RoW}$</td>
<td>0.441</td>
<td>Labor share of output = 55.9%, PWT9.0</td>
</tr>
<tr>
<td>$\theta^{RoW}$</td>
<td>1.5</td>
<td>Elasticity of Intertemporal Substitution = 0.75</td>
</tr>
<tr>
<td>$\rho_{\varepsilon}^{RoW}$</td>
<td>0.949</td>
<td>Cross-country average, see text</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon}^{RoW}$</td>
<td>0.276</td>
<td>Cross-country average, see text</td>
</tr>
<tr>
<td>$L^{RoW}$</td>
<td>152.6</td>
<td>Size of the RoW labor force</td>
</tr>
<tr>
<td>$\mu^{RoW}$</td>
<td>126.8</td>
<td>Total RoW Employment/Average Employment ratio</td>
</tr>
<tr>
<td>$\beta^{RoW}$</td>
<td>0.958</td>
<td>$r = 2.79%$, T-bills real interest rate, average for 1990-2007</td>
</tr>
<tr>
<td>$Z^{RoW}$</td>
<td>0.31</td>
<td>Canadian Capital/World Capital = 2.26%, PWT9.0</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.19</td>
<td>Income inequality ratio, pre-post Gov. $\frac{\text{Var}[\log(y^\text{Post})]}{\text{Var}[\log(y^\text{Pre})]} = 46%$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.02</td>
<td>Borrowing and lending interest rates gap = 2%</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.827</td>
<td>Estimate on OECD data, see Table 5</td>
</tr>
</tbody>
</table>

Table 2: Calibration of the Two-Region Model, Parameters Set Externally for the RoW and Parameters Set in Equilibrium
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_D$ - Interest rate on Public Debt</td>
<td>0.0279</td>
<td>T-bills real interest rate, average for 1990-2007</td>
</tr>
<tr>
<td>$\gamma_g$ - Public Good</td>
<td>0.128</td>
<td>$g$/GDP = 12.8%, 2017 value</td>
</tr>
<tr>
<td>$\rho_{g,D}$ - G/GDP and D/GDP Correlation</td>
<td>${ - }$</td>
<td>OLS and FE estimates, see Table 4</td>
</tr>
</tbody>
</table>

Calibrated in Equilibrium:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ - Discount factor</td>
<td>0.9882</td>
<td>C.A. = 0, avg for 1990-2007</td>
</tr>
<tr>
<td>$\eta_{OLS}$ - Consumption weight in Utility</td>
<td>0.717</td>
<td>Max votes for the status-quo D/Y, see text</td>
</tr>
<tr>
<td>$\theta_{OLS}$ - Curvature of Utility Function</td>
<td>1.685</td>
<td>Relative Risk Aversion = 1.5</td>
</tr>
<tr>
<td>$\xi$ - Government Transfers</td>
<td>0.19</td>
<td>Income inequality ratio, pre-post Gov. $\frac{\text{Var}[\log(y^{Post})]}{\text{Var}[\log(y^{Pre})]} = 46%$</td>
</tr>
</tbody>
</table>

Alternative Calibration:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{FE}$ - Cons. Weight in Utility</td>
<td>0.730</td>
<td>Max votes for the status-quo D/Y, see text</td>
</tr>
<tr>
<td>$\theta_{FE}$ - Curvature of Utility Function</td>
<td>1.697</td>
<td>Relative Risk Aversion = 1.5</td>
</tr>
</tbody>
</table>

Table 3: Calibration of the Public Goods Model, all the other parameters are set to their benchmark value.
<table>
<thead>
<tr>
<th>Variable</th>
<th>FE</th>
<th>OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>netgdp</td>
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</table>

The dependent variable is the Public Debt/GDP ratio. Standard errors in parentheses.

Table 4: The (D/Y, G/Y) Correlation in the OECD Data
References


Appendix A - The OLG Model and its Recursive Representation

A - Stationary Equilibrium

The economy is a production economy with an endogenous asset distribution, where a government collects taxes to finance both an exogenously given stream of public expenditures and a pension scheme. The model is an extension of the Huggett (1996) economy, appropriately modified to allow for several sources of heterogeneity in labor income. Beside the workers, there is a representative firm that produces the final output relying on a Cobb-Douglas production function on aggregate labor and aggregate capital.

Time is discrete. The economy is populated by finitely lived agents facing an age-dependent death probability \( \pi_j^d \). Age is denoted with \( j \) and there are \( J \) overlapping generations, each consisting of a continuum of agents. At age \( J_R \) all agents that are still alive become retirees. Regarding the mortality risk, I consider two different environments. In the benchmark case, there is a perfectly competitive annuity market, where the agents share their mortality risk. It follows that in this version of the model there are no accidental bequests, as the average asset holdings of the individuals that die are zero. In an alternative case, there are accidental bequests denoted by \( b \), which are distributed uniformly to all the surviving agents, and the newborns.

Preferences: Agents’ preferences are assumed to be time-separable and represented by the utility function \( U(\cdot) \). Agents’ utility is defined over stochastic consumption \( \{c_j\}_{j=1}^J \) sequences: their aim is to choose how much to consume \( (c_j) \) and how much to save in an interest bearing asset \( (a_{j+1}) \) in each period of their lives, in order to maximize their objective function. The agents’ problem can be defined as:

\[
\max_{\{c_j,a_{j+1}\}_{j=1}^J} \mathbb{E}_1 U(c_1, \ldots, c_J) = \max_{\{c_j,a_{j+1}\}_{j=1}^J} \mathbb{E}_1 \sum_{j=1}^J \beta^{j-1} \left[ \prod_{s=1}^j \left( 1 - \pi_s^d \right) \right] u(c_j)
\]

where \( \mathbb{E}_1 \) represents the expectation operator over the idiosyncratic sequences of shocks, and \( \beta > 0 \) is the subjective discount factor. In the benchmark formulation, I assume that \( u(c_j) = \frac{c_j^{1-\theta} - 1}{\theta} \), that is the per-period utility function is strictly increasing in consumption, strictly concave, satisfies the Inada conditions, and has a constant relative risk aversion. The labor supply is fixed and equal to the time endowment multiplied by the value of the efficiency units.

Endowments: Agents differ in their labor endowments \( \epsilon_{j,s,\varepsilon,\alpha,\beta} \). There are five channels that contribute to the determination of the total efficiency units that the workers supply in the labor market. First, there is an age component \( j \), which interacts with one of the fixed effects, namely the growth rate of individual productivity. Second, there are two correlated fixed effects, \( f_\alpha \) and \( f_\beta \), that are jointly normally distributed and are observed at labor market entry. Finally, there are two time-varying shocks, one being a stochastic component \( \varepsilon \) that follows a stationary AR(1) process with normally distributed shocks \( \nu^\varepsilon \), and the other one being a stochastic component \( s \) that follows a random walk process RW with normally distributed shocks \( \nu^s \). The income dynamics of the life-cycle for a generic individual \( i \) are represented as follows:
\begin{align*}
\epsilon_{i,j} &= f_{\alpha,i} + f_{\beta,i} \ast (j-1) + \epsilon_{i,j} + s_{i,j} \\
\epsilon_{i,j} &= \rho_{j} \epsilon_{i,j-1} + \nu_{i,j} \\
s_{i,j} &= s_{i,j-1} + \nu_{i,j} \\
(f_{\alpha,i}, f_{\beta,i}) &\sim iid \ N(0,0; \sigma_{\alpha}^2, \sigma_{\beta}^2), \nu_{i,j} &\sim iid \ N(0, \sigma_{\epsilon}^2), \nu_{i,j} &\sim iid \ N(0, \sigma_{\epsilon}^2)
\end{align*}

Since the regression analysis is performed on the log transformation of labor earnings, the total efficiency units a worker is endowed with is the exponential transformation of the product of all the components described above. It follows that labor earnings are \( y_{j}^w = w \exp(\epsilon_{j,s,e,f_{\alpha},f_{\beta}}) \). After the common retirement age \( J_R \), the labor endowment drops to zero, and the agents receive a pension \( y_{j}^w \) above. It follows that labor earnings are a subset of the popular Tauchen/Hussey procedure. The transition function of the auto-regressive component of the labor endowment shock is represented by the matrix \( \Pi \) while that of retirees with \( \Pi_R \). Income taxes are levied, with a proportional tax rate \( \tau \), to finance interest payments on public debt \( rD \), public expenditure \( G \), and public transfers \( TR \). Agents can insure against their mortality risk. As a consequence, on average agents die with zero wealth. Newborns enter the economy with a zero asset endowment and with the average realization of the stochastic component of labor earnings, which is normalized to 1.

Now the problem of the agents in their recursive representation is defined, then I provide a formal definition of the equilibrium concept used in this model, the recursive competitive equilibrium. The individual state variables are: age \( j \in J = \{1, ..., J\} \), the level fixed effect \( f_{\alpha} \in \mathcal{F}_{\alpha} = \{f_{\alpha,\min}, ..., f_{\alpha,\max}\} \), the slope fixed effect \( f_{\beta} \in \mathcal{F}_{\beta} = \{f_{\beta,\min}, ..., f_{\beta,\max}\} \), the persistent shock component of the labor endowment \( s \in \mathcal{S} = \{s_{\min}, ..., s_{\max}\} \), the auto-regressive shock component of the labor endowment \( \epsilon \in \mathcal{E} = \{\epsilon_{\min}, ..., \epsilon_{\max}\} \) and asset holdings \( a \in \mathcal{A} = [-b, a] \). Notice that \( \epsilon \) is discretized with the Rouwenhorst method, using a 5-state Markov chain, while \( s \) is discretized using a 5-state grid whose elements are functions of the s.d., and are changing over the working age, as the variance of the random walk increases linearly. This method is an extension for a life-cycle set-up of the popular Tauchen/Hussey procedure. The transition function of the auto-regressive component of the labor endowment shock is represented by the matrix \( \Pi \) while that of retirees with \( \Pi_R \). The transition function of the auto-regressive component of the labor endowment shock is represented by the matrix \( \Pi \) while that of retirees with \( \Pi_R \). The transition function of the auto-regressive component of the labor endowment shock is represented by the matrix \( \Pi \) while that of retirees with \( \Pi_R \). The transition function of the auto-regressive component of the labor endowment shock is represented by the matrix \( \Pi \) while that of retirees with \( \Pi_R \).

### A.1 - Problem of the Agents

The model is solved backwards, starting from the terminal age \( J \) and with the assumption that the terminal utility value is zero, i.e. \( V_{J+1} = 0 \).

**Problem of the retirees.** The value function of an age-\( j \) retired agent whose current asset holdings are equal to \( a \) is denoted with \( V_{j}^R(a, f_{\alpha}, f_{\beta}) \). The problem of these agents can be represented as follows:

\[
V_{j+1}^R(a, f_{\alpha}, f_{\beta}) = \max_{s, \epsilon, f_{\alpha}, f_{\beta}} \left\{ f_{\alpha} \times (f_{\beta} \ast (j-1)) \times \epsilon \times s \right\} \Phi_j,
\]

and are normalized to add up to 1.
\[
V^R_j(a, f_\alpha, f_\beta) = \max_{c, a'} \left\{ u(c) + \beta \left( 1 - \pi^d_j \right) V^R_{j+1}(a', f_\alpha, f_\beta) \right\}
\]

\[
\text{s.t.}
\]
\[
(1 + \tau_c)c + a' = [(1 + r)/(1 - \pi^d)]a + (1 - \tau)\bar{y}_R(a, f_\alpha, f_\beta) + tr + b
\]
\[
c \geq 0, \quad a' \geq 0
\]

The value functions are indexed also by the two individual effects, as the pension benefits are a fixed replacement rate of the average labor earnings. Because of the related computational burden, the transitory shocks do not enter in the computation of the pension, as they would require keeping track of another continuous state variable. Instead, for the pension computation I consider the average labor earnings arising from the permanent components of the labor efficiency units and the unconditional means of the other stochastic elements. Since the AR1 component is mean reverting, and the dispersion of the RW component is not too wide, this simplification should not heavily affect the results. In the budget constraint notice the presence of the after-tax pension payment \((1 - \tau)\bar{y}_R(a, f_\alpha, f_\beta)\), the public transfer \(tr\) and (possibly) the accidental bequest \(b\). In the model with annuities, there are no accidental bequest, but the gross interest rate is adjusted by the average survival probability \((1 - \pi^d)\).

**Problem of the workers.** The value function of a working-age agent whose current asset holdings are equal to \(a\), whose current efficiency units shock is \(\varepsilon\) and whose fixed effects are \(f_\alpha\) and \(f_\beta\) is denoted with \(V_j(a, s, \varepsilon, f_\alpha, f_\beta)\). The problem of these agents can be represented as follows:

\[
V_j(a, s, \varepsilon, f_\alpha, f_\beta) = \max_{c, a'} \left\{ u(c) + \beta \left( 1 - \pi^d_j \right) \sum_{\varepsilon', s'} \pi(\varepsilon', \varepsilon) \pi(s', s) V_{j+1}(a', s', \varepsilon', f_\alpha, f_\beta) \right\}
\]

\[
\text{s.t.}
\]
\[
a \geq 0 : (1 + \tau_c)c + a' = [(1 + r)/(1 - \pi^d)]a + (1 - \tau)w_{\varepsilon_j, s, \varepsilon, f_\alpha, f_\beta} + tr + b
\]
\[
a < a < 0 : (1 + \tau_c)c + a' = [(1 + r^b)/(1 - \pi^d)]a + (1 - \tau)w_{\varepsilon_j, s, \varepsilon, f_\alpha, f_\beta} + tr + b
\]
\[
a_0 = 0, \quad c \geq 0, \quad a' > a
\]

Non-retired agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they can still be alive, and with probability \(\pi(\varepsilon', \varepsilon)\) they transit from their current efficiency units \(\varepsilon\) to the value \(\varepsilon'\). These agents pay income taxes \(\tau w_{\varepsilon_j, s, \varepsilon, f_\alpha, f_\beta}\) to finance total government outlays. They also pay a proportional tax \(\tau_R\) on their labor earnings to finance the pension scheme. Finally, they are born with the average shock \(\bar{\varepsilon}\), with no wealth and are subject to an exogenous borrowing constraint, \(a \leq 0\).
A.2 - Recursive Stationary Equilibrium

Definition 1: For given public policies \( \{\tau, \tau_c, \phi_R, G/Y, TR/Y, D/Y\} \) a recursive stationary equilibrium is a set of home decision rules, \( \{c_j (a, s, \varepsilon, f_\alpha, f_\beta), a'_j (a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R} \) and \( \{c^R_j (a, f_\alpha, f_\beta), a'^R_j (a, f_\alpha, f_\beta)\}_{j=J_R}^J \), foreign decision rules, \( \{c_{\text{RoW}} (a, \varepsilon), a'_{\text{RoW}} (a, \varepsilon)\} \), value functions, \( \{V_j (a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R} \) and \( \{V^R_j (a, f_\alpha, f_\beta)\}_{j=J_R}^J \), prices \( \{r, w, w_{\text{RoW}}\} \), and a set of stationary distributions, \( \{\mu_j (a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R} \) and \( \{\mu^R_j (a, f_\alpha, f_\beta)\}_{j=J_R}^J \), and \( \mu_{\text{RoW}} (a, \varepsilon) \), such that:

- Given relative prices \( \{r, w\} \), taxes and pension benefits \( \gamma_R (f_\alpha, f_\beta) \), the individual home policy functions \( \{c_j (a, s, \varepsilon, f_\alpha, f_\beta), a'_j (a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R} \) and \( \{c^R_j (a, f_\alpha, f_\beta), a'^R_j (a, f_\alpha, f_\beta)\}_{j=J_R}^J \) solve the household problems (1)-(2), and \( \{V_j (a, s, \varepsilon, f_\alpha, f_\beta)\}_{j=1}^{J_R} \) and \( \{V^R_j (a, f_\alpha, f_\beta)\}_{j=J_R}^J \) are the associated value functions.

- Given relative prices \( \{r, w_{\text{RoW}}\} \), the individual foreign policy functions \( \{c(a, \varepsilon), a'_{\text{RoW}} (a, \varepsilon)\} \), solve the household problem.

- Given relative prices \( \{r, w\} \), \( K/L \) solves the domestic firm’s problem.

- Given relative prices \( \{r, w_{\text{RoW}}\} \), \( K_{\text{RoW}} / L_{\text{RoW}} \) solves the foreign firm’s problem.

- The labor markets are in equilibrium, and the labor inputs \( L \) and \( L_{\text{RoW}} \) correspond to the total supply of labor efficiency units in both regions.

\[
L = \sum_{j=1}^{J_R-1} \Phi_j \int_{A \times S \times \varepsilon \times F \times F \times F \times F} c_{j, s, \varepsilon, f_\alpha, f_\beta} d\mu_j (a, s, \varepsilon, f_\alpha, f_\beta)
\]

\[
L_{\text{RoW}} = \int_{A \times \varepsilon} c_{\text{RoW}} d\mu_{\text{RoW}} (a, \varepsilon)
\]

- The global asset market clears

\[
K + K_{\text{RoW}} + D = \sum_{j=1}^{J_R-1} \Phi_j \int_{A \times S \times \varepsilon \times F \times F \times F \times F} a'_j (a, s, \varepsilon, f_\alpha, f_\beta) d\mu_j (\cdot) + \sum_{j=J_R}^{J} \Phi_j \int_{A \times F \times F \times F} a'^R_j (a, f_\alpha, f_\beta) d\mu^R_j (\cdot) + \int_{A \times \varepsilon} a'_{\text{RoW}} (a, \varepsilon) d\mu_{\text{RoW}} (a, \varepsilon)
\]

- The pension system is budget-balanced, and the payroll tax \( \tau_R \) is set to satisfy this condition.

- The government’s intertemporal budget constraint is satisfied, that is total tax revenues \( T \) (obtained from taxing labor income, asset income, pensions, and consumption) minus total government expenditure (the sum of government purchases \( G \) and public transfers \( TR \)) are equal to the interest costs of public debt, namely public debt is constant over time.

\[
\frac{D}{Y} = \left( \frac{T - G - TR}{Y} \right)
\]
• The stationary distributions \( \{ \mu_j (a, s, \varepsilon, f_\alpha, f_\beta), \mu^R_j (a, f_\alpha, f_\beta), \mu^{RoW} (a, \varepsilon) \} \) satisfy

\[
\mu_{j+1} (a', s', \varepsilon', f_\alpha, f_\beta) = \int \nu (a, s, \varepsilon, f_\alpha, f_\beta, j, a', s', \varepsilon') \, d\mu_j (a, s, \varepsilon, f_\alpha, f_\beta) \tag{3}
\]

\[
\mu^R_{j+1} (a', f_\alpha, f_\beta) = \int \nu^R (a, f_\alpha, f_\beta, j, a') \, d\mu^R_j (a, f_\alpha, f_\beta) \tag{4}
\]

\[
\mu^{RoW} (a', \varepsilon') = \int \nu^{RoW} (a, \varepsilon, a', \varepsilon') \, d\mu^{RoW} (a, \varepsilon) \tag{5}
\]

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the above three equations (3), (4), and (5), where \( \nu (\cdot) \), \( \nu^R (\cdot) \) and \( \nu^{RoW} (\cdot) \) are the transition functions.

### A.3 - Welfare Measures

• The ex-ante welfare measure \( W \) is the expected utility of a new-born in the steady-state, namely the value function of the age 1 individuals, evaluated at the initial conditions and integrate with respect to the fixed effect:

\[
W = \int_{\mathcal{F}_\alpha \times \mathcal{F}_\beta} V_1 (a = 0, s, \varepsilon = \bar{s}, f_\alpha, f_\beta) \, d\mu_1 (a = 0, s = \bar{s}, \varepsilon = \bar{s}, f_\alpha, f_\beta) \tag{6}
\]

• The social welfare measure \( W^S \) is utilitarian, i.e. it weights the agents’ lifetime utilities by their mass in the steady-state

\[
W^S = \sum_{j=1}^{J_R} \Phi_j \int_{\mathcal{A} \times \mathcal{S} \times \mathcal{E} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} V_j (a, s, \varepsilon, f_\alpha, f_\beta) \, d\mu_j (a, s, \varepsilon, f_\alpha, f_\beta) + \sum_{j=J_R}^{J} \Phi_j \int_{\mathcal{A} \times \mathcal{F}_\alpha \times \mathcal{F}_\beta} V^R_j (a, f_\alpha, f_\beta) \, d\mu^R_j (a, f_\alpha, f_\beta) \tag{7}
\]

• The consumption based welfare measure \( \varpi \) is the percentage increase in consumption in all states of the world that makes welfare in the counterfactual economy \( W^1 (\varpi) \) equal to welfare in the baseline one \( W^0 \)

\[
W^0 = W^1 (\varpi)
\]

\[
\varpi = \left( \frac{W^1}{W^0} \right)^{\frac{1}{\gamma}} - 1 \tag{8}
\]
Appendix B - Estimation and Empirical Targets

• The estimation was performed with STATA SE 15.1.

• The public version of the Labour Force Survey data can be downloaded from the following website:
  
  http://dli-idd-nesstar.statcan.gc.ca/webview/

• The Canadian Life Tables can be downloaded from the following website:
  
  https://www150.statcan.gc.ca/n1/pub/84-537-x/84-537-x2018002-eng.htm

• The public debt and gdp data were obtained from the OECD and the PWT9.0 databases.

• The CPI and the net public debt data were obtained from the CANSIM database.

• The national net lending or net borrowing was obtained from the CANSIM database, series v62305900, and was adjusted for inflation using the GDP deflator, series v1997756.
Appendix C - The Solution Algorithm

This algorithm represents the computational procedure used to solve the two-region model:

1. Set the home public debt/GDP ratio to its long-run value.
2. Generate a discrete grid over the asset space for the home economy $[-b, ..., a_{\text{max}}]$ and for the RoW $[-b_{\text{RoW}}, ..., a_{\text{RoW}}_{\text{max}}]$.
3. Generate a joint discrete grid over the two individual effects $[f_{a,\text{min}}, ..., f_{a,\text{max}}] \times [f_{\beta,\text{min}}, ..., f_{\beta,\text{max}}]$.
4. Generate a discrete grid over the AR(1) income shocks with the Rouwenhorst method for the home economy $[\varepsilon_{\text{min}}, ..., \varepsilon_{\text{max}}]$ and for the RoW $[\varepsilon_{\text{RoW},\text{min}}, ..., \varepsilon_{\text{RoW},\text{max}}]$.
5. Generate a discrete grid over the random walk income shocks with a life-cycle adaptation of Tauchen/Hussey’s method $[s_{\text{min}}, ..., s_{\text{max}}]$.
6. Guess the interest rate $r_0$.
7. Guess the home income tax rate $\tau_0$.
8. Guess the home pension benefits $\overline{y}_{R,0} (f_{\alpha}, f_{\beta})$.
9. Get the home capital demand $K_0$ and wages $w_0$.
10. Get the home saving functions $a'_j (a, s, \varepsilon, f_{\alpha}, f_{\beta})$, $a'_R (a, f_{\alpha}, f_{\beta})$ and the home value functions $V_j (a, s, \varepsilon, f_{\alpha}, f_{\beta})$, $V'^R_j (a, f_{\alpha}, f_{\beta})$.
11. Get the home stationary distributions $\mu_j (a, s, \varepsilon, f_{\alpha}, f_{\beta})$, $\mu'_R (a, f_{\alpha}, f_{\beta})$.
12. Get the RoW capital demand $K_{0\text{RoW}}$ and wages $w_{0\text{RoW}}$.
13. Get the RoW saving functions $a'_{\text{RoW}} (a, \varepsilon)$.
14. Get the RoW stationary distributions $\mu_{\text{RoW}} (a, \varepsilon)$.
15. Get the aggregate world capital supply and check the world asset market clearing; Get $r_1$.
16. Update $r'_0, \tau'_0$ and $\overline{y}'_{R,0} (f_{\alpha}, f_{\beta})$ (with a relaxation method).
17. Iterate until the global asset market clearing and aggregate consistency of the pensions are satisfied.
18. Compute the home ex-ante welfare $W$ of a new-born and the social welfare $W^S$.
19. Get the home consumption functions $c_j (a, s, \varepsilon, f_{\alpha}, f_{\beta})$, $c^R_j (a, f_{\alpha}, f_{\beta})$, the RoW consumption functions $c_{\text{RoW}} (a, \varepsilon)$ and check the final good market clearing.
20. Adjust the parameters $\beta, \beta_{\text{RoW}}, A_{\text{RoW}}, \xi$ until the empirical targets are matched.
21. Set the public debt/GDP ratio to a counterfactual value on an appropriate grid and repeat steps 2-18 for all the points in the grid.
Figure 10: Joint bivariate normal density of the correlated level (\(\alpha\)) and slope (\(\beta\)) fixed effects. For ease of comparison with the joint density \(P(\alpha, \beta)\), the marginal densities \(P(\alpha)\) and \(P(\beta)\) are multiplied by an appropriate normalizing factor.
Figure 11: Demographics and discretization of the income process.
Figure 12: Equilibrium outcomes for the closed economy (with accidental bequests).
Appendix D.1 - The SOE Model with an Exogenous Interest Rate Rule

As a robustness analysis, I solved the SOE version of the model, closing it by assuming an ad-hoc rule for the determination of the domestic interest rate. In order to assign an interest rate in the counterfactual cases, with a changing public debt, I exploit the empirical relationship estimated on a Panel of OECD Countries over the 1990-2015 period. In particular, I estimated the interest rate elasticity to public debt using a reduced form regression reported in Table 5. As for the dependent variable, I consider the long-run interest rate on Treasury bills (a variable available in the OECD database), while the main variable of interest is the log of the Public Debt/GDP ratio (logdgdp). In order to reflect the features of the theoretical model, I consider a parsimonious regression, which tries to control for aggregate shocks (with a set of time dummies), and other macroeconomic outcomes, such as the log of GDP (logrgdp) and log of the price level (logcpi). Arguably, there are sources of endogeneity that might bias the estimates. In particular, current public debt mechanically increases with the current interest rate, as the cost of servicing the debt increases, which can potentially be financed with more debt. This leads to an obvious issue of reverse causality. In order to circumvent this problem, I instrument the current public debt with its lagged value. The past value of public debt is predetermined, hence it does not respond to the future value of the rate of return of treasury bills. At the same time, the high autocorrelation of debt ensures that this is not an instance of a weak instrument: the correlation between the endogenous regressor and the instrument is 0.988. The system GMM estimation of the panel model (FE-IV) leads to my preferred estimate of 0.629, which is somewhat lower compared to the basic fixed effects (FE) estimate.

Table 5: Partial Correlation between Interest rates and Public Debt/GDP.

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</table>

Standard errors in parentheses

25 Notice that I cannot consider the log of the inflation rate, because there are some deflationary events. I consider the CPI instead, which provides the same information without suffering from this problem.