Overlapping Generations Models of Graded Age-Group Societies: Economics Meets Ethnography*

by

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Abstract

Across much of the world, tribal societies are organized into age groups. In these age-group societies, social and economic relations between individuals are regulated by well-established rules governing transitions through the lifecycle. In this paper we detail the relationship between the classic formulations in Stewart's (1977) *Fundamentals of Age-Group Systems* and the overlapping generations (OLG) model. We establish that OLG models can accurately represent a large subset of actual age-group societies called graded age-set societies. Thus, an OLG model bears a close resemblance to reality.

Keywords: Age Sets, Graded Age-Group Systems, Lifecycle, Overlapping Generations Model.

JEL Classification: J10, P00, Z10.

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The overlapping generations (OLG) model is arguably the paradigm general equilibrium model in economics.[1] It captures lifecycle analysis and market incompleteness, essential features of realistic economies. In a typical OLG model, an individual's life span is divided into two, or possibly three, stages. Each stage differs from the other(s) by the economic choices open to the individual in that stage: savings/consumption; work/leisure. An economic model consisting of citizens of different generations, simultaneously consuming and producing, generates a dynamic path for the economy as a whole.

Economists view OLG models as stylized models, with each period describing a general "stage of life". Typically, the periods are of the same discrete length and agents within a generation are synchronized in their transitions through various stages of life. Though the model has provided numerous theoretical insights, it is difficult to apply directly because in most modern economies the sequencing of generations is asynchronous across families.[2] Unable to point to any actual societies that describe the stylized model, economists tend to view and teach the OLG model as a useful fiction.

This paper establishes that the stylized OLG model accurately captures anthropologists' descriptions of actual societies. These societies are found around the world and belong to what ethnographers call *age-group societies*.[3] Individuals in these societies live by rules governing social relations that are based on chronological age and/or maturity level. Spencer (1997) describes the overarching premise of such societies as the respect for age. This contrasts with other premises such as honor, associated with the integrity of kinship, and purity, associated with status and caste.

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[1] The OLG model originated with Allais (1947) and Samuelson (1958). Azariadis (1993), Brock (1990) and Geanakoplos (1987) survey theoretical applications of the model. Distinct from the Arrow-Debreu model, not all agents can interact with each other – agents are "born" and "die" and therefore have temporal existence. Thus markets are incomplete. For this reason, Hahn and Solow (1995) prefer the OLG model to the Arrow-Debreu model in macroeconomics.

[2] Most applied researchers have avoided the daunting task of tracking the lifecycles of heterogeneous populations and have relied on simpler models. A notable except is Auerbach and Kotilikoff (1988) who simulate a 58-generation OLG model.

[3] According to Ritter (1980) and Stewart (1977) age-set systems have existed in Africa, North America, South America Central Asia, Taiwan, New Guinea, Switzerland and Albania. Africa has the greatest number and variety of well-documented systems.

In these real age-group societies, allocations are made on the basis of age; the associated status is formalized by categories called *age grades*. An individual's position in the age grade structure governs their work assignment, marriage matching, parental relationships and property inheritance. Unlike modern market societies prices play a smaller role in allocating people and resources.

The definitive integrative work in this area is Stewart's *Fundamentals of Age-Group Systems* (1977). Of this work Spencer (1997) says: "This provides a pioneering and unsurpassed attempt to place a world-wide range of these systems within a logical framework, devising a method of analysis and identifying the implications of the rules that characterize them." Stewart's book details rules on the *age-group model* and the *age-grade system* as well as on societies incorporating both of these rules called the *graded age-group system*. Age groups describe the generational composition of society whereas age grades detail the lifecycle behavior. Graded age groups describe both aspects.

Stewart develops an empirically relevant prototype model that we term a *standard graded age-set system.* He describes other age-group models as variants of this system. As economists we were struck by the similarities between Stewart's systems and the OLG model. Our investigations revealed exact correspondences between the models.

In this paper we provide four propositions which detail the relationship between the OLG model and the classifications in Stewart. We show that the standard OLG model can be interpreted/understood as an age-set model (see Table 1), an age-grade system (see Table 2), and a graded-age set system. We further show that the standard graded age-set system is equivalent to the standard OLG system. Thus, the well-known properties of the OLG model can be applied to modeling these societies.

Stewart's work points the way to using the OLG model as a serious demographic framework. His work provides "recruitment" rules for collecting individuals of disparate ages into generations. He also creates a taxonomy of social systems based on deviations from the prototype. Our work provides a link with Stewart's and generates additional insights/questions about the workings of graded age-group societies.

The paper proceeds to first define a standard OLG system as a generalization of OLG models. Section 2 presents Stewart's age set model and age grade system. These systems are compared to the OLG system in Section 3. Sections 4 and 5 extend the comparison to variants of the graded age-set system. Section 6 concludes.

1. Overlapping Generations Models

Auerbach and Kotlikoff's (1998, p33) undergraduate text description of the overlapping generations model is typical:

> Inhabitants of this country live for just two time periods. In their first period, they are young and go to work. In their second, they are old and retired. At the end of their second period, they pass away. All individuals born at the same time are identical.
>
> Although each generation dies after two periods, the economy is ongoing. At the beginning of each period, a new generation is born, and the previous young generation passes from youth into old age. Hence, in each period, there is always a set of young people and a set of old people.
>
> A period of time in our model refers, in real time, to roughly 30 years of adulthood. You should think of each generation's "youth" as corresponding to the 30 years between ages 20 and 50 and its old age as corresponding to the 30 years between ages 50 and 80. Admittedly, it would be more realistic to have 60 or so periods of life – one for each year of adulthood – but this would overly complicate the model without altering its basic insights.

After explaining the demography, Auerbach and Kotlikoff describe utility, production and markets. Their Figure 1 provides a time schematic of this economy. The beauty of the OLG model is that it not only details each agent's lifecycle but also analyzes the interactions between agents at different points in their lifecycle. For example, in period 2, generation 1 is old and sells (dissaves) its capital stock to buy consumption goods. In period 2, generation 2 is young and buys the capital stock from its elders to save for their old age (in period 3). This structure is the simplest possible characterization.[4] The simple model can be expanded in a number of ways. For example, models have been developed with N-period lived agents, where N can be uncertain and/or infinite (see Geanakopolos (1987)).

(**Figure 1** here)

Demography is the distinguishing feature of OLG models. In almost all OLG models, the demography provides the structure for the time periods. The usual convention is to index agents by the time period in which they are "born" into the economy. Thus, "generation j" agents are "born", or brought into the economy, in period j. In all subsequent periods, these agents are identified as generation j -- at least, until they die, at which point they disappear from the model. As agents live $N>1$ periods, living generations overlap at any point in time.

A time index and an age index are used to track generations. Denote generation j agents alive at time t by the set A_{at}, where the index $a=t-j+1$, $1 \leq a \leq N$ gives the age of the agents. Hence, those born in period t are age 1. Those who die at the end of period t are age N: they were born N-1 periods prior to the current generation and are from generation $j = t - (N-1)$. At time t the set of all agents alive is $A_t = \{A_{1t}, A_{2t}, \ldots, A_{Nt}\}$.

These features are common to most OLG models. As we could not find a general description in the literature, we formalize these features in the following definition.

Definition. The *standard OLG system* consists of the following six elements.

1. (Time) Real time is partitioned into discrete time periods which are indexed by whole numbers: t, t+1, …, T-1, T, where t is the beginning period for the economy and T is the end period.

[4] Interestingly, for most economic analyses the two-period lived demography provides a rich characterization. This is because two periods is sufficient to capture the incomplete markets in OLG models. Economists have mostly used the OLG model to explore the implications of incomplete markets. See Bullard (1992) for discussion and references.

2. (Agents and Generations) At the beginning of each period $j=t,\ldots,T$, the set of agents A_{1j} agents are born; the first subscript indicates that it is the first period of the agents' lives. The longest-lived member of generation j lives $N>1$ periods and dies at the end of period $j+N-1$. Population changes only by birth and death. This is a "closed economy" where there is no migration.[5]

3. (Endpoints) In a *perpetual* economy neither the beginning nor end is finite: $t=-\infty$ and $T=\infty$. Here endpoints need not be specified because the full demographic structure is described by elements 1 and 2. However, if the economy has either a beginning or end, or both, the demography must be specified at the end point(s). The most common specification is of an *ongoing* economy with a beginning but no end: $t > -\infty$ and $T = \infty$. Here the convention is that the economy begins with a full cross-section of generations representing the complete lifecycle: $A_t = \{A_{1t}, A_{2t}, A_{3t}, \ldots A_{Nt}\}$, where agents $A_{2t}, A_{3t}, \ldots A_{Nt}$ are often described as "pre-existing agents". Such an ongoing economy is a perpetual economy from time t onward. Finally, the economy can have a finite endpoint $T<\infty$. In this case there is no convention for specifying the demography after period T. Hereafter, we assume a perpetual economy (unless otherwise noted).

4. (Period length) The discrete time divisions correspond to periods during which agents from at least one living generation face a substantive choice or life passage. Each period is short enough that relevant choices or life passages are captured.

5. (Lifecycle stages) Each agent passes through a progression of $C > 1$ distinct totally-ordered stages, $\{ \Sigma_{\acute{o}} \}_{\acute{o}=1,\ldots,C}$.[6] This sequence of stages is referred to as the lifecycle. Each stage consists of at least one whole period, so $C \leq N$; stages $\Sigma_2,\ldots, \Sigma_{C-1}$ consist of whole periods or multiples thereof; the beginning and end stages $\Sigma_{1,}$ and Σ_C need not be

[5] See Obstfeld and Rogoff (1996) for references to OLG models that specify the interactions between two of more economies.

[6] Blanchard's (1985) continuous time OLG model is a notable exception. All living agents have an equal probability of dying in the next interval of time. Agents are replaced, born into the economy, at the same rate as agents die. This OLG model does not capture standard features of the lifecycle where most agents can expect to live through established life passages.

of integer length. If a stage consists of two or more periods the periods must be contiguous. Stages are contiguous and usually contain all N periods. The lifecycle stage of agents A_{at} is a function of their age: $\Sigma_{ót}(a)$. It follows that agents who are age 1 are in stage 1, $\Sigma_{1t}(1)$, and those of age N are in stage C, $\Sigma_{Ct}(N)$.[7]

6. (Stationarity) There is a unique mapping from agent age to lifecycle stage independent of the generation or the time period: $\Sigma_{ó}(a) = \Sigma_{ót}(a)$ for all j and t. This implies that generations transit through the stages such that the cross section of generations in stages is stationary. Note that at time t generations $j = t\text{-}N+1, \ldots, t\text{-}a+1, \ldots, t$ are living and stages are assigned $\Sigma_{1t}(1), \ldots, \Sigma_{ót}(a), \ldots, \Sigma_{Ct}(N)$. As the assignment is the same for an arbitrary t, the cross section is stationary.

Elements 1-3 are the bare bones description of the OLG system. Elements 4-6 describe how the system is used to address substantive economic issues. Element 4 simplifies modeling by keeping the number of periods to the minimum required to capture the relevant activities. Element 5 describes the existence of different life stages for each agent. Economists do not use the term “stages”. Instead, more descriptive terms like youth, student, working age, retirement etc. are typically used. Where such language is not used, stages are implicit. Element 6 describes the stationary cross sectional lifecycle structure of the system. Stationarity enables us to model an essentially dynamic system in terms of a representative individual and representative time period.

The elements of the standard OLG system are found in most OLG models. A particular OLG model details all the characteristics of each agent at each age and stage through the specification of endowments, preferences, technology, information, and societal constraints. The solution for the model fully describes the interactions between and within the overlapping generations. We use the more general OLG system below.

2. Age Sets and Age Grades

[7] Since $C \leq N$, two or more ages may map into the same stage. Hence, the function $\Sigma_{ót}(a)$ is not generally invertible (i.e. there is no unique mapping from stage to age).

Age sets and age grades describe different demographic dimensions. Age sets describe the generations in society whereas age grades detail the lifecycle behavior.

2.1 *The Age-Set Model*

Stewart (1977) developed a formal analysis of age group systems, drawing on ethnographic studies of societies across the world. His work abstracted from many of the details of the specific societies to produce a general framework. The original definition of an *age set* comes from Radcliffe-Brown (1929): "(a) recognized and sometimes organized group consisting of persons… who are of the same age…Once a person enters a given age-set, whether at birth or by initiation, he remains a member of the same set for the remainder of his life". Stewart says a society "has an age-set system when it has a number of age sets with no members in common and distinct mean ages."

Stewart defines an age-set model in terms of 8 characteristics.[8]

(**Table 1** here)

The characteristics reveal that age sets are much like generations in the OLG model: they are totally ordered, non-overlapping, groups of individuals of roughly the same age. Membership in an age set identifies an individual with his generational cohort.

2.2 *The Age-Grade System*

Stewart also details the characteristics of age grades. In contrast to age sets, age grades identify individuals' social roles, according to their stage in the lifecycle. For example, elders usually have very different rights and responsibilities from those available to children. Stewart identifies these collections of rights and responsibilities

[8] Foner and Kertzer (1978) use different criteria to designate age-set societies and use a definition much like Stewart's definition of graded age-group systems. See Stewart (1977) for an extensive discussion of earlier definitions.

with *rule-sets*, and provides the following list of constraints on the *transition rules* which govern the assignment of rule-sets to individuals or groups:

(**Table 2** here)

Later we examine transition rules that link both age sets and age grades to define a *graded age-set system*; this last is the ultimate focus of the paper.

3. Comparisons of Economic and Ethnographic Systems

There are clearly similarities between Stewart's systems and OLG models. In particular, generations resemble age sets and lifecycle stages resemble age grades. In this section we formally show that Stewart's systems are general enough to include the "economy" constructed by the standard OLG system.

3.1 *Age Sets and the OLG System*

Proposition 1. The standard OLG system is an age-set model.

Proof. To prove this proposition, we show that the standard OLG system satisfies each of Stewart's eight characteristics in Table 1.

(1) In an OLG model a group is a generation. A new generation j is formed at the beginning of each new period $j = t,\ldots,T$. Generation j consists of all agents born into the economy in period j. As periods are discrete there is a total ordering on these groups.

(2) Recruitment is sequential and hence non-overlapping. (It is the generations that overlap not the recruitment into generations.)

(3) By construction any OLG model has at least two generations/groups (that overlap as the name implies).

(4) In the OLG system group j dissolves when the last member of generation j dies. At least one member of each generation lives N periods, hence no generation disappears from the model before the previous generation.

(5) In the OLG system agents enter the economy at "birth" which occurs at the beginning of a period. Hence both the minimum enrollment age and the basic enrollment age are 1.

(6) Follows from explanation to (1)

(7) (i)This is true in the standard OLG system because it is a closed economy and agents cannot leave the economy other than by dying. (ii) This follows from (4); a group only dissolves when the last member of that group dies.

(8) Follows from (4) and (7).

This proposition implies that the OLG system can be used to directly study a subset of age-set models.

3.2 *Age Grades and the OLG System*

Proposition 2. The standard OLG system is an age-grade system.

Proof. The standard OLG system is an age-grade system where $C = n$ and Σ_1 corresponds to G_1 , Σ_2 to G_2, …, Σ_C to G_n. This construction explicitly establishes (1)-(6) of Stewart's definition listed in Table 2.

Individuals move through the sequence of grades in a one-to-one correspondence with the contiguous lifecycle stages.

4. The OLG System as a Graded Age-Set System

Stewart writes (p.129) "When age-groups are a prominent feature of the social organization of a society, they almost always operate in conjunction with age-grades." Stewart's combines age groups and age grades in a graded-age group system by imposing one additional constraint.[9]

[9] Bernardi (1985) implies that these two systems are almost always integrated: "The formation of class and promotion in terms of grades are two aspects of the same phenomenon of formal institutionalization: the recruitment of candidates to a class implies, in fact, their promotion to an initial age grade. Thus the relationship that is set in motion by recruitment and promotion not only brings about the structure of a special grouping; it also implies the chronological succession of classes in the grades, so that the emerging structure is rightly called an 'age class system'."

Definition. (*Integration constraint)*: "An age-grade system is integrated with an age-group system if the transition rules of the age-grade system are such that it never happens that a member of group *S* is assigned a higher grade than a member of a group $S+k$, $k \geq 1$." (Stewart p. 135)

This constraint requires that an individual never be assigned a higher age grade than someone who belongs to a senior age group. The notation $S+k$ refers to the *k*'th age group inaugurated before *S*, and $S-k$ is the *k*'th age group inaugurated after *S*. The minus sign indicates a junior age group and the plus sign a senior age group.

Definition. A *graded age-group system* is an age-group system combined with an age-grade system that satisfies the integration constraint.[10]

In this section we look at a subset of the graded age-group systems.

Definition. The *graded age-set system* is a graded age-group system that satisfies the age-set model.

Proposition 3. The standard OLG system is a graded age-set system.

Proof. The standard OLG system satisfies the integration constraint because lifecycle stages coincide with grades and all agents from a generation transit the lifecycle stages together. Thus, this proposition follows directly from Propositions 1 and 2.

The standard OLG system is a type of graded age-set system. We now turn to identify which type.

5. The Graded Age-Set System as an OLG System

We seek conditions under which the converse of Proposition 3 is true: the conditions under which the standard OLG system completely describes a graded age-set system. Stewart's analysis provides a guide to these conditions as well as to the remarkable variety of actual societies. He simplifies the graded age-set system to an empirically important prototype and uses this as the reference for societies that vary from it. We refer to this prototype as a *standard graded age-set system* and show that it is equivalent to the standard OLG model.

5.1. *Constraints and Level Rules for Simplifying Graded Age-Set Systems*

Graded age-set systems can display complicated patterns; for example, more than one age-set and/or grade sequence, sequences that have time gaps, and individuals who belong to more than one group and/or sequence over time.[11] Stewart examines a number of societies that have these features, but notes that: "Very often there is only one sequence so that the system and sequence are identical."(Stewart, p. 29) He also says "… sequence assignment rules are always, or almost always, such that an individual cannot join more than one sequence". (Stewart, p.121) Stewart's prototype is a system with a single sequence without gaps and where age sets coincide with grades.

Based on empirical observation, Stewart introduces two additional constraints. Though he provides examples of societies that violate the constraints, he asserts that violations are uncommon. (Stewart p.142)

[10] See Stewart p.135. According to Stewart, the integration constraint is hardly ever violated. Violations can be found in societies where age grades are independent of the age-set system. This occurs amongst the Banapas Bedik and the Kikuyu.

[11] There can exist more than one age-set sequence when they are defined on collections of groups of different individuals. Thus, males and females may belong to different groups and hence different sequences. Stewart mentions that there is the possibility of an individual belonging to more than one sequence but only one group in each sequence. For example, a society may have sequences that have time gaps and do not overlap; an individual may alternate between groups in these sequences. Moreover, it is the case in many societies that some individuals are not part of age sets proper - for example, in many tribes women are assigned to the age set of their brothers before marriage, and of their husbands after marriage.

Definition. The *group unity constraint* requires that all members of a group are at all times assigned the same grade.

Definition. The *grade-filling constraint* requires that each grade in the sequence be always occupied.

Stewart refers to these constraints together with the integration constraint as the three *combination constraints*. Stewart writes, “Indeed, systems that meet all three combination constraints are found around the world.” (Stewart, p.142)

By the group-unity constraint, all members of a single age-set are assigned to the same grade. Observe that more than one age-set can inhabit the same age grade. However, the integration constraint ensures that only adjacent age-sets inhabit the same grade and that junior age-sets are never assigned higher grades.

The *grade filling constraint* requires that each of grades $G_1 \ldots, G_i, \ldots, G_n$ be occupied by at least one age-set at all times. Thus the grade structure is stationary over time -- for any two (arbitrary) points in time, those members of the tribe in the age grade system can be partitioned into the same grade structure.

Though the grade structure is stationary, the above constraints are not sufficient to ensure that the graded age-set structure is stationary over time. For example, a grade maybe occupied by one group now and two groups later.

To develop a more specific transition rule, Stewart introduces *level numbers*. When newly enrolled, an age set is assigned level number 1. This age set is assigned level number 2 when the next age set is enrolled and so on. Thus, the level number for an age set tracks the number of enrollments since that age set was inaugurated. Level rules provide a mapping from level numbers to age grades.[12]

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[12]In graded age-set societies the initiation into an age set usually marks an important life passage and hence coincides with a new behavior rule set: (Stewart, p.145) “…a single ceremony or set of

Definition. “A *level rule* is a transition rule that assigns a grade to an age group or disassigns one from it according to the level number of that age group. A level rule that assigns a grade is necessarily accompanied by one that disassigns the preceding grade (except in the case of the entry rule); and a level rule that disassigns a grade is necessarily accompanied by one that assigns the next grade (except in the case of the exit rule).” (Stewart, p.136)

Definition. “If an age-grade system has only one sequence of level rules, then that system uses *simple level rules*.” (Stewart p.137)

With simple level rules, the mapping from level numbers to grades is unique but not necessarily complete. Stewart p.138 states that “…simple level rules are overwhelmingly the most common type of transition rule. All the Taiwanese systems for which I have information, and indeed almost all the non-African systems that I know of, operate solely with simple level rules (except that some have exit rules that are not level rule, e.g., where the last grade is disassigned only at death).”[13]

Stewart describes the Changki Ao Nagas society as an example of his prototype society. The Changki assign simple level rules where the level number *x* assigns each age set the grade x, for *x* =1, …, 9. Level numbers beyond 9 assign grade 9. Figure 2, taken from Stewart p.136, describes this assignment. The Changki inaugurate a new age set every 3 years. The enrolment age is 12 and the nine grades are indicated by the Roman numerals in the boxes. Consider the situation at the time of the enrollment of age set *S*-11. A cross section of the society is given by the bottom row. Age-set *S*-11 is assigned level number 1 and grade I, age set *S*-10 is assigned level number 2 and grade II, and so on until level 9 is reached; thereafter age sets stay at grade IX.

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simultaneous ceremonies marks both the inauguration of a new group and any shifts in grade that this necessitates. This is indeed what happens in the great majority of systems with level rules”.

[13] With *complex level rules* different age sets will be assigned different grades for the same level number. This necessarily results in different numbers of age sets occupying grades at different points in time. Hence, the age set structure is not stationary. Stewart (p.139) asserts that complex level rules describe some societies in West Africa, but nowhere else.

(**Figure 2** here)

As is evident from the figure, if all the transitions are governed by simple level rules then the combination constraints are satisfied. In this case, simple level rules also necessarily assign to a given grade $G_1,\ldots, G_{n-1}$ a number of age sets which is constant over time. The exception is the last grade G_n where the number of sets may vary when disassignment is by death or infirmity. Otherwise, age sets transit the grade structure with new inaugurations in the same fashion over time. Simple level rules describe societies where age sets and grades are stationary in model time. When the inaugurations are at constant intervals, societies are also stationary in real time.

5.2. *Real Time*

Stewart describes "recruitment rules" for inaugurating individuals into age sets in real time using the following timeline diagram (p.35). This diagram identifies the length of three real time intervals relevant to the age set S created, or "inaugurated", at time t_S.[14] Age set *S* cannot recruit members prior to this inauguration date. The length of the *inauguration interval* for age set *S*, v_S, is the length of the interval between the inauguration of *S* and the inauguration of junior age set *S*-1. The length of the recruitment period for this age set, r_S, represents the time interval during which members join this set. Typically, all members are enrolled during a single ceremony, which may take a day or more; in some societies, recruitment continues throughout the inauguration interval.

(**Figure 3** here)

The time interval between the end of the recruitment period for the previous set (set *S* +1) and the end of the recruitment period for set *S* is called the *cessation interval* for set *S*, c_S. All individuals who join age set *S* during its recruitment period must reach

[14] Stewart says on p.113, "In fact, in all age-groups systems that I know of the rules governing the enrolment of an individual in a group are the precisely the same for all groups." Thus, this schematic is a generally applicable.

the minimum enrollment age during c_S. Thus, the oldest person in age set *S* will have reached the minimum enrollment age immediately after age set *S*+1 ceased recruiting, while the youngest member of age set *S* reaches the minimum enrollment age just before age set *S* ceases recruiting.

Stewart introduces this notation/vocabulary to identify discrete points in time at which individuals of disparate ages are collected into groups.[15] The construct helps determine the age range of an age-set in real time as well as in terms of the discrete intervals that coincide with inauguration intervals.

Assumption: The duration of the inauguration interval is constant over time.

This assumption requires that recruitment take place at regular intervals. According to Stewart (p.206-207) this is almost always the case in systems that determine inauguration dates by rules. In other "informal" inauguration systems, one or more members of the society determine inauguration.[16] Even here the evidence is that, in many cases, those who operate the system have a definite interval in mind.

5.3 *The Standard Graded Age-Set System and the Standard OLG System*

Definition. A *standard graded age-set system* is a representation of a graded age-set system that uses simple level rules to assign all grades at regular inauguration intervals.

[15]Further, if we consider the age set to be inaugurated at time t_S, at any time $t < t_S$ we can identify the individuals who *will* become members of that age set from the minimum enrollment age and the cessation period. This identification will be necessary when we later consider the possibility that individuals may be assigned to a grade prior to joining an age set.

[16]Foner and Kertzer (1978:1087-1090) point out that when transitions to different grades bring about different rights and powers, conflicts over the timing of transitions arise. These conflicts may be resolved within weeks, in which case we may view the inauguration interval as being

The Changki Ao Nagas society (described earlier in Figure 2) is an example of a standard graded age-set system.

The standard graded age-set system has the following essential features:

(i) The length of the inauguration interval is constant over time.

(ii) When an individual is enrolled into an age set he/she enters the first grade.

(iii) Once in the grade system an individual can pass to the next grade only at the beginning of a new inauguration interval.

(iv) At any two points in time, t and $t+\Delta$, individuals who have passed the same time since inauguration are assigned to the same grade.

Feature (ii) follows Stewart's description and provides a convenient common reference interval with which to track groups transiting the grades. It also accords well with observation: enrollment ages are typically low. Some societies have natal enrollment systems, but this is uncommon, because "…children have to be several years old before they can recognize the existence of rights and duties or undertake any significant joint activities." (Stewart, p. 225-226) Often "real grades" start with inauguration at puberty but putative grades like childhood are defined relative to it.

Feature (iii) restricts grades $G_2,\ldots, G_n$ to start at the beginning of inauguration intervals. This assumption permits a grade to be of any duration that is a multiple of the inauguration interval. However, the end grade G_n need not be of integer length. Grades such as elderhood may encompass several inauguration intervals and may be open ended.

Whereas features (i) and (iii) require individuals to transit the grade structure at regular intervals, feature (iv) requires age sets to transit the grade structure in the same way over time. Together these features yield an age-set and grade structure that is stationary across time, and matches the standard OLG system.

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effectively constant over time; when the resolution takes a much longer period of time, this assumption becomes less tenable.

Proposition 4. The standard OLG system is equivalent to the standard graded age-set system.

Proof. We first show that the standard graded age-set system satisfies the six elements that define the standard (perpetual) OLG system.

(1) (Time) Individuals are enrolled in age sets at the beginning the inauguration intervals. These enrollments partition real time into discrete periods that can be represented by whole numbers.

(2) (Agents and Generations) Enrollment in age set S at the beginning of interval v_S (see p.14) corresponds to the beginning of period j for generation j in the OLG system. More generally, individuals in age set S alive in interval v_{S-k} correspond to generation j agents in period $t=j+k$: A_{at}, where $a = k+1 = t - j + 1$.

(3) (Endpoints) This is a perpetual system so that endpoints do not have to be specified.

(4) (Period Length) The constant inauguration interval demarcates the period. For the enrolled age set this is a substantive life passage. As grades do not change within the period (features (iii) and (iv)), no substantive life choices are omitted by using this discrete period time unit.

(5) (Lifecycle) In the standard age-grade system each individual passes through $n > 1$ distinct totally ordered grades $G_1, G_2, \ldots, G_n$. This sequence corresponds to agents passing through $C = n$ totally ordered lifecycle stages where G_1 corresponds to Σ_1 , G_2 to Σ_2 ,…, G_n to Σ_C. Let N be the total number of inauguration intervals or equivalently whole periods in the graded age-set system. Then by feature (iii) each grade, like each stage, corresponds to at least one whole period so $C = n \le N$. If a grade corresponds to two or more periods, the periods must be contiguous. Periods are contiguous by (1). Grades (like stages) are contiguous and the grade sequence usually contains all N whole periods. Grades are contiguous from the no-interval constraint of the age-grade system.

(6) (Stationarity) Features (ii) and (iv) imply there is a unique mapping from age set to grade according to the duration from enrollment, independent of the enrollment period j. This mapping is the same as that in the OLG system when grades and stages correspond as above implying that the structure of the system is stationary.

Thus, the standard graded age-set system is a standard OLG system. It remains to show the converse. The converse follows immediately as a corollary to Propositions 2 and 3: since the mapping from generations to stages is unique and stationary, the standard OLG system is a standard graded age-set system. This establishes equivalence.

This proposition establishes that a prototype graded age-set system and a prototype OLG system are isomorphic. Since Stewart's prototype describes a large subset of actual age-group societies, the OLG system bears a close resemblance to reality.

5.4 *OLG Models and Graded Age-Group Societies*

OLG models can also be used to describe more general age-group societies. Relaxing feature (i) allows inauguration intervals to be of different durations. The standard graded age-set system without regular inauguration intervals is stationary in model time (except for the number of age sets in the last grade). It follows that there is an equivalent OLG system in model time where age sets correspond with generations and grades with stages. Level numbers or, equivalently, periods measure duration.

Relaxing feature (ii) allows us to capture societies where grades begin before inauguration. To do this we assign level numbers to intervals prior to inauguration. Suppose age set *S* is assigned the first grade at the beginning of interval v_{S+e}, where *e* is the number of intervals to enrollment for age set *S*.[17] We can now track all individuals who will be or are already enrolled in age set *S* by the level number sequence –*e*+1, -*e*+2, … 1, …*N*-*e*, where –*e*+1 is the number corresponding to the first grade and 1 is the number for the grade assigned at inauguration. The correspondence to the standard OLG system follows by aligning generation *j* with the level number –*e*+1.

[17] The first grade may start part way through the prior interval v_{S+e+1}. Level numbers capture the first inauguration after the assignment of the first grade.

Feature (iii) is violated if "time rules" specify transitions different from level rules.[18] Time rules specify a fixed transition interval to the next grade. When these intervals are of the same duration, the analysis can be modified to replace inauguration intervals with time rule intervals. Except for the inaugurations, time rules describe transitions that can be captured by the standard OLG system.

Feature (iv) is violated by complex level rules that permit the number of age sets occupying a grade to vary over time. Stewart mentions that most often complex level rules consist of cyclical sequences of level rules. For example amongst the Nawdeba of Togo there are five level rules that are repeated in a cycle. (Stewart p.139-40). The cyclical feature allows this society to be straightforwardly tracked by an OLG model. The grade assignment displays stationary periodicity.

Feature (iv) is also violated by simple level rules that form an incomplete mapping from level numbers to grades. Then some grades may be assigned on the basis of criteria other than age and are nonstationary. The nonstationarity will be localized as long as the simple level rules bound the grades that are assigned on the other basis. If this other basis can be modeled, then we have a complete temporal description.

Other more pronounced departures from the standard graded age-set system involve "deviant" systems. Where such systems have regular features they may be amenable to description by OLG models. For example, Engineer, Roth and Welling (2000) use an OLG model to simulate how the institution of "sepaade" among the Rendille of Northern Kenya affects demography.[19] Sepaade is the custom whereby the marriage of the females from every third age set is delayed until middle age. This institution induces cycles of three and six periods in otherwise steady state demographic simulations.

[18] Stewart p.142 mentions that a few societies use time rules. Time rules are transition rules that assign grades according to fixed time intervals independent of the next inauguration. This can lead to violation of the grade-filling constraint.

[19] Stewart (1977) and Roth (1993) describes the Rendille as an age-group society that deviates from the standard system in two respects: sepaade and climbing. Climbing is where children born to very old fathers are accepted into a more senior age set than their ages would otherwise allow.

6. Conclusion

This paper details the demographic correspondence between the structure of the OLG model and Stewart's age set, age grade, and graded age-set systems. We show that the correspondence is exact for "standard" versions of the models. In particular, all three of Stewart's systems describe the standard OLG system. Conversely, the standard OLG system provides an exact description of a large important subset of graded age-group societies that we termed standard graded age-set societies. Moreover, the OLG model can be readily generalized to capture many nonstandard features of graded age-set societies.

We believe this paper provides a bridge between economics and ethnography. Stewart's work is of interest to economists not only because it documents actual societies that fit the stylized OLG model but also because it provides an analytical guide to the vast variety of age-group systems. Stewart follows the analytic method of building an empirically important prototype model and describing the essentials of how other systems deviate from it. His work is a taxonomic guide to an important class of human societies. It also suggests interesting structures for economic analysis.

Age-group societies provide a revealing glimpse of archetypical social relations. They suggest fascinating questions. Why do societies organize themselves along age group lines? Are age groups systems institutions for controlling the young or restricting population or accommodating the ecology? Do individuals in age groups form natural coalitions? What types of age groups and institutional structures are stable?[20] These questions remain for future research.

[20] Ritter (1980) surveys anthropologists' attempts to answer this question. In the economics literature Engineer and Bernhardt (1992) look at the incentive compatibility conditions between two-period lived generations, and Engineer, Esteban and Sákovics (1997) examine the core of a similar OLG model. Generally speaking it is not incentive compatible for the young to support the old and the core of the model is empty in the absence of institutions. These papers do not examine the more complex relations found in age-group societies.

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Appendix: List of Symbols

Category	Notation/Index
OLG System:	
Time periods	$t = \tau, \tau+1, \ldots, T-1, T$
Each generation lives	N periods
Generation index	j
Generation j agents alive at time t	A_{at}
Age of the agent	$a=t-j+1 \leq N$
Lifecycle stages	$\Sigma_{1,} \Sigma_2,\ldots, \Sigma_{C-1}, \Sigma_C$, $C \leq N$
Age Sets and Age Grade Systems:	
Age set index	S
The group k age sets younger than S	S-k
Grades	$G_{1,} G_2 ,\ldots, G_i ,\ldots, G_{n-1}, G_n$, $n \leq N$

Table 1:

THE AGE-SET MODEL

A collection of groups in a society constitutes an age-set sequence when the groups are governed by rules such that in that society they generate an unbounded number of groups with the following characteristics:

1. **The ordering characteristic.** There is a total ordering on the groups given by the order in which they begin recruiting.

2. **The no-overlapping characteristic.** Each group ceases permanently to recruit members before the next one starts.

3. **The two-group characteristic.** There are always at least two groups in existence.

4. **The dissolution characteristic.** No group dissolves before one that began recruiting before it.

5. **The enrolment characteristic.** There is a certain age (the **enrolment age**) that has the following properties:
   No individual joins a group before reaching this age: it is the **minimum enrolment age**.
   Any individual who has not yet joined a group, but who is going to join one, and who has reached this age, joins a group as soon as there is one recruiting members: it is the **basic enrolment age**.

6. **The single membership characteristic.** No individual is at any time a member of more than one group.

7. **The no-resigning characteristic.** A member only leaves a group under one of the following circumstances:
   (i) When he leaves the society, or
   (ii) When the group is dissolving.

8. **The non-rejoining characteristic.** A member who has left a group because he has left the society or because the group is dissolving does not again join a group.

Stewart (1977, p.28)

Table 2:

THE AGE-GRADE SYSTEM

An age-grade is one of a finite collection of not less than two rule-sets. The rule-sets are all different from one another, and the collection is totally ordered, say G_1, G_2,…,G_n. The rule-sets are assigned and disassigned to persons by rules that meet the following constraints:

1. **The first-grade constraint.** No set is assigned to a person before G_1.

2. **The last-grade constraint.** No set is assigned to a person after G_n.

3. **The sequential-assignment constraint.** If the highest set a person has been assigned is G_i (i=1, 2, …, n-1), and that person is assigned another set, then the set assigned is G_{i+1}.

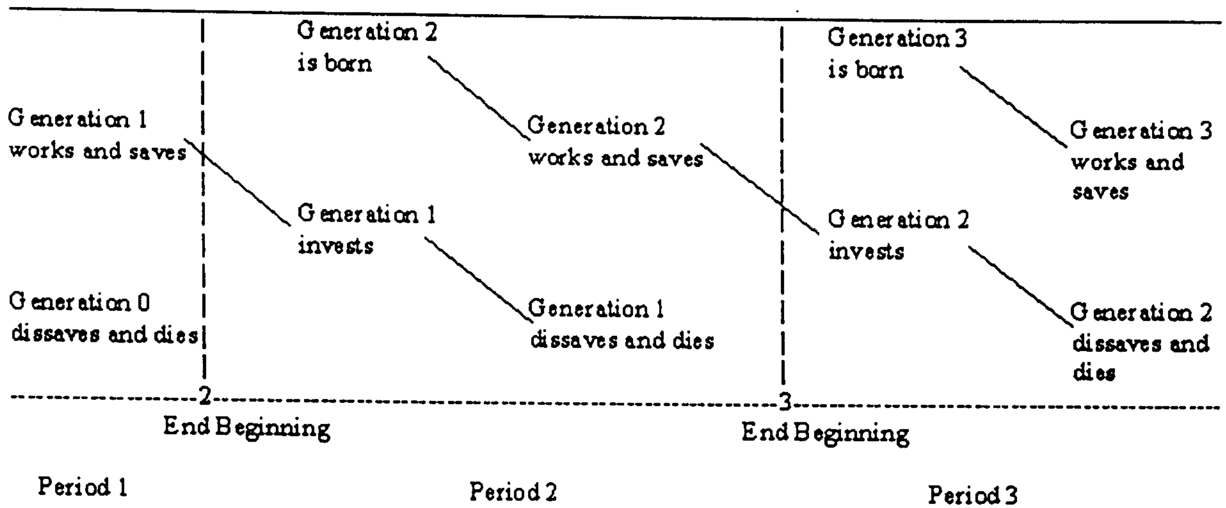
4. **The whole-sequence constraint.** An individual who has been assigned some G_i (i=1, 2, …, n-1) will eventually be assigned G_n if (but not only if) he survives to the maximum life span.

5. **The unique-assignment constraint.** No person is at any time assigned more than one set.

6. **The no-interval constraint.** If a person who is disassigned a set is assigned another, then the two events occur simultaneously.

Stewart (1977, p.130)

Two period OLG Model



(modified Figure 2.1 from Auerbach, A. and L. Kotlikoff (1988))

Figure 1

	S	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11
	I											
3	II	I										
6	III	II	I									
9	IV	III	II	I								
12	V	IV	III	II	I							
15	VI	V	IV	III	II	I						
18	VII	VI	V	IV	III	II	I					
21	VIII	VII	VI	V	IV	III	II	I				
24	IX	VIII	VII	VI	V	IV	III	II	I			
27		IX	VIII	VII	VI	V	IV	III	II	I		
30			IX	VIII	VII	VI	V	IV	III	II	I	
33				IX	VIII	VII	VI	V	IV	III	II	I
	S	S-1	S-2	S-3	S-4	S-5	S-6	S-7	S-8	S-9	S-10	S-11

Figure 2

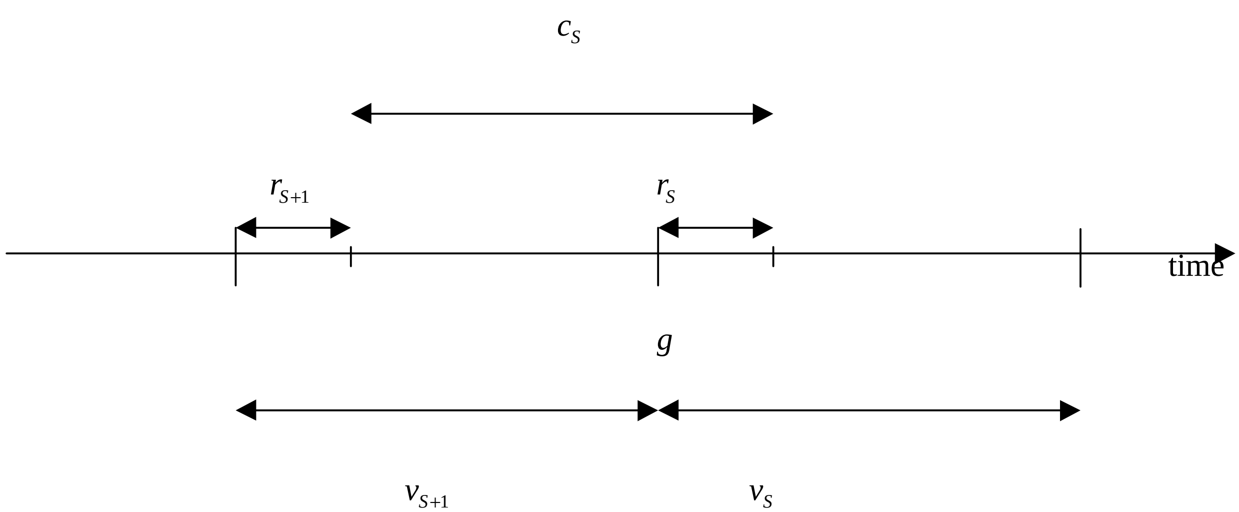


Figure 3: Timeline

Stewart (1977, p.35)