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TRESTING FOR STRUCTURAL CHANGE IN REGRESSION: AN EMPIRICAL LIKELIHOOD APPROACH

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Abstract

In this paper we derive an empirical likelihood type Wald (ELW)test for the problem testing for structural change in a linear regression model when the variance of error term is not known to be equal across regimes. The sampling properties of the ELW test are analyzed using Monte Carlo simulation. Comparisons of these properties of the ELW test and of three other commonly used tests (Jayatissa, Weerahandi, and Wald) are conducted. The finding is that the ELW test has very good power properties.

Keywords: Empirical likelihood, Wald test, Monte Carlo simulation, power and size, structural change

JEL Classifications: C12, C15, C16

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1 Introduction

There has been a great deal of interest in testing for the equality of regression coefficients (*i.e.*, the absence of structural change) in two linear regressions when the disturbance variances are unequal. Suppose these two linear regression models satisfy the classical assumptions such as normality, homoscedasticity, and serial independence for the error terms, and the two error terms are also independent of each other. The usual Chow test (Chow, 1960) is mostly often used by researchers to test for the problem of structural change. However, the Chow test assumes equal disturbance variances for the models. Toyoda (1974) showed that the usual Chow test of the coefficients of two regression models is misleading if the two variances are unequal and the sample sizes are small. The Behrens-Fisher problem is just the case when there is only one regressor, the constant term, in each of the regression models.

The first constructive test, in the literature, for the problem of structural change in the linear regression model when the error variance may also change was developed by Jayatissa (1977). We refer to it as the J test. The J test is an exact test whose test statistic has an exact F distribution if the null hypothesis is true. Watt (1979) and Honda (1982) proposed a Wald test for this problem and provided evidence that the Wald test is preferred to the J test when the number of regressors is greater than one. The Wald test is an asymptotic test, of course.

Weerahandi (1987) developed another exact test which makes use of the empirical significance level, the p-value. We refer to this test here as the WEE test. Zaman (1996) highly recommended the WEE test and discussed the test in detail because Weerahandi's approach introduced a new idea to the econometrics testing literature.

The main objective of this study is to develop a new solution to the problem of testing for structural change in a linear regression model when the variance of the error term is not necessarily constant. The approach that we take is the maximum empirical likelihood method (EL). The EL method is a non-parametric technique that was developed recently by Owen (1988, 1990, 1991). The EL method has obvious merits. It utilizes the likelihood function without specifically assuming the form of the underlying data distribution. It utilizes side information, through moment equations, which maximizes the efficiency of the method. Using the EL approach, one is able to effectively avoid possible mis-specification problems that one often faces in parametric approaches and the problem of lack of efficiency in other non-parametric approaches.

The test we propose in this study is a empirical likelihood type Wald (EL_W) test. The empirical likelihood (EL) approach allows us to make the best use of the information in hand. It also provides a way to tie the estimation and testing issues together nicely. In addition, the empirical likelihood approach provides us with a practical tool to conduct a test for the problem of structural change and for normality of the underlying data distributions simultaneously. We provide a detailed analysis of the sampling properties for the EL_W test. We also conduct a power comparison for the EL_W test and the conventional tests that we have mentioned above. Monte Carlo simulation is employed to compute the empirical size and the size-adjusted critical values in finite samples. The empirical powers of the tests are computed using these size-adjusted critical values to ensure that every test is being considered at the same actual significance level.

The outline of the rest of the paper is as follows. Section 2 provides a brief introduction of the existing tests that we have mentioned. Section 3 presents the set-up of the EL approach that we use. Section 4 presents the Monte Carlo experiment. Section 5 discusses the associated results. A summary and our conclusions are provided in Section 6.

2 Tests for structural change under heteroscedasticity

Suppose there are two classical linear regressions. We wish to test for the equality of the two coefficient vectors when the disturbance variances are not known to be equal.

$$Y_i = X_i\beta_i + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma_i^2 I_{n_i}), \quad i = 1, 2 \tag{1}$$

where Y_i and X_i are $n_i \times 1$ and $n_i \times k$ observation matrices, β_i are $k \times 1$ coefficient vectors, and ε_i are $n_i \times 1$ error vectors. We assume that $E(\varepsilon_1\varepsilon_2') = 0$ and that each of the regressor matrices is non-random and of full column rank.

The least squares estimators of β_i are:

$$\hat{\beta}_i = (X_i'X_i)^{-1}X_i'Y_i, \; i = 1, \; 2. \tag{2}$$

The least squares residual vectors are

$$\hat{\varepsilon}_i = M_i Y_i, \tag{3}$$

where $M_i = I_{n_i} - X_i(X_i'X_i)^{-1}X_i'$; $i = 1,\ 2$. The matrix M_i can be decomposed into Z_iZ_i', where Z_i is the $n_i \times (n_i - k)$ eigenvector matrix of M_i and has the properties: $Z_i'X_i = 0$ and $Z_i'Z_i = I_{n_i-k}$; $i = 1,\ 2$.

A type of BLUS residual vector (Theil, 1965 and 1968) can be formed using the Z_i matrix:

$$\varepsilon_i^* = Z_i'\hat{\varepsilon}_i; i = 1,\ 2. \tag{4}$$

The BLUS residual vectors ε_i^* are distributed as: $\varepsilon_i^* \sim N(0, \sigma_i^2 I_{n_i-k})$. These residuals are independent and identically distributed if the error terms are from normal distributions.

The difference of the two least squares estimators of the β_i vectors is distributed as follows:

$$\hat{\beta}_1 - \hat{\beta}_2 \sim N(\delta, \Sigma), \tag{5}$$

where $\delta = \beta_1 - \beta_2$, and $\Sigma = \sigma_1^2(X_1'X_1)^{-1} + \sigma_2^2(X_2'X_2)^{-1}$. A solution to the problem of testing for structural change is just a test of the hypothesis that $H_0:\ \beta_1 - \beta_2 = 0$ based on the estimated covariance matrices.

There are two types of solutions to this problem of testing for structural change: the exact and asymptotic tests. The Jayatissa and the Weerahandi tests are exact tests in which the exact distributions of the test statistics are known under the null hypothesis. The Wald test and the empirical likelihood type test are asymptotic ones, where the asymptotic null distributions are known but the actual null distributions in finite samples are unknown. We have chosen the Jayatissa test, the Weerahandi test, and the Wald test for comparison purposes. The reason that these tests are chosen is that they are the most commonly used tests in the econometrics literature associated with testing for this type of structural change.

2.1 Jayatissa test (J)

Jayatissa (1977) proposed the J test in which the test statistic has an exact central F distribution under the null hypothesis of no structural change. The J test has been the corner stone and benchmark in the literature on testing regression vector equality in the presence of heteroscedasticity. The virtue of this test is that the probability of an incorrect rejection of the null hypothesis does not depend on the values of the nuisance parameters, the variances $\sigma_i^2, i = 1,\ 2$.

The J test makes use of the transformed regression residual vectors, ε_i^*, $i = 1,\ 2$, as in (5), and the decomposition of the matrices $(X_i'X_i)^{-1} = Q_i'Q_i$ where the Q_i are $k \times k$ matrices. However, if the numbers of observations from the two regressions are not equal suppose $(n_1 > n_2)$, then the vector ε_1^* is truncated to have a length of n_2. If n_2/k is not an integer, then the two residual vectors are truncated again in order to form the J test statistic.

The criticisms arise from the fact that the J test does not use all of the data efficiently. It involves throwing away some of the information by truncating the residual vectors. It also lacks uniqueness, for there are different methods that could be used to decompose the matrices $(X_i'X_i)^{-1}$. In addition to this, the J test requires the minimum sample size, *i.e.*, $\min((n_1 - k)/k, (n_2 - k)/k) > 1$. Watt (1979) and Honda (1982) have discussed these issues in more detail.

2.2 Weerahandi test (WEE)

Weerahandi (1987) introduced a new approach to testing for structural change. The WEE test yields a particular type of exact solution to the problem. It is an exact test based on the observed level of significance, the p-value.

The test is to reject the null hypothesis of no structural change if the p-value is too small, for instance, smaller than a preassigned significance level. The computational work associated with the construction of the WEE test is moderate. It requires only a one-dimensional numerical integration over the quantity $R = \frac{\hat{\varepsilon}_1'\hat{\varepsilon}_1}{\sigma_1^2} / (\frac{\hat{\varepsilon}_1'\hat{\varepsilon}_1}{\sigma_1^2} + \frac{\hat{\varepsilon}_2'\hat{\varepsilon}_2}{\sigma_2^2})$, which is distributed as $\text{Beta}((n_1 - k)/2,\ (n_2 - k)/2)$ under the null hypothesis. The observed significance level is obtained from the formula:

$$p = 1 - E^R(F_{k,T}(V)) \tag{6}$$

where $F_{k,T}$ is the cumulative F-distribution with degrees of freedom $(k,\ T)$,

$$V = \frac{T}{k}\hat{\delta}'(\frac{SSR_1}{R}(X_1'X_1)^{-1} + \frac{SSR_2}{1-R}(X_2'X_2)^{-1})^{-1}\hat{\delta}, \tag{7}$$

$T = n_1 + n_2 - 2k$, $\hat{\delta} = \hat{\beta}_1 - \hat{\beta}_2$, and SSR_i are the sums of squared residuals, $i = 1,\ 2$.

The WEE test performs well for small samples. The two parameters of the Beta distribution that are involved in computing the WEE test statistic depend on the sample sizes $(n_1,\ n_2)$. When the sample sizes are large, these two parameters become large; the integration over the space for R, which is $(0,\ 1)$, yields a result very close to zero; and then the calculated p-value becomes close to one. Thus, the WEE test fails to reject any hypothesis when the sample sizes are large.

The p-value approach is useful for some problems with nuisance parameters, such as the problem of structural change with the σ_i^2 as nuisance parameters. The probability of an incorrect rejection of the null hypothesis depends on the observations and the nuisance parameters. It is not fixed in advance. To make testing on the basis of the p-value comparable to the fixed level testing, we can choose to reject the null hypothesis whenever the p-value is less than the preassigned nominal significance levels. Weerahandi's p-value approach often yields a useful and clear solution while the fixed level testing does not (Zaman, 1996, p. 247).

2.3 Wald test

Watt (1979) and Honda (1982) proposed a Wald test under the inequality of the two variances. The test statistic has the form:

$$w = (\hat{\beta}_1 - \hat{\beta}_2)'(\hat{\sigma}_1^2(X_1'X_1)^{-1} + \hat{\sigma}_2^2(X_2'X_2)^{-1})^{-1}(\hat{\beta}_1 - \hat{\beta}_2) \tag{8}$$

where $\hat{\sigma}_i^2 = \frac{\hat{\varepsilon}_i'\hat{\varepsilon}_i}{n_i - k}$, $i = 1, 2$ are the usual unbiased least squares estimators of the variances of the error terms. The Wald test is obviously easy to compute and the test statistic has an asymptotic distribution of $\chi^2_{(k)}$.

Watt (1979) and Honda (1982) provided comparisons of the size and the power of the Wald test and the J test. They pointed out that only when the number of regressors is one, $k = 1$, is the J test preferred to the Wald test. For $k > 1$, the Wald test always outperforms the J test in terms of higher power. The limitation of these two studies are essentially two-fold. First, when the power of the Wald test was calculated, the number of rejections was counted with reference to the asymptotic distribution of the test rather than the actual distribution of the test in finite samples. Second, Watt(1979) and Honda (1982) considered an "ad hoc $W2$" test, this being the Wald test applied at the 2.5% significance level, but used in this case to approximate a 5% level test.

3 Empirical likelihood approach

3.1 Empirical likelihood method in a regression model

The theory associated with applying the EL method to a regression model for the estimation of the coefficient vector β was established by Owen (1990 and 1991). As illustrated in Mittelhammer *et al.* (2000, p. 306), the unbiased moment equations used in the EL approach in the context of regression are of the form:

$$E(h(Y, \beta)) = E(X'(Y - X\beta) = 0. \tag{9}$$

This is the case when the number of moment equations equals the number of the parameters. The solution from the equation system solves the maximization problem of the empirical likelihood with the weights $p_j = n^{-1}$, where j denotes the jth observation, and the likelihood function achieves its maximum, $L(F_n) = n^{-n}$. The EL estimator of the coefficient vector β is precisely the same as the least squares estimator, since the moment equations coincide with those equations used in the least squares estimation method.

The EL estimator β^{EL} is more efficient than the least square estimator β^{LS} when heteroscedasticity presents. In the context of the classical linear regression model with the assumptions of homoscedasticity and a multivariate normal distribution for the error term, $\hat{\beta}^{LS}$ is unbiased and most efficient. When the homoscedasticity assumption is dropped, $\hat{\beta}^{LS}$

is still unbiased but it is inefficient. The variance of $\hat{\beta}^{LS}$ is no longer consistently estimated by $(X'X)^{-1}\hat{\sigma}^2$. However, the asymptotic covariance matrix estimator using the EL approach is still asymptotically efficient even under heteroscedasticity.

The EL estimated covariance matrix $\hat{\Sigma}$ of the EL estimator $\hat{\beta}$ has the form:

$$\hat{\Sigma} = [n^{-1}(X'X)^{-1}(\sum_{j=1}^{n} \hat{p}_j(y_j - x_j'\hat{\beta})^2 x_j' x_j)(X'X)^{-1}]^{-1}. \tag{10}$$

There is a close analogy between $\hat{\Sigma}^{-1}$ and White's (1980) heteroscedasticity-robust estimate of the covariance matrix of $\hat{\beta}^{LS}$. So the EL method is able to capture the information associated with the possible presence of heteroscedasticity.

When the regressor matrix X is non-stochastic, the EL approach to the regression model actually becomes more complicated than when random regressors are allowed. The set of moment equations for each observation has the form:

$$h(y_j, \beta) = x_j'(y_j - x_j\beta), \quad \text{for} \quad j = 1, \ldots, n. \tag{11}$$

It is unbiased, $E(h(y_j, \beta)) = 0$, but the covariance matrix of $h(y_j,\ \beta)$ varies with each observation:

$$cov(h(y_j, \beta)) = \sigma^2 x_j' x_j; j = 1,\ \ldots,\ n. \tag{12}$$

That is, the $h(y_j,\ \beta)$ are not identically distributed for all j.

Theorem 2 in Owen (1991) provides a solution to this situation when the data are not identically distributed. We denote: $cov(h(y_j, \beta)) = \Phi_j$, and $V_n = n^{-1}\sum_{j=1}^{n} \Phi_j$. ξ^S and ξ^L are the smallest and largest eigenvalues of V_n. The following assumptions are made:

1. $\lim_{n\to\infty} P(0 \in ch\{h(y_1,\ \beta),\ \ldots,\ h(y_n,\ \beta)\}) = 1$, where $ch\{\}$ denotes the convex hull of the data;
2. $n^{-2}\sum_{j=1}^{n} E\frac{\|h(y_j,\beta)\|^4}{\xi^{L2}} \to 0$, as $n \to \infty$;
3. $\frac{\xi^S}{\xi^L} \geq c > 0$, $\forall n \geq k$;

Under these assumptions, minus two times the log empirical likelihood ratio function,

$$-2\log R(\beta) = -2(\log L(\hat{\beta}^c) - \log L(\hat{\beta})^u), \tag{13}$$

has a limiting distribution of $\chi^2_{(d)}$, where d is the number of restrictions.

This theorem enable us to relax the assumption that the data are i.i.d. in the standard EL approach. This theorem is essential for us to handle the regression models with non-random regressors. The Lindeberg-Levy central limit theorem is replaced by the Lindeberg-Feller central limit theorem to deal with the asymptotics in this non-i.i.d. case. The largest eigenvalue of V_n is used to scale the problem. With this theory, we are able to set up the EL approach for the problem of testing for structural change in a regression model.

3.2 The EL$_W$ test

For the two linear regression models in the problem of structural change, the EL estimators, $\hat{\beta}_i$, $i = 1,\ 2$, of the coefficient vectors are the same as the least squares estimators. From the regression model using the $\hat{\beta}_i$'s, we obtain two least squares residual vectors: $\hat{\varepsilon}_i = Y_i - X_i\hat{\beta}_i$, and these residual vectors are distributed as: $\hat{\varepsilon}_i \sim N(0,\ M_i\sigma_i^2)$.

The objective of this section is to develop a EL$_W$ test for the equality of the two coefficient vectors. The null hypothesis is:

$$H_0:\ \beta_1 = \beta_2.$$

We know that the distribution of $\hat{\delta} = \hat{\beta}_1 - \hat{\beta}_2$ is $N(0,\ \sigma_1^2(X_1'X_1)^{-1} + \sigma_2^2(X_2'X_2)^{-1})$ under the null hypothesis and under the classical assumptions for each of the two regression models. Suppose the X_i matrices are non-stochastic. Then the possible efficiency gain of the EL approach comes from the EL estimators of σ_i^2's. We hope that the EL estimators of the σ_i^2's would be more efficient than the least squares estimators given the fact that the EL approach utilizes both the likelihood functions and the information available in terms of the data distribution and the equality of two coefficient vectors.

The data that we have are the two sets of least squares residuals $\hat{\varepsilon}_i$, $i = 1,\ 2$. As the OLS residuals are not independently distributed, we first transform the OLS residuals $\hat{\varepsilon}_i$ into a type of BLUS residuals, $\varepsilon_i^* \sim N(0,\ \sigma_i^2 I_{n_i-k})$, as in equation (5). The Z_i are the $n_i \times (n_i - k)$ eigenvector matrices of M_i matrices corresponding to the unit roots. The BLUS residual vectors $\varepsilon_i^* = Z_i'\hat{\varepsilon}_i$ have the distribution $N(0, \sigma_i^2 I_{n_i-k})$, for $i = 1,\ 2$.

The data transformation technique that was described in Dong (2004) is applied here to the two sets of the residuals. We transform the residual vector ε_1^* to a vector V_1 that has the same distribution as ε_2^*; we combine the two sets of the residuals V_1 and ε_2^* to form a full set of residuals that are i.i.d.; then we apply the EL approach to the full set of residuals.

The EL approach that we develop here allows us to achieve three objectives sequentially. (i) Obtain the EL estimators of the two variance parameters that are more efficient; (ii) Construct an EL_W test for the structural change problem; (iii) Conduct an ELR test for the normality of the disturbance terms in the presence of possible heteroscedasticity.

The steps associated with implementing the EL approach to the problem of testing for structural change are as follows.

Step 1. Transform the residual vector ε_1^* to have the same distribution as of the residual vector ε_2^* using the formula:

$$V_1 = \varepsilon_1^*(\rho^2)^{\frac{1}{2}} \tag{14}$$

where $\rho^2 = \sigma_2^2/\sigma_1^2$. Then, $V_1 \sim N(0,\ \sigma_2^2 I_{n_1-k})$. Let

$$V_2 = \varepsilon_2^*. \tag{15}$$

Stacking the two vectors, V_1 and V_2 on top of each other, we get the full set of the residuals $V = \{v_1,\ v_2,\ \ldots,\ v_T\}'$, where $T = n_1 + n_2 - 2k$. The residual vector V has a distribution $N(0,\ \sigma_2^2 I_T)$.

Assign a probability parameter p_j to v_j, the jth element of the residual vector V. The empirical likelihood function that is supported on the data is formed by $\prod_{j=1}^T p_j$. Maximizing the empirical likelihood function $\prod_{j=1}^T p_j$ subject to the probability constraints and the moment constraints is the conventional EL method. The Lagrangian function of the log empirical likelihood function has the form:

$$G = T^{-1}\sum_{j=1}^{T}\log p_j - \eta(\sum_{j=1}^{T} p_j - 1) - \lambda'\sum_{j=1}^{T} p_j h(v_j,\theta) \tag{16}$$

where $E[h(v_j,\theta)] = 0$ is the set of the first four unbiased moment equations for the residual vector V. The empirical version of $E[h(v_j,\theta)] = 0$ has the form:

$$\sum_{j=1}^{T} p_j v_j = 0 \tag{17}$$

$$\sum_{j=1}^{T} p_j v_j^2 - \sigma_2^2 = 0 \tag{18}$$

$$\sum_{j=1}^{T} p_j v_j^3 = 0 \tag{19}$$

$$\sum_{j=1}^{T} p_j v_j^4 - 3\sigma_2^4 = 0. \tag{20}$$

The parameter vector is $\theta = (\rho^2,\ \sigma_2^2)'$. The optimal value of the Lagrangian multiplier η is unity. The p_j's have the expression:

$$p_j = T^{-1}(1 + \lambda' h(v_j, \theta))^{-1},\ j = 1,\ 2,\ \ldots,\ T. \tag{21}$$

Note that the elements in the first portion of the vector V are functions of the parameters; the first order derivative of these elements with respect to the parameter has the form:

$$\frac{\partial v_j}{\partial \theta} = (\varepsilon_{1j}^*(\rho^2)^{-0.5} 0.5,\ 0)', \tag{22}$$

for $j = 1,\ 2,\ \ldots,\ n_1 - k$. The first order conditions of the Lagrangian function with respect to the parameters have the form:

$$\sum_{j=1}^{n_1-k} p_j(\lambda_1 + 2\lambda_2\varepsilon_{1j}^* + 3\lambda_3\varepsilon_{1j}^{*2} + 4\lambda_4\varepsilon_{1j}^{*3})(\rho^2)^{-0.5} 0.5 = 0 \tag{23}$$

$$\sum_{j=1}^{T} p_j(\lambda_2 + 6\lambda_4\sigma_2^2) = 0. \tag{24}$$

Solving the equation system of the four moment equations and the two first order conditions, we get the EL estimators $\tilde{\sigma}_2^2$, $\hat{\rho}^2$, and $\hat{\lambda}$. Then we obtain the probability parameter estimators, the $\hat{p_j}$'s, using the formula of the p_j's, for $j = 1,\ 2,\ \ldots,\ T$. The estimator of the parameter σ_1^2 can be recovered from: $\tilde{\sigma}_1^2 = \tilde{\sigma}_2^2 / \hat{\rho}^2$.

Step 2. With the EL estimators of the variance parameter in hand, we can conduct an EL_W test for the problem of structural change. The EL_W test statistic has the form:

$$EL_W = (\hat{\beta}_1 - \hat{\beta}_2)'(\tilde{\sigma}_1^2(X_1'X_1)^{-1} + \tilde{\sigma}_2^2(X_2'X_2)^{-1})^{-1}(\hat{\beta}_1 - \hat{\beta}_2) \tag{25}$$

where $\tilde{\sigma}_i^2$ are the EL estimators of σ_i^2, $i = 1, 2$. The test statistic has an asymptotic distribution of $\chi^2_{(k)}$ under the null hypothesis.

Step 3. Testing for normality

With the estimators of the probability parameters, $\hat{p}_j$'s, we can easily set up an empirical likelihood ratio test for normality in the error terms. The log empirical likelihood ratio statistic has the form:

$$-2\log R(\hat{\theta}) = -2\sum_{j=1}^{T} \log T\hat{p}_j, \tag{26}$$

and it has a limiting distribution of $\chi^2_{(2)}$. This is a ELR test for normality in the context of the problem of structural change. That is, the EL approach we described above can be used to test for normality in a regression model in the presence of heteroscedasticity.

In a general situation, we may be interested in using the technique that is described here to test for normality of the two underlying populations. If we are satisfied with the results and accept the null hypothesis that the underlying populations are normal, then, we can continue to test for the problem of structural change in regression. While it is recognized that this can give rise to "preliminary test" distortions (Giles and Giles, 1993), this is an issue that we do not pursue further here. Alternatively, the settings here are similar to that in Dong (2004), we can conduct the two tests, testing for normality and structural change, simultaneously so as to avoid the pretesting issue.

4 Monte Carlo experiments

The design of our Monte Carlo experiments is based on the regression models:

$$Y_1 = \beta_{11} + \beta_{12}x_1 + \varepsilon_1 \tag{27}$$

$$Y_2 = \beta_{21} + \beta_{22}x_2 + \varepsilon_2. \tag{28}$$

The disturbance vector ε_i has a distribution of $N(0,\ \sigma_i^2 I_{n_i})$; n_i is the number of observations for the ith model, $i = 1,\ 2$. The error terms in the two regression models are independent of each other and they are independent of the regressor variables x_i. The variance ratio parameter of the two error terms is $\rho^2 = \sigma_2^2/\sigma_1^2$. The true value of the parameter ρ^2 changes according the values $\{0.1,\ 0.5,\ 1,\ 2,\ 10\}$. The true value of the parameter σ_1^2 equals unity, the true value of the parameter σ_2^2 varies with the parameter ρ^2.

The number of replications is chosen to be $5,000$. The sample size pair $(n_1,\ n_2)$ ranges from $(20,\ 10)$, $(60,\ 30)$, $(100,\ 50)$, to $(250,\ 125)$. The ratio of the sample sizes is kept constant at two.

The coefficient vector for the first regression model is $\beta_1 = \begin{pmatrix} \beta_{11} \\ \beta_{12} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. $\beta_2 = \begin{pmatrix} \beta_{21} \\ \beta_{22} \end{pmatrix}$ is the coefficient vector of the second linear regression model in the problem. Under the null hypothesis, we have $\beta_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Otherwise, this vector is $\beta_2 = \begin{pmatrix} 1 \\ \beta_{22} \end{pmatrix}$, where β_{22} varies with the non-centrality parameter, δ.

In computing the power of the test, the non-centrality parameter is chosen to be:

$$\delta = [(\beta_1 - \beta_2)'(\sigma_1^2(X_1'X_1)^{-1} + \sigma_2^2(X_2'X_2)^{-1})^{-1}(\beta_1 - \beta_2)]^{\frac{1}{2}}. \tag{29}$$

With only two regressors in our setting, this can be simplified to

$$\delta = [(1 - \beta_{22})^2 d]^{\frac{1}{2}}, \tag{30}$$

where

$$d = \left(\frac{\sigma_1^2}{\sum_{t=1}^{n_1} x_{1t}^2 - (\sum_{t=1}^{n_1} x_{1t})^2} + \frac{\sigma_2^2}{\sum_{t=1}^{n_2} x_{2t}^2 - (\sum_{t=1}^{n_2} x_{2t})^2}\right). \tag{31}$$

Thus, the parameter $\beta_{22} = 1 - \delta/d^{\frac{1}{2}}$. It changes with the non-centrality parameter. The true value of δ is varied according to $\{1,\ 2,\ 3,\ 4\}$.

4.1 Regressor matrix X

The regressor matrix has the form of $X = \{1,\ x\}$ in our setting. It is kept fixed in each replication. That is, X is non-stochastic. The x vector is designed to come from two different generating processes. The first case is when the x vector is generated following a stationary AR(1) process as follows:

$$x_t = \rho_1 x_{t-1} + u_t, \quad t = 1, \ldots, n \tag{32}$$

where $x_0 = 0$ and the u_t are distributed i.i.d. $N(0,\ 1-\rho_1^2)$. The true value of the parameter ρ_1 equals 0.5. The results are invariant to this value. We first generate the x vector of size $n+300$, and then we discard the first 300 observations so as to eliminate the effect of x_0. We keep the remainder of the x vector fixed and partition it into the sample sizes $(n_1,\ n_2)$ as needed. We then join the x vector with a column of ones to form the regressor matrices. The regressor matrices X_1 and X_2 are non-stochastic.

The second case is when the x vector is generated from a uniform distribution: $U(0,\ 1)$. Steps corresponding to those described above are taken to ensure that the matrices X_1 and X_2 are non-stochastic. In this second setting, the true values of the elements of the β_i's are chosen to be different from the first setting. The true values of the coefficient vectors are $\beta_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ under the null hypothesis. Under the alternative hypothesis, the true values of the vector β_2 is $\beta_2 = \begin{pmatrix} 0 \\ \beta_{22} \end{pmatrix}$, and $\beta_{22} = 1 - \delta/d^{\frac{1}{2}}$ varies with the non-centrality parameter as was described in the first case.

5 Results

The tables in the Appendix provide complete comparisons of the sizes, the size-adjusted critical values, and the powers for the four tests: the EL_W test, the Wald test by Watt (1979) and Honda (1982), the WEE test, and the J test, across a wide range of situations. We have chosen to use the four nominal significance levels: 10%, 5%, 2%, 1% to provide a broad picture of the sampling properties of the tests. As described in Section 6.3, the parameter ρ^2 varies according to the values of $\{0.1,\ 0.5,\ 1,\ 2,\ 10\}$ and the sample size pair increases following the pattern of $(n_1,\ n_2) = \{(20,\ 10),\ (60,\ 30),\ (100,\ 50),\ (250,\ 125)\}$.

In each replication, the same data set is used in the application of all of the four tests. The J test is an exact test with a known distribution in finite samples, but we have simulated the sizes and the size-adjusted critical values, and we use these critical values to compute the powers for the J test to make them comparable with those for the other two asymptotic tests. For the WEE test, only the sizes are provided as the WEE test is an exact test that uses the observed significance level, the p-value, for inference purposes. The concept of the size-adjusted critical value is not applicable for the WEE test. In computing the powers for the WEE test, the observed significance levels, rather than the size-adjusted critical values, under the alternative hypotheses are used.

The first group of tables, Tables 1 to 13, provides the results for the case when the regressor x is generated following an AR(1) process, as described in the previous section. Tables 1 to 3 present the sizes and the size-adjusted critical values for the three tests: the EL_W test, the Wald test, and the J test.

The sizes of the EL_W test are slightly lower than the nominal significance levels. These sizes converge to the correct nominal levels as the sample size pair $(n_1,\ n_2)$ grows. For example, the size changes from 3.74% to 4.72% when the sample pair grows from $(20,\ 10)$ to $(250,\ 125)$ at the nominal significance level of 5%, with $\rho^2 = 0.1$. The size distortion of a test is the difference between the actual size of the test and the nominal significance level. The size distortion of the EL_W test is small and it changes with the value of the parameter ρ^2; it actually grows as the parameter ρ^2 varies from 0.1 to 10. For instance, the size distortion is -1.26% at $\rho^2 = 0.1$ and it is 21.6% at $\rho^2 = 10$ for the sample pair $(n_1,\ n_2) = (20,\ 10)$ and at $\alpha = 5\%$. The size distortion is the worst when $\rho^2 = 10$. However, the size distortion is within the range of our expectation for the EL-type tests and it tends to vanish as the sample sizes grow.

The size of the Wald test has the same patterns as the EL_W test. The size of the test converges to the correct nominal level as the sample size pair grows. The size slightly increases with the value of the ρ^2 parameter. For the cases when $\rho^2 = 2$ and $\rho^2 = 10$, the size distortion of the test is less severe than that of the EL_W test. In another words, the size of the Wald test is better for the cases when $\rho^2 = 2$ and $\rho^2 = 10$ than is that of the EL_W test.

The size-adjusted critical values of both the EL_W and the Wald tests are associated with the regressor matrices that we use. They are not universally applicable to the situation with a different regressor matrix X. To assist researchers in using the EL_W test, we will provide a small library in Gauss code that contains two procedures, in future work. One is used to take in a general regressor matrix X and to compute the actual size and size-adjusted critical values; another is used to take in the regressor matrix and size-adjusted critical values and to calculate the power of the EL_W test.

The sizes of the J test and the WEE test are very close to their nominal levels. In addition, they are robust against changes in the value of ρ^2 and changes in the sample size. These outcomes result from the fact that both of the J test and the WEE test are exact tests. The WEE test is not applicable for the sample pair (250, 125) for the reasons explained in Section 2.2.

Tables 4 to 13 provide the power comparisons for the four selected tests. The powers of the three tests, the EL_W, the Wald, and the J test are computed using the respective size-adjusted values. That is, we compare the powers of the tests at the same actual levels. The power of the WEE test is computed using the observed significance level, the p-value. We reject the null hypothesis when the p-value is smaller than the same actual levels used for other tests.

The power of the EL_W test grows as the non-centrality parameter δ increases, given the sample size pair and the value of the parameter ρ^2. For example, the power grows from 16.44% to 56.82% as δ increases from 1 to 4, given $\rho^2 = 0.1$, the sample size pair $(n_1,\ n_2) = (20,\ 10)$, and at the actual size level of $\alpha = 5\%$.

The power of the EL_W test increases as the sample size grows, given the the values of δ and ρ^2. For example, the power increases from 56.82% to 70.5% when $(n_1,\ n_2)$ increases from $(20,\ 10)$ to $(250,\ 125)$, given $\delta = 4$, $\rho^2 = 0.1$, and at the actual size level of $\alpha = 5\%$. The power of the test changes very little as the parameter ρ^2 varies, given the sample size pair, the non-centrality parameter, and the actual size level.

The power performances of the four tests that we consider are very similar. For the case when $(n_1,\ n_2) = (20,\ 10)$ and $\rho^2 = 0.1$, the power comparison in descending order is: the Wald test, the WEE test, the EL_W test, and the J test. For the case of larger sample size pair $(n_1,\ n_2) = (250,\ 125)$ with $\rho^2 = 0.1$, the order is: the Wald test, the EL_W test, and the J test. The WEE test is not applicable in the cases whenever $(n_1,\ n_2) = (250,\ 125)$. The order of the power for the tests changes when the parameter ρ^2 varies, but the difference among them is minor. All the four tests have good power properties in testing for the problem of structural change in the regression model.

Table 14 to 17 provide the sizes and the size-adjusted critical values for the four tests when the regressor x is obtained from a Uniform distribution and kept fixed in each replication. The values of the parameter ρ^2 and the sample size pair $(n_1,\ n_2)$ change in the same way as described in the first case. Table 18 to 27 illustrate the power comparisons for the tests of the cases when the non-centrality parameter δ varies. The comparison results are similar to those in the first case.

6 Summary and conclusions

The problem of structural change in regressions has attracted considerable interest in the literature. There have been several well known tests developed, such as Jayatissa test and Weerahandi test. In this paper, we derived a empirical likelihood type Wald (EL_W) test by applying the maximum empirical likelihood method to this problem of testing for the absence of structural change in a regression model. The fact that the maximum empirical likelihood method can be used to this problem demonstrates the flexibility of the EL method.

Using the Monte Carlo simulation technique, we have provided a detailed analysis for the sampling properties of the EL_W test, and we have conducted a comparison of these sampling properties for the EL_W test and other conventional tests. The comparisons are made across the full dimensions of the parameter space, including different designs of the regressor matrices. Our results indicate that the EL_W test is as powerful as the other tests. Overall, the EL approach provides efficient testing procedures and the EL_W test has good power properties.

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Appendix: Tables of structural change

Table 1: Size and Size Adjusted Critical Values, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 0.1$				$\rho^2 = 0.5$			
$(n_1,\ n_2)$	(20, 10)	(60, 30)	(100, 50)	(250, 125)	(20, 10)	(60, 30)	(100, 50)	(250, 125)
EL-W test at nominal levels:								
10%	0.0730	0.0730	0.0704	0.0840	0.0858	0.0980	0.0888	0.0972
5%	0.0374	0.0383	0.0371	0.0416	0.0462	0.0552	0.0448	0.0480
2%	0.0180	0.0160	0.0160	0.0154	0.0228	0.0240	0.0154	0.0208
1%	0.0110	0.0092	0.0090	0.0090	0.0128	0.0130	0.0080	0.0104
Size-Adjusted Critical Values:								
10%	3.9241	4.0676	4.0206	4.3125	4.3119	4.5473	4.3753	4.5177
5%	5.4822	5.3601	5.3769	5.6873	5.7367	6.1951	5.7596	5.9109
2%	7.6182	7.2349	7.2311	7.3603	8.0932	8.2175	7.3601	7.9367
1%	9.3748	8.8698	8.7977	9.1190	10.185	10.005	8.6382	9.2574
Wald test :								
10%	0.1280	0.1090	0.1032	0.1040	0.1283	0.1090	0.0995	0.0994
5%	0.0728	0.0620	0.0522	0.0526	0.0728	0.0584	0.0509	0.0490
2%	0.0370	0.0252	0.0231	0.0225	0.0342	0.0294	0.0194	0.0216
1%	0.0214	0.0124	0.0138	0.0126	0.0190	0.0180	0.0110	0.0100
Size-Adjusted Critical Values:								
10%	5.2138	4.8003	4.6500	4.7040	5.2206	4.7854	4.5933	4.5964
5%	7.0280	6.4164	6.1223	6.0811	6.9428	6.4037	6.0578	5.9422
2%	9.3464	8.3834	8.3975	7.9536	9.0290	8.9807	7.7791	8.0267
1%	11.1943	10.0002	9.9763	9.7063	10.8544	10.451	9.3119	9.2080
J test :								
10%	0.0982	0.1070	0.0985	0.1056	0.0984	0.1042	0.1010	0.0927
5%	0.0480	0.0488	0.0478	0.0522	0.0492	0.0562	0.0488	0.0466
2%	0.0200	0.0216	0.0218	0.0222	0.0194	0.0240	0.0180	0.0220
1%	0.0076	0.0092	0.0102	0.0118	0.0108	0.0150	0.0088	0.0112
Size-Adjusted Critical Values:								
10%	5.3889	2.8113	2.5242	2.4551	5.3855	2.7956	2.5564	2.3237
5%	9.3401	3.7716	3.3628	3.1857	9.5194	4.0153	3.3741	3.1102
2%	18.475	5.4456	4.7514	4.319	18.4115	5.7280	4.5575	4.2390
1%	27.3093	6.4371	5.6666	5.1832	32.1514	7.3551	5.4789	5.0438
WEE test :								
10%	0.0948	0.0992	0.0961	–	0.0859	0.0960	0.0930	–
5%	0.0463	0.0516	0.0488	–	0.0386	0.0496	0.0463	–
2%	0.0162	0.0200	0.0216	–	0.0122	0.0225	0.0162	–
1%	0.0072	0.0102	0.0112	–	0.0060	0.0124	0.0078	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with larger sample sizes.

Table 2: Size and Size-Adjusted Critical Values, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 1$				$\rho^2 = 2$			
Sample size:	(20, 10)	(60, 30)	(100, 50)	(250, 125)	(20, 10)	(60, 30)	(100, 50)	(250, 125)
EL-W test at nominal levels:								
10%	0.1196	0.1240	0.1090	0.1168	0.1638	0.1584	0.1318	0.1308
5%	0.0651	0.0724	0.0598	0.0620	0.1108	0.0960	0.0796	0.0746
2%	0.0325	0.0352	0.0278	0.0276	0.0662	0.0462	0.0360	0.0398
1%	0.0200	0.0196	0.0160	0.0148	0.0446	0.0292	0.0188	0.0225
Size-Adjusted Critical Values:								
10%	4.9605	5.1270	4.8045	4.9273	6.3331	5.8789	5.3248	5.3178
5%	6.7269	6.9166	6.4057	6.4209	8.7909	7.6497	7.1144	7.0886
2%	9.1073	9.1581	8.4132	8.6556	12.1633	10.4332	9.1453	9.5255
1%	11.360	10.9984	10.5341	10.1076	14.7807	12.5284	10.4337	11.0906
Wald test :								
10%	0.1258	0.1114	0.1022	0.1044	0.1368	0.1206	0.1086	0.1072
5%	0.0734	0.0606	0.0566	0.0538	0.0834	0.0644	0.0578	0.0592
2%	0.0364	0.0284	0.0242	0.0231	0.0448	0.0292	0.0228	0.0282
1%	0.0208	0.0152	0.0148	0.0118	0.0298	0.0172	0.0102	0.0132
Size-Adjusted Critical Values:								
10%	5.1752	4.8401	4.6440	4.6913	5.4119	5.0095	4.7581	4.7812
5%	7.0252	6.4137	6.2856	6.1137	7.5290	6.5304	6.2902	6.4230
2%	9.2898	8.5605	8.3555	8.0896	10.4506	8.9098	8.0356	8.4791
1%	11.0669	10.0879	10.0416	9.6211	13.3108	10.7222	9.2518	10.0419
J test:								
10%	0.1014	0.1096	0.1000	0.1052	0.0964	0.1036	0.1000	0.1046
5%	0.0458	0.0550	0.0526	0.0540	0.0500	0.0548	0.0518	0.0584
2%	0.0191	0.0224	0.0222	0.0228	0.0202	0.0222	0.0218	0.0256
1%	0.0084	0.0106	0.0095	0.0106	0.0118	0.0108	0.0094	0.0138
Size-Adjusted Critical Values:								
10%	5.5241	2.8717	2.5468	2.4589	5.2738	2.8077	2.5473	2.4564
5%	9.0994	3.9826	3.5018	3.2342	9.5241	3.9280	3.4469	3.2940
2%	18.0297	5.5664	4.7575	4.3470	19.0044	5.5976	4.7747	4.3624
1%	26.4114	6.8010	5.6176	5.0388	35.5952	6.8281	5.5962	5.2980
WEE test :								
10%	0.0806	0.0972	0.0960	–	0.0878	0.1028	0.0985	–
5%	0.0386	0.0496	0.0504	–	0.0422	0.0504	0.0504	–
2%	0.0126	0.0202	0.0208	–	0.0156	0.0224	0.0172	–
1%	0.0056	0.0094	0.0110	–	0.0082	0.0108	0.0080	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with larger sample sizes.

Table 3: Size and Size-Adjusted Critical Values, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 10$			
Sample size:	(20, 10)	(60, 30)	(100, 50)	(250, 125)
EL-W test at nominal levels:				
10%	0.2960	0.2046	0.1754	0.1544
5%	0.2174	0.1330	0.1092	0.0890
2%	0.1442	0.0822	0.0598	0.0454
1%	0.1070	0.0546	0.0378	0.0254
Size-Adjusted Critical Values:				
10%	9.5921	7.0449	6.2411	5.7256
5%	14.1282	9.5328	8.4754	7.4453
2%	20.9046	13.7238	11.4052	9.9942
1%	26.0257	16.1886	13.7603	11.8519
Wald test :				
10%	0.1542	0.1172	0.1074	0.1086
5%	0.0952	0.0670	0.0558	0.0572
2%	0.0562	0.0324	0.0272	0.0216
1%	0.0396	0.0196	0.0152	0.0098
Size-Adjusted Critical Values:				
10%	5.8456	4.9903	4.7658	4.8786
5%	8.2013	6.7091	6.2682	6.2549
2%	11.5971	9.1366	8.6370	8.0732
1%	14.5764	10.845	9.8575	9.1729
J test:				
10%	0.0948	0.1026	0.1000	0.1060
5%	0.0488	0.0508	0.0482	0.0532
2%	0.0191	0.0222	0.0176	0.0222
1%	0.0094	0.0126	0.0078	0.0088
Size-Adjusted Critical Values:				
10%	5.2953	2.7676	2.5447	2.4775
5%	9.3683	3.8351	3.3853	3.2253
2%	18.1224	5.5513	4.5569	4.3196
1%	29.8879	7.2861	5.2979	4.8404
WEE test :				
10%	0.0938	0.1006	0.0978	–
5%	0.0466	0.0512	0.0488	–
2%	0.0170	0.0218	0.0228	–
1%	0.0086	0.0102	0.0088	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with larger sample sizes.

Table 4: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 0.1$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2528	0.4384	0.5802	0.6984	0.3154	0.5114	0.6682	0.7808
5%	0.1644	0.3006	0.4372	0.5682	0.2092	0.3788	0.5352	0.6792
2%	0.0948	0.1780	0.3008	0.4074	0.1180	0.2410	0.3664	0.523
1%	0.0616	0.1222	0.2198	0.3074	0.0688	0.1608	0.2664	0.3978
Wald test :								
10%	0.2890	0.5016	0.6438	0.7688	0.3184	0.5266	0.6924	0.8004
5%	0.1812	0.3602	0.5062	0.6480	0.2124	0.3880	0.5522	0.6954
2%	0.0966	0.2322	0.3660	0.5074	0.1262	0.2662	0.3992	0.5562
1%	0.0674	0.1702	0.2756	0.4060	0.0792	0.1852	0.3000	0.4474
J test :								
10%	0.2150	0.3217	0.4218	0.5224	0.2940	0.4758	0.6200	0.7382
5%	0.1146	0.1952	0.2524	0.3234	0.1932	0.3486	0.4818	0.6162
2%	0.0458	0.0910	0.1170	0.1646	0.0976	0.1974	0.3036	0.4348
1%	0.0286	0.0522	0.0702	0.1019	0.0678	0.1456	0.2298	0.3396
WEE test :								
10%	0.3022	0.4940	0.6478	0.7516	0.3096	0.5168	0.6954	0.7976
5%	0.1942	0.3526	0.5092	0.6310	0.2034	0.3852	0.5646	0.6932
2%	0.1034	0.2164	0.3370	0.4704	0.1130	0.2592	0.4140	0.5498
1%	0.0640	0.1434	0.2508	0.3534	0.0709	0.1834	0.3166	0.4412

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 5: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 0.1$							
$(n_1,\ n_2)$	(100, 50)				(250, 150)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3366	0.5145	0.6778	0.7986	0.3258	0.5322	0.6824	0.8014
5%	0.2166	0.3822	0.5452	0.6898	0.2114	0.3938	0.5578	0.7050
2%	0.1134	0.2454	0.3836	0.5394	0.1278	0.2676	0.4182	0.5716
1%	0.0672	0.1620	0.2878	0.4138	0.0708	0.1810	0.3026	0.4408
Wald test :								
10%	0.3466	0.5302	0.6956	0.8136	0.3232	0.5397	0.6904	0.8070
5%	0.2272	0.4016	0.5732	0.7072	0.2188	0.4082	0.5696	0.7148
2%	0.1156	0.2472	0.4024	0.5506	0.1298	0.2764	0.4302	0.5832
1%	0.0678	0.1714	0.3002	0.4438	0.0796	0.1862	0.3222	0.4600
J test:								
10%	0.3286	0.4994	0.6574	0.7808	0.3104	0.5192	0.6742	0.7916
5%	0.2142	0.3684	0.5288	0.6646	0.2148	0.3960	0.5510	0.6929
2%	0.1090	0.2178	0.3492	0.4852	0.1130	0.2452	0.3926	0.5334
1%	0.0704	0.1540	0.2674	0.3796	0.0720	0.1674	0.2965	0.4274
WEE test :								
10%	0.3192	0.5430	0.6856	0.8058	–	–	–	–
5%	0.2146	0.4134	0.5740	0.7082	–	–	–	–
2%	0.1198	0.2723	0.4194	0.5679	–	–	–	–
1%	0.0758	0.1932	0.3217	0.4700	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 6: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 0.5$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2918	0.4362	0.5921	0.7054	0.3230	0.5135	0.6848	0.8016
5%	0.1988	0.3182	0.4584	0.5814	0.2064	0.3640	0.5402	0.6866
2%	0.1016	0.1876	0.3020	0.4168	0.1130	0.2368	0.3884	0.5326
1%	0.0604	0.1206	0.2084	0.3044	0.0662	0.1544	0.2808	0.4128
Wald test :								
10%	0.3192	0.4842	0.6542	0.7622	0.3248	0.5175	0.6894	0.8058
5%	0.2022	0.3478	0.5158	0.6444	0.2116	0.3794	0.5530	0.7018
2%	0.1180	0.2348	0.3708	0.5102	0.1056	0.2164	0.3736	0.5224
1%	0.0728	0.1642	0.2826	0.4088	0.0660	0.1554	0.2846	0.4214
J test:								
10%	0.2326	0.3226	0.4292	0.5130	0.3000	0.4672	0.6298	0.7510
5%	0.1224	0.1770	0.2516	0.3196	0.1772	0.3084	0.4616	0.5896
2%	0.0516	0.0834	0.1214	0.1572	0.0878	0.1696	0.2738	0.4028
1%	0.0210	0.0370	0.0606	0.0782	0.0500	0.0994	0.1794	0.2752
WEE test :								
10%	0.2700	0.4812	0.6212	0.7392	0.3222	0.5185	0.6770	0.8018
5%	0.1660	0.3390	0.4756	0.6018	0.2120	0.3850	0.5457	0.6888
2%	0.0792	0.1900	0.3118	0.4324	0.1144	0.2534	0.3872	0.5340
1%	0.0428	0.1254	0.2132	0.3236	0.0732	0.1758	0.2910	0.4272

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 7: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 0.5$							
$(n_1,\ n_2)$	(100, 50)				(250, 125)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3230	0.5290	0.6924	0.7956	0.3406	0.5368	0.6952	0.8234
5%	0.2156	0.4064	0.5674	0.6944	0.2340	0.4102	0.5726	0.7310
2%	0.1312	0.2770	0.4304	0.5662	0.1254	0.2602	0.4162	0.5816
1%	0.0882	0.1966	0.3412	0.4726	0.0866	0.1958	0.3292	0.4848
Wald test :								
10%	0.3322	0.5407	0.7042	0.8098	0.3430	0.5400	0.6954	0.8266
5%	0.2238	0.4154	0.5848	0.7094	0.2384	0.4158	0.5812	0.7348
2%	0.1358	0.2868	0.4436	0.5788	0.1258	0.2642	0.4214	0.5878
1%	0.0866	0.1960	0.3442	0.4782	0.0903	0.2072	0.3428	0.5030
J test:								
10%	0.3056	0.4996	0.6566	0.7680	0.3410	0.5387	0.6939	0.8238
5%	0.2058	0.3796	0.5286	0.6614	0.2264	0.4006	0.5584	0.7138
2%	0.1130	0.2410	0.3738	0.5082	0.1192	0.2532	0.3936	0.5496
1%	0.0736	0.1692	0.2864	0.4082	0.0767	0.1776	0.3026	0.4440
WEE test :								
10%	0.3368	0.5336	0.6896	0.8020	–	–	–	–
5%	0.2260	0.3996	0.5698	0.6996	–	–	–	–
2%	0.1300	0.2662	0.4188	0.5604	–	–	–	–
1%	0.0806	0.1874	0.3217	0.4626	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 8: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 1$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2914	0.4582	0.6004	0.7154	0.3197	0.5158	0.6802	0.7796
5%	0.1876	0.3230	0.4616	0.5762	0.1981	0.3724	0.5374	0.6667
2%	0.1044	0.1991	0.3120	0.4216	0.1080	0.2374	0.3886	0.5228
1%	0.0596	0.1318	0.2090	0.3050	0.0626	0.1578	0.2868	0.4136
Wald test :								
10%	0.3170	0.5018	0.6652	0.7675	0.3270	0.5264	0.6852	0.7884
5%	0.2058	0.3595	0.5244	0.6400	0.2120	0.3904	0.5552	0.6818
2%	0.1228	0.2366	0.3731	0.4966	0.1114	0.2428	0.3982	0.5312
1%	0.0808	0.1708	0.2824	0.3978	0.0716	0.1728	0.3044	0.4382
J test:								
10%	0.2178	0.3134	0.4306	0.5038	0.2800	0.4726	0.6153	0.7250
5%	0.1324	0.1845	0.2768	0.3368	0.1719	0.3156	0.4594	0.5814
2%	0.0562	0.0792	0.1292	0.1668	0.0906	0.1864	0.2985	0.4018
1%	0.0366	0.0484	0.0842	0.1094	0.0560	0.1224	0.2098	0.2962
WEE test :								
10%	0.2708	0.4652	0.6213	0.7326	0.3138	0.5076	0.6818	0.7886
5%	0.1690	0.3207	0.4766	0.5852	0.2054	0.3778	0.5467	0.6854
2%	0.0842	0.1844	0.3076	0.4180	0.1206	0.2484	0.3872	0.5380
1%	0.0494	0.1144	0.2046	0.3034	0.0728	0.1724	0.2868	0.4340

Notes to the table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 9: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 1$							
$(n_1,\ n_2)$	(100, 50)				(250, 125)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3380	0.5394	0.6850	0.7907	0.3178	0.5185	0.6942	0.7950
5%	0.2196	0.3996	0.5554	0.6770	0.2088	0.3934	0.5754	0.6998
2%	0.1268	0.2600	0.4078	0.5306	0.1080	0.2534	0.4006	0.5437
1%	0.0696	0.1535	0.2854	0.3963	0.0738	0.1820	0.3078	0.4464
Wald test :								
10%	0.3502	0.5544	0.6986	0.8074	0.3250	0.5264	0.7052	0.8020
5%	0.2248	0.4110	0.5674	0.6908	0.2124	0.4006	0.5874	0.7072
2%	0.1312	0.2660	0.4160	0.5400	0.1156	0.2660	0.4220	0.5570
1%	0.0766	0.1732	0.3158	0.4300	0.0746	0.1865	0.3206	0.4566
J test:								
10%	0.3268	0.5215	0.6632	0.7658	0.3081	0.5090	0.6802	0.7854
5%	0.2064	0.3718	0.5248	0.6350	0.2008	0.3758	0.5492	0.6814
2%	0.1116	0.2306	0.3605	0.4756	0.1012	0.2486	0.3956	0.5225
1%	0.0728	0.1634	0.2744	0.3744	0.0728	0.1806	0.3110	0.4318
WEE test :								
10%	0.3252	0.5180	0.6818	0.8054	–	–	–	–
5%	0.2252	0.3886	0.5584	0.7028	–	–	–	–
2%	0.1280	0.2530	0.4192	0.5629	–	–	–	–
1%	0.0835	0.1766	0.3120	0.4578	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 10: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 2$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2630	0.4188	0.5500	0.6788	0.3000	0.4960	0.6344	0.7712
5%	0.1590	0.2778	0.3966	0.5292	0.1991	0.3741	0.5162	0.6656
2%	0.0754	0.1580	0.2406	0.3524	0.1080	0.2376	0.3654	0.5080
1%	0.0460	0.0990	0.1610	0.2504	0.0674	0.1584	0.2723	0.4069
Wald test :								
10%	0.3138	0.4916	0.6375	0.7622	0.3090	0.5132	0.6518	0.7870
5%	0.1872	0.3348	0.4756	0.6156	0.2060	0.3844	0.5334	0.6766
2%	0.0938	0.1996	0.3042	0.4430	0.1076	0.2370	0.3706	0.5162
1%	0.0482	0.1120	0.1922	0.3062	0.0638	0.1596	0.2688	0.4058
J test:								
10%	0.2260	0.3396	0.4290	0.5308	0.2914	0.4726	0.6098	0.7392
5%	0.1130	0.1872	0.2436	0.3206	0.1780	0.3260	0.4516	0.5921
2%	0.0478	0.0774	0.1106	0.1526	0.0910	0.1865	0.2814	0.4122
1%	0.0218	0.0344	0.0526	0.0694	0.0528	0.1290	0.2006	0.3090
WEE test :								
10%	0.2728	0.4596	0.6072	0.7332	0.3258	0.5178	0.6812	0.7910
5%	0.1592	0.3154	0.4568	0.5914	0.2218	0.3834	0.5542	0.6796
2%	0.0812	0.1752	0.2944	0.4114	0.1222	0.2396	0.3970	0.5264
1%	0.0468	0.1070	0.1956	0.2965	0.0734	0.1666	0.3094	0.4196

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 11: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 2$							
$(n_1,\ n_2)$	(100, 50)				(250, 125)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3120	0.5128	0.6724	0.7947	0.3086	0.5110	0.6844	0.7802
5%	0.1988	0.3714	0.5400	0.6792	0.1928	0.3756	0.5406	0.6647
2%	0.1244	0.2488	0.3962	0.5464	0.0956	0.2372	0.3766	0.5072
1%	0.0886	0.1892	0.3207	0.4676	0.0634	0.1652	0.2926	0.4198
Wald test :								
10%	0.3272	0.5340	0.6934	0.8108	0.3187	0.5195	0.6934	0.7902
5%	0.2148	0.3898	0.5668	0.7060	0.1946	0.3806	0.5524	0.6774
2%	0.1342	0.2718	0.4300	0.5834	0.1038	0.2444	0.3943	0.5298
1%	0.0970	0.2110	0.3449	0.4960	0.0622	0.1676	0.3018	0.4286
J test:								
10%	0.3120	0.5088	0.6640	0.7828	0.3106	0.5108	0.6808	0.7804
5%	0.2008	0.3696	0.5188	0.6610	0.1948	0.3736	0.5376	0.6596
2%	0.1058	0.2268	0.3536	0.4868	0.1036	0.2346	0.3906	0.5164
1%	0.0748	0.1628	0.2760	0.4022	0.0624	0.1534	0.2800	0.4036
WEE test :								
10%	0.3246	0.5098	0.6872	0.7924	–	–	–	–
5%	0.2140	0.3776	0.5628	0.6874	–	–	–	–
2%	0.1168	0.2520	0.4136	0.5484	–	–	–	–
1%	0.0720	0.1810	0.3144	0.4452	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1,1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 12: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 10$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2698	0.4334	0.5852	0.6822	0.3066	0.5002	0.6350	0.7432
5%	0.1578	0.2788	0.4026	0.5074	0.2002	0.3750	0.5008	0.6203
2%	0.0709	0.1496	0.2312	0.3187	0.0980	0.2210	0.3248	0.4314
1%	0.0386	0.0926	0.1506	0.2198	0.0596	0.1548	0.2452	0.3434
Wald test :								
10%	0.2910	0.4588	0.6236	0.7348	0.3236	0.5292	0.6692	0.7868
5%	0.1830	0.3250	0.4634	0.5748	0.2116	0.3920	0.5346	0.6690
2%	0.0984	0.1968	0.3026	0.4060	0.1084	0.2442	0.3720	0.4994
1%	0.0556	0.1276	0.2026	0.2954	0.0654	0.1702	0.2770	0.4008
J test:								
10%	0.2206	0.3330	0.4366	0.5222	0.3032	0.4910	0.6336	0.7403
5%	0.1162	0.1920	0.2662	0.3164	0.1954	0.3502	0.4784	0.5986
2%	0.0482	0.0900	0.1283	0.1528	0.0893	0.1950	0.2942	0.4002
1%	0.0254	0.0462	0.0664	0.0826	0.0476	0.1086	0.1874	0.2652
WEE test :								
10%	0.2670	0.4568	0.5846	0.7034	0.3126	0.5038	0.6686	0.7866
5%	0.1613	0.3020	0.4346	0.5588	0.2082	0.3642	0.5380	0.6762
2%	0.0774	0.1706	0.2684	0.3662	0.1150	0.2304	0.3832	0.5198
1%	0.0468	0.1070	0.1780	0.2484	0.0748	0.1612	0.2838	0.4114

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 13: Power Comparison, Case 1. regressor $x \sim AR(1)$

	$\rho^2 = 10$							
$(n_1,\ n_2)$	(100, 50)				(250, 125)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3110	0.4950	0.6604	0.7710	0.3120	0.5148	0.6664	0.7776
5%	0.1976	0.3584	0.5188	0.6524	0.2114	0.4004	0.5514	0.6746
2%	0.1064	0.2300	0.3574	0.4944	0.1224	0.2610	0.3956	0.5188
1%	0.0634	0.1572	0.2728	0.3850	0.0806	0.1896	0.3060	0.4222
Wald test :								
10%	0.3282	0.5238	0.6948	0.8034	0.3140	0.5185	0.6776	0.7922
5%	0.2180	0.3918	0.5704	0.6969	0.2116	0.4069	0.5672	0.6934
2%	0.1104	0.2474	0.3980	0.5330	0.1302	0.2842	0.4298	0.5560
1%	0.0806	0.1892	0.3237	0.4538	0.0912	0.2240	0.3570	0.4840
J test:								
10%	0.3134	0.4940	0.6684	0.7762	0.3172	0.5188	0.6744	0.7856
5%	0.2034	0.3646	0.5312	0.6637	0.2084	0.3948	0.5500	0.6744
2%	0.1136	0.2384	0.3872	0.5058	0.1160	0.2617	0.4042	0.5152
1%	0.0772	0.1776	0.3114	0.4252	0.0872	0.2068	0.3340	0.4566
WEE test :								
10%	0.3282	0.5258	0.6864	0.8042	–	–	–	–
5%	0.2212	0.3916	0.5544	0.6994	–	–	–	–
2%	0.1258	0.2610	0.4008	0.5508	–	–	–	–
1%	0.0804	0.1850	0.2970	0.4422	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 14: Size and Size-Adjusted Critical Values, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 0.1$				$\rho^2 = 0.5$			
$(n_1,\ n_2)$	(20, 10)	(60, 30)	(100, 50)	(250, 125)	(20, 10)	(60, 30)	(100, 50)	(250, 125)
EL-W test at nominal levels:								
10%	0.0660	0.0718	0.0782	0.0916	0.1102	0.0902	0.0978	0.1014
5%	0.0374	0.0354	0.0383	0.0472	0.0648	0.0420	0.0470	0.0480
2%	0.0180	0.0120	0.0150	0.0196	0.0308	0.0172	0.0214	0.0196
1%	0.0122	0.0047	0.0060	0.0088	0.0190	0.0090	0.0106	0.0108
Size-Adjusted Critical Values:								
10%	3.7750	3.8969	4.1094	4.4349	4.8774	4.4135	4.5564	4.6314
5%	5.2994	5.3515	5.4618	5.8975	6.6703	5.6767	5.8985	5.9091
2%	7.4650	6.9453	7.1417	7.7820	8.9708	7.4310	7.9519	7.7614
1%	9.7809	7.9671	8.4766	8.9547	10.9034	8.9247	9.2294	9.3805
Wald test :								
10%	0.1198	0.1080	0.1062	0.1104	0.1180	0.1036	0.1038	0.1019
5%	0.0674	0.0564	0.0542	0.0594	0.0672	0.0534	0.0520	0.0480
2%	0.0316	0.0231	0.0212	0.0246	0.0342	0.0208	0.0216	0.0206
1%	0.0196	0.0122	0.0114	0.0138	0.0196	0.0108	0.0106	0.0102
Size-Adjusted Critical Values:								
10%	4.9596	4.7238	4.6984	4.8243	4.9918	4.6876	4.6647	4.6487
5%	6.7220	6.2828	6.1744	6.3649	6.7224	6.1036	6.0306	5.9311
2%	9.1258	8.0539	8.0013	8.2947	9.1083	7.9028	8.1867	7.8829
1%	10.7119	9.6299	9.4989	9.8070	10.8487	9.2949	9.3284	9.2365
J test:								
10%	0.0978	0.0998	0.0984	0.1116	0.0970	0.0992	0.1036	0.0984
5%	0.0504	0.0488	0.0497	0.0602	0.0516	0.0450	0.0490	0.0484
2%	0.0176	0.0196	0.0185	0.0242	0.0182	0.0158	0.0182	0.0198
1%	0.0094	0.0086	0.0098	0.0132	0.0090	0.0074	0.0076	0.0100
Size-Adjusted Critical Values:								
10%	5.3412	2.7328	2.5295	2.5358	5.2671	2.7255	2.5849	2.3612
5%	9.6550	3.7804	3.3983	3.3486	9.6543	3.7144	3.3958	3.1068
2%	17.3237	5.2890	4.5679	4.3895	17.2202	4.8989	4.5007	4.1591
1%	29.0342	6.4562	5.5921	5.2787	28.3156	6.1546	5.4440	4.9743
WEE test :								
10%	0.0842	0.0950	0.0988	–	0.0764	0.0916	0.0958	–
5%	0.0383	0.0488	0.0494	–	0.0346	0.0429	0.0462	–
2%	0.0134	0.0170	0.0188	–	0.0115	0.0144	0.0200	–
1%	0.0068	0.0092	0.0090	–	0.0050	0.0076	0.0082	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 15: Size and Size-Adjusted Critical Values, Case 2. regressor $x \sim U(0,\ 1)$

$(n_1,\ n_2)$	$\rho^2 = 1$ (20, 10)	(60, 30)	(100, 50)	(250, 125)	$\rho^2 = 2$ (20, 10)	(60, 30)	(100, 50)	(250, 125)
EL-W test at nominal levels:								
10%	0.1686	0.1250	0.1208	0.1140	0.2184	0.1468	0.1424	0.1296
5%	0.1056	0.0662	0.0634	0.0596	0.1486	0.0859	0.0859	0.0714
2%	0.0630	0.0278	0.0298	0.0258	0.0940	0.0438	0.0404	0.0325
1%	0.0429	0.0144	0.0156	0.0138	0.0686	0.0256	0.0254	0.0176
Size-Adjusted Critical Values:								
10%	6.1735	5.0768	4.9858	4.8674	7.5273	5.5747	5.5483	5.2841
5%	8.6057	6.5641	6.5777	6.4220	10.7483	7.5126	7.2799	6.7662
2%	12.4093	8.3297	8.6862	8.4486	14.9988	10.0055	9.8494	8.8505
1%	14.8839	9.8460	10.4269	10.1336	17.6106	11.7867	11.382	10.6668
Wald test :								
10%	0.1354	0.1162	0.1072	0.0964	0.1436	0.1140	0.1100	0.1062
5%	0.0792	0.0602	0.0522	0.0520	0.0884	0.0578	0.0578	0.0520
2%	0.0414	0.0228	0.0238	0.0204	0.0512	0.0274	0.0252	0.0185
1%	0.0260	0.0128	0.0114	0.0104	0.0320	0.0146	0.0136	0.0100
Size-Adjusted Critical Values:								
10%	5.42350	4.9473	4.7550	4.5587	5.6213	4.8503	4.8098	4.7030
5%	7.40080	6.3320	6.1357	6.0367	7.8950	6.3658	6.3470	6.0525
2%	10.1167	8.1745	8.1948	7.8897	10.3846	8.5207	8.2424	7.7123
1%	12.7445	9.7742	9.5291	9.2376	13.3478	9.9669	9.7975	9.2089
J test :								
10%	0.1022	0.1076	0.0995	0.0954	0.0982	0.106	0.1032	0.0976
5%	0.0522	0.0524	0.0484	0.0476	0.0466	0.0509	0.0524	0.0488
2%	0.0216	0.0188	0.0198	0.0188	0.0176	0.0216	0.0230	0.0194
1%	0.0092	0.0104	0.0094	0.0100	0.0102	0.0092	0.0126	0.0092
Size-Adjusted Critical Values:								
10%	5.5808	2.8581	2.5453	2.3395	5.4016	2.8224	2.5889	2.3602
5%	9.8341	3.893	3.3770	3.0961	9.0499	3.8455	3.4679	3.1198
2%	20.0004	5.2806	4.6482	4.0770	17.0544	5.5745	4.8583	4.0907
1%	30.1025	6.7266	5.6375	4.9625	30.9161	6.5544	6.0671	4.8730
WEE test :								
10%	0.0846	0.1014	0.0980	–	0.0900	0.0966	0.1008	–
5%	0.0383	0.0476	0.0468	–	0.0446	0.0470	0.0514	–
2%	0.0150	0.0166	0.0190	–	0.0132	0.0184	0.0188	–
1%	0.0078	0.0086	0.0090	–	0.0078	0.0094	0.0098	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 16: Size and Size-Adjusted Critical Values, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 10$			
$(n_1,\ n_2)$	(20, 10)	(60, 30)	(100, 50)	(250, 125)
EL-W test at nominal levels:				
10%	0.3336	0.2074	0.1802	0.1446
5%	0.2606	0.1332	0.1116	0.0844
2%	0.1922	0.0762	0.0606	0.0408
1%	0.1486	0.0526	0.0388	0.0225
Size-Adjusted Critical Values:				
10%	11.9227	7.0128	6.2510	5.6206
5%	17.1694	9.4087	8.4526	7.3271
2%	25.1885	12.9078	11.2381	9.7585
1%	32.9852	15.8220	13.4227	11.9317
Wald test :				
10%	0.1618	0.1180	0.1104	0.1002
5%	0.1084	0.0622	0.0596	0.0496
2%	0.0634	0.0304	0.0258	0.0200
1%	0.0440	0.0176	0.0130	0.0098
Size-Adjusted Critical Values:				
10%	6.2595	4.9847	4.7794	4.6069
5%	8.7252	6.5938	6.4056	5.9851
2%	12.9218	8.9203	8.3061	7.7744
1%	16.0356	10.5972	10.1592	9.0934
J test:				
10%	0.1046	0.1000	0.1019	0.0982
5%	0.0568	0.0442	0.0484	0.0450
2%	0.0228	0.0160	0.0191	0.0182
1%	0.0126	0.0082	0.0100	0.0080
Size-Adjusted Critical Values:				
10%	5.6697	2.7335	2.5698	2.3715
5%	10.4569	3.6688	3.3904	3.0405
2%	21.2871	5.1100	4.6080	4.0787
1%	36.7252	6.3380	5.6577	4.8048
WEE test :				
10%	0.1042	0.0998	0.0982	–
5%	0.0500	0.0488	0.0509	–
2%	0.0198	0.0198	0.0188	–
1%	0.0092	0.0095	0.0106	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 17: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 0.1$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3006	0.4820	0.6150	0.7148	0.3323	0.5274	0.6697	0.7902
5%	0.1886	0.3292	0.4754	0.5832	0.2084	0.3850	0.5276	0.6590
2%	0.1024	0.2039	0.3170	0.4178	0.1200	0.2556	0.3910	0.5278
1%	0.0526	0.1194	0.2004	0.2934	0.0868	0.1984	0.3176	0.4510
Wald test :								
10%	0.3166	0.5264	0.6782	0.7764	0.3380	0.5368	0.6806	0.8016
5%	0.2012	0.3822	0.5392	0.6606	0.2202	0.4074	0.5558	0.6914
2%	0.1100	0.2334	0.3736	0.5106	0.1310	0.2856	0.4176	0.5679
1%	0.0738	0.1748	0.2936	0.4204	0.0858	0.1978	0.3184	0.4612
J test :								
10%	0.2148	0.3286	0.4332	0.5294	0.3128	0.4926	0.6316	0.7460
5%	0.1044	0.1766	0.2466	0.3288	0.1998	0.3439	0.4806	0.6040
2%	0.0472	0.0876	0.1174	0.1786	0.1070	0.2128	0.3230	0.4324
1%	0.0220	0.0468	0.063	0.0964	0.0698	0.1444	0.2368	0.3284
WEE test :								
10%	0.2754	0.4840	0.6340	0.7512	0.3190	0.528	0.6854	0.7964
5%	0.1660	0.3464	0.4954	0.6200	0.2144	0.3966	0.5654	0.6906
2%	0.0880	0.2120	0.3340	0.4518	0.1200	0.2572	0.4158	0.5510
1%	0.0524	0.1398	0.2390	0.3404	0.0740	0.1816	0.3232	0.4460

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 18: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 0.1$							
$(n_1,\ n_2)$	(100, 50)				(250, 150)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3152	0.5302	0.6889	0.7970	0.3197	0.5218	0.6806	0.7892
5%	0.2076	0.3996	0.5628	0.6929	0.2039	0.3840	0.5440	0.6788
2%	0.1266	0.2644	0.4170	0.5506	0.1120	0.2518	0.3996	0.5308
1%	0.0874	0.1896	0.3192	0.4466	0.0782	0.1894	0.3071	0.4448
Wald test :								
10%	0.3190	0.5360	0.6939	0.8070	0.3197	0.5286	0.6840	0.7927
5%	0.2124	0.4076	0.5668	0.7016	0.2066	0.3906	0.5530	0.6836
2%	0.1312	0.2774	0.4296	0.5622	0.1196	0.2610	0.4132	0.5492
1%	0.0890	0.1994	0.3276	0.4650	0.0750	0.1848	0.3076	0.4480
J test:								
10%	0.3094	0.5000	0.6664	0.7774	0.3038	0.5054	0.6622	0.7794
5%	0.2052	0.3764	0.5358	0.6496	0.1988	0.3746	0.5312	0.6618
2%	0.1180	0.2422	0.3734	0.5030	0.1124	0.2410	0.3824	0.5110
1%	0.0724	0.1664	0.2710	0.3894	0.0704	0.1656	0.2822	0.4068
WEE test :								
10%	0.3262	0.5320	0.7008	0.7988	–	–	–	–
5%	0.2186	0.4036	0.5718	0.7004	–	–	–	–
2%	0.1298	0.2678	0.4292	0.5692	–	–	–	–
1%	0.0820	0.1864	0.3306	0.4674	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1,1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 19: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 0.5$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2954	0.4742	0.6160	0.7422	0.3316	0.5410	0.6834	0.8114
5%	0.1858	0.3396	0.4700	0.6054	0.2344	0.4160	0.5742	0.7191
2%	0.1090	0.2176	0.3312	0.4656	0.1408	0.2846	0.4370	0.5910
1%	0.0722	0.1530	0.2446	0.3554	0.0876	0.2058	0.3398	0.4886
Wald test :								
10%	0.3240	0.5240	0.6770	0.7966	0.3323	0.5414	0.6852	0.8154
5%	0.2110	0.3872	0.5336	0.6732	0.2248	0.4124	0.5702	0.7158
2%	0.1176	0.2552	0.3800	0.5190	0.1364	0.2874	0.4366	0.5896
1%	0.0777	0.1822	0.2898	0.4222	0.0892	0.2138	0.3482	0.4962
J test:								
10%	0.2128	0.3348	0.4346	0.5374	0.3034	0.4834	0.6256	0.7604
5%	0.1132	0.1824	0.2556	0.3322	0.1948	0.3494	0.4952	0.6240
2%	0.0570	0.0872	0.1356	0.1855	0.1174	0.2336	0.3660	0.4812
1%	0.0282	0.0482	0.0724	0.1060	0.0728	0.1524	0.2592	0.3622
WEE test :								
10%	0.2662	0.4518	0.6250	0.7398	0.3068	0.5125	0.6774	0.7914
5%	0.1608	0.3128	0.4778	0.5974	0.1988	0.3754	0.5516	0.6852
2%	0.0800	0.1796	0.2996	0.4104	0.1096	0.2476	0.4020	0.5242
1%	0.0448	0.1106	0.1928	0.2890	0.0696	0.1678	0.3054	0.4252

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 20: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 0.5$							
$(n_1,\ n_2)$	(100, 50)				(250, 125)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3353	0.5356	0.7078	0.8038	0.3372	0.5410	0.6979	0.8178
5%	0.2302	0.4218	0.5996	0.7098	0.2304	0.4220	0.5920	0.7284
2%	0.1258	0.2784	0.4360	0.5688	0.1368	0.2834	0.4502	0.5910
1%	0.0882	0.2076	0.3464	0.4764	0.0835	0.1874	0.3352	0.4778
Wald test :								
10%	0.3292	0.5316	0.7050	0.8024	0.3392	0.5402	0.7006	0.8206
5%	0.2248	0.4138	0.5890	0.7036	0.2286	0.4232	0.5914	0.7296
2%	0.1202	0.2612	0.4180	0.5512	0.1340	0.2782	0.4440	0.5876
1%	0.0852	0.1988	0.3482	0.4698	0.0884	0.1998	0.3469	0.4898
J test:								
10%	0.3074	0.4982	0.6656	0.7644	0.3388	0.5387	0.6924	0.8129
5%	0.1990	0.3662	0.5336	0.6528	0.2236	0.4102	0.5699	0.7086
2%	0.1148	0.2382	0.3874	0.5078	0.1252	0.2552	0.4152	0.5648
1%	0.0722	0.1648	0.2829	0.3986	0.0786	0.1774	0.3142	0.4502
WEE test :								
10%	0.3210	0.5264	0.6830	0.8036	–	–	–	–
5%	0.2150	0.3992	0.5586	0.7054	–	–	–	–
2%	0.1208	0.2693	0.4176	0.5774	–	–	–	–
1%	0.0736	0.1952	0.3242	0.4678	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 21: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 1$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2774	0.4536	0.6024	0.7160	0.3278	0.5250	0.6726	0.7934
5%	0.1676	0.3066	0.4416	0.5574	0.2262	0.3996	0.5588	0.6954
2%	0.0828	0.1638	0.2636	0.3756	0.1442	0.2834	0.4452	0.5856
1%	0.0522	0.1124	0.1930	0.2890	0.1002	0.2026	0.3540	0.4920
Wald test :								
10%	0.2974	0.4806	0.6464	0.7576	0.3202	0.5182	0.6717	0.7906
5%	0.1858	0.3414	0.4940	0.6250	0.2252	0.3934	0.5642	0.6962
2%	0.1032	0.2046	0.3328	0.4606	0.1332	0.2710	0.4372	0.5734
1%	0.0556	0.1336	0.2326	0.3352	0.0910	0.1826	0.3343	0.4694
J test:								
10%	0.1981	0.3098	0.4152	0.5030	0.2950	0.4644	0.6124	0.7432
5%	0.1054	0.1708	0.2428	0.3126	0.1888	0.3204	0.4832	0.6098
2%	0.0432	0.0675	0.1014	0.1400	0.1104	0.2026	0.3268	0.4448
1%	0.0242	0.0376	0.0614	0.0816	0.0651	0.1232	0.2194	0.3206
WEE test :								
10%	0.2664	0.4402	0.5956	0.7221	0.3232	0.5070	0.6652	0.7848
5%	0.1606	0.3002	0.4472	0.5748	0.2120	0.3721	0.5427	0.6756
2%	0.0842	0.1712	0.2832	0.3970	0.1190	0.2424	0.3867	0.5284
1%	0.0420	0.1076	0.1910	0.2824	0.0730	0.1676	0.2926	0.4252

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1,1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 22: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 1$							
$(n_1,\ n_2)$	(100, 50)				(250, 125)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3362	0.5350	0.6844	0.7967	0.3390	0.5416	0.6878	0.8080
5%	0.2248	0.4140	0.5620	0.6962	0.2274	0.4062	0.5629	0.6996
2%	0.1318	0.2800	0.4192	0.5639	0.1228	0.2668	0.4198	0.5564
1%	0.0854	0.2050	0.3180	0.4626	0.0762	0.1870	0.3176	0.4416
Wald test :								
10%	0.3280	0.5294	0.6780	0.7917	0.3492	0.5494	0.6959	0.8149
5%	0.2248	0.4122	0.5620	0.6956	0.2310	0.4118	0.5689	0.7066
2%	0.1266	0.2706	0.4040	0.5540	0.1258	0.2770	0.4308	0.5706
1%	0.0884	0.2008	0.3252	0.4686	0.0834	0.2052	0.3412	0.4744
J test:								
10%	0.3196	0.5130	0.6494	0.7688	0.3416	0.5374	0.6899	0.8012
5%	0.2088	0.3744	0.5185	0.6538	0.2254	0.4048	0.5598	0.6954
2%	0.1112	0.2332	0.3585	0.4878	0.1232	0.2716	0.4194	0.5466
1%	0.0684	0.1586	0.2630	0.3786	0.0792	0.1816	0.3111	0.4346
WEE test :								
10%	0.3234	0.5118	0.6996	0.8076	–	–	–	–
5%	0.2104	0.3830	0.5788	0.7040	–	–	–	–
2%	0.1252	0.2518	0.4238	0.5618	–	–	–	–
1%	0.0792	0.1738	0.3302	0.4546	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 23: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 2$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2942	0.4540	0.5961	0.7151	0.3286	0.5182	0.6778	0.7992
5%	0.1736	0.3020	0.4350	0.5644	0.2136	0.3780	0.5450	0.6876
2%	0.0908	0.1794	0.2806	0.3968	0.1204	0.2532	0.3990	0.5386
1%	0.0630	0.1293	0.2154	0.3190	0.0840	0.1816	0.3108	0.4402
Wald test :								
10%	0.3046	0.4760	0.6246	0.7410	0.3276	0.5128	0.6802	0.8014
5%	0.1868	0.3226	0.4688	0.5970	0.2210	0.3836	0.5586	0.7024
2%	0.1138	0.2186	0.3358	0.4616	0.1230	0.2498	0.4062	0.5442
1%	0.0651	0.1364	0.2304	0.3276	0.0850	0.1858	0.3168	0.4576
J test:								
10%	0.2226	0.3376	0.4258	0.5205	0.2896	0.4624	0.6268	0.7428
5%	0.1266	0.2008	0.2650	0.3390	0.1855	0.3246	0.4774	0.6056
2%	0.0538	0.0946	0.1303	0.1748	0.0932	0.1784	0.2964	0.4120
1%	0.0228	0.0484	0.0572	0.0872	0.0651	0.1282	0.2226	0.3318
WEE test :								
10%	0.2816	0.4462	0.5946	0.7168	0.3328	0.5192	0.6727	0.7836
5%	0.1648	0.3114	0.4346	0.5688	0.2200	0.3834	0.5416	0.6690
2%	0.0859	0.1778	0.2744	0.3792	0.1178	0.2450	0.3782	0.5150
1%	0.0496	0.1062	0.1772	0.2716	0.0750	0.1754	0.2798	0.4136

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 24: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 2$							
$(n_1,\ n_2)$	(100, 50)				(250, 125)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3232	0.5174	0.6882	0.7906	0.3256	0.5292	0.6796	0.7952
5%	0.2200	0.3932	0.5610	0.6936	0.2292	0.4082	0.5672	0.6946
2%	0.1196	0.2514	0.4056	0.5520	0.1300	0.2832	0.4156	0.5564
1%	0.0832	0.1962	0.3320	0.4634	0.0790	0.1956	0.3136	0.4390
Wald test :								
10%	0.3170	0.5242	0.6899	0.8004	0.3353	0.5426	0.6954	0.8026
5%	0.2144	0.3900	0.5659	0.6982	0.2314	0.4226	0.5824	0.7032
2%	0.1220	0.2644	0.4172	0.5686	0.1408	0.2984	0.4432	0.5828
1%	0.0776	0.1902	0.3252	0.4616	0.0910	0.2132	0.3432	0.4730
J test:								
10%	0.3088	0.4950	0.6632	0.7682	0.3392	0.5440	0.6896	0.8006
5%	0.1962	0.3595	0.5302	0.6530	0.2258	0.4138	0.5669	0.6912
2%	0.0958	0.2178	0.3454	0.4836	0.1290	0.2778	0.4230	0.5480
1%	0.0508	0.1336	0.2340	0.3605	0.0848	0.1986	0.3217	0.4468
WEE test :								
10%	0.3300	0.5410	0.6750	0.7940	–	–	–	–
5%	0.2146	0.4170	0.5436	0.6909	–	–	–	–
2%	0.1252	0.2738	0.4012	0.5486	–	–	–	–
1%	0.0801	0.1896	0.3030	0.4434	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.
The WEE test is not applicable with large sample sizes.

Table 25: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 10$							
$(n_1,\ n_2)$	(20, 10)				(60, 30)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.2662	0.4030	0.5446	0.6450	0.2996	0.4938	0.6254	0.7478
5%	0.1628	0.2732	0.3982	0.4970	0.1986	0.3604	0.5002	0.6260
2%	0.0800	0.1570	0.2436	0.3298	0.1058	0.2180	0.3536	0.4708
1%	0.0429	0.0952	0.1544	0.2242	0.0650	0.1474	0.2640	0.3646
Wald test :								
10%	0.2808	0.4270	0.5820	0.6939	0.3114	0.5100	0.6622	0.7810
5%	0.1734	0.2970	0.4390	0.5500	0.2048	0.3731	0.5332	0.6700
2%	0.0784	0.1678	0.2640	0.3605	0.1088	0.2372	0.3832	0.5130
1%	0.0502	0.1086	0.1824	0.2634	0.0680	0.1668	0.2920	0.4058
J test:								
10%	0.2154	0.3050	0.4062	0.4874	0.2985	0.4812	0.6238	0.7506
5%	0.1112	0.1671	0.2344	0.2908	0.1950	0.3546	0.5006	0.6258
2%	0.0412	0.0694	0.1014	0.1290	0.1030	0.2204	0.3434	0.4480
1%	0.0206	0.0364	0.0514	0.0644	0.0641	0.1426	0.2486	0.3366
WEE test :								
10%	0.2842	0.4556	0.5846	0.7018	0.3227	0.5220	0.6674	0.7832
5%	0.1702	0.3071	0.4296	0.5450	0.2158	0.3830	0.5460	0.6654
2%	0.0842	0.1698	0.2610	0.3634	0.1138	0.2438	0.3942	0.5195
1%	0.0492	0.1092	0.1676	0.2474	0.0730	0.1654	0.2945	0.4076

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1, 1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1, 2, 3, 4\}$.

Table 26: Power Comparison, Case 2. regressor $x \sim U(0,\ 1)$

	$\rho^2 = 10$							
$(n_1,\ n_2)$	(100, 50)				(250, 125)			
δ:	1	2	3	4	1	2	3	4
EL-W test at actual size levels:								
10%	0.3200	0.5172	0.6608	0.7760	0.3187	0.5220	0.6812	0.7966
5%	0.2128	0.3842	0.5250	0.6566	0.2158	0.3980	0.5636	0.6918
2%	0.1264	0.2466	0.3878	0.5210	0.1206	0.2604	0.4134	0.5360
1%	0.0818	0.1762	0.2992	0.4222	0.0712	0.1776	0.3002	0.4242
Wald test :								
10%	0.3266	0.5328	0.6782	0.7966	0.3318	0.5452	0.7070	0.8169
5%	0.2142	0.3956	0.5464	0.6842	0.2260	0.4206	0.5996	0.7158
2%	0.1346	0.2634	0.4124	0.5528	0.1378	0.2862	0.4528	0.5870
1%	0.0766	0.1774	0.3006	0.4316	0.0908	0.2112	0.3544	0.4956
J test:								
10%	0.3071	0.5098	0.6538	0.7732	0.3248	0.5342	0.6948	0.8044
5%	0.2092	0.3782	0.5235	0.6578	0.2260	0.4188	0.5866	0.7044
2%	0.1176	0.2366	0.3654	0.4954	0.1258	0.2678	0.4306	0.5634
1%	0.0706	0.1544	0.2693	0.3776	0.0810	0.1912	0.3342	0.4670
WEE test :								
10%	0.3094	0.5195	0.6826	0.8064	–	–	–	–
5%	0.2044	0.3864	0.5534	0.6959	–	–	–	–
2%	0.1120	0.2506	0.4000	0.5488	–	–	–	–
1%	0.0718	0.1848	0.3004	0.4432	–	–	–	–

Notes to table:
Number of replications is 5,000. Sample sizes are the pair (n_1, n_2). $\rho^2 = \sigma_2^2/\sigma_1^2$.
The true values of β_i, $i = 1, 2$:
H_0: $\beta_1 = \beta_2 = (1,1)'$, H_a: $\beta_2 = (1, \beta_{22})'$, where β_{22} varies according to $\delta = \{1,2,3,4\}$.
The WEE test is not applicable with large sample sizes.

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