Applying the RESET Test in Allocation Models : A Cautionary Note

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Abstract: We discuss some issues which arise if the RESET test is used to validate the functional form of the equations in an allocation model. The simple application of this test, equation, by equation, is inappropriate on various grounds. The appropriate formulation of the RESET test in a full systems context requires care. This is discussed and illustrated with Engel curves for expenditure on alcoholic beverages in Australia. The same principles apply to other "variable addition tests" used in econometrics.

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I. Introduction

Allocation models form an important class of models which have wide application in economics. Such models comprise a system of regression equations which explain how some aggregate is "allocated" among its component parts. The aggregate is taken as "given", and so is generally treated as an exogenous variable. Allocation models, which were first discussed formally by Nicholson (1949), are characterized by the fact that the data satisfy the constraint that the sum of the "dependent" variables exactly equals a linear combination of the regressors, at every sample point. Examples of situations where such models arise are consumer demand systems (including systems of Engel curves as a special case when there is no price variation); asset demand models; and certain trade models. Bewley (1986) provides a comprehensive discussion of the associated literature.

It is well known that the "adding up" feature of the data imposes exact restrictions on the coefficients of an allocation model. In addition, the error covariance matrix will be singular, but the parameter estimates will (generally) be invariant to the choice of equation to "drop" from the model prior to joint systems estimation (*e.g.*, Barten (1969), Powell (1969)). When each equation has identical regressors, so that joint estimation of the system is identical to Ordinary Least Squares (OLS), it is also well known that the latter estimator yields *unrestricted* coefficient estimates which automatically satisfy these restrictions, and that the OLS *predicted* values for the dependent variables *automatically* sum to the *actual* value of the aggregate variable at each sample point.[1]

The special features of an allocation model also have implications for other aspects of inference. For example, Berndt and Savin (1975) showed that if the model's error structure follows a vector ARMA process, then quite stringent restrictions must be imposed on the latter's parameters if the invariance of the parameter estimates to the choice of deleted equation is to be assured. Giles (1988) extended these results to other forms of autocorrelation in the errors. The appropriate construction of certain diagnostic tests is also an issue in the context of allocation models, but is not one which appears to have attracted specific discussion to date. In this paper we illustrate this point with respect to the well known "RESET Test" (Ramsey (1969)) for mis-specification of the regressors and/or functional form. Specifically, we argue that care has to be taken over the implementation of such tests in the allocation model context, and we demonstrate this point with an empirical application.

II. Background Theory

First, consider a static allocation model with k identical regressors in each equation. If there are m "components" to be allocated and n observations, the m-equation model may be written

$$Y = XB + U \quad (1)$$

where Y and U are $(n\times m)$; X is $(n\times k)$ and non-stochastic, and B is $(k\times m)$. The contemporaneous error covariance matrix is

$$n^{-1} E(U'U) = \Omega. \quad (2)$$

In general, the adding-up characteristic of an allocation model implies that

$$Y\iota = X\theta, \quad (3)$$

where $\iota = (1, 1, ...,1)'$ is $(m\times 1)$ and θ is a $(k\times 1)$ vector of known constants. For example, if the sum of the n dependent variables equals the first regressor, then $\theta = (1, 0, 0, ...,0)'$. So, from (1) and (3), $B\iota = \theta$ and $U\iota = 0$.

The OLS estimator of B is $B^* = (X'X)^{-1}X'Y$, so that $B^*\iota = \theta$, from (3). The *estimated* coefficients automatically satisfy the restrictions on the true parameters. Further, the predictions from the model are given by $Y^* = XB^*$, so $Y^*\iota = X\theta = Y\iota$. That is, the *predicted* components satisfy the adding-up constraint at all sample points. Finally, $\Omega\iota = n^{-1}E(U'U)\iota = 0$, so the error covariance matrix is singular.

Now, consider the corresponding model with (at least some) different regressors in different equations. Following Bewley (1986, pp.21-22), the model may then be written as:

$$\text{vec}(Y) = \text{diag}(X_i)\,\beta + \text{vec}(U) \quad (4)$$

where diag(X_i) is an (n × K) block-diagonal matrix whose i'th diagonal block is the (n × k_i) matrix X_i ; and β is a (K × 1) vector, where $K = \sum k_i$. Let the number of *distinct* regressors appearing in the full system be d, and let X be the corresponding (n × d) matrix of observations. Defining S_i to be a suitable (d × k_i) "selection matrix", we can write $X_i = XS_i$, and the full system can be expressed as:

$$vec(Y) = (I \otimes X) B_0' + vec(U) \qquad (5)$$

where $B_0 = (S_1\beta_1, S_2\beta_2, ..., S_m\beta_m)$, and $\beta' = (\beta_1', \beta_2', ..., \beta_m')$. Note that B_0 is (d × m), with a number of zero elements, and each β_i is (k_i × 1).

For this model the adding-up constraint is as in (3), and this again implies that $B_0\iota = \theta$ and $U\iota = 0$. For this more general model, the single-equation OLS estimates will *not* generally satisfy the adding-up restrictions. Accordingly, the OLS results lack the uniqueness and invariance that they enjoy in the context of model (1). In addition, it is essential that each regressor *must* appear in at least two equations of the system, if the latter is to be logically consistent with the adding-up constraints. We illustrate this point in the next section, where it has crucial implications for the construction of the RESET test in such models.

III. Applying the RESET Test

The usual application of the RESET test to the i'th equation of (1), *taken in isolation*, would involve obtaining Y_i^*, the i'th column of Y*; then subsequently regressing Y_i (the i'th column of Y) on the columns of X and on p powers of Y_i^*; and testing if the coefficients of the latter regressors are jointly zero. It follows from the Milliken-Graybill (1970) Theorem that the usual test statistic will be *exactly* F-distributed with p and (n-k-p) degrees of freedom under the null hypothesis, if the errors are independent, homoskedastic, and normally distributed[2].

However, testing the specification of each equation of the system (1) separately is unsatisfactory in several respects. First, notice that the powers of Y_i^* differ from equation to equation, because each equation has a different dependent variable. So, when we consider the full system of "augmented" equations for the application of the RESET test, we now have (some) different regressors in the different equations, as in model (5) above. As noted already, this has adverse implications for a single-equation/OLS approach. Second, if the original characteristic of the allocation model is to be preserved, we *cannot* have powers of completely different Y_i^* variables in the different equations. To see this, suppose we apply the RESET test with just the square of each Y_i^* as the "extra" regressor in the i'th equation, and consider the case where m = 2. A simple illustrative augmented model for the application of the RESET test would be of the form (say):

$$Y_{1j} = \alpha_1 + \beta_1 x_j + \gamma_1 Y_{1j}^{*2} + u_{1j}$$

$$Y_{2j} = \alpha_2 + \beta_2 x_j + \gamma_2 Y_{2j}^{*2} + u_{2j} \qquad (6)$$

with $Y_{1j} + Y_{2j} = x_j$; j = 1, 2, ..., n. As a result of the latter adding-up restriction, model (6) must satisfy : $\alpha_1 + \alpha_2 = 0$; $\beta_1 + \beta_2 = 1$; $\gamma_1 = \gamma_2 = 0$; and $u_{1j} + u_{2j} = 0$.

The RESET test would usually then involve *testing* if $\gamma_1 = 0$ and $\gamma_2 = 0$ (either separately or jointly), but we see that in fact these are *not* testable restrictions - they are restrictions which *must* hold exactly in an allocation model. In the context of this example, the obvious "solution" is to include *both* Y_{1j}^{*2} and Y_{2j}^{*2} in *both* of the equations of (6), and then test for their joint significance, either in one equation at a time, or jointly in both equations simultaneously.[3]

With regard to this last point, it is generally the case that the functional form of each equation in an allocation model is the same. For instance, this comes about if a system of demand equations or Engel curves is derived from a constrained utility-maximization problem. Then, the functional form of the equations to be estimated depends on the functional form of the underlying utility function. If the model specification is at fault, it will need to be remedied across all of the equations in the system if the underlying "economic sense" of the model is to be preserved.

In short, in the case of an allocation model, the application of the RESET test needs to be viewed within the context of the full system. Proper account must be taken of the cross-equation restrictions associated with the coefficients of both the original regressors and also the "augmenting" powers of the prediction vectors that form the basis for the RESET test. In practice, this means that *all* of the latter augmentation terms should appear in *all* of the equations of the system.

IV. An Illustrative Application

To illustrate the importance of these points, we have undertaken a small empirical application. We have used Australian alcohol expenditure data[4] reported by Goldschmidt (1990) to estimate systems of Engel curves for three expenditure categories: Beer, Wine and Spirits. There are 242 cross-section observations, each relating to average expenditures over groups of households in 1975/76. Information on the numbers of households per group is available, so the data can be "weighted" to compensate for the heteroskedasticity that may be induced by the use of "grouped data" (*e.g.*, Kakwani (1977), Giles and Hampton (1985)).

Let e_{ij} be expenditure on the i'th beverage by group j, and let E_j be the corresponding total expenditure (on alcoholic beverages). The first functional form that we have considered is the basic Linear model:

$$e_{ij} = \alpha_i + \beta_i E_j + u_{ij} \;; \qquad i = 1, 2, 3 \;;\; j = 1,2, \ldots\ldots, 242 \qquad (7)$$

where[5] the restrictions $\sum \alpha_i = 0$; $\sum \beta_i = 1$ ensure "Engel aggregation". The second is the Working-Leser model:

$$w_{ij} = \alpha_i + \beta_i \ln(E_j) + u_{ij} \;; \qquad i = 1, 2, 3 \;;\; j = 1, 2, \ldots\ldots, 242 \qquad (8)$$

where $w_{ij} = (e_{ij} / E_j)$, and the aggregation restrictions[6] are $\sum \alpha_i = 1$; $\sum \beta_i = 0$. The third is the Addilog model of[7] Bewley (1982):

$$\ln(w_{ij} / w_j^{+}) = \alpha_i + \beta_i \ln(E_j) + u_{ij} \;;\quad i = 1, 2, 3 \;;\; j = 1, 2, ..., 242 \qquad (9)$$

where $\ln(w_j^{+}) = (\sum \ln(w_{ij})) / m$, and $\sum \alpha_i = \sum \beta_i = \sum u_{ij} = 0$.

The first of these models is quite restrictive, but its functional form is consistent with the Linear Expenditure System of demand equations (Stone (1954)) and with the Rotterdam demand model (Theil (1965)). The second model, due to Working (1943) and Leser (1963), incorporates a more flexible functional form which has performed well in several comparative empirical applications (*e.g.*, Giles and Hampton (1995), Dissanayake and Giles (1988)). It may also be derived from the Almost Ideal Demand System of Deaton and Muellbauer (1980), and was used[8] with this data set by Goldschmidt (1990). The third model generally performs well (*e.g.*, Bewley (1982)) when some of the goods have saturation levels at moderate levels of total expenditure.

Augmenting each equation of (7) to apply (a simple version of) the RESET test, we have[9]:

$$e_{ij} = \alpha_i + \beta_i E_j + \gamma_{i1} e_{1j}^{*2} + \gamma_{i2} e_{2j}^{*2} + \gamma_{i3} e_{3j}^{*2} + u_{ij} \;, \qquad (10)$$

i = 1, 2, 3 ; j = 1, 2, ..., 242; and the RESET tests involve testing $H_{0i} : \gamma_{i1} = \gamma_{i2} = \gamma_{i3} = 0$; for each of i =1, 2, 3. Similarly, augmenting each equation of (8) to apply the RESET test, we have:

$$w_{ij} = \alpha_i + \beta_i \ln(E_j) + \gamma_{i1} e_{1j}^{*2} + \gamma_{i2} e_{2j}^{*2} + \gamma_{i3} e_{3j}^{*2} + u_{ij} \;, \qquad (11)$$

where i = 1, 2, 3 ; j = 1, 2, ..., 242; and the RESET tests involve the same null hypotheses as for equation (10). Augmenting each equation of (9) to apply the RESET test, we have:

$$\ln(w_{ij} / w_j^{+}) = \alpha_i + \beta_i \ln(E_j) + \gamma_{i1} e_{1j}^{*2} + \gamma_{i2} e_{2j}^{*2} + \gamma_{i3} e_{3j}^{*2} + u_{ij} \;, \qquad (12)$$

i = 1, 2, 3 ; j = 1, 2, ..., 242; and the RESET tests are applied as above.

Under the null of no mis-specification of the functional form, the usual RESET statistics are F-distributed with 3 and 237 (or, more generally, pm and (n - pm - k)) degrees of freedom, as systems estimation collapses to OLS in models (10) to (12). The corresponding Wald test statistics for these restrictions are asymptotically Chi square with three (or, more generally, pm) degrees of freedom, in each of the above cases if the restrictions associated with the RESET framework are tested equation by equation. More generally, if *all*[10] of the restrictions in all (m-1) equations of the system were tested concurrently, the Wald version of the RESET test statistic would be asymptotically Chi square with pm(m-1) degrees of freedom, bearing in mind that one of the m equations has to be deleted in view of the singular error covariance matrix[11] . It is important to note that the augmented models (10) to (12) are constructed merely to provide an environment for the application of the RESET tests. There is no suggestion that the e_{ij}* terms are part of the economic model - the parameter estimates that would actually be used would be based on (7), (8) or (9).

Table 1 shows these estimates after weighting the data to allow for the differing numbers of households per "group". Table 2 reports the results of applying the RESET tests (wrongly) on the basis of single-equation estimation with different "augmentation" variables in each equation. The results of applying the RESET tests (properly), in the manner discussed above, appear in Table 3. All of the computations were undertaken with the SHAZAM (1993) package. White's (1980) heteroskedasticity-consistent estimator of the error covariance matrix was used in the construction of the RESET (Wald) tests, as there was evidence of remaining heteroskedasticity in the regression residuals[12].

In Table 1, the Linear model exhibits the best R^2 values, but Akaike's Information Criterion[13] favours the Addilog model. The (inappropriate) results in Table 2 probably favour the Addilog model, on balance, though if the results based on p = 3 are ignored the Linear model is also well supported. The effect of applying the RESET test properly can be seen by comparing the p-values in Table 3 with their counterparts in Table 2. There are many obvious differences. Now the Working-Leser model is probably favoured, on balance, especially if the results based on p = 3 are ignored.

V. Conclusions

We have shown that care must be taken when applying certain standard specification tests, such as the RESET test, in the context of allocation models. It is important that these tests be implemented properly if the fundamental economic properties of such models are not to be violated in the process. As we have demonstrated, applying the RESET test properly, or inappropriately, can produce markedly different results. The principles outlined in this paper have more general application than to the RESET test. In particular, many other mis-specification tests in econometrics can be interpreted and constructed as "variable addition tests" (*e.g.*, Pagan and Hall (1983), Pagan (1984)). In all such cases, care must be taken in their application to allocation models, for precisely the reasons we have discussed.

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Footnotes

1. See Worswick and Champerdowne (1954-55), and Bewley (1986). In fact, these results also hold if any Instrumental Variables estimator is applied to each equation of the system (*e.g.*, Denton (1978) and Giles and Hampton (1985)).

2. Under more general conditions, p times the RESET F-statistic coincides with the usual Wald statistic for testing the p zero restrictions on the augmentation variables, and this is asymptotically χ^2 with p degrees of freedom.

3. Of course, the parameter estimates for joint systems estimation would be identical to those from single-equation OLS estimation in this case.

4. The data are from Australian Bureau of Statistics (1976).

5. The various summations which follow are all over $i = 1, 2, ..., m$.

6. As was noted in section 2 above, the disturbances in each model satisfy $\sum u_{ij} = 0$, but this is not a restriction that needs to be imposed in the context of estimation, of course.

7. It is based on earlier work by Leser (1941) and Houtthakker (1960).

8. Goldschmidt also allows for differences in household composition and occupations.

9. For illustrative purposes here we have taken p = 1, so the squared prediction vector forms the single augmenting variable. Note that e_{ij}^* itself cannot be included as an "augmenting" regressor as this would lead to perfect multicollinearity. In Tables 2 and 3 we allow for p = 1, 2, 3.

10. If (m-1) >2 there are various subsets of restrictions that could be tested jointly.

11. Recall, though, that our formulation of the RESET specification testing problem ensures that the results are invariant to the choice of equation to be deleted.

12. The Wald version of the RESET test is asymptotically valid with heteroskedastic errors as long as the error covariance matrix is consistently estimated, but the RESET F-tests are invalid in this case. With homoskedastic errors the Likelihood Ratio test would be a natural (asymptotically equivalent) alternative to the Wald test. However, it cannot be made robust to heteroskedasticity in the way that the latter test can.

13. The AIC values have been corrected for the different forms of the dependent variables, as in Giles and Hampton (1985, p.455).

Table 1. *Parameter estimates* [a, b]

	Linear			Working-Leser			Addilog		
i	α_i (s.e.)	β_i (s.e.)	R^2 [AIC]	α_i (s.e.)	β_i (s.e.)	R^2 [AIC]	α_i (s.e.)	β_i (s.e.)	R^2 [AIC]
1	-0.407 (0.260)	0.734 (0.043)	0.882 [-1072.4]	0.542 (0.053)	0.062 (0.026)	0.641 [-1330.6]	0.651 (0.157)	0.244 (0.079)	0.357 [-1603.8]
2	0.305 (0.258)	0.116 (0.045)	0.255	0.199 (0.034)	-0.019 (0.019)	0.176	-0.427 (0.175)	-0.110 (0.100)	0.146
3	0.102 (0.200)	0.150 (0.030)	0.462	0.258 (0.049)	-0.043 (0.022)	0.307	-0.224 (0.193)	-0.134 (0.100)	0.038

[a] The "beer", "wine" and "spirits" expenditure categories correspond to i = 1, 2, 3.

[b] White's (1980) "heteroskedasticity-consistent" standard errors appear in parentheses and Akaike's Information Criterion (AIC) appears in brackets. R^2 is the usual (single-equation) coefficient of determination.

Table 2. *Single-equation RESET Wald - tests* [a, b]

		Linear		Working-Leser		Addilog	
i	p	Wald	p-value	Wald	p-value	Wald	p-value
1	1	0.626	(0.43)	1.840	(0.18)	0.416	(0.52)
	2	1.184	(0.55)	4.960	(0.09)	5.277	(0.17)
	3	9.828	(0.02)	12.333	(0.01)	4.398	(0.22)
2	1	0.003	(0.96)	0.009	(0.92)	0.506	(0.48)
	2	1.540	(0.46)	0.242	(0.89)	3.228	(0.20)
	3	6.225	(0.10)	0.345	(0.95)	3.525	(0.32)
3	1	2.406	(0.12)	3.129	(0.08)	2.141	(0.14)
	2	2.088	(0.35)	3.186	(0.20)	2.206	(0.33)
	3	4.176	(0.17)	5.142	(0.16)	3.831	(0.28)

[a] When p = 1, the squared prediction vector is tested and the RESET statistic is asymptotically χ^2_1. When p = 2, the squared and cubed prediction vectors are tested, and the RESET statistic is asymptotically χ^2_2. When p = 3, the second, third and f
asymptotically χ^2_3.

[b] The tests are constructed using White's (1980) heteroskedasticity-consistent estimator of the error covariance matrix.

Table 3. *Full-system RESET Wald-tests* [a, b]

		Linear		Working-Leser		Addilog	
i	p	Wald	p-value	Wald	p-value	Wald	p-value
1	1	1.800	(0.61)	1.851	(0.60)	0.535	(0.91)
	2	10.346	(0.11)	6.169	(0.40)	11.146	(0.08)
	3	58.211	(0.00)	24.015	(0.00)	26.362	(0.00)
2	1	6.455	(0.09)	4.609	(0.20)	3.046	(0.38)
	2	15.951	(0.01)	9.464	(0.15)	6.958	(0.32)
	3	37.394	(0.00)	17.576	(0.04)	14.668	(0.10)
3	1	4.550	(0.21)	3.560	(0.31)	4.256	(0.24)
	2	9.372	(0.15)	9.012	(0.17)	12.579	(0.05)
	3	33.343	(0.00)	13.708	(0.13)	36.268	(0.00)

[a] When p = 1, 2, 3 the RESET statistic is asymptotically χ_3^2, χ_6^2, χ_9^2.

[b] The tests are constructed using White's (1980) heteroskedasticity-consistent estimator of the error covariance matrix.