

Department of Economics

## LABOR MARKETS, UNEMPLOYMENT AND OPTIMAL INFLATION

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### **Abstract**

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**JEL Classifications:** E40 E30

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## Abstract

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# 1 Introduction

The optimal inflation rate and the relationship between inflation and unemployment are classic issues in macroeconomics and central policy concerns. Though it is widely accepted that inflation is a monetary phenomenon, there is no generally accepted theory of unemployment. Economists hold widely divergent views regarding the structure and functioning of labor markets. In addition, the labor market institutions such as unions and unemployment insurance vary widely across countries and time. The differences in structure, functioning, and institutions of labor markets raise important positive and normative questions such as (i) Does the relationship between inflation and unemployment vary across different labor markets? (ii) Does the optimal inflation rate depend on the structure of the labor market? (iii) Is the Friedman rule which requires that the rate of growth of money supply be equal to the rate of discount, optimal in imperfect labor markets?

In this paper, I examine these questions in a search-theoretic monetary model with frictions in labor market. In particular, I embed four widely used wage setting mechanisms: individual Nash bargaining, wage posting, efficiency wage, and union bargaining in the search-theoretic monetary model of Shi (1997). Search-theoretic monetary models provide micro foundations for monetary economies based on search theory with explicit descriptions of meetings, specialization, and information, which lead to the lack of ‘double coincidence of wants’ ( see Kiyotaki and Wright 1993, Lagos and Wright 2005, Shi 1995, 1997). Fiat money alleviates the problem of ‘double coincidence of wants’ and it emerges as medium of exchange.

The model developed has two markets – goods market and labor market. Both markets are characterized by search frictions. In the goods market, buyers and sellers are assumed to be price takers (Walrasian). Friction in the goods market is generated by assuming that only a fixed fraction of buyers and sellers are able to enter the goods market. In the labor market, workers who want to find jobs have to search for suitable vacancies. Similarly, firms who want to hire workers have to open vacancies and search for workers. In this general set-up, I consider four wage-setting mechanisms: individual Nash bargaining, wage posting, efficiency wage, and union bargaining.

In the search and matching labor market models, Nash bargaining and wage posting are widely used (see below). Efficiency wage and union models are normally posed as alternative to search and matching models. In this paper, I treat them not as an alternative to search and matching models

rather as alternative wage-setting mechanisms which can be embedded in the search and matching framework. Whatever be the wage setting mechanism, workers who want to find jobs and firms who want to hire workers have to engage in costly search. In the individual Nash bargaining the matched worker and firm bargain over wage bilaterally. In the union model, a union bargains with firm on behalf of matched workers. In the case of efficiency wage, productivity of a matched worker depends on the wage paid. A firm takes into account this relationship at the time of setting wages. In all three above cases, wages are determined after matches are formed (*ex-post*). In the wage posting model, firms post wages before workers search for jobs. Workers observe posted wages and decide where to apply (see Moen 1997, Julien *et. al.* 2000, Mortensen and Wright 2002).

My paper relates to Shi (1998) and Berentsen, Menzio, and Wright(2006) who embed the search and matching model of Mortensen and Pissarides in the search-theoretic monetary framework. Shi (1998) integrates the search-theoretic model of Shi (1997) and the Mortensen and Pissarides model. Berentsen, Menzio, and Wright(2006) integrate the Lagos-Wright model (Lagos and Wright 2005) and the Mortensen and Pissarides model. Compared to these papers, I consider a wider set of wage setting mechanisms. Additionally, in their models goods market is non-Walrasian. Finally, the focus of Shi (1998) is on the propagation of monetary shocks.

Rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the optimal decisions of agents. Section 4 derives the efficient allocations. Section 5 analyzes the properties of stationary symmetric monetary equilibrium under different wage setting mechanisms. It analyzes the effects of inflation and its welfare implications qualitatively. This is followed by concluding remarks.

## 2 The Economy

Time is discrete. Consider an economy comprised of a continuum of infinitely-lived households and firms. There is also a continuum of goods with measure one. The type of good a household would like to consume is determined by a uniformly distributed *i.i.d.* random shock every period. A firm  $i$  produces good  $i$ . Goods can be stored only by its producing firms as inventory.<sup>1</sup> Each

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<sup>1</sup>The assumption that a good can be stored only by its producers rules out commodity money. The possibility of storage (or inventory) ensures that a firm's opportunity cost to

household is endowed with a single share in a fully diversified portfolio of claims on the profit earnings by firms. For simplicity assume that these shares and claims on inventory cannot be used to buy goods in the goods market.<sup>2</sup> Firms maximize their profits and repatriate their profits to their owners (households) every period.

There are two markets: the goods market and the labor market.<sup>3</sup> Both markets are characterized by frictions. Agents in both the markets are brought together randomly through the meeting processes described later. Random meeting implies a particular household or a firm in the goods market cannot be relocated in future. The assumption that a household's desired consumption good is randomly determined every period coupled with the fact that an individual firm or household cannot be relocated in future in the goods market rules out credit arrangements and exchanges must be *quid-pro-quo*. Thus, money is used as a medium of exchange. In addition, frictions in goods market and random preference imply that money is used as a means of wage payment. This issue is further discussed in section 5.1. Dividends are also paid in terms of money.

Each household is endowed with  $\hat{M}_0$  units of fiat money at time zero. At the beginning of each subsequent period, each household receives  $(g-1)\hat{M}_{t-1}$  units of fiat money from the government as a lump-sum transfer, where  $\hat{M}_t$  is the post-transfer per-household average holding of fiat money at time  $t$  in the economy ( $\hat{M}_t = \hat{M}_{t-1} + (g-1)\hat{M}_{t-1} = g\hat{M}_{t-1}$ ). The government plays no role in the economy, other than making lump-sum transfers to the households. In what follows, as a convention, the variables, which are taken as given by a particular household/firm, are denoted with superscript “ $\hat{\phantom{x}}$ ”.

Due to random meeting, individual agents in these markets face uncertainty in their matching outcomes. This generates non-degenerate distributions of money holding, wages, employment, and inventories, which makes the model analytically intractable and numerically challenging. In order to make these distributions degenerate and the analysis tractable, following Shi

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selling goods in the goods market is strictly positive.

<sup>2</sup>In such models, this restriction is needed so that these claims do not replace money as a medium of exchange. One can assume that households can easily counterfeit such claims and sellers in the goods market cannot verify these claims (Aruba, Waller, and Wright 2006, Berentsen, Menzio, and Wright 2006).

<sup>3</sup>The share market is suppressed as there will no trading in shares in equilibrium. The issues of dividend payments and trading in shares are further discussed in Section 3.1.

(1997, 1998), I use the construct of large households and firms.<sup>4</sup>

Each household is assumed to comprise of unit measures of two types of members: buyers and workers. Workers are of two types – unemployed and employed. Each type of members in the household play a distinct role. Buyers buy the household's desired consumption goods in the goods market. Unemployed workers search for suitable jobs. Employed workers work. Let  $e_t^h$  and  $u_t$  denote the measures of employed workers and unemployed workers in a household respectively at time  $t$  ( $e_t^h + u_t = 1$ ).

Members of the household do not have independent preferences. Rather, the household prescribes the trading strategies for each type of members to maximize the overall household utility. The members of the household share equally in the utility generated by the household consumption. With this modeling device, the decisions of different household types are identical in a symmetric equilibrium, except for the types of goods they consume. Thus, I can analyze the behavior of a representative household.

Similarly, a firm consists of a large number of managers. Managers recruit workers and sell goods in the goods market. Assume that unit measure of managers (called recruiters) are engaged in recruiting activities and unit measure (called sellers) in selling activities. Just as in the case of households, these agents do not have independent preferences, but undertake activities in order to maximize firm's profit.<sup>5</sup> Large number of managers implies that idiosyncratic risks faced by individual managers are smoothed within the firm. With this construction of firms, the decisions of firms of different types are identical in a symmetric equilibrium, except for the types of goods they produce. Thus, I can analyze the behavior of a representative firm.

## 2.1 Trading and Price Determination in the Goods Markets

The pricing mechanism adopted is Walrasian pricing (price taking) and is same as competitive equilibrium analyzed in Rocheteau and Wright (2003) in the search-monetary framework. The buyers and sellers in the goods market

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<sup>4</sup>An alternative framework that produces degenerate distributions of money holding and prices is examined by Lagos and Wright (2005).

<sup>5</sup>These managers need not be unpaid. One can assume that a *fixed number* of workers are required for managerial activities. These employees are chosen randomly at the beginning of every period from the existing pool of employees.

are assumed to be price takers. The price is determined by a Walrasian auctioneer, which equates total demand and total supply of goods.

To generate friction in the goods market, it is assumed that buyers and sellers queue to enter the market and only a fraction of them are able to enter the market. Suppose that only a fixed fraction  $\xi$  ( $0 < \xi < 1$ ) of buyers and sellers are able to enter the market. Since, measures of buyers in a household and sellers in a firm are normalized to one, the measures of buyers in a household and sellers of a firm who are able to trade are equal to  $\xi$  in any time period.

Buyers and sellers inside the market trade at the announced price,  $\hat{p}_t(i)$ , for good  $i$ . Denote average price by  $\hat{p}_t$ . Since, I am going to focus on symmetric equilibrium in which  $\hat{p}_t(i) = \hat{p}_t$ , I will drop the good index  $i$  from prices of individual goods in the rest of the paper.

## 2.2 Trading and Wage Determination in the Labor Market

In the labor market, workers and vacancies are matched through a matching function, which relates the flow of hiring to the average per-firm measure of vacancies,  $\hat{v}_t$ , and average per-household unemployed workers,  $\hat{u}_t$ . The matching function  $\mathcal{M}(\hat{v}_t, \hat{u}_t)$  is assumed to be concave, increasing, and is subject to constant returns to scale. Denote the labor market tightness by  $\hat{\theta}_t \equiv \frac{\hat{v}_t}{\hat{u}_t}$ . Then the aggregate matching rate of unemployed workers is given by

$$\frac{\mathcal{M}(\hat{v}_t, \hat{u}_t)}{\hat{u}_t} = \mu(\hat{\theta}_t); \mu'(\hat{\theta}_t) > 0, \lim_{\hat{\theta}_t \rightarrow 0} \mu(\hat{\theta}_t) \rightarrow 0 \quad (2.1)$$

where  $\mu'(\hat{\theta}_t)$  denotes the first derivative of the matching function with respect to  $\hat{\theta}_t$ . Similarly, the aggregate matching rate of vacancies is given by

$$\frac{\mu(\hat{\theta}_t)}{\hat{\theta}_t}; \lim_{\hat{\theta}_t \rightarrow 0} \frac{\mu(\hat{\theta}_t)}{\hat{\theta}_t} = \infty, \lim_{\hat{\theta}_t \rightarrow \infty} \frac{\mu(\hat{\theta}_t)}{\hat{\theta}_t} = 0, \frac{d(\mu(\hat{\theta}_t)/\hat{\theta}_t)}{d\hat{\theta}_t} < 0. \quad (2.2)$$

Assume that a newly formed match starts producing from the next period and continues until the match is dissolved. A match can dissolve due to idiosyncratic exogenous shocks, in which case an employee becomes unemployed. Assume that in each period fraction  $\sigma$  of a firm's existing matches

are exogenously dissolved. This also implies that in any time period, fraction  $\sigma$  of employed workers in a household become unemployed.

I consider several wage determination processes in section 5. In particular I analyze four wage determination processes: individual Nash bargaining, wage posting (directed search), efficiency wage, and union bargaining.

## 3 Optimal Decisions of the Household and the Firm

### 3.1 Timing

The representative household at the beginning of period  $t$  enters with  $M_t$  units of post-transfer money,  $u_t$ , unemployed workers, and,  $e_t^h$ , employed workers. Similarly, the representative firm of type  $x$  enters with,  $e_t^f$ , employees and inventory of goods denoted by  $i_t$ .

At the beginning of period  $t$ , the firm produces goods using its existing employees,  $e_t^f$ . It distributes available goods, which includes current output,  $f(e_t^f)$ , plus the inventory,  $i_t$ , carried from the previous period, equally among sellers. Thus each seller receives  $f(e_t^f) + i_t$  units of goods. It chooses the quantity of produced goods,  $q_t$ , to be sold by sellers at price,  $p_t$ . It also chooses next period employment level,  $e_{t+1}^f$ , and the inventory level,  $i_{t+1}$ , and the measure of vacancies,  $v_t$ .

The preference shock for the household is realized. Assume that at the time of the realization of preference shock employed workers are separated from the household and the household cannot convey this information to employed workers only after they have received their wages. This assumption captures the notion that individuals are not fully sure of their desired consumption goods at the time of working but relatively more sure at the time of shopping. This assumption ensures that the wages are paid only in terms of money (see section 5.1).

After the shock the household distributes available money balance,  $M_t$ , equally among buyers and chooses consumption for period  $t$ ,  $c_t$ , a new money balance for the next period,  $M_{t+1}$ , and the amount of money to be spent by buyers,  $m_t$ . Note that each buyer receives  $M_t$  units of money as the measure of buyers is unity.

After these decisions, individual agents go to their respective markets. Buyers and sellers who are successful in entering the goods market trade.



After trading in the goods market, buyers come back to the household with the purchased goods and any residual nominal money balances. Sellers come back with their nominal sales receipts and any unsold stock of goods. The firm pays wages to its employees and employees return to their respective households with their nominal wage receipts.

Similarly, in the labor market unemployed workers search for suitable jobs and recruiters for suitable workers. Match dissolutions take place. Recall that the newly hired employees start working from the next period. Also only the existing matches receive the idiosyncratic exogenous shocks and not the new ones. Trading in the labor market and the exogenous dissolution of matches determine the next period's measure of employed workers,  $e_{t+1}^h$ , the measure of unemployed workers,  $u_{t+1}$ , and the measure of employees of firms,  $e_{t+1}^f$ .

At the end of the labor market session workers go back to their respective households and consumption takes place. Since, firms make strictly positive profit, there is an issue of how dividends are paid. Assume that at the end of the labor and the goods market sessions and after consumption, there is an asset market session, where households buy and sell shares and receive dividends. The asset market is competitive.<sup>6</sup> Since, goods cannot be stored by the households and consumption has already taken place, households of a particular type do not have any incentive to buy only the shares of firms of their own type and dividends are paid only in terms of money. Also in equilibrium no trading in shares takes place.

After the end of the asset market session, the wage receipts of employed workers, the dividend received, and any residual money balances brought back by the buyers are added to the household nominal money balance for the next period. The unsold stock of goods are carried as inventory to the next period by the firm. Time moves to the next period  $t + 1$ .

### 3.2 The Optimal Decisions of the Household

Assume that the representative household maximizes the discounted sum of utilities from the sequence of consumption less the disutility incurred by

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<sup>6</sup>Berentsen, Menzio, and Wright (2006) who integrate search and matching labor market model in the Lagos-Wright monetary framework also assume that the profits earned by firms in the decentralized labor and goods markets are paid to the households in the centralized market. The shares cannot be used to buy goods in the decentralized goods market.

employed workers from working.<sup>7</sup> The household's inter-temporal utility is represented by

$$\sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left[ U(c_t) - \phi e_t^h \right], \quad U'(c_t) > 0, \quad U''(c_t) < 0, \quad \lim_{c_t \rightarrow 0} U'(c_t) > 0 \quad (3.1)$$

where  $r$ ,  $U(c_t)$ , and  $\phi$ , are the rate of time-preference, the utility derived from consumption  $c_t$ , and the disutility from working respectively.

The money spent by an individual buyer in a match satisfies the following inequality

$$\hat{m}_t \leq M_t. \quad (3.2)$$

Recall that the measure of matched buyer in the household is  $\xi$ . Then consumption,  $c_t$ , satisfies the following inequality

$$c_t \leq \xi \frac{m_t}{\hat{p}_t}. \quad (3.3)$$

Denote the nominal dividends received by the household as  $\hat{\pi}_{Mt}$  and nominal wage of an employed worker as  $\hat{p}_t w_t$  ( $w_t$  is the real wage; nominal wage divided by the average price level,  $\hat{p}_t$ ) at time  $t$ . Then the budget constraint of the representative household is given by

$$M_{t+1} \leq M_t + (g-1)\hat{M}_t + \hat{\pi}_{Mt} + \hat{p}_t e_t^h w_t - \xi \hat{m}_t. \quad (3.4)$$

The term on the left hand side is the post-transfer money holding at the beginning of period,  $t+1$ . The first term on the right hand side is the nominal money balances of the household at time  $t$ , the second term is the lump-sum monetary transfer at the beginning of period  $t+1$ , and the third term is the dividend received at time  $t$ . The fourth term is the total nominal wage payment received by the employed workers of the household. The final term is the money spent by the matched buyers at time  $t$ .

Next I set up the optimization problem of the household. Taking the labor market tightness,  $\hat{\theta}_t$ , the prices in the goods market,  $\hat{p}_t$ , the optimal choices of firms and other households, and the initial conditions  $\{M_0, e_0^h, u_0\}$  as given, the representative household of type  $x$  chooses the sequence  $\{c_t, m_t, M_{t+1}, e_{t+1}^h, u_{t+1}\}, \forall t \geq 0$  to solve the following problem.

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<sup>7</sup>The disutility from search in the goods and the labor markets is normalized to zero.

### Household Problem (PH)

$$\max_{c_t, m_t, M_{t+1}, e_{t+1}^h, u_{t+1}} \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left[ U(c_t) - \phi e_t^h \right] \quad (3.5)$$

subject to the constraints on money spent by an individual buyer in a match (3.2), the household's consumption (3.3), the budget constraint (3.4), and the laws of motion of the employed workers,  $e_t^h$ , and unemployed workers,  $u_t$ ,

$$e_{t+1}^h \leq (1 - \sigma)e_t^h + \mu(\hat{\theta}_t)u_t, \quad (3.6)$$

$$u_{t+1} \leq \sigma e_t^h + (1 - \mu(\hat{\theta}_t))u_t, \quad (3.7)$$

The left hand side of (3.6) is the measure of employed worker at time  $t+1$ . The first term on the right hand side is the measure of employed workers at the beginning of the period  $t$  who remain in the same pool at the end of the period. An employed worker leaves this pool due to exogenous dissolution of the match. The second term is the measure of unemployed workers who receive wage offers. (3.7) can be interpreted similarly.

Turning to the optimal choices of  $c_t$  and  $M_{t+1}$ , consumption,  $c_t$ , is given by the equality constraint (3.3). Denote the Lagrangian multiplier associated with the constraint on the nominal money balance of an individual buyer (3.2) by  $\lambda_t$ . Then the first order condition for the optimal choices of  $M_{t+1}$  is given by

$$\omega_{Mt} = \frac{1}{1+r} [\omega_{Mt+1} + \xi \lambda_{t+1}] \quad (3.8)$$

The first order condition has the usual interpretation. The right hand side of (3.8) is the discounted expected marginal benefit from carrying an additional unit of money next period. If the household carries one additional unit of money next period, then it relaxes the budget constraint (3.4) as well as the constraint on the nominal balances of matched buyers (3.2). Note that only  $\xi$  fraction of buyers are matched.

The optimal choice of spending by an individual buyer,  $m_t$ , satisfies

$$\lambda_t = \frac{U'(c_t)}{\hat{p}_t} - \omega_{Mt}. \quad (3.9)$$

$\lambda_t$  can be interpreted as the net surplus generated by a matched buyer for the household from an additional unit of expenditure. Spending of an additional unit of money increases the household's utility by  $\frac{U'(c_t)}{\hat{p}_t}$ , but at the same time it tightens the budget constraint. For a matched buyer to get positive surplus *i.e.*,  $\lambda_t > 0$ , the constraint on the spending of a matched buyer given in (3.2) must be binding.

Denote the langrangian multipliers associated with the measures of employed workers and unemployed workers by  $\Omega_{et}^h$  and  $\Omega_{ut}$  respectively. The first order conditions for the optimal choices of  $e_{t+1}^h$  and  $u_{t+1}$  are given by

$$\Omega_{et}^h = \frac{1}{1+r} [\hat{p}_{t+1} \hat{w}_{t+1} \omega_{Mt+1} - \phi + (1-\sigma)\Omega_{et+1}^h + \mu(\hat{\theta}_{t+1})\Omega_{ut+1}] \quad (3.10)$$

and

$$\Omega_{ut} = \frac{1}{1+r} [\mu(\hat{\theta}_{t+1})\Omega_{et+1}^h + (1-\mu(\hat{\theta}_{t+1}))\Omega_{ut+1}]. \quad (3.11)$$

The right hand side of (3.11) is the discounted benefit to the household for having one additional employed worker. The first two terms in the brackets is the net flow of benefits and the last term is the continuation value. (3.12) can be interpreted similarly.

### 3.3 The Optimal Decisions of the Firm

Suppose that the representative firm has a production function given by

$$f(e_t^f), f'(e_t^f) > 0, f''(e_t^f) < 0, \lim_{e_t^f \rightarrow 0} f'(e_t^f) = \infty \quad (3.12)$$

where  $e_t^f$  is the measure of employees at time  $t$ .

Recall that firms have to engage in costly recruitment activity. Suppose that in order to hire workers, firms have to advertise with advertising agencies and pay to them in terms of money. These advertising agencies are owned by the households and operate in a competitive market. A short-cut way to model this recruitment process is to assume that firms incur cost in terms of disutility of the representative household (or owner). This obviates the need to explicitly model the behavior of advertising agencies. Let  $k$  be the cost

of creating and maintaining one unit of vacancy per period in terms of the disutility of the representative household (or owner).<sup>8</sup>

In the goods market at time  $t$ , the measure of matched sellers is  $\xi$  and each seller receives  $\hat{p}_t q_t$  units of money (under symmetry) for selling  $q_t$  units of goods at price  $\hat{p}_t$ . Thus total sales in nominal terms is  $\xi \hat{p}_t q_t$ . The profit of the firm in nominal terms at time  $t$ ,  $\pi_{Mt}$ , is given by

$$\pi_{Mt} = \xi \hat{p}_t q_t - e_t^f \hat{p}_t \hat{w}_t \quad (3.13)$$

where the second term in the right hand side is the total nominal wage payments made by the firm. Wage payments are made whether goods are sold or not.

Suppose that the firm creates  $v_t$  vacancies at time  $t$ . Then, the profit of the representative firm in utility terms at time  $t$ ,  $\pi_t$  is given by

$$\pi_t = \pi_{Mt} \hat{w}_{Mt} - k v_t. \quad (3.14)$$

The representative firm maximizes profit given in (3.14). Given the ownership structure of firms, this assumption is reasonable.

Let  $\delta \in (0, 1)$  be the rate of depreciation for the inventory. The total units of goods sold by the matched sellers are  $\xi \hat{q}_t$  and thus the evolution of inventory,  $i_t$ , over time is given by

$$i_{t+1} \leq (1 - \delta)(f(e_t^f) + i_t - \xi q_t) \quad (3.15)$$

where  $f(e_t^f) + i_t - \xi q_t$  is the total quantity of unsold goods at the end of the goods market session. Note that the units of goods sold by a matched seller,  $q_t$ , satisfies the following inequality:

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<sup>8</sup>This assumption can be justified as follows. Suppose advertising agencies operate in a competitive market and charge price  $p_t^A$  per vacancy per unit of time in terms of money. Each advertising agency employs one person (different from production workers) and can at most handle one advertisement per unit of time. Suppose that a household incurs disutility  $k$  per non-production worker. Then if we allow free-entry, the price charged per-unit of vacancy advertised satisfies  $p_t^A \hat{w}_{Mt} = k$ . Thus, if a firm advertises  $v_t$  vacancies, the payment in terms of money is  $p_t^A v_t$ . Since, the firm maximizes utility of its owners, in the utility terms the cost of advertising is  $p_t^A v_t \hat{w}_{Mt} = k v_t$ . By assuming that vacancy cost is incurred in terms of utility, I avoid explicitly modeling the behavior of advertising agencies and non-production workers. Alternatively, one can assume that a firm incurs vacancy cost in terms of lost production. In this case, number of vacancies directly affect the terms of trade in the goods market.

$$q_t \leq f(e_t^f) + i_t. \quad (3.16)$$

Taking as given the labor market tightness,  $\hat{\theta}_t$ , the prices in the goods market,  $\hat{p}_t$ , the job-acceptance strategies of workers, and the initial conditions  $\{i_0, e_0^f\}$ , the firm's problem is to choose the sequence of  $\{i_{t+1}, e_{t+1}^f, v_t, q_t\}$   $\forall t \geq 0$  to maximize

**Firms' Problem (PF)**

$$\max_{i_{t+1}, e_{t+1}^f, q_t, v_t} \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \pi_t \quad (3.17)$$

subject to the law of motion of inventory (3.15), the quantity constraint on sellers (3.16) and the law of motion of employees,  $e_t^f$ , given by

$$e_{t+1}^f \leq (1 - \sigma)e_t^f + \frac{\mu(\hat{\theta}_t)}{\hat{\theta}_t} v_t. \quad (3.18)$$

The term on the left hand side of (3.18) is the measure of employees at the beginning of the period  $t + 1$ . The right hand side has two parts. The first part is the measure of employees of the firm at the beginning of the period  $t$  who do not leave the firm due to exogenous shocks. The second part gives the measure of new matches formed.

Denote the Lagrangian multiplier associated with the law of motion of inventory (3.15) by  $\Omega_{it}$  and the Lagrangian multiplier associated with the quantity constraint of the matched seller (3.16) by  $\Omega_{qt}$ . The first-order condition for the choice of next period inventory,  $i_{t+1}$ , is

$$\Omega_{it} = \frac{1}{1+r} \left[ (1 - \delta)\Omega_{it+1} + \xi\Omega_{qt+1} \right]. \quad (3.19)$$

(3.19) equates the marginal cost of carrying an addition unit of inventory to its discounted expected marginal benefit. If the firm carries one extra unit of inventory next period, then it relaxes the next period constraints on the inventory and the quantity of goods that can be sold by matched sellers.

The optimal choice of  $q_t$  satisfies

$$\Omega_{qt} = \hat{\omega}_{Mt}\hat{p}_t - (1 - \delta)\Omega_{it}. \quad (3.20)$$

$\Omega_{qt}$  can be interpreted as the net surplus generated by a matched seller to the firm by selling an additional unit of good. An additional unit of good

sold by a matched seller increases the firm's profitability in the utility terms by  $\hat{\omega}_{Mt}\hat{p}_t$ . At the same time it reduces the inventory carried to the next period by  $(1 - \delta)$ , the value of which is  $(1 - \delta)\Omega_{it}$ . For a matched seller to get positive surplus *i.e.*,  $\Omega_{qt} > 0$ , the constraint on the amount of goods sold by a matched seller given in (3.16) must be binding.

Denote the Lagrangian multiplier associated with the constraint on the measure of employees (3.18) by  $\Omega_{et}^f$ . The first order conditions associated with the optimal choices of  $v_t$  is

$$k = \frac{\mu(\hat{\theta})}{\hat{\theta}}\Omega_{et}^f \quad (3.21)$$

(3.21) equates the marginal cost of creating vacancy with the expected marginal benefit.

The first order conditions associated with the optimal choices of  $e_{t+1}^f$  is

$$\Omega_{et}^f = \frac{1}{1+r} [\{\xi\Omega_{qt+1} + (1-\delta)\Omega_{it+1}\}f'(e_{t+1}^f) - \hat{p}_{t+1}w_{t+1}\hat{\omega}_{Mt+1} + (1-\sigma)\Omega_{et+1}^f]. \quad (3.22)$$

Using (3.19), (3.22) can be written as

$$\Omega_{et}^f = \frac{1}{1+r} [(1+r)\Omega_{it}f'(e_{t+1}^f) - \hat{p}_{t+1}w_{t+1}\hat{\omega}_{Mt+1} + (1-\sigma)\Omega_{et+1}^f]. \quad (3.23)$$

(3.23) can be interpreted as follows. An additional new employee increases the production next period by his marginal product,  $f'(e_{t+1}^f)$ , which is equivalent to increasing inventory this period by  $f'(e_{t+1}^f)$ . The firm pays real wage  $w_{t+1}$ . Thus the value of net gain from an additional new employee is  $(1+r)\Omega_{it}f'(e_{t+1}^f) - \hat{p}_{t+1}w_{t+1}\hat{\omega}_{Mt+1}$ . A match can dissolve with probability  $\sigma$ . Thus the continuation value of match is  $(1-\sigma)\Omega_{et+1}^f$ .

### 3.4 Goods Market and Price Determination

Now I turn to the determination of prices in the goods market. Since the goods market is Walrasian, the demand for goods should be equal to supply. Recall that only fraction  $\xi$  of buyers and sellers are able to enter the goods market. Thus, the total demand is  $\frac{\xi m_t}{\hat{p}_t}$  and total supply is  $\xi q_t$ . The price,  $\hat{p}_t$ , is given by

$$q_t = \frac{m_t}{\hat{p}_t}. \quad (3.24)$$

Before analyzing the monetary equilibrium with the alternative wage determination processes, it is convenient to derive social optimal allocations. These allocations can be derived independently of the assumed market structure, prices, and wages. I analyze monetary equilibrium in section 5.

## 4 Welfare

I assume that the social planner faces same trading frictions in goods and labor markets as households and firms. It chooses the sequence of  $\{c_t, \theta_t, e_{t+1}\}$  in order to maximize the representative household utility net of disutility cost of vacancy subject to resource constraints. The social planning problem is

$$\max_{c_t, \theta_t, e_{t+1}} \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t [U(c_t) - \phi e_t - k(\theta_t(1 - e_t))] \quad (4.1)$$

subject to constraints on consumption

$$c_t \leq \xi[f(e_t) + i_t], \quad (4.2)$$

and laws of motion of inventory and employment

$$i_{t+1} \leq (1 - \delta)(1 - \xi)[f(e_t) + i_t], \quad (4.3)$$

.

$$e_{t+1} \leq (1 - \sigma)e_t + (1 - e_t)\mu(\theta_t). \quad (4.4)$$

Denote the elasticity of matching function with respect to labor market tightness as  $\eta(\theta) \equiv \frac{\mu'(\theta)\theta}{\mu(\theta)}$ . Then in the steady state in which output, consumption, and employment are constant over time, the social optimal level of labor market tightness is given by

$$k = \frac{\mu'(\theta)}{r + \sigma + \mu(\theta)(1 - \eta(\theta))} [Af'(e)U'(c) - \phi] \quad (4.5)$$

where  $A$  is a constant given by



$$A \equiv \frac{\xi(1+r)}{r+\delta+\xi(1-\delta)}.$$

$c$ ,  $i$  and  $e$  are given by (4.2), (4.3), and (4.4) respectively. It can be easily be shown that there exists a unique level of labor market tightness,  $\theta$ , which solves (4.5). Denote social optimal levels of labor markets tightness, consumption, output, and employment by  $\theta^s$ ,  $c^s$ ,  $f(e^s)$ , and  $e^s$  respectively.

In the next section, I derive stationary monetary equilibrium under alternative wage setting processes. I analyze efficiency properties of these wage setting processes and the effects of changes in the growth rate of money supply. In particular, I consider four wage setting processes: (i) individual Nash bargaining, (ii) wage posting, (iii) efficiency wage, and (iv) union bargaining.

## 5 Symmetric Stationary Monetary Equilibrium

This paper restricts its attention to a symmetric and stationary monetary equilibrium. First, I require that all households have common marginal value of money,  $\omega_{Mt}$ , consumption,  $c_t$ , and employment,  $e_t^h$ . Similarly, all firms have identical employment level,  $e_t^f$ , and choose identical levels of inventory,  $i_{t+1}$ , and vacancies,  $v_t$ . Secondly, prices in the goods market,  $\hat{p}_t$ , are identical across markets. Thirdly, real wages,  $\hat{w}_t$ , are identical across matches. Fourthly, consumption, inventory, unemployment, employment, and vacancies are constant over time. Finally, money has value *i.e.*, the marginal value of real money balance,  $\hat{p}_t\omega_{Mt}$ , is strictly positive.

Given the definition  $\hat{p}_t \equiv \frac{\dot{M}_t}{q_t}$ ,  $\forall t$ , in the stationary and symmetric equilibrium the average price level,  $\hat{p}_t$ , will grow at the rate equal to the money creation rate *i.e.*, inflation rate

$$\frac{\hat{p}_{t+1}}{\hat{p}_t} = g \quad \forall t. \quad (5.1)$$

Denote the real money balance,  $M \equiv \frac{M_t}{\hat{p}_t}$ , the buyer's surplus per purchase,  $\hat{p}_t\lambda_t = \lambda$ , the real money balance with a buyer,  $m \equiv \frac{m_t}{\hat{p}_t}$ , and the marginal value of real money balances,  $\Omega_M \equiv \hat{p}_t\omega_{Mt}$ . From now on, I drop the subscript  $t$  from real variables.

For a symmetric stationary monetary equilibrium to exist, the surplus of both the seller and the buyer in a match must be positive (*i.e.*,  $\Omega_q, \lambda > 0$ ). If a matched seller does not receive positive surplus (*i.e.*,  $\Omega_q = 0$ ), then (3.20) implies that the marginal value of inventory must grow at the rate  $\frac{1+r}{1-\delta}$  (*i.e.*,  $\frac{\Omega_{it+1}}{\Omega_{it}} = \frac{1+r}{1-\delta}$ ). Also if the matched buyer does not receive a positive surplus ( $\lambda = 0$ ), then (3.8) implies that the marginal value of money,  $\Omega_M = 0$ . Thus in a symmetric stationary monetary equilibrium buyers' nominal cash balance expenditure constraint (3.2) and sellers' quantity constraints (3.16) will be binding.

Next I consider wage determination.

## 5.1 Individual Nash Bargaining

In this section, I assume that wages are determined by bargaining between a matched worker and a firm. In the current environment, wages are paid only in terms of money. This is the consequence of my assumptions that a household's preference is idiosyncratic and employed workers do not know the household's desired consumption good at the time of receipt of wages.<sup>9</sup>

Suppose that bargaining weight of the firm is  $\alpha \in (0, 1)$ . Then wage,  $w_{t+1}$  solves<sup>10</sup>

$$\max_{w_{t+1}} \Omega_{et}^f \alpha \Omega_{et}^h 1 - \alpha. \quad (5.2)$$

The first order condition is

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<sup>9</sup>This can be shown very easily. Suppose there are  $N$  goods. Then if a firm pays wages in terms of goods,  $w_g$ , the net current flow of surplus to the household,  $S_g$ , is

$$S_g \equiv \frac{1}{N} U'(c) w_g - \phi.$$

In case, the wage,  $w_m$ , is paid in terms of money the net current flow of surplus to the household,

$$S_m \equiv \Omega_M w_m - \phi.$$

The household will supply labor when wages are paid in terms of good only when  $S_g \geq 0$ . When wages are paid in terms of money the condition is  $S_m \geq 0$ . In all the equilibrium I consider in the economy  $S_m > 0$ . But if  $N$  is large (which is the assumption here) then  $S_g < 0$ . In the case where  $N$  is not large and  $S_g > 0$  ruling out payment of wages in terms of good would require that  $S_m > S_g$ .

<sup>10</sup>Here, I have assumed that the marginal value of vacancy to the firm is zero, which is going to be the case as the firm chooses optimal level of vacancy.

$$\frac{1-\alpha}{\alpha}\Omega_{et}^f = \Omega_{et}^h \quad (5.3)$$

Next I define a symmetric stationary monetary equilibrium (SSME) with individual Nash bargaining.

**Definition:** A symmetric stationary monetary equilibrium (SSME) with individual Nash bargaining is defined as a collection of the household's choice variables  $X^h \equiv \{c, M, e^h\}$ , the firm's choice variables,  $X^f \equiv \{i, e^f, v\}$ , the prices in the goods market,  $\hat{p}_t$ , and real wage in the labor market,  $\hat{w}$ , and the aggregate variables  $\hat{X}^h$  and  $\hat{X}^f$ , such that

(i) Given aggregate variables,  $\hat{X}^h$  and  $\hat{X}^f$ , and the prices in the goods market,  $\hat{p}_t$ , and real wage in the labor market,  $\hat{w}$ , the household's choice variables  $X^h$  solve (PH);

(ii) Given aggregate variables,  $\hat{X}^h$  and  $\hat{X}^f$ , the prices in the goods market,  $\hat{p}_t$ , and the real wage in the labor market,  $\hat{w}$ , the firm's choice variables  $X^f$  solve (PF);

(iii) the prices in the goods market,  $\hat{p}_t$ , satisfies (3.24);

(iv) the real wage in the labor market,  $\hat{w}$ , satisfies (5.3);

(v) aggregate variables are equal to the relevant household's and firm's variables,  $\hat{X}^h = X^h$ ,  $\hat{X}^f = X^f$ ; and

(vi) the marginal value of real money balances,  $\Omega_M$ , is strictly positive and finite.

From now on I suppress “ $\hat{\phantom{x}}$ ” from the aggregate variables. Since every employed worker is an employee, in the symmetric stationary equilibrium,  $e^h = e^f$ . From now on I also suppress superscripts  $h$  and  $f$  from these variables, and let  $e$  denote the employment level. (3.6) implies that employment,  $e$ , and the distribution of real wage earnings,  $G(w)$ , are given by

$$e = \frac{\mu(\theta)}{\sigma + \mu(\theta)}. \quad (5.4)$$

(3.2), (3.3), (3.15), (3.16), and (3.24) imply that consumption  $c$  is given by

$$c = \frac{\xi}{\delta + \xi(1 - \delta)} f(e). \quad (5.5)$$

(3.8), (3.9), and (5.1) together give the expression for the marginal value of real money balance,  $\Omega_M$ ,

$$\Omega_M = \frac{\xi}{(1 + r)g - 1 + \xi} U'(c). \quad (5.6)$$

(3.19) and (3.20) imply that the marginal value of inventory,  $\Omega_i$ , is given by

$$\Omega_i = \frac{\xi}{r + \delta + \xi(1 - \delta)} \Omega_M. \quad (5.7)$$

Using (3.20), (5.6), and (5.7), one can show that for any  $\Omega_M > 0$ , the surplus to the seller is strictly positive *i.e.*,  $\Omega_q > 0$ . Also (3.9) and (5.6) show that the growth rate of money supply,  $g$ , must exceed the rate of discount,  $\frac{1}{1+r}$ , in order to ensure that a matched buyer gets positive surplus *i.e.*,  $\lambda > 0$ . (3.19), (3.20), and (3.23) imply that the marginal value of an employee,  $\Omega_e^f$ ,

$$\Omega_e^f(w) = \frac{1}{r + \sigma} \left[ \frac{\xi(1 + r)}{r + \delta + \xi(1 - \delta)} f'(e) - w \right] \Omega_M. \quad (5.8)$$

(3.10), (3.11), (5.3) and (5.8) imply that the real wage,  $w$ , is given by

$$w = \frac{1}{r + \sigma + (1 - \alpha)\mu(\theta)} \left[ \frac{\xi(1 + r)(1 - \alpha)(r + \sigma + \mu(\theta))}{r + \delta + \xi(1 - \delta)} f'(e) + \frac{\alpha(r + \sigma)\phi}{\Omega_M} \right]. \quad (5.9)$$

Then (3.21), (5.6), (5.8), and (5.9) imply that the equilibrium level of labor market tightness,  $\theta$ , implicitly solves

$$k = \frac{\mu(\theta)}{\theta} \frac{\alpha}{r + \sigma + (1 - \alpha)\mu(\theta)} \left[ \frac{\xi}{(1 + r)g - 1 + \xi} AU'(c) f'(e) - \phi \right]. \quad (5.10)$$

Using (5.4) and (5.5), equation (5.10) can be reduced to a single equation in the labor market tightness,  $\theta$ . The existence of equilibrium depends crucially on whether (5.10) has a non-trivial solution or not.

**Lemma 1:** Equation (5.10) has a unique and finite solution,  $0 < \theta < \infty$  under the conditions that  $\lim_{c \rightarrow 0} U'(c) > 0$ ,  $\lim_{e \rightarrow 0} f'(e) > 0$ ,  $\lim_{\theta \rightarrow 0} \mu(\theta) = 0$ ,  $\lim_{\theta \rightarrow 0} \frac{\mu(\theta)}{\theta} = \infty$ ,  $\mu'(\theta) > 0$ , and  $\frac{d(\mu(\theta)/\theta)}{d\theta} < 0$ .

The existence and uniqueness of SSME with individual Nash bargaining follows directly from lemma 1.

**Proposition 1.** Under assumption 1 and conditions specified in lemma 1, there exists a unique SSME with individual Nash bargaining characterized by equations 5.4-5.10.

Next I discuss the effects of higher growth rate of money supply on equilibrium variables. The following proposition summarizes the impact of higher growth rate of money supply on output, consumption, and employment.

**Proposition 2.** Higher growth rate of money supply,  $g$ , reduces output,  $f(e)$ , consumption,  $c$ , employment,  $e$ , the marginal value of real money balance,  $\Omega_M$ , and increases the real wage  $w$ .

The intuition for proposition 2 is quite simple. The marginal cost of creating vacancy (left hand side of (5.10)) is independent of the growth rate of money supply  $g$ , but the marginal benefit from creating vacancy (the right hand side of (5.10)) is declining in the growth rate of money supply. The reason is that higher growth rate erodes the value of real money balance for given consumption level which increases real wage and reduces profit. Thus labor market tightness falls leading to lower output, consumption, and employment.

From (5.6), it is clear that higher growth rate of money supply directly reduces the marginal value of real money balance. But since consumption also falls, this raises the marginal value of real money balance. Overall, the first effect dominates the second and the marginal value of real money balance falls with higher growth rate of money. The fall in the marginal value of real money balance,  $\Omega_M$ , along with the increase in the marginal product of labor leads to higher real wage.

**Proposition 3.** Under the individual Nash bargaining at the Friedman Rule ( $(1+r)g = 1$ ),

(i) The social optimal consumption,  $c^s$ , output,  $f(e^s)$ , employment,  $e^s$ , and labor market tightness,  $\theta^s$  coincide with market consumption,  $c$ , output,  $y$ ,

employment,  $e$ , and labor market tightness,  $\theta$ , when the elasticity of matching function with respect to labor market tightness,  $\eta(\theta)$ , equals bargaining power of firms,  $\alpha$ , *i.e.*  $\eta(\theta) = \alpha$ ;

(ii) When  $\eta(\theta) < \alpha$ , then  $c^s < c$ ,  $f(e^s) < f(e)$ ,  $e^s < e$ , and  $\theta^s < \theta$ ;

(iii) When  $\eta(\theta) > \alpha$ , then  $c^s > c$ ,  $f(e^s) > f(e)$ ,  $e^s > e$ , and  $\theta^s > \theta$ .

The intuition for these results is quite simple. In the model, inefficiencies arise in both demand side and supply side of the goods market. The inefficiency on the demand side arises due to binding real money balance constraint on buyers which leads to quantity of goods traded being inefficient. The Friedman rule maximizes the real value of money, thus making the monetary constraint on buyers non-binding and inducing the efficient quantity of trade.<sup>11</sup>

The inefficiency on supply side is due to search frictions in the labor market which leads to trading externalities. Due to these externalities firms do not hire efficient number of workers resulting in inefficient supply of goods. In the model, an increase in vacancies raises the matching probability of unemployed workers, but reduces the matching probability of vacancies. Thus, one more vacancy in the labor market makes unemployed workers better off, but it makes other firms worse-off. The measure of positive externality caused by an additional vacancy on unemployed workers is given by  $\eta(\theta)$ . The negative externality caused by an additional vacancy is measured by  $\eta(\theta) - 1$ . Similarly, an increase in unemployed workers reduces the matching probability of unemployed workers, but increases the matching probability of vacancies. Thus, one more unemployed worker in the labor market makes other unemployed workers worse off, but makes firms better-off.

The *ex-post* Nash bargaining rule in general does not internalize these externalities. The reason is that wage is determined by bargaining between matched workers and recruiters who ignore the effects of their choices on unmatched workers and recruiters. Only under certain parametric restriction known as the Hosios condition (Hosios 1990) the resulting labor market tightness is efficient. The condition is that the elasticity of the matching function with respect to vacancies be equal to the firm's share of surplus created by the job,  $\eta(\theta) = \alpha$ .

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<sup>11</sup>If we endogenize the number of buyers and sellers in the goods market (*i.e.* make  $\xi$  endogenous), then the Friedman rule may not optimal as it may not lead to inefficient number of trades.

When the monetary growth rate is given by the Friedman rule ( $g = \frac{1}{1+r}$ ), and  $\eta(\theta) = \alpha$  inefficiencies on both the demand and supply sides of the goods market are internalized. Thus market allocations coincide with social optimal allocations. When  $\eta(\theta) \neq \alpha$  then the market allocations are sub-optimal. In the case  $\eta(\theta) < \alpha$ , firms get higher return from creating vacancies compared to the social return and thus they create too many vacancies. Opposite is the case when  $\eta(\theta) > \alpha$ .

As discussed above, efficiency in the labor market is achieved only when  $\eta(\theta) = \alpha$ . In the given environment there is no reason to believe that this condition is satisfied as the elasticity of the matching function depends on the properties of the matching technology and the firm's share of surplus on the bargaining environment. However, a literature (directed search) has emerged, which shows that under certain wage posting mechanism Hosios condition is automatically satisfied (*e.g.* Moen 1997, Julien et. al. 2000, Mortensen and Wright 2002). In other words, wage posting picks the efficient distribution of bargaining power. One interpretation of the directed search models (*e.g.* Moen 1997) is that there are many sub-markets characterized by wages and waiting time (labor market tightness). Firms and workers observe these wages and waiting time and choose where to go, which sorts them in appropriate sub-markets. The resulting labor market tightness is efficient.

Next I analyze the question whether the Friedman rule is optimal when search is directed. Here, I change the way wages are determined. Rest of the structure of the economy remains the same.

## 5.2 Wage Posting

In this section, I embed the wage posting model of Mortensen and Wright (2002) in the monetary search framework. Suppose that there are competing market makers who can open sub-markets. These market makers post wages,  $w$ . After observing wages firms and workers choose to enter any sub-market. Within any sub-market matching is random and depends on measure of firms and workers in the sub-market. Thus each sub-market is characterized by wage,  $w$ , and labor market tightness,  $\theta$ . In equilibrium, the set of sub-markets is complete in the sense that there is no sub-market that could be opened that would make some workers and firms better off.

A market maker can make a profit if he can design a sub-market if he can make unemployed workers (or firms) better off without making firms (or unemployed workers) worse off. Thus the market maker chooses  $(w, \theta)$  to

maximize the value of unemployed workers,  $\Omega_{ut}$ , subject to the first order condition associated with the optimal choice of vacancies given in (3.21). In the steady-state the solution of competitive search is equivalent to

$$\max_{w,\theta} \frac{\mu(\theta)}{r + \sigma + \mu(\theta)} (w\Omega_M - \phi) \quad (5.11)$$

subject to the free entry condition

$$k = \frac{1}{1+r} \frac{\mu(\theta)}{\theta} [Af'(e)\Omega_M - w\Omega_M]. \quad (5.12)$$

Using the first order conditions, one can show that

$$w\Omega_M = \frac{1}{r + \sigma + (1 - \eta(\theta))\mu(\theta)} [(1 - \eta(\theta))(r + \sigma + \mu(\theta))Af'(e)\Omega_M + \eta(\theta)(r + \sigma)\phi] \quad (5.13)$$

and

$$k = \frac{\mu'(\theta)}{r + \sigma + (1 - \eta(\theta))\mu(\theta)} [Af'(e)\Omega_M - \phi]. \quad (5.14)$$

One can easily show that there exists a unique SSME. The qualitative effects of higher growth rate of money on allocations and real wage are same as before.

**Proposition 4.** In the SSME with wage posting, higher growth rate of money supply reduces output, consumption, and employment, and raises real wage under the condition that  $\frac{d\eta(\theta)}{d\theta} \leq 0$ .

Comparing (4.5) and (5.14), it is immediately clear that the social and the market levels of labor market tightness coincide. Thus we have the following proposition.

**Proposition 5.** In the SSME with wage posting, the Friedman rule is optimal,  $c = c^s$ ,  $e = e^s$ ,  $f(e) = f(e^s)$ , and  $\theta = \theta^s$ .

Intuition is that the Friedman rule removes the source of inefficiency on the demand side and the directed search on the supply side of the goods market. Next I consider efficiency wage.



### 5.3 Efficiency Wage

There are many versions of efficiency wage model. The basic idea behind these models is that the productivity of workers depends on real wage paid. Higher real wage induces greater labor input per worker. The positive relationship between real wage and labor input prevents firms from paying reservation wage to workers.

To analyze the effects of higher monetary growth rate and efficiency implications, I embed the shirking model developed by Shapiro and Stiglitz (1984) in the monetary framework. Suppose that there are two levels of effort 0 and 1. Exerting effort equal to 1 leads to disutility equal to  $\phi$ . In case of shirking, no disutility is incurred but the shirking worker gets fired with probability  $\chi$ .

In the stationary environment, the marginal value of employed worker who does not shirk,  $\Omega_e^h$ , is given by

$$r\Omega_e^h = (1+r)(w\Omega_M - \phi) - \sigma(\Omega_e^h - \Omega_u^h) \quad (5.15)$$

where  $\Omega_u^h$  is the marginal value of unemployed worker.

$$r\Omega_U^h = \mu(\theta)(\Omega_E^h - \Omega_U^h). \quad (5.16)$$

The marginal value of employed worker who shirks,  $\Omega_S^h$ , is given by

$$r\Omega_S^h = (1+r)w\Omega_M - (\sigma + \chi)(\Omega_S^h - \Omega_U^h). \quad (5.17)$$

A firm will set wage such that an employed worker is indifferent between shirking and not-shirking *i.e.*,  $\Omega_S^h = \Omega_E^h$ . This condition together with (5.15-5.17) gives equilibrium wage,  $w$

$$w = \frac{\phi}{\Omega_m} \left[ 1 + \frac{r + \sigma + \mu(\theta)}{\chi} \right]. \quad (5.18)$$

(5.18) shows that employed workers get premium over their reservation wage,  $\frac{\phi}{\Omega_M}$ . The premium exists due to the inability of firms to perfectly monitor workers' effort level and does not depend on the productivity of workers. The size of premium is negatively related to the detection probability,  $\xi$ , and positively related to the matching probability of workers,  $\mu(\theta)$ .

(3.21), (3.23) and (5.18) imply that the equilibrium level of labor market tightness  $\theta$  solves

$$k = \frac{\mu(\theta)}{\theta} \frac{1}{r + \sigma} \left[ Af'(e)\Omega_M - \phi\left(1 + \frac{r + \sigma + \mu(\theta)}{\chi}\right) \right] \quad (5.19)$$

where  $\Omega_M$  and  $e$  are given by (5.6) and (5.4) respectively. One can easily show that there exists a unique SSME. The following proposition summarizes the effects of higher growth rate of money supply on market allocations.

**Proposition 6.** In the SSME with the efficiency wage, higher growth rate of money supply reduces labor market tightness, output, consumption, and employment.

A higher growth rate of money reduces labor market tightness, output, consumption, and employment for reasons discussed earlier. The higher growth rate of money supply may raise or lower real wage. Higher growth rate of money supply by reducing the value of real money balance increases real wage directly. But the matching rate of workers also falls, which by reducing the wage premium has the opposite effect.

Next I consider the welfare implications.

**Proposition 7.** In the SSME with the efficiency wage, at the Friedman Rule,  $(1 + r)g = 1$ ,:

- (i) The social optimal allocations coincide with market allocations,  $c^s = c$ ,  $f(e^s) = f(e)$ ,  $e^s = e$ , and  $\theta^s = \theta$ , when the detection rate of shirking workers,  $\chi = \infty$ , and the elasticity of matching function with respect to labor market tightness,  $\eta(\theta) = 1$ ;
- (ii) When  $\eta(\theta) < 1$  and  $\chi = \infty$ , then  $c^s < c$ ,  $f(e^s) < f(e)$ ,  $e^s < e$ , and  $\theta^s < \theta$ ;
- (iii) When  $\eta(\theta) = 1$  and  $\chi < \infty$ , then  $c^s > c$ ,  $f(e^s) > f(e)$ ,  $e^s > e$ , and  $\theta^s > \theta$ ;
- (iv) When  $\eta(\theta) < 1$  and  $\chi < \infty$ , then results are ambiguous and  $c^s \gtrless c$ ,  $f(e^s) \gtrless f(e)$ ,  $e^s \gtrless e$ , and  $\theta^s \gtrless \theta$ ;
- (v) When  $\eta(\theta) = 0$  and  $\chi > 0$ , then  $c^s < c$ ,  $f(e^s) < f(e)$ ,  $e^s < e$ , and  $\theta^s < \theta$ .

In the case of efficiency wage not only do we have search externalities, but also deal with shirking problem. To avoid shirking, firms pay worker premium over their reservation wage. This premium disappears when the detection rate of shirking workers,  $\chi = \infty$ . But when  $\chi = \infty$  workers receive

only their reservation wage, firms create too many vacancies. This happens because in the market firms take matching rate of vacancy as given and ignore the effects of their vacancy creation on the matching rate of vacancies. As discussed above the extent of negative externality is given by  $\eta(\theta) - 1$ . This negative externality disappears only when  $\eta(\theta) = 1$ . This explains part(i) and (ii) of the proposition.

When  $\eta(\theta) = 1$  and  $\chi < \infty$ , then there is no search externality but firms pay wage premium. Thus they create too little vacancies compared to the social optimum. This explains part (iii) of the proposition. Part (iv) follows from parts (ii) and (iii). Finally, when  $\eta(\theta) = 0$  the negative externality is very severe and thus firms create too many vacancies.

Next I consider union bargaining.

## 5.4 Union Bargaining

Here, I assume that once an unemployed worker accepts a match, he allows union to bargain over wage on his behalf. I also assume that there are many independent decentralized unions. Each firm negotiates with a single union, which ignores the effects of its negotiation on labor market outcomes. I consider the case of right to manage, where unions and firms bargain over wages, but firms are free to choose employment level taking as given the negotiated wage. I consider only symmetric case where  $\omega_{Mt} = \hat{\omega}_{Mt}$  and  $e_t^f = e_t^h = e_t$ .

The total surplus to union at time  $t + 1$  is

$$\omega_{Mt+1} p_{t+1} w_{t+1} e_{t+1}. \quad (5.20)$$

The total surplus to the firm at time  $t + 1$  is

$$((f(e_{t+1}) + i_{t+1}) - e_{t+1} w_{t+1}) p_{t+1} \omega_{t+1}. \quad (5.21)$$

Then, wage  $w_{t+1}$  solves

$$\max_{w_{t+1}} [((f(e_{t+1}) + i_{t+1}) - e_{t+1} w_{t+1}) p_{t+1} \omega_{t+1}]^\alpha [\omega_{Mt+1} p_{t+1} w_{t+1} e_{t+1}]^{1-\alpha}. \quad (5.22)$$

The first order condition is

$$\frac{1-\alpha}{\alpha} [((f(e_{t+1}) + i_{t+1}) - e_{t+1} w_{t+1}) p_{t+1} \omega_{t+1}] = \omega_{Mt+1} p_{t+1} w_{t+1} e_{t+1}. \quad (5.23)$$

Let  $B \equiv \frac{\xi}{\delta + \xi(1-\delta)}$ , then in the steady state wage,  $w$ , is given by

$$w = (1 - \alpha)B \frac{f(e)}{e} \Omega_M + \alpha \frac{\phi}{\Omega_M}. \quad (5.24)$$

(5.24) shows that wage,  $w$ , is the weighted average of the average product of labor and the reservation wage. By putting (5.24) in (3.23) and using (3.21), one can derive equation which solves for the optimal level of labor market tightness,  $\theta$ , which is given by

$$k = \frac{\mu(\theta)}{\theta} \frac{1}{r + \sigma} \left[ \left( A - \frac{(1 - \alpha)B}{\zeta(e)} \right) f'(e) \Omega_M - \alpha \phi \right] \quad (5.25)$$

where  $\zeta(e) \equiv \frac{f'(e)e}{f(e)}$  is the elasticity of production function with respect to employment.  $\Omega_M$  and  $e$  are given by (5.6) and (5.4) respectively.

**Proposition 8.** Under the assumption that  $\frac{d\zeta(e)}{de} \leq 0$  and  $A > \frac{(1-\alpha)B}{\zeta(1)}$  there exists a unique SSME with unions.

The first parametric restriction ensures that the right hand side of (5.25) is downward sloping. The second restriction ensures that firms make strictly positive profit for some  $e \in (0, 1)$ . Thus firms will have incentive to create strictly positive level of vacancies. The effects of higher growth rate of money is summarized below.

**Proposition 9.** In the SSME with unions, higher growth rate of money supply reduces output, consumption, labor market tightness, and employment.

The intuition for proposition 9 is same as before. Next I consider efficiency implications.

**Proposition 10.** In the SSME with unions, at the Friedman Rule,  $(1+r)g = 1$ ,

(i) The social optimal allocations coincide with market allocations,  $c^s = c$ ,  $f(e^s) = f(e)$ ,  $e^s = e$ , and,  $\theta^s = \theta$ , when the bargaining weight of firms,  $\alpha = 1$ , and the elasticity of matching function with respect to labor market tightness,  $\eta(\theta) = 1$ ;

(ii) When  $\eta(\theta) < 1$  and  $\alpha = 1$ , then  $c^s < c$ ,  $f(e^s) < f(e)$ ,  $e^s < e$ , and  $\theta^s < \theta$ ;

- (iii) When  $\eta(\theta) = 1$  and  $\alpha < 1$ , then  $c^s > c$ ,  $f(e^s) > f(e)$ ,  $e^s > e$ , and  $\theta^s > \theta$ ;
- (iv) When  $\eta(\theta) < 1$  and  $\alpha < 1$ , then results are ambiguous and  $c^s \gtrless c$ ,  $f(e^s) \gtrless f(e)$ ,  $e^s \gtrless e$ , and  $\theta^s \gtrless \theta$ ;
- (v) When  $\eta(\theta) = 0$  and  $0 < \alpha \leq 1$ , then  $c^s < c$ ,  $f(e^s) < f(e)$ ,  $e^s < e$ , and  $\theta^s < \theta$ .

To explain these results, it is useful to abstract from search frictions in the labor market. Consider a model in which wages are determined by bargaining between a union and a firm. Then the firm hires workers till the marginal product of workers equals negotiated wage,  $f'(e) = w$ . It is easy to show that the wages are given by (5.24) and employment by

$$f'(e) = (1 - \alpha)B \frac{f(e)}{e} \Omega_M + \alpha \frac{\phi}{\Omega_M}. \quad (5.26)$$

The social optimal level of employment (with no friction in the labor market) is given by

$$f'(e) = \frac{\phi}{\Omega_M}. \quad (5.27)$$

The comparison of (5.26) and (5.27) shows that the market and the social levels of labor market tightness coincide only when  $\alpha = 1$ . For any  $\alpha < 1$  firms hire relatively few workers, since workers are paid more than their reservation wage.

But when search frictions are introduced results are more complicated due to trading externalities. When  $\alpha = 1$  but  $\eta(\theta) < 1$  firms create too many vacancies relative to the social optimum due to congestion externalities. Only when  $\eta(\theta) = 1$  the congestion externalities disappear. This explains part (i) and (ii) of the proposition. Now consider part (iii) of the proposition. When  $\eta(\theta) = 1$  then there is no search externalities and with  $\alpha < 1$  firms create too little vacancies. Part (iv) follows from part (ii) and (iii). Finally, when  $\eta(\theta) = 0$  the congestion externalities is very high and firms create too many vacancies.

To conclude, a higher monetary growth reduces output, employment, consumption, and labor market tightness under all wage setting mechanisms. Real wage increases only under individual Nash bargaining and wage posting. Under efficiency wage and union bargaining real wage may rise or fall.

The Friedman rule is optimal only under wage posting. In general, monetary growth rate higher than the Friedman rule is likely to be optimal in economies with relatively low elasticity of matching probability with labor market tightness,  $\eta(\theta)$ . These are the economies with relatively high negative externalities.

## 6 Conclusion

In the paper, I developed a search-theoretic monetary model with imperfect labor markets. I studied effects of inflation on output, employment, real wages and welfare under different wage setting rules: individual bargaining, wage posting, efficiency wage, and union bargaining. It finds that generally Friedman rule is not optimal except under wage posting. Higher inflation reduces output and employment. It raises real wage under individual and union bargaining. It may raise or lower real wage under efficiency wage.

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