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SHARE THE GAIN, SHARE THE PAIN? ALMOST TRANSFERABLE UTILITY, CHANGES IN PRODUCTION POSSIBILITIES, AND BARGAINING SOLUTIONS*

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Abstract

Consider a two-person economy in which allocative efficiency is independent of distribution but the cardinal properties of the agents' utility functions precludes transferable utility (a property I call "Almost TU"). I show that Almost TU is a necessary and sufficient condition for both agents to either benefit or to lose with any change in production possibilities under generalized utilitarian bargaining solutions (of which the Nash Bargaining solution is a special case) and under the Kalai-Smorodinsky solution. I apply the result to household decision-making in the context of the Rotten Kid Theorem and discuss other applications.

Keywords: Axiomatic bargaining, resource monotonicity, transferable utility, risk aversion

JEL Classifications: C71, D63, D13

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1 Introduction

Consider a two-person economy with production, in which the welfare of each agent is determined by either a general utilitarian bargaining solution or by the Kalai-Smorodinsky solution. How does a change in the production possibility set impact both agents' utilities? I find that both agents' welfare changes in the same direction with any change in the production possibility set if utility is Almost Transferable. I also show the reverse: unless Almost Transferable Utility holds, a change in the production possibility set can make one agent better off and the other agent worse off.

Chun and Thomson (1988) show that an expansion of the utility possibility set due to an increase in one of many goods in a two-person exchange economy does not in general increase both agents' utilities under these bargaining solutions. In the one-good economy, however, both agents benefit from an increase in the endowment of the good. In this paper I show that Chun and Thomson's result for the one-good economy can be applied to an economy in which many goods are produced if utility is Almost Transferable.

Bergstrom (1989, p.1147) defines transferable utility (TU) as the condition that any redistribution of utilities among agents along the utility possibility frontier does not affect the sum of agents' utilities. Put differently, the utility possibility frontier with TU is a simplex. A necessary condition for TU is that agents' preferences must be such that allocative efficiency is independent of the distribution of welfare. If agents have what I call "Almost Transferable Utility" they have utility functions that share the ordinal characteristics with utility functions necessary for TU, but the cardinal characteristics are more general. Thus the distinction between TU and Almost TU is irrelevant if for a distribution mechanism only ordinal properties of agents' utility functions matter (e.g. market mechanism). In contrast, bargaining solutions and the maximization of social welfare functions typically lead to different consumption bundles for individuals depending on which cardinal properties are imposed on their utility functions even if ordinal properties of their utility functions are preserved.[1] Thus any results in the context of maximizing a social welfare function or in the context of applying a cooperative bargaining solution that rely on TU are less general than if they are shown to hold with Almost TU as well.[2]

My result has also implications for Gary Becker (1974)'s Rotten Kid The-

[1] MasCollel et al. (1995, p. 831) take as given that individuals have cardinal utility functions, when they state:"[...] whereas a policy maker may be able to identify individual cardinal utility functions (from revealed risk behavior, say), it may actually do so but only up to a choice of origins and units."

[2] The assumption that agents are strictly risk averse also rules out TU, but it does not rule out Almost TU. In the context of bargaining solutions, Nash (1950) explicitly states that agents have von Neumann-Morgenstern utility functions and thus cardinal properties of agents' utility functions matter. In Rubinstein et al. (1992)'s restatement of the Nash Bargaining problem in terms of preferences, choice under uncertainty plays an essential role. However, Kalai and Smorodinsky (1975) do not mention that utility functions are of the von Neumann-Morgenstern type, and neither do textbooks when they introduce the Nash bargaining problem (e.g., Moulin 1988; MasColell et al. 1995).

orem. Bergstrom (1989) formalizes the game that rotten kids play with their altruistic parent: Children take actions that impact their own utility as well as the utility of the other family members. The parent has a fixed amount of money to its disposal and, after observing its kids' actions, the parent determines monetary transfers to its off-spring by maximizing its altruistic utility function. Children thus take into account how the parent will react to their actions when they choose their own actions. The Rotten Kid Theorem states that even if the children are completely selfish and care only about their own consumption, they will behave as if they are maximizing the parent's altruistic utility function. In order for the Rotten Kid Theorem to hold, Bergstrom (1989) shows that his weak assumption that the parent treats each kid's utility as a normal good in its utility function implies the very strong requirement of TU on children's utility functions. In contrast, my result here shows that in order for the Rotten Kid Theorem to hold, the stronger assumption that the altruistic parent's utility function is generalized utilitarian implies the less strong requirement of Almost TU on children's utility functions.

Finally, the paper's technique can also be applied to consumer choice. If the consumer's budget set is strictly convex because prices of a good are a function of the quantity of the good and go up with a higher quantity purchased (e.g. the expenditure function of electricity usage with a progressive tax on the use of electricity), then an increase in income will increase the consumption of both goods provided the individual utility function defined over consumption goods takes on the equivalent form to the generalized utilitarian social welfare function.

The next section introduces the model, section 3 presents the main result, and section 4 discusses further applications of my result and concludes.

2 The Model

Two agents, Ava ($i = A$) and Bob ($i = B$), produce $L \geq 2$ goods to consume. The production possibility set is given by the convex set $Y \subset \mathbb{R}^L_+$. Denote the consumption bundle of agent i by x_i and denote the set of feasible consumption bundles by X where $X \subset \mathbb{R}^L_+ \times \mathbb{R}^L_+$. It contains at least one private good and may contain public goods. In the case of all private goods for any $(x_A, x_B) \in X$, it must be that $\Sigma x_i \in Y$. If good l is a public good both agents consume the same amount of the good, i.e. $x_{lA} = x_{lB} = x_l$, and consumption is non-rival such that $x_l = y_l$. Thus in the case of M private goods and $L - M$ public goods, for any pair $(x_A, x_B) \in X$, it must be that $((\Sigma x_{1i}, .., \Sigma x_{Mi}), (x_{(L-M)}, ...x_L)) \in Y$. Ava's utility function is given by $u_A(x_A)$, and Bob's utility function is given by $u_B(x_B)$. Agents' utility functions are assumed to be continuous, concave and twice differentiable.

The utility possibility set – denoted by U – contains all the utility pairs that arise from feasible consumption bundles. It is given by the convex set $U = \left\{(u_A, u_B) \in \mathbb{R}^2 : u_A^{-1}(u_A(x_A)) + u_B^{-1}((u_B(x_B))) \in X(Y)\right\}$. The utility possibility frontier – denoted by ∂U – is the set of all the Pareto efficient utility pairs. That is, $\partial U = \{(u_A, u_B) \in U : \nexists (u'_A, u'_B) \in U \text{ and } (u'_A, u'_B) \geq (u_A, u_B)\}$. A vector $y \in Y$ is allocative efficient if it is associated with a Pareto efficient allocation (x_A, x_B). If allocative efficiency is independent of distribution, then there exists *one* $y \in Y$ such that, in the case of all private goods, for any Pareto efficient (x_A, x_B), $x_A + x_B = y$; and such that, in the case of M private goods and $L-M$ public goods, for any Pareto efficient (x_A, x_B), $((\Sigma x_{1i}, .., \Sigma x_{Mi}), (x_{(L-M)}, ...x_L)) = y$.

There is *Transferable Utility* if for any given Y, there exists a utility representation u_i for each agent such that

$$\partial U = \{(u_A, u_B) \in U : u_A + u_B = \lambda\}$$

for some λ (Bergstrom 1989). Note that for any Y, resource and technological constraints only play a role in the size of λ: If agents have transferable utility (TU), allocative efficiency is independent of distribution (Bergstrom and Cornes 1983, and Bergstrom and Varian 1985).[3]

Whether allocative efficiency is independent of distribution, depends on the ordinal properties of agents' utility functions only. If agents' utility functions are such that allocative efficiency is independent of distribution, but cardinal properties of their utility functions prohibit the particular utility representation that would lead to TU, then it must still be true that there exists a positive monotonic transformation of u_A and u_B such that

$$f_A(u_A) + f_B(u_B) = \lambda.$$

The transformation $f_i(u_i)$ just does not represent an agent's utility anymore.

Definition 1 *Almost Transferable Utility. There is* Almost Transferable Utility *if, for any given Y, agents' utility functions are such that the utility possibility frontier is of the form*

$$\partial U = \{(u_A, u_B) \in U : f_A(u_A) + f_B(u_B) = \lambda\}$$

for some increasing, twice differentiable and convex function $f_i(.)$, and for some λ.

An example in which Almost TU and TU are violated is a utility possibility frontier given by $U = \{(u_A, u_B) \in \mathbb{R}^2_+ : \lambda_B u_A + \lambda_A u_B = \lambda_A \lambda_B\}$, where λ_i is the highest utility of agent i given Y, even if for a particular Y, $\lambda_A = \lambda_B$.

Definition 2 *Bargaining solutions. Let $d = (d_A, d_B)$ be the threatpoint in the bargaining problem, where $d \in U$, and $d \notin \partial U$. (i) A generalized utilitarian bargaining solution (GUBS) is the unique utility vector u_{GU} that maximizes $g_A(u_A - d_A) + g_B(u_B - d_B)$ s.t. $(u_A, u_B) \in \partial U$ and where $g_i(\cdot)$ is a strictly*

[3] Bergstrom and his co-authors give an exhaustive list of agents' utility functions that lead to TU. Agents' utility functions must allow the indirect utility representation of the Gorman Polar Form in an economy with only private goods (Bergstrom and Varian 1985) and a form dual to the Gorman Polar form in an economy with public and private goods (Bergstrom and Cornes 1981 and 1983).

concave function. (ii) The Nash bargaining solution (NBS) is the unique utility vector u_N that maximizes $(u_A - d_A)(u_B - d_B)$ s.t. $(u_A, u_B) \in \partial U$. (iii) Let $\bar{u}_A$ be the highest utility Ava receives if Bob receives d_B and let $\bar{u}_B$ be the highest utility Bob receives if Ava receives d_A. The Kalai-Smorodinsky solution (KSS) is the unique utility vector u_K that equalizes relative utility gains: $(u_A - d_A)/(\bar{u}_A - d_A) = (u_B - d_B)/(\bar{u}_B - d_B)$ and $(u_A, u_B) \in \partial U$.

Note that the problem of maximizing a general utilitarian welfare function $\max_{u_A, u_B} g_A(u_A) + g_B(u_B)$ s.t. $(u_A, u_B) \in \partial U$ is formally equivalent to applying a generalized utilitarian bargaining solution with $d = (0, 0)$. As pointed out by Chun and Thomson (1988) the Nash bargaining solution is an example of a generalized utilitarian bargaining solution by setting $g_i(u_i - d_i) = \ln(u_i - d_i)$. Also note that by requiring $g_i(\cdot)$ to be a strictly concave function, I rule out the utilitarian social welfare function, $u_A + u_B$. I make this assumption to rule out the possibility of multiple solutions in the case of TU.

3 Result

In order to determine if the utilities of both agents change in the same direction with any change in the production possibility set, I first present a lemma to establish that, for any change in the production possibility set, the utility possibility frontier associated with the production possibility set before the change and the utility possibility frontier associated with the production possibility set after the change can never cross if Almost TU holds. I then show that under any generalized utilitarian bargaining solution the utilities of both agents must change in the same direction with any change in the production possibility set if Almost TU holds. I prove the result under KSS, too. Finally, I show that if allocative efficiency is not independent of distribution and therefore neither TU or Almost TU hold, a change in the production possibility set can make one agent better off and the other agent worse off.

If agents have transferable utility (TU), allocative efficiency is independent of distribution (Bergstrom and Cornes 1983, and Bergstrom and Varian 1985). This implies that any Pareto efficient allocation (x_A, x_B) is associated with the same $y \in Y$. Since every Pareto efficient allocation (x_A, x_B) must lead to $u_A(x_A) + u_B(x_B) = \lambda$, it must be the case that λ does not depend on (x_A, x_B), but on y only. This result provides a convenient way of solving for the utility possibility frontier in the case of TU. TU holds for any Y and therefore we can compare how λ changes if we restrict Y to contain a single vector of goods: A change from a certain bundle of total amounts of goods to a different bundle either shifts the utility possibility frontier parallel inwards or outwards, so that the utility possibility frontiers associated with the two bundles can never cross each other (e.g. Bergstrom 1989). Call the utility possibility frontier associated with a fixed bundle of goods produced the "restricted utility possibility frontier." Then, if different combinations of goods can be produced, the utility possibility frontier can be found by picking the point in the production possibility set that is associated with the restricted utility possibility frontier that lies the furthest out of all the restricted utility possibility frontiers generated by a given $y \in Y$, or put differently by maximizing $\lambda(y)$ subject to $y \in Y$.

Example 1 Finding the utility possibility frontier with TU. *a) Two private goods.* Suppose $u_i = (x_{1i} x_{2i})^{1/2}$. Then $\partial U = \{(u_A, u_B) \in \mathbb{R}^2_+ : u_A + u_B = \lambda\}$, where $\lambda = \max_{y \in Y} (y_1 y_2)^{1/2}$. *b) A private and a public good.* Suppose preferences over a private good and a public good are quasi-linear in the public good x_2, such that $u_i = x_{1i} + h_i(x_2)$, where $h_i(\cdot)$ is a strictly concave function. Then the utility possibility frontier consists of all the points on the line from $(u_A = h_A(y_2), u_B = y_1 + h_B(y_2))$ to $(u_A = y_1 + h_A(y_2), u_B = h_B(y_2))$ where the vector (y_1, y_2) is found by $\arg\max_{y \in Y} y_1 + h_A(y_2) + h_B(y_2)$, and $\lambda = \max_{y \in Y} y_1 + h_A(y_2) + h_B(y_2)$.

In order to have Almost TU utility functions need to satisfy the same ordinal characteristics as utility functions leading to TU. Since allocative efficiency is a purely ordinal concept, allocative efficiency requires that the same point in the production possibility set be picked for any given utility functions that share the same ordinal properties, namely $\arg\max_{y \in Y} \lambda(y)$. To illustrate this point, consider the following modification to example 1.

Example 2 Finding the utility possibility frontier with Almost TU. *a) Two private goods.* Suppose $u_i = (x_{1i} x_{2i})^{1/3}$ then $\partial U = \{(u_A, u_B) \in \mathbb{R}^2_+ : u_A^{3/2} + u_B^{3/2} = \lambda\}$, where $\lambda = \max_{y \in Y} (y_1 y_2)^{1/2}$. *b) A private and a public good.* Suppose the cardinal utility function of agent i over a private good (x_{1i}) and a public good (x_2) is given by $u_i = (x_{1i} + h_i(x_2))^{\alpha_i}$, where $h_i(\cdot)$ is a strictly concave function and $\alpha_i \in (0, 1)$. Then the utility possibility frontier consists of the endpoints $(u_A = (h_A(y_2))^{\alpha_A}, u_B = (y_1 + h_B(y_2))^{\alpha_B})$ and $(u_A = (y_1 + h_A(y_2))^{\alpha_A}, u_B = (h_B(y_2))^{\alpha_B})$ and of all points (u_A, u_B) between these endpoints for which $u_A^{1/\alpha_A} + u_B^{1/\alpha_B} = \lambda$, where the vector (y_1, y_2) is found by $\arg\max_{y \in Y} y_1 + h_A(y_2) + h_B(y_2)$, and $\lambda = \max_{y \in Y} y_1 + h_A(y_2) + h_B(y_2)$.

Next consider what happens if Almost TU holds and the production possibility set changes.

Lemma (Expansion and contraction of the utility possibility set): Given Almost TU, any change in production possibilities of the economy results in an expansion or contraction of the utility possibility set.

Proof. Consider a change from Y to Y'. Since allocative efficiency is given by $\max_{y \in Y} \lambda(y)$, λ increases or decreases with a change from Y to Y'. Hence the utility possibility set (UPS) associated with Y is either entirely contained in the UPS associated with Y' or the UPS associated with Y is entirely contained in the UPS associated with Y'. ∎

Proposition 1 Under any generalized utilitarian bargaining solution or the Kalai-Smorodinsky solution, any change in production possibilities either benefits both agents or makes both agents worse off if and only if agents' utility functions satisfy Almost TU.

Proof. For sufficiency, Chun and Thomson (1988) show that if agents have concave utility functions over one good only, and this good's supply increases, both agents benefit under GUBS and KSS. In what follows it will be useful to work with $v_i(x_i) = f_i(u_i(x_i))$, that is, $v_i(x_i)$ is a monotonic transformation of $u_i(x_i)$ and so preserves the ordinal properties of $u_i(x_i)$, but not necessarily the cardinal properties. Express $u_i = f_i^{-1}(v_i)$, where $f_i^{-1}(\cdot)$ is a concave and increasing function. With this notation any GUBS is found by first solving $\max_{v_A, v_B} g_A(f_A^{-1}(v_A) - d_A) + g_B(f_B^{-1}(v_B) - d_B)$ s.t. $v_A + v_B = \lambda$ and then finding the corresponding u_A and u_B from $u_i = f_i^{-1}(v_i)$. Similarly, the KSS is found by solving $\frac{f_A^{-1}(v_A)-d_A}{f_A^{-1}(\lambda - f_B(d_B))-d_A} = \frac{f_B^{-1}(v_B)-d_B}{f_B^{-1}(\lambda - f_A(d_A))-d_B}$ and $v_A + v_B = \lambda$ for v_A and v_B and then finding the corresponding u_A and u_B from $u_i = f_i^{-1}(v_i)$. Presented in this way a change in λ plays the same role as a change in the only good provided in the one-good economy of Chun and Thomson (1988) and thus their proofs apply. For completeness I prove necessity here using my notation and accounting for $d \neq (0,0)$.

For GUBS, the utility shares must satisfy

$$\frac{\partial g_A}{\partial \left(f_A^{-1}(v_A) - d_A\right)} \frac{df_A^{-1}}{dv_A} = \frac{\partial g_B}{\partial \left(f_B^{-1}(v_B) - d_B\right)} \frac{df_B^{-1}}{dv_B}. \tag{1}$$

The left hand side of (1) depends on v_A only, and the right hand side of (1) depends on v_B only. Since $f_i^{-1}(\cdot)$ is concave, $\frac{d^2 f_i^{-1}}{dv_i^2} \leq 0$. Since $g_i(\cdot)$ is strictly concave and $f_i^{-1}(\cdot)$ is increasing $\frac{\partial g_i}{\partial\left(f_i^{-1}(v_i)-d_i\right)}$ is decreasing in v_i. A change in λ changes v_A and v_B. An increase in λ increases v_B, and therefore $\frac{df_B^{-1}}{dv_B}$ does not increase and $\frac{\partial g_B}{\partial\left(f_B^{-1}(v_B)-d_B\right)}$ decreases. Thus an increase in v_B decreases the right hand side of (1). Since the left hand side of (1) must equal the right hand side, $\frac{\partial g_A}{\partial\left(f_A^{-1}(v_A)-d_A\right)}\frac{df_A^{-1}}{dv_A}$ must also decrease. This, however can only happen if v_A also increases. An increase in λ increases both v_A and v_B and therefore increases both u_A and u_B. Similarly, a decrease in λ decreases both u_A and u_B.

For KSS, the utility shares must satisfy

$$\frac{f_A^{-1}(v_A) - d_A}{f_A^{-1}\left(\lambda - f_B\left(d_B\right)\right) - d_A} = \frac{f_B^{-1}\left(v_B\right) - d_B}{f_B^{-1}\left(\lambda - f_A\left(d_A\right)\right) - d_B}.$$

Taking the derivative with respect to λ (both v_A and v_B will change with a change in λ) on both sides gives us a second equation

$$\delta_A = \delta_B$$

where

$$\delta_i = \frac{\frac{df_i^{-1}}{dv_i}\frac{dv_i}{d\lambda}}{f_i^{-1}\left(\lambda - f_j\left(d_j\right)\right) - d_i} - \frac{\left(f_i^{-1}\left(v_i\right) - d_i\right)\frac{df_i^{-1}}{d(\lambda - f_j(d_j))}}{\left[f_i^{-1}\left(\lambda - f_j\left(d_j\right)\right) - d_i\right]^2}.$$

An increase in λ increases the bliss utility of both agents, that is $\frac{df_i^{-1}}{d(\lambda - f_j(d_j))} > 0$. I show that an increase in λ increases the utility of both agents, that is $u_A > 0$ and $u_B > 0$ using a proof by contradiction. Suppose an increase in λ does not increase Ava's utility, i.e. $\frac{df_A^{-1}}{dv_A}\frac{dv_A}{d\lambda} \leq 0$. Then $\frac{dv_A}{d\lambda} \leq 0$, and $\delta_A < 0$. But since

$$v_A + v_B = \lambda,$$

an increase in λ needs to satisfy

$$\frac{dv_A}{d\lambda} + \frac{dv_B}{d\lambda} = 1.$$

Thus $\frac{dv_B}{d\lambda} \geq 1$ and Bob is better off, i.e. $\frac{df_B^{-1}}{dv_B}\frac{dv_B}{d\lambda} > 0$. If this also implies that $\delta_B > 0$, there is a contradiction. First, in order to sign δ_B, divide δ_B by $f_B^{-1}\left(v_B\right) - d_B$ and multiply by $f_B^{-1}\left(\lambda - f_A\left(d_A\right)\right) - d_B$. Then the expression to sign in order to determine the sign of δ_B is

$$\frac{\frac{df_B^{-1}}{dv_B}\frac{dv_B}{d\lambda}}{f_B^{-1}\left(v_B\right) - d_B} - \frac{\frac{df_B^{-1}}{d(\lambda - f_A(d_A))}}{f_B^{-1}\left(\lambda - f_A\left(d_A\right)\right) - d_B}.$$

Since $d \notin \partial U$, KSS always gives a utility to Bob that is lower than his bliss utility, i.e. $f_B^{-1}\left(v_B\right) < f_B^{-1}\left(\lambda - f_A\left(d_A\right)\right)$. Hence $v_B < \lambda - f_A\left(d_A\right)$ and due to the concavity of $f_B^{-1}\left(\cdot\right)$,

$$\frac{df_B^{-1}}{dv_B} \geq \frac{df_B^{-1}}{d\left(\lambda - f_A\left(d_A\right)\right)}.$$

Moreover,

$$\frac{\frac{df_B^{-1}}{dv_B}}{f_B^{-1}\left(v_B\right) - d_B} > \frac{\frac{df_B^{-1}}{d(\lambda - f_A(d_A))}}{f_B^{-1}\left(\lambda - f_A\left(d_A\right)\right) - d_B}.$$

Since $\frac{dv_B}{d\lambda} \geq 1$, $\delta_B > 0$, a contradiction. Both agents gain with an increase in λ and lose with a decrease in λ.

For necessity, note first that if the UPS changes such that the old and the new UPF intersect, any GUBS and KSS can lead to one agent being better off and the other agent being worse off. Figure 1 illustrates this case using KSS, but any GUBS can lead to a qualitatively similar graph. Second, I show that if allocative efficiency is not independent of distribution, and therefore Almost TU does not hold, a change in the production possibility set (PPS) may cause the UPF associated with the PPS before the change and the UPF associated with the PPS after the change to cross. I assume here for simplicity that Ava and Bob have identical and well-behaved preferences over two private goods and that the Inada conditions hold. Moreover, $u(0, x_{i2}) = u(x_{i1}, 0) = 0$ for all $x_{i1}, x_{i2} \geq 0$. It is straightforward to redo the proof with different utility functions, more than two goods and/or public goods. At the intercepts of the UPF, one agent receives nothing and the other agent receives the maximum utility given the PPS. Thus the $y \in Y$ associated with the intercept of the UPF is found where Ava's (or equivalently Bob's) indifference curve is tangent to the production possibility frontier (PPF). See Figure 2, point A. If both agents have equal utility on the UPF, both agents must receive the same amount of both goods. Figure 2 shows that point A is not efficient if both agents receive equal utility, because Ava's marginal rate of substitution (MRS) when she receives half of the total amount of each good is not the same as the marginal rate of transformation (MRT) at point A. The same is true for Bob. Thus changing the product mix by moving down along the PPF leads to another efficient allocation associated with the point on the UPF at which both agents have equal utility. Now consider a change of the PPS so that point A is on the new PPF and has the same slope at point A as Ava's indifference curve has if she receives half of the amounts of both goods. Point A on the new PPF is now the efficient allocation when Ava's and Bob's utility are equal. This implies that the point on the new UPF at which both agents enjoy equal utility lies inside the old UPS. This also means that the new PPF cuts the old PPF from above in point A and would therefore yield a new UPF with higher intercepts than the old PPF just like the two UPFs drawn in Figure 1. ■

Remark More than two agents. The proof for GUBS goes through for any number of agents, but the proof for KSS does not go through if more than two agents are considered (for a counterexample with three agents see Chun and Thomson 1988).

3.1 The Game Rotten Kids Play

So far, I have considered exogenous changes in the production possibility set and their impact on the welfare of both agents. Now suppose that Y is fixed, but agents, or better the children in the rotten kid terminology, choose their actions non-cooperatively, so that some $y \in Y$ will be the outcome of their chosen actions. Utilities of the children are determined by maximizing the parent's altruistic preferences subject to the utility possibility set restricted by y. If each $y \in Y$ generates a restricted utility possibility frontier that is Almost TU, then there exists one $y \in Y$ that pushes the restricted utility possibility frontier the furthest out, and there do not exist any two $y, y' \in Y$ such that the restricted utility possibility frontier associated with y and the restricted utility possibility frontier associated with y' cross. If the parent's altruistic utility function takes on the form of a general utilitarian social welfare function, then proposition 1 tells us that all children will benefit from choosing actions that result in the $y \in Y$ that shifts the restricted utility possibility frontier the furthest out. Put differently, every child has a dominant strategy of choosing the action that pushes the restricted utility possibility frontier the furthest out given any action of the other children and so each child behaves as if it would maximize the altruistic utility function of the parent. The Rotten Kid Theorem holds. Bergstrom (1989)'s proof of the Rotten Kid Theorem requires TU, because he assumes that the parent treats every child's utility as a normal good in its altruistic utility function: Only if the restricted utility possibility frontier is a straight line and shifts out parallel are all the children guaranteed to benefit. In comparison to Bergstrom, I can weaken the requirement of TU to Almost TU by imposing a stronger but not unreasonable condition on the parent's altruistic utility function.

3.2 Application to Individual Choice

The technique of the proof of Proposition 1 can also be used to answer the question of whether the demand of all goods $(x_1, ..., x_L)$ goes up when there is an increase in income I and the budget constraint is convex and of the form $\sum_{l=1}^{L} e_l(x_l) = I$, where $e_l(\cdot)$ is a convex function. Suppose $u(x) = \sum_{l=1}^{L} g_l(x_l)$, then

$$\max_x \sum_{l=1}^{L} g_l(x_l) \text{ s.t. } \sum_{l=1}^{L} e_l(x_l) = I$$

is equivalent to

$$\max_e u(e_l) = \sum_{l=1}^{L} g_l\left(e_l^{-1}(e_l)\right) \text{ s.t. } \sum_{l=1}^{L} e_l = I$$

and then once solved for e, one can find x by applying $x_l = e_l^{-1}(e_l)$. That is, the problem is transformed from the commodity space into the expenditure space, because in the expenditure space we can work with a linear constraint. Since the objective function of e is formally equivalent to the generalized utilitarian function, Chun and Thomson (1988)'s result applies and an increase in I will increase the expenditure of all commodities and thus increase the demand of all commodities. Such an analysis is, for example, relevant in public economics if commodities are taxed with a progressive tax rate.

4 Conclusion

I focus on agents' utility functions for which allocative efficiency is independent of distribution, a necessary condition for TU, but allow for agents to be strictly risk averse or to have cardinal utility functions, conditions that preclude TU. I call such a condition Almost TU. The assumption of Almost TU in the context of applying cooperative bargaining solutions or maximizing social welfare functions is a less restrictive assumption than TU. The result presented in this paper can be interpreted as good news or bad news. The good news is, that under the standard bargaining solutions and social welfare functions, the result that everybody's utility is affected in the same direction for any change in the production possibility set can be generalized from TU to Almost TU. The bad news is that without allocative efficiency being independent of distribution (or without Almost TU) agents do no longer share the pain or gain caused by any change in the production possibility set.

This result is important for anyone applying these standard solution concepts from cooperative game theory. For example, research on family economics frequently applies bargaining rules – most often the NBS – to analyze intrafamily distribution. Husband and wife are assumed to cooperate to achieve Pareto efficiency, but disagree on which point on the utility possibility frontier to settle. This conflict is resolved by applying a bargaining rule. In this literature parameters that change the threatpoint without changing the utility possibility set (McElroy (1990) refers to them as extrahousehold environmental parameters) have received substantial attention (Lundberg and Pollak 1996, Lundberg et al. 1997, Rubalcava and Thomas 2000, Chiappori et al. 2002), but policies that have the potential to affect the threatpoint as well as the utility possibility set are more difficult to analyze. Examples of such policies affecting both are parental leave policies, policies subsidizing child care, and family taxation. The result presented here establishes conditions under which such policy changes are guaranteed to change each family member's welfare in the same direction provided the threatpoint remains the same. Put differently, Almost Transferable Utility allows us to decompose the impact of a family policy into a "utility possibility set" effect (family members share the gain or the pain) and a "threatpoint" effect (different family members may experience changes in their utility at the threatpoint in opposing directions).

The result is also useful for economists who combine elements of non-cooperative behavior and cooperative bargaining in their models. Somewhat similar to the game that rotten kids play, suppose agents apply a generalized utilitarian bargaining solution or the Kalai-Smorodinsky solution to distribute goods once they are produced by agents' actions in the economy. This procedure implies that the threatpoint is determined *before* agents choose their actions. Under what conditions do agents have an incentive to produce the efficient amount of these goods? I show that if Almost TU holds any change in one agent's action given the other agents' actions changes the utility of all agents in the same direction. Hence it is in everybody's best interest to push the restricted utility possibility frontier as far out as possible. Agents' actions, although chosen non-cooperatively, will lead to Pareto efficiency. Put differently, if Almost TU does not hold, this procedure can lead to inefficient outcomes.[4]

[4] Note that such a procedure is different from so-called ex-post bargaining models: In these, agents first produce goods non-cooperatively and bargain over the distribution only once the goods have been produced. Thus the relevant threatpoint is determined *after* agents have chosen their actions. Given Almost TU, inefficiency in the ex-post bargaining case occurs because agents' actions not only affect the utility possibility set but also the threatpoint in the bargaining stage. Note also that the hold-up problem is an example of ex-post bargaining.

5 Figures

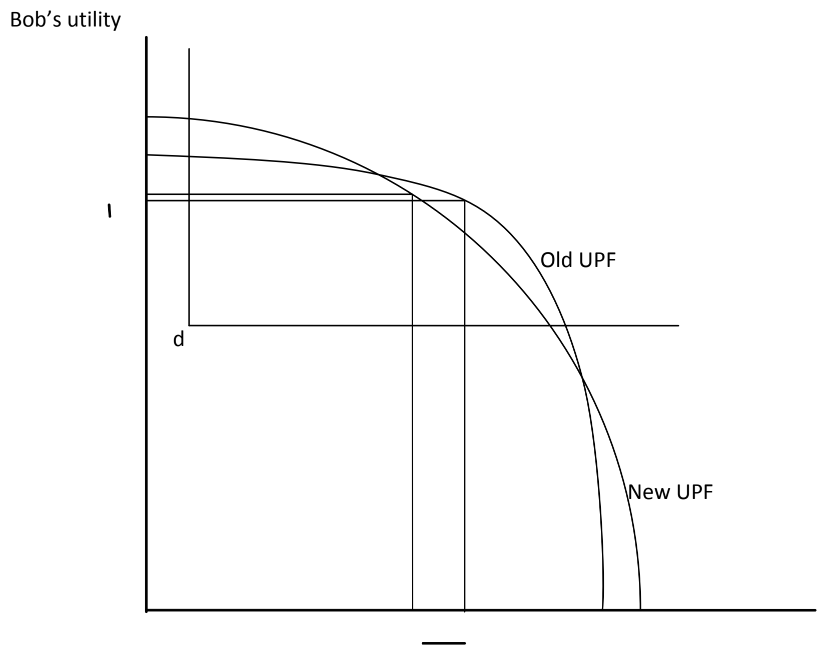

Figure 1: Change in UPS makes Bob better off and Ava worse off.

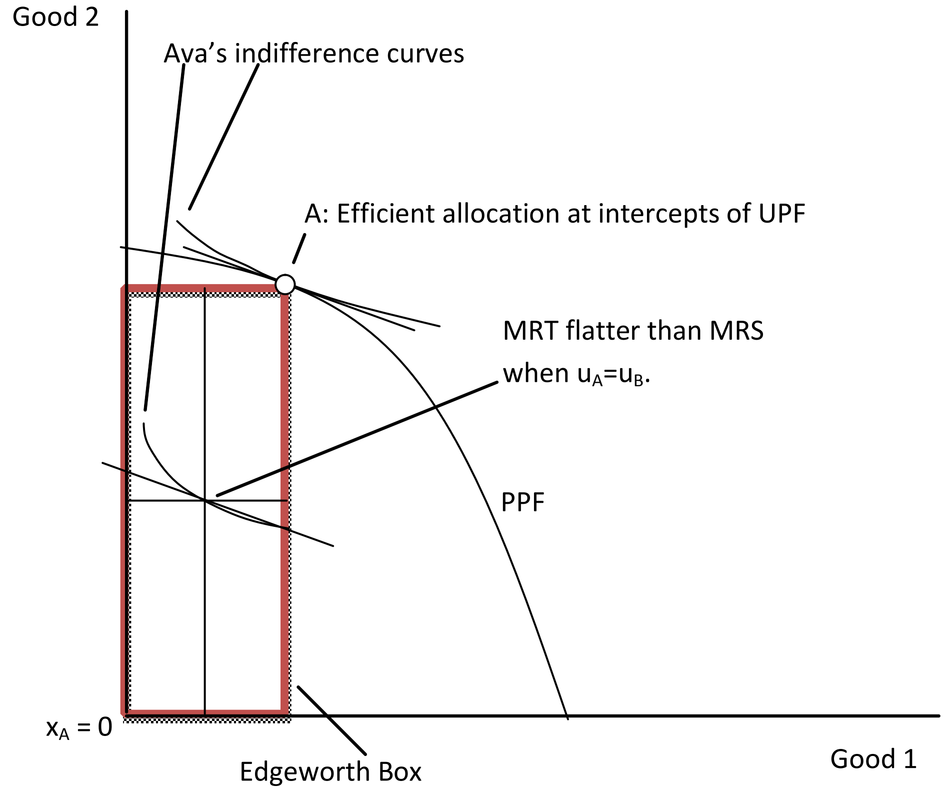

Figure 2

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