Bank Loan Portfolio and Monetary Transmission*

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Abstract

We develop a tractable theoretical framework of money, banking and finance to study the macroeconomic implications of the bank portfolio choice between commercial and collateralized household loans, as well as the transmission of monetary policy through bank decisions. In our model with fractional reserve banking, monetary policy is transmitted through an interest-rate channel and a bond-supply channel. We prove that steady-state inflation is ultimately determined by the interaction of labor demand and supply, which justifies why central banks closely monitor labor market conditions when making policy decisions. Fractional reserve banking can be welfare-improving. Our quantitative study illustrates how various short-run disturbances and long-run policy changes alter the bank loan distribution, which results in crowding-out effects between construction and production and has direct implications for overall financial risk. Finally, contractionary monetary policy reduces loan volumes, pivots bank lending toward household mortgage loans, and mitigates financial risk, and *vice versa* for expansionary policy.

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1 Introduction

We construct a tractable theoretical framework of money, banking and finance to study (i) the macroeconomic impact of the bank portfolio choice between commercial and collateralized household loans and (ii) the transmission of monetary policy through bank lending decisions. Our study is motivated by two sets of empirical observations on bank lending: the complexity of bank lending risk and the crowding-out effects of bank loans.

Household mortgage loans and commercial and industrial loans are twin pillars of bank lending, making up over 70% of total loans in the U.S. in the last twenty years.[1] However, these two loan types accrue different risks, and commercial and industrial loans are generally riskier than real estate loans (see evidence provided in Section 5). Banks manage lending risks by adjusting the composition of their loan portfolios. For example, Bidder et al. (2021) and Dell'Ariccia et al. (2021) find evidence that in an effort to cope with loss, banks curtail commercial lending and reallocate lending to other assets, such as mortgages. Therefore, the need to better assess financial risks calls for a theoretical framework that allows for differential risks across loan types and for banks to choose the composition of their loan portfolios.

On a related but separate note, there has been a growing body of empirical work on the crowding-out effects of bank lending. Some find that banks in housing booms prioritize mortgage lending over commercial lending, causing reduced investment and related real activities (*e.g.*, see Section 2 for a discussion of Chakraborty et al. (2018), Fieldhouse (2019), Chakraborty et al. (2020), Suh and Yang (2020), and Li et al. (2022)). Nevertheless, the literature does not have a consensus on crowding out. For example, Bezemer et al. (2020) document a positive effect of mortgage credit expansion on business credit growth in advanced economies but a negative effect in emerging and developing economies. The seemingly contradicting empirical evidence calls for a theoretical framework that connects the bank portfolio choice with real activities, to better understand whether bank lending leads to competing or complementary effects between the production and construction sectors.

[1] According to data obtained from FDIC.

To address these observations, we design our framework to accommodate a bank portfolio choice, household demand for collateralized loans, entrepreneurial demand for risky business loans, production and construction sectors, and monetary policy transmission. Monetary policy is transmitted through an *interest-rate* channel and a *bond-supply* channel. Our model delivers a wide range of analytical and quantitative results that shed light on both the monetary policy effects on bank lending and the implications of bank portfolio decisions for overall financial risks and real consequences of loan crowding out.

Given the complex nature of the desired model, we strive to maintain tractability to obtain insights into how various elements work jointly to shape macroeconomic results. To this end, we model safe household collateralized loans in the spirit of Kiyotaki and Moore (1997) and risky business loans *à la* Carlstrom and Fuerst (1997). For the former, there are patient and impatient households in our model who, respectively, become savers and borrowers in equilibrium. Household loans must be collateralized by assets, such as capital and housing, subject to a loan-to-value (henceforth LTV) constraint. For the latter, there are entrepreneurs who operate risky investment projects to produce capital goods. They borrow from banks with a fixed interest rate. If the project fails, the entrepreneur will default on their debt and be audited by the bank.

These modeling choices improve tractability in two aspects: First, we abstract away from the risks associated with collateralized household debt and only model commercial debt as risky. This allows us to reduce model complexity without losing the key empirical observation that commercial debt is generally riskier than household debt collateralized by houses. Next, the Calstrom-Fuerst structure generates the result that only the aggregate entrepreneurial capital stock matters for solving the equilibrium despite idiosyncratic project outcomes. This allows the analysis of our model to remain close to the representative-agent style.

Another factor that helps improve analytical tractability is that we model monetary transmission by imposing a proportional reserve requirement on banks. Policy measures, such as the required reserve ratio and the interest rate paid on reserves, directly impact banks' choices of the amounts of deposit to take in and to lend out, as long as the reserve requirement puts a binding constraint on lending. When bank lending is constrained, monetary policy affects the tightness of this constraint and moves equilibrium lending rates. The lending rates subsequently transmit policy effects to the rest of the economy, including asset prices and optimal decisions. This mechanism applies to a broad context of regulations that limit bank lending for various reasons, such as reserve requirements, capital requirements, premiums on deposit insurance, Basel regulations, etc. The key takeaway here is that constrained bank lending is a simple and effective way for monetary policy to be transmitted to the rest of the economy. Finally, we further improve analytical tractability by assuming quasi-linear household preferences.

With all of the above factors, our model is tractable to the extent that solving the steady-state equilibrium boils down to one equation and one unknown. We solve the steady state in two steps: First, given the inflation rate, all of the over forty variables have closed-form solutions. Next, the steady-state inflation rate is solved from the labor market clearing condition. We provide a complete list of solutions in Appendix A.4. Since it is solved by the clearing of the labor market, the steady-state inflation rate of our model is influenced, but not directly controlled, by the monetary authority, unlike many other monetary models in the literature. Moreover, our solution algorithm, that is, the order in which we solve for the steady-state variables in Appendix A.4, illustrates that it is not a coincidence that inflation is ultimately pinned down by the labor market condition. The labor market clearing condition is the only equilibrium condition that involves (directly and indirectly) the optimal choices of all decision-makers. In particular, bank decisions determine interest rates and entrepreneurs' decisions affect capital prices, both of which matter for the labor demand by construction and production firms and the labor supply of household borrowers and savers. Since inflation permeates every aspect of the economy, it takes the labor market to bring all of its economic influences together to fully determine the inflation level in equilibrium. This result corroborates the fact that central banks closely monitor labor market conditions when making policy decisions.

In addition to theoretical analysis, we calibrate our framework to the U.S. economy and quantitatively evaluate long-run policy impacts and short-run dynamics in response to economic disturbances. Our main results are summarized as the following:

First, instead of imposing houses as collateral on household debt, we allow households to choose both capital and houses as collateral. We theoretically prove that household borrowers find it optimal to use only housing as collateral for their loans. That is, house-backed debt endogenously arises as the only form of household collateralized debt in a steady state with constrained bank lending. This result indicates a strong connection with the empirical fact that most household debt (about 73%) is in the form of mortgages.[2]

Second, all else equal, inflation tightens the reserve constraint and raises real lending rates, which renders a dual effect on the economy: on one hand, aggregate labor demand tends to decrease with inflation as higher interest rates suppress demand for capital and housing due to worsening financial conditions. On the other hand, aggregate labor supply rises with inflation as higher interest rates exacerbate the financial burden on household borrowers, prompting them to work more. A policy change that shifts labor demand up (or labor supply down) is inflationary, and *vice versa*.

Third, we show that various short-run disturbances to the economy can cause the crowding out of loan types and have implications for overall financial risk. As previously mentioned, the phenomenon of a crowding-out effect between mortgage and commercial lending has been widely documented. Our quantitative study illustrates that financial crowding out may lead to real crowding out between goods production through capital investment and housing construction through demand-driven price changes. For example, our model suggests that adverse TFP and labor supply shocks favor the housing market, increase the collateralized loan/ commercial loan (CA/C) ratio, and reduce financial risk, while adverse construction, housing demand, and LTV shocks favor the goods market, decrease the CA/C ratio, and increase the financial risk.

Fourth, we find that the expansion of one loan type is not always associated with the reduction of the other, which is consistent with the findings by Bezemer et al. (2020). In particular, our model suggests that long-run monetary and macroprudential (in the form of changing the LTV requirement) policies, as well as short-run monetary shocks, move both loan types in the same direction (*i.e.*, either both increase or decrease in volume). Nevertheless, in

[2] Data obtained from FRBNY Consumer Credit Panel.

such a case, crowding out still exists but is subtle and manifests itself only in relative terms. For example, a contractionary monetary policy, one that raises real interest rates, causes a reduction in both collateralized (CA) and commercial (C) loans but a rise in the ratio of CA relative to C loans. As a result, contractionary monetary policy mitigates the overall financial risk as banks pivot toward collateralized loans while reducing commercial loan default risk. These findings are in line with the empirical evidence put forward by Bidder et al. (2021) that banks exposed to negative shocks tighten credit for both business loans and mortgages while expanding credit to mortgages and thus rebalancing the portfolio to have less risk. In addition, we show that due to the crowding-out effect of loan redistribution, the production sector suffers more than the housing market upon a tighter monetary policy in the long run. Moreover, the difference in the changes in production and construction sectors declines as monetary policy becomes tighter and tighter. Finally, contractionary monetary policy mitigates financial risks and fractional reserve banking can be welfare-improving.

The rest of this paper is organized as follows. Section 2 sketches the related literature. Section 3 presents the model environment, defines the equilibrium, and characterizes the steady state. Section 4 conducts quantitative studies of long-run policy effects and short-run responses to various disturbances. Section 5 provides empirical support to our findings. Section 6 concludes the paper.

2 Related Literature

Our paper directly speaks to the empirical research on the crowding-out effects of bank lending on the economy. Chakraborty et al. (2018) document that active banks in robust housing markets prioritize mortgage lending over commercial lending, leading to reduced investment for borrowing firms. This finding suggests that housing price appreciation can have adverse effects on the real economy. Fieldhouse (2019) shows that U.S. housing credit policies subsidizing an expansion in residential mortgage lending unintentionally crowd out commercial lending and related real activity. Moreover, Chakraborty et al. (2020) show that the US Federal Reserve's mortgage backed security purchases as a quantitative easing effort boosted mortgage origination for beneficiary banks but reduced commercial lending and borrowing firms' investment. Using a new disaggregated bank credit data set, Bezemer et al. (2020) find a positive effect of mortgage credit expansion on business credit growth in advanced economies and a negative effect in emerging and developing economies. Suh and Yang (2020) find international firm-level evidence that large housing price booms are detrimental to investment, suggesting a possible reallocation of resources from the production sector to the housing sector during those phases. Bidder et al. (2021) study how banks cope with loss and find that banks that are exposed to shocks tighten credit for both business loans and mortgages while expanding credit to mortgages to be securitized, and thus rebalance the portfolio to have less risk. Dell'Ariccia et al. (2021) exploit heterogeneity in bank exposure to the compositional shift from tangible to intangible capital and show that exposed banks curtail commercial lending and reallocate lending to other assets, such as mortgages. Li et al. (2022) find Australian evidence that crowding out of business loans towards housing loans in response to increased opportunities in strong housing markets and curtailed business investment.

Our paper complements the above empirical literature by providing a theoretical structure that demonstrates how a loan reallocation by banks can generate crowding-out effects in the real sector and impact overall financial risk. Moreover, we identify the financial crowding-out effects as either absolute or relative in nature, depending on the source of changes/shocks to the economy. An *absolute crowding out* occurs when the volume of one type of loan is reduced while the other increases. An absolute crowding out is more obvious to identify empirically. Nevertheless, there can also be *relative crowding out*, which takes place when shocks or policy changes cause both types of loans to grow or shrink in volume, coupled with a change in loan distribution (*e.g.*, the ratio of collateralized loans relative to commercial loans). Relative crowding out is more subtle in nature and may not have drawn attention empirically. Our VAR framework in Section 5 provides support for the relative crowding out that is caused by the monetary policy. Our paper also suggests that the finding by Bezemer et al. (2020) of a positive effect of mortgage credit expansion on business credit growth in advanced economies need not be evidence against financial crowding out. Further empirical work in line with Section 5 searching for clues of relative crowding out could become fruitful endeavors.

Our paper clearly belongs to the vast literature on banking, and is closest in relation to the theoretical subdivision that studies the macroeconomic implications of bank lending decisions in a monetary context. To name a few, Berentsen et al. (2007) show that bank-like financial intermediaries can help improve the allocation and that when credit rationing occurs, increasing the rate of inflation can be welfare-improving. Sun (2007), Corbae and D'Erasmo (2021), Dong et al. (2021), Head et al. (2022), Wang et al. (2022), Chiu et al. (2023), Altermatt and Wang (2024) address the consequences of imperfect competition in the banking industry. Bech and Monnet (2016) and Williamson (2019) study central bank intervention in the context of interbank lending.

Our unique angle relative to the above papers is that we incorporate differential loan risks and endogenize bank decisions over collateralized and commercial loans.[3] In our model, banks' optimal loan distribution choice directly affects capital and housing investments, which then influence production, construction, household consumption, savings, and so on. In addition, the tractability of our theoretical structure allows for insights into how labor market interactions affect inflation. Moreover, tractability offers analytical and quantitative convenience that renders a rich set of results on both long-run policy effects and short-run dynamic responses that arise from banks' needs to adjust their loan portfolios.

There have been previous papers in the macro-finance literature that have modeled financial frictions with differential loan types. Lombardo and McAdam (2012) study the financial market frictions in a model of the euro area. They model the financial constraints faced by households through limited enforceability and collateralized debt (Iacoviello (2005) and those faced by firms through costly state verification and default risk (*e.g.*, Bernanke et al. (1999)). Clerc et al. (2015) analyze macroprudential policies in a dynamic general equilibrium model where household, firm, and bank debt are all subject to default risk. Rawat (2017) studies the

[3] See Dia and VanHoose (2017) for a review of efforts to apply developments in bank modeling to augment macroeconomic models.

interaction between firm and household credit constraints over the business cycle. The model combines household debt in the spirit of Kiyotaki and Moore (1997) and business debt as in Bernanke et al. (1999). Yoo (2017) evaluates the relative effectiveness of a policy to inject capital into banks versus a policy to relieve households of mortgage debt. The paper combines household debt *à la* Iacoviello (2005), business debt in the costly state verification (CSV) setup of Gale and Hellwig (1985), and bank leverage constraint following Gerali et al. (2010). Note that banks in Yoo (2017) do not make a portfolio choice but instead only choose the amounts of deposit and wholesale loans. In contrast to our model, all of these models are either in real terms or nominal with price rigidities. Moreover, none of these models allow for a portfolio choice made by banks.

Song (2021) examines how the credit supply mechanisms in the financial intermediation sector influence monetary policy. In the model, endogenous default of mortgage and business loans and prepayment of household mortgages influence the costs of supplying credits by the financial intermediaries. The intermediary optimizes loan portfolio composition given the cost variations with frictions. Our paper differs from Song (2021) in several aspects: Topic-wise, Song studies whether monetary policy's effectiveness is enhanced or reduced by the credit supply channel, while we focus on the crowding-out effects of bank loan portfolio choice and how monetary policy makes its impact on the economy through the bank choice. Approach wise, on the banking side, the bank portfolio choice in Song (2021) is driven by the differential costs arising from loan defaults and an adjustment cost to the portfolio. In contrast, we do not consider such costs but instead focus on the bank's portfolio choice when faced with a reserve requirement and various market interest rates on depositing and lending. On the monetary side, Song (2021) takes the New-Keynesian style with nominal rigidities, whereas in our model monetary policy is transmitted in a simple mechanism through the proportional reserve requirement.

Finally, our paper adds to a recent strand of the macro-finance literature that theoretically investigates how monetary policy affects financial stability through its impact on asset prices (*e.g.*, Caballero and Simsek (2019), Caballero and Simsek (2022), and Caballero and Simsek

(2024)). Although our model does not directly address financial stability, it provides insights into how risk is accumulated in the financial sector. In particular, we show that the amount of financial risk is determined not only through risks associated with each type of loan but also through banks' loan distribution choices. For example, a rise in the riskiness of one type of loan does not necessarily lead to worsening overall risk if there is a simultaneous increase in the relative amount of safer loans. Such an insight would not have been obtained in a model with a single loan type. Therefore, modeling loans of various risk types is critical for gauging the overall financial risk.

3 The Model

The economy is populated by patient and impatient households, entrepreneurs, banks, production firms, construction firms, and a central bank. Banks accept deposits from households to make *commercial loans* to finance entrepreneurial projects and *collateralized loans* to finance household investments. Entrepreneurial projects produce capital goods. Households and entrepreneurs own the total capital stock. Construction firms build and sell houses to households. Both production and construction firms hire labor from households and entrepreneurs. Production firms also rent capital. All banks, production, and construction firms are competitive.

The commercial loans are subject to a default risk from the bank's perspective. The bank verifies the project output by incurring a monitoring cost if the entrepreneur defaults on the repayment. Any hidden output will be forfeited. Capital and housing owned by households can be used to collateralize their loans.

Banks are required to hold at least a fraction $\bar{R}_t \in (0,1)$ of their deposits as reserves. The central bank pays interest on bank reserves at a gross nominal rate of R_t^b. In each t, the central bank issues a one-period nominal bond B_t that will mature in $t+1$. These bonds are used to finance reserve interest:

$$B_t = \left(R_{t-1}^b - 1\right) p_{t-1} S_{t-1}, \tag{1}$$

where S_t denotes the real aggregate bank reserves in period t. The central bank issues money to cover bond payments:

$$M_t - M_{t-1} = R^g_{t-1} B_{t-1}. \tag{2}$$

Equations (1) and (2) ensure the consolidated budget balances for the central bank. That is,

$$M_t - M_{t-1} + B_t = R^g_{t-1} B_{t-1} + \left(R^b_{t-1} - 1\right) p_{t-1} S_{t-1}. \tag{3}$$

Money is publicly recognized because the central bank accepts its currency for bank reserves and bond transactions.

Timing. The timing of events in period t is the following: 1) Aggregate shocks are realized; 2) Households and entrepreneurs supply labor to production and construction firms. Households and entrepreneurs rent capital to production firms; 3) Production and construction take place, after which capital and housing depreciate; 4) Wages and rents are paid to households and entrepreneurs. Previous collateralized loans are repaid, and banks pay interest on previous deposits. Households make new deposits in banks. Banks put up reserves in their central-bank accounts; 5) The central bank pays interest on previously-held government bonds and makes money injections. The government issues new bonds and pays interest on previously held bank reserves; 6) Banks lend to households and entrepreneurs. The former invests in housing, nominal bonds and capital, and the latter invests in projects to produce capital goods; 7) Project outcomes are realized, and commercial loans are repaid or defaulted on; 8) Households and entrepreneurs consume.

Households. The economy is populated by a measure $(1 - \varrho)$ of infinitely-lived households. A fraction of α among them is considered as *patient households* with a discount factor of $\beta^1 < 1$, and the rest is considered as *impatient households* with a discount factor of $\beta^2 < \beta^1$. Let $j = 1, 2$ denote the type of households. Each household has the periodic preference, $u\left(c_{j,t}, h_{j,t}, l_{j,t}\right)$, where $c_{j,t}$ is consumption, $h_{j,t}$ is housing services, and $l_{j,t}$ is hours worked.

Let R^g_t be the gross nominal interest rate on bonds, R^d_t be the gross nominal interest rate on bank deposits, and R_t^m be the gross nominal interest rate for the collateralized loan contract between the households and the bank between t and $t+1$. Moreover, q_t^k and q_t^h respectively are real capital and real housing prices, w_t is the real wage rate, and r_t^k is the rental rate of capital. δ^k and δ^h represent the capital and housing depreciation rates, respectively. $\Pi_{j,t}$ is a household's total dividend income, including dividends from production firms, banks, and construction firms. Let $\pi_t = \frac{p_t}{p_{t-1}}$ denote the gross inflation rate, where p_t is the nominal price.

Taking prices, wage rate, rental rate, interest rates, dividends, and policy ($q_t^k, q_t^h, w_t, r_t^k, R_t^g, R_t^d$, $R_t^m, \Pi_{j,t}, \xi_t$) as given, a representative type-j household chooses consumption of the final goods ($c_{j,t}$), capital ($k_{j,t}$) and housing ($h_{j,t}$) investments, hours worked ($l_{j,t}$), deposits ($d_{j,t}$), collateralized debt ($m_{j,t}$), and bond holdings ($b_{j,t}$) to solve the following maximization problem:

$$\max_{(c_{j,t},k_{j,t},h_{j,t},l_{j,t},d_{j,t},m_{j,t},b_{j,t})} E\sum_{t=0}^{\infty}\beta^j\left(\ln c_{j,t} + \varphi_t \ln h_{j,t} - \gamma_t l_{j,t}\right),$$

where φ_t represents the household's preferences for housing services and γ_t is a shock to labor supply, both of which follow the AR(1) processes below:

$$\ln \varphi_t = (1-\rho_\varphi)\ln\bar{\varphi} + \rho_\varphi \ln \varphi_{t-1} + \varepsilon_{\varphi,t} \tag{4}$$

$$\ln \gamma_t = \rho_\gamma \ln \gamma_{t-1} + \varepsilon_{\gamma,t}, \tag{5}$$

where $\rho_\varphi \in (-1,1)$ and $\rho_\gamma \in (-1,1)$ are the persistence parameters, and $\varepsilon_{\varphi,t}$ and $\varepsilon_{\gamma,t}$ are *i.i.d.* standard normal processes. The maximization problem is subject to:

(i) the budget constraint,

$$\begin{aligned} & c_{j,t} + q_t^k\left[k_{j,t} - \left(1-\delta^k\right)k_{j,t-1}\right] + q_t^h\left[h_{j,t} - \left(1-\delta^h\right)h_{j,t-1}\right] + d_{j,t} + b_{j,t} \\ & + \frac{R_{t-1}^m}{\pi_t}m_{j,t-1} = w_t l_{j,t} + r_t^k k_{j,t-1} + \frac{R_{t-1}^d}{\pi_t}d_{j,t-1} + \frac{R_{t-1}^g}{\pi_t}b_{j,t-1} + m_{j,t} + \Pi_{j,t}; \end{aligned} \tag{6}$$

(ii) the collateral constraint for household loans,

$$R_t^m m_{j,t} \leq \xi_t E_t\left\{\left[q_{t+1}^k k_{jt} + q_{t+1}^h h_{j,t}\right]\pi_{t+1}\right\}; \tag{7}$$

and (iii) the regularity conditions such as $c_{j,t} > 0$, $k_{jt}, h_{j,t}, l_{j,t} \geq 0$.

The budget constraint in (7) is rather standard. The left-hand side of this condition is the total household expenditure in a given period, which includes consumption, investments in capital, housing and bonds, bank deposits, and loan payments. The right-hand side is the household's total income from wages, rentals, deposits, bonds, new loans, and dividends. The collateral constraint in (7) stipulates that the amount of new debt must not exceed a proportion, ξ_t, of the expected value of all collaterals consisting of household capital and housing holdings. Therefore, the parameter ξ_t represents a stochastic *loan-to-value ratio* (LTV) to households, serving as a collateral constraint shock that follows the AR(1) process below:

$$\ln \xi_t = (1 - \rho_\xi) \ln \bar{\xi} + \rho_\xi \ln \xi_{t-1} + \varepsilon_{\xi,t}, \tag{8}$$

where $\rho_\xi \in (-1, 1)$ is the persistence parameter, and $\varepsilon_{\xi,t}$ is *i.i.d.* standard normal process. $\bar{\xi}$ is a macroprudential policy parameter controlled by the central bank.

Let $\lambda_{j,t}$ be the multiplier of the collateral constraint. For each type-j household, the optimality conditions are given by:

$$\begin{aligned}
\gamma_t c_{j,t} &= w_t && (9)\\
q_t^k &= \beta^j E_t \left[\frac{c_{j,t}}{c_{j,t+1}} \left(r_{t+1}^k + \left(1 - \delta^k\right) q_{t+1}^k \right) \right] + c_{j,t} \lambda_{j,t} \xi_t E_t (q_{t+1}^k \pi_{t+1}) && (10)\\
q_t^h &= E_t \left[\frac{\varphi_t c_{j,t}}{h_{j,t}} + \beta^j \frac{c_{j,t}}{c_{j,t+1}} q_{t+1}^h \left(1 - \delta^h\right) \right] + c_{j,t} \lambda_{j,t} \xi_t E_t (q_{t+1}^h \pi_{t+1}) && (11)\\
1 &\geq \beta^j R_t^d E_t \left[\frac{1}{\pi_{t+1}} \frac{c_{j,t}}{c_{j,t+1}} \right], \quad d_{j,t} \geq 0 && (12)\\
1 &\leq R_t^m E_t \left[\beta^j \frac{1}{\pi_{t+1}} \frac{c_{j,t}}{c_{j,t+1}} + \lambda_{jt} c_{j,t} \right], \quad m_{j,t} \geq 0 && (13)\\
1 &\geq \beta^j R_t^g E_t \left[\frac{1}{\pi_{t+1}} \frac{c_{j,t}}{c_{j,t+1}} \right], \quad b_{j,t} \geq 0 && (14)\\
0 &= \lambda_{jt} E_t \left[\xi_t \left(q_{t+1}^k k_{jt} + q_{t+1}^h h_{j,t} \right) \pi_{t+1} - R_t^m m_{j,t} \right], \quad \lambda_{jt} \geq 0. && (15)
\end{aligned}$$

To ensure banking is in equilibrium, it requires $R_t^d \geq 1$ for all t so that households are willing to deposit money in banks, *i.e.*, $d_{j,t} > 0$ for some $j = 1, 2$.

Entrepreneurs. There are infinitely-lived risk-neutral entrepreneurs of measure ϱ, each with preferences given by $E_0 \sum_{t=0}^{\infty} (\beta^e)^t c_t^e$, where c_t^e is the consumption of the entrepreneur and β^e is the discount factor such that $\beta^e < \beta_1$. Entrepreneurs supply labor inelastically to production and construction firms. Each entrepreneur is endowed with a project every period that utilizes consumption goods to produce capital goods in a random fashion. All projects have a duration of one period. The project opportunity vanishes by the end of a period, and the entrepreneur will be endowed with another opportunity in the next period. With an investment of i_t, the project produces $\omega_t i_t$ units of capital goods, where $\omega_t \sim \Phi(\cdot)$ is *i.i.d.* across entrepreneurs and over time with non-negative support, $E(\omega_t) = 1$ and density $\phi(\cdot)$. The realization of the project outcome ω_t is *private information* of the entrepreneur, and the bank must incur a monitoring cost to observe the true outcome.

Optimal contracting decision. While collateralized loans are intertemporal, commercial loans are intratemporal in nature.[4] Entrepreneurs use internal funds and funds borrowed from banks, both in terms of consumption goods, to produce capital goods. After the project outcome is realized, the entrepreneur repays the loan by the end of the period. The layout of the debt contract for the commercial loan is in the spirit of Carlstrom and Fuerst (1997).

Consider an entrepreneur with a net worth of n_t. For an investment i_t, the entrepreneur will need to borrow $\max[i_t - n_t, 0]$. By investing i_t units of consumption goods, the entrepreneur's project produces $\omega_t i_t$ units of capital goods. Let R_t be the real gross commercial loan rate. That is, the entrepreneur pays R_t units of capital goods for each unit of consumption goods borrowed. The entrepreneur has limited liability to the loan; after the project outcome is realized, the entrepreneur either makes the repayment according to R_t or defaults on the loan. Upon default, the bank will verify and forfeit all of the actual project output. The monitoring cost per project is equal to $\mu q_t^k i_t$ units of consumption goods, where $\mu \in (0, 1)$. The repayment

[4]This assumption is for analytical convenience and is not critical for obtaining results.

measured in units of consumption goods can be summarized as:

$$\begin{cases} R_t\left(i_t - n_t\right) & \text{if no default} \\ \omega_t i_t & \text{if default.} \end{cases}$$

Given R_t, there exists a critical value $\bar{\omega}_t$ such that

$$R_t\left(i_t - n_t\right) = \bar{\omega}_t i_t. \tag{16}$$

The entrepreneur will default if the realization of the project outcome is

$$\bar{\omega}_t < \bar{\omega}_t\left(R_t\right) \equiv \frac{1}{i_t} R_t\left(i_t - n_t\right). \tag{17}$$

Therefore, $\bar{\omega}_t$ is the *default threshold.* The lower this threshold, the less likely a commercial loan default.

Given the above contract of commercial loans, the expected income of an entrepreneur with a net worth n_t is given by:

$$\begin{aligned} & q_t^k \int_{\bar{\omega}_t}^{\infty} \left[\omega_t i_t - R_t\left(i_t - n_t\right)\right] d\Phi\left(\omega_t\right) \\ = \;& q_t^k i_t \left\{ \int_{\bar{\omega}_t}^{\infty} \omega_t d\Phi\left(\omega_t\right) - \bar{\omega}_t\left[1 - \Phi\left(\bar{\omega}_t\right)\right] \right\} \equiv q_t^k i_t f\left(\bar{\omega}_t\right). \end{aligned} \tag{18}$$

Moreover, the expected payoff of the bank for a loan with the borrower's net worth being n_t is given by:

$$\begin{aligned} & q_t^k \int_{\bar{\omega}_t}^{\infty} \left[R_t\left(i_t - n_t\right)\right] d\Phi\left(\omega_t\right) + q_t^k \int_0^{\bar{\omega}_t} \left(\omega_t i_t - \mu i_t\right) d\Phi\left(\omega_t\right) \\ = \;& q_t^k i_t \left\{ \bar{\omega}_t\left[1 - \Phi\left(\bar{\omega}_t\right)\right] + \int_0^{\bar{\omega}_t} \omega_t d\Phi\left(\omega_t\right) - \mu\Phi\left(\bar{\omega}_t\right) \right\} \equiv q_t^k i_t g\left(\bar{\omega}_t\right). \end{aligned} \tag{19}$$

Given $E(\omega_t) = 1$, it is important to note from the above two equations that

$$f(\bar{\omega}_t) + g(\bar{\omega}_t) = 1 - \mu\Phi(\bar{\omega}_t). \tag{20}$$

Thus, on average, $\mu\Phi(\bar{\omega}_t)$ of the produced capital is destroyed by monitoring and the rest is distributed between the entrepreneur $f(\bar{\omega}_t)$ and the bank $g(\bar{\omega}_t)$.

We assume that the entrepreneurs offer loan contracts to competitive banks. Given net worth n_t, an entrepreneur chooses the size of investment (i_t) and the interest rate R_t through choosing $(\bar{\omega}_t)$ according to (17), to solve the following contract design problem to maximize her expected payoff of borrowing:

$$\begin{aligned} & \max_{(i_t,\bar{\omega}_t)} \left\{ q_t^k i_t f(\bar{\omega}_t) \right\} \\ s.t. \qquad & q_t^k i_t g(\bar{\omega}_t) \geq R_t^c (i_t - n_t), \end{aligned} \tag{21}$$

where R_t^c is the expected gross intratemporal loan rate. Note that R_t^c differs from R_t in two aspects: First, R_t is the commercial loan rate specified in the contract. Yet the contract may be defaulted on and thus R_t represents a risky rate. In contrast, R_t^c is essentially a risk-free rate, which is the expected rate after taking into account for the potential default. Secondly, R_t is the rate that converts a loan of consumption goods into a payment of capital goods in return, whereas R_t^c is a rate in terms of consumption goods only. Let λ_t be the Lagrangian multiplier. The first-order conditions for the above contracting problem are given by:

$$\begin{aligned} q_t^k f(\bar{\omega}_t) + \lambda_t \left[q_t^k g(\bar{\omega}_t) - R_t^c \right] &= 0 \\ q_t^k i_t f'(\bar{\omega}_t) + \lambda_t q_t^k i_t g'(\bar{\omega}_t) &= 0 \\ q_t^k i_t g(\bar{\omega}_t) - R_t^c (i_t - n_t) &= 0, \end{aligned} \tag{22}$$

where

$$f'(\bar{\omega}_t) = -\left[1 - \Phi(\bar{\omega}_t)\right] < 0 \tag{23}$$

$$g'(\bar{\omega}_t) = 1 - \Phi(\bar{\omega}_t) - \mu\phi(\bar{\omega}_t) > 0. \tag{24}$$

Eliminating the multiplier yields

$$q_t^k f(\bar{\omega}_t) = \frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}\left[q_t^k g(\bar{\omega}_t) - R_t^c\right], \tag{25}$$

which is the condition that determines the choice of $\bar{\omega}_t$. It is obvious that the threshold $\bar{\omega}_t$ depends on the capital price, q_t^k, and the intra-temporal loan rate, R_t^c, but not the net worth n_t. This is a convenient result that makes aggregating more tractable. Accordingly, the optimal intratemporal lending rate does not depend on n_t, either, because (16) and (22) together imply

$$R_t = \frac{\bar{\omega}_t i_t}{i_t - n_t} = \frac{\bar{\omega}_t R_t^c}{q_t^k g(\bar{\omega}_t)}. \tag{26}$$

The optimal investment (size) is solved from (22) and is linear in n_t:

$$i_t\left(q_t^k, n_t, R_t^c\right) = \frac{n_t}{1 - \frac{q_t^k g\left(\bar{\omega}_t\left(q_t^k, R_t^c\right)\right)}{R_t^c}}. \tag{27}$$

The aggregate new investment, given net worth n_t, across all entrepreneurs is

$$\varrho i_t\left(q_t^k, n_t\right)\left[1 - \mu\Phi\left(\bar{\omega}_t\left(q_t^k, R_t^c\right)\right)\right] = \frac{1 - \mu\Phi\left(\bar{\omega}_t\left(q_t^k, R_t^c\right)\right)}{1 - \frac{q_t^k g\left(\bar{\omega}_t\left(q_t^k, R_t^c\right)\right)}{R_t^c}}\varrho n_t. \tag{28}$$

Other entrepreneurial decisions. At the end of each period, when the project outcome has been realized, the entrepreneur takes her available income at that point and decides on consumption and savings. That is, this decision comes after she has borrowed (if necessary) and invested in the project, and then finally repaid or defaulted on the loan, depending on the project outcome. The entrepreneur's internal funds n_t are her period-t wage and rental income, as given by

$$n_t = w_t + \left[r_t^k + q_t^k\left(1-\delta^k\right)\right] k_{t-1}^e. \tag{29}$$

Given n_t, the entrepreneur borrows $i_t - n_t$ if necessary and invests in her project. Depending on the realization of ω_t, the entrepreneur either makes the repayment or defaults, and thus, has all project output forfeited. In particular, the entrepreneur has the following end-of-period income depending on the realized value of ω_t:

$$\begin{cases} q_t^k\left[\omega_t i_t - R_t\left(i_t - n_t\right)\right], & \text{if } \omega_t \geq \bar{\omega}_t \\ 0, & \text{if } \omega_t < \bar{\omega}_t \end{cases} \tag{30}$$

If the income is zero, then trivially $c_t^e = k_t^e = 0$ for the current period. For a positive income, equation (16) implies that the entrepreneurial income reduces to $q_t^k\left(\omega_t - \bar{\omega}_t\right) i_t$. Then, the entrepreneur solves the following utility maximization problem in recursive form, taking prices and transfers $\left\{q_t^k, w_t, r_t^k\right\}$ as given:

$$\begin{aligned} V\left(k_{t-1}^e, \omega_t\right) &= \max_{\left(c_t^e, k_t^e\right)} \left\{c_t^e + \beta^e E_t V\left(k_t^e, \omega_{t+1}\right)\right\} \\ & \quad s.t. \;\; c_t^e + q_t^k k_t^e = q_t^k\left(\omega_t - \bar{\omega}_t\right) i_t, \end{aligned} \tag{31}$$

where i_t is given by (27) and n_t by (29). The expectations are taken over the random processes for the aggregate states $\left\{A_{t+1}, A_{t+1}^h\right\}$ for goods production and construction, respectively, and idiosyncratic state ω_{t+1}. It is straightforward to derive the following Euler equation for any solvent entrepreneur:

$$q_t^k = \beta^e E_t\left[\frac{q_{t+1}^k\left[r_{t+1}^k + q_{t+1}^k\left(1-\delta^k\right)\right] f\left(\bar{\omega}_{t+1}\right)}{1 - \frac{q_{t+1}^k g\left(\bar{\omega}_{t+1}\right)}{R_{t+1}^c}}\right], \tag{32}$$

where $\bar{\omega}_t$ solves the following:

$$q_t^k\left[1 - \mu\Phi\left(\bar{\omega}_t\right) - \frac{\mu\phi\left(\bar{\omega}_t\right) f\left(\bar{\omega}_t\right)}{1 - \Phi\left(\bar{\omega}_t\right)}\right] = R_t^c, \tag{33}$$

according to equations (16), (18), (20), (23), (24) and (25). Note that equation (32) is independent of n_t, and therefore, the equation holds for all solvent entrepreneurs.

Banking sector. The banking sector is competitive with measure one of the banks owned by patient households. They take deposits from households and make loans to households and entrepreneurs in the form of collateralized and commercial loans, respectively. For a commercial loan, the bank will verify (by incurring the monitoring cost) and forfeit any hidden output if the entrepreneur defaults on the repayment. The collateralized loan, however, is pledged by the amount of capital and housing owned by the borrowing household. Finally, it is important to recall that commercial loans are intratemporal, and collateralized loans are one-period loans.

Banks are required to hold a fraction $\bar{R}_t \in (0,1)$ of their deposits as reserves. The central bank pays interests on all reserves at a gross nominal rate of R_t^b. Given policy rates, bank decisions involve interest rates, $\left(R_t^d, R_t^m, R_t^c\right)$, which are nominal interest rates for deposits, collateralized loans, and commercial loans, respectively.[5] The profit maximization problem of a representative bank is given by:

$$\Pi_t^B = \max_{(C_t, CA_t, D_t, S_t)} E_t \left[R_t^c C_t + \beta^1 \frac{c_{1,t}}{c_{1,t+1}} \left(\frac{R_t^m CA_t}{\pi_{t+1}} + \frac{R_t^b S_t}{\pi_{t+1}} - \frac{R_t^d D_t}{\pi_{t+1}} \right) \right]$$

where C_t and CA_t are respectively the total amount of commercial and collateralized loans, S_t is the real reserves held by the bank, and D_t is the total amount of deposits accepted by this bank in real terms. The term $\beta^1 \frac{c_{1,t}}{c_{1,t+1}}$ represents the bank's discount factor, given the fact that the patient households are the bank owners. The above problem is subject to (i) the *balance sheet condition*, which ensures that the total amount of the deposit is sufficient to cover the total amount of loans made:

$$C_t + CA_t + S_t \leq D_t \tag{34}$$

and (ii) the *cash reserve requirement*, $S_t \geq \bar{R}_t D_t$. It is straightforward that (34) must hold with equality given any $R_t^d \geq 1$. Then, we set up the Lagrangian and use the binding (34) to

[5] Since the expected commercial loan rate, R_t^c, is an intra-temporal rate, the nominal and real levels of this rate are identical.

eliminate C_t in the objective function. Let λ_t^B be the multiplier associated with $S_t \geq \bar{R}_t D_t$. The first-order conditions for interior choices are:

$$
\begin{aligned}
R_t^c - \beta^1 \frac{c_{1,t}}{c_{1,t+1}} \frac{R_t^d}{\pi_{t+1}} - \lambda_t^B \bar{R}_t &= 0 && \text{(35a)} \\
\beta^1 \frac{c_{1,t}}{c_{1,t+1}} \frac{R_t^m}{\pi_{t+1}} - R_t^c &= 0 && \text{(35b)} \\
\beta^1 \frac{c_{1,t}}{c_{1,t+1}} \frac{R_t^b}{\pi_{t+1}} + \lambda_t^B - R_t^c &= 0. && \text{(35c)}
\end{aligned}
$$

Given conditions (35a) - (35c), the bank profit is zero, i.e., $\Pi_t^B = 0$ for all t.

Production sector. There is a perfectly competitive production sector with the following technology: $Y_t = A_t \left(L_t^y\right)^\nu \left(K_t^y\right)^{1-\nu}$, where A_t is the stochastic productivity, which is realized at the beginning of t and follows a AR(1) process,

$$
\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{A,t}, \tag{36}
$$

where $\rho_A \in (-1, 1)$ is the persistence parameter, and $\varepsilon_{A,t}$ is *i.i.d.* standard normal process. K_t and L_t are respectively capital and labor inputs. Capital depreciates at the rate of δ^k, immediately after the production of consumption goods. Labor supplied by entrepreneurs and households are perfectly substitutable for production. As is standard, firm optimal decisions are such that

$$
\begin{aligned}
w_t &= \nu A_t \left(L_t^y\right)^{\nu-1} \left(K_t^y\right)^{1-\nu} && \text{(37)} \\
r_t^k &= (1-\nu) A_t \left(L_t^y\right)^\nu \left(K_t^y\right)^{-\nu}. && \text{(38)}
\end{aligned}
$$

Construction sector. The housing sector is also competitive. A measure one of construction companies produce housing according to the following technology:

$$
Y_t^h = A_t^h L_t^h, \tag{39}
$$

where A_t^h is the stochastic construction productivity and follows an AR(1) process,

$$\ln A_t^h = \rho_{A^h} \ln A_{t-1}^h + \varepsilon_{A^h,t}, \tag{40}$$

while $\rho_{A^h} \in (-1, 1)$ is the persistence parameter, and $\varepsilon_{A^h,t}$ is *i.i.d.* standard normal process. The optimal construction decision is such that $w_t = q_t^h A_t^h$.

3.1 Equilibrium

Definition 1 *A competitive equilibrium with money and banking consists of*

$$\{c_{j,t}, k_{j,t}, h_{j,t}, l_{j,t}, d_{j,t}, m_{j,t}, b_{j,t}, c_t^e, k_t^e, Z_t, i_t, n_t, C_t, CA_t, D_t, S_t, Y_t, K_t^y, L_t^y,$$
$$H_t, Y_t^h, L_t^h, \lambda_{j,t}, \lambda_t^B, \bar{\omega}_t, R_t^c, R_t, R_t^d, R_t^g, R_t^m, q_t^k, q_t^h, w_t, r_t^k, \pi_t, M_t, B_t, \Pi_t\}_{j=1,2}$$

for all t such that given policy $\left(\bar{R}_t, R_t^b, \bar{\xi}_t\right)$, (i) All decisions are optimal; (ii) All markets clear; (iii) Zero profit of all competitive firms and banks; (iv) Consistency: The laws of motion for capital and housing stocks follow

$$K_{t+1}^y = \left(1 - \delta^k\right) K_t^y + \varrho i_t \left[1 - \mu \Phi\left(\bar{\omega}_t\right)\right] \tag{41}$$

$$H_t = \left(1 - \delta^h\right) H_{t-1} + A_t^h L_t^h; \tag{42}$$

(v) Central bank operations are such that (1) and (2) are satisfied.

Given the above definition of equilibrium, we now list the market-clearing conditions. First, note that given the linear investment and monitoring technologies, only the first moment of the wealth distribution across entrepreneurs affects aggregate outcomes. Denote Z_t as the aggregate entrepreneurial capital stock. Next, define C_t^e as the average entrepreneurial consumption, N_t as the average entrepreneurial net worth, and I_t as the average entrepreneurial investment.

Then aggregating the budget constraints across all entrepreneurs solves for

$$Z_t = \varrho \left[f\left(\bar{\omega}_t\right) I_t - \frac{C_t^e}{q_t^k} \right], \tag{43}$$

where

$$\begin{aligned} I_t &= \frac{N_t}{1 - \frac{q_t^k g(\bar{\omega}_t)}{R_t^c}} && (44) \\ N_t &= w_t + \left[r_t^k + q_t^k \left(1 - \delta^k\right)\right] \frac{Z_{t-1}}{\varrho} && (45) \end{aligned}$$

given (27) and (29). Given (43), the market-clearing conditions of labor, capital, housing, goods, deposits, bonds, collateralized loans, and commercial loans are

$$\begin{aligned} L_t^y + L_t^h &= (1-\varrho)\left[\alpha l_{1,t} + (1-\alpha) l_{2,t}\right] + \varrho && (46) \\ K_t^y &= (1-\varrho)\left\{\left[\alpha k_{1,t-1} + (1-\alpha) k_{2,t-1}\right] + Z_{t-1}\right. && (47) \\ H_t &= (1-\varrho)\left\{\left[\alpha h_{1,t} + (1-\alpha) h_{2,t}\right]\right. && (48) \\ Y_t &= (1-\varrho)\left[\alpha c_{1t} + (1-\alpha) c_{2t}\right] + \varrho C_t^e + \varrho I_t && (49) \\ D_t &= (1-\varrho)\left[\alpha d_{1t} + (1-\alpha) d_{2t}\right] && (50) \\ \frac{B_t}{p_t} &= (1-\varrho)\left[\alpha b_{1t} + (1-\alpha) b_{2t}\right] && (51) \\ CA_t &= (1-\varrho)\left[\alpha m_{1t} + (1-\alpha) m_{2t}\right] && (52) \\ C_t &= \varrho\left(I_t - N_t\right) && (53) \end{aligned}$$

Condition (47) is for clearing the capital market. The LHS is the capital demand from productive firms at the beginning of period t. The RHS is the capital supply from households and entrepreneurs also at the onset of t. Note that the notation is such that $k_{j,t-1}$ and Z_{t-1} denote the amount of capital holdings at the end of $t-1$ and thus at the beginning of t.

3.2 Steady State

A *steady state* is an equilibrium in which the real terms remain constant over time. Therefore, the steady state inflation rate is such that:

$$\pi = \frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{M_t}. \tag{54}$$

There can be two types of steady states, respectively with $\lambda^B > 0$ and $\lambda^B = 0$. We refer to the former as a steady state with *constrained lending* because the representative bank is constrained in the amount to lend to the private sector as the reserve constraint binds. In contrast, the one with $\lambda^B = 0$ is a steady state with *unconstrained lending*.

The steady-state version of conditions (35a) - (35c) yield

$$R^c = \frac{\beta^1}{\pi} R^d + \lambda^B \bar{R} = \frac{\beta^1}{\pi} R^m = \frac{\beta^1}{\pi} R^b + \lambda^B. \tag{55}$$

If $\lambda^B = 0$, then $\frac{\pi}{\beta^1} R^c = R^m = R^d = R^b$. That is, there are no spreads between the nominal lending rate, the nominal deposit rate, and the reserve rate. If $\lambda^B > 0$, banks only maintain the required amount of reserves, *i.e.*, $S = \bar{R}D$. Moreover, we have $\frac{\pi}{\beta^1} R^c = R^m > R^d > R^b$. In this case, the reserve rate R^b paid by the central bank is less than the deposit rate R^d, and therefore, banks incur a strict loss by keeping reserves as required. A direct consequence is that the lending rate R^m must be above the deposit rate so that banks earn a strictly positive profit from lending to balance off the loss from the required reserves.

Finally, the steady-state real bond balance being non-negative requires that the policy rate satisfy $R^b \geq 1$, as is shown in (A.72) of Appendix A. We will maintain this assumption for the rest of the paper. We have the following theorem on the properties of the steady state, and a detailed proof is available in Appendix A:

Theorem 1 *Provided that $R^b \geq 1$, a steady state with money and banking has the following properties: (I) Impatient households do not hold bank deposits, government bonds or capital,* *i.e.,* $d_2 = b_2 = k_2 = 0$*. The nominal saving rates satisfy* $R^d = R^g = \pi/\beta^1$ *for any given* π*. The nominal and real loan rates are such that* $R^m = \pi R^c/\beta^1$*; (II) There exists a unique steady state with a binding bank reserve constraint, i.e.,* $\lambda^B > 0$*, iff there exists a unique solution,* π_{ss}*, to*

$$L_{ss}^y(\pi) + L_{ss}^h(\pi) = (1-\varrho)\left[\alpha l_1^{ss}(\pi) + (1-\alpha)\, l_2^{ss}(\pi)\right] + \varrho, \tag{56}$$

and that conditions (A.79) to (A.83) and (A.85) are satisfied. Functions $L_{ss}^h(\pi)$*,* $L_{ss}^y(\pi)$*,* $l_1^{ss}(\pi)$*, and* $l_2^{ss}(\pi)$ *are specified respectively by (A.57), (A.63), (A.76), and (A.77) in Appendix A. Provided that* $\lambda^B > 0$*, all real variables depend on monetary policy* $(\bar{R}, R^b)$*, except for* $m_1 = \lambda_1 = d_2 = b_2 = k_2 = 0$*. Nominal interest rates are such that* $R^m > R^d > R^b$ *and the real loan rate is such that* $R^c > 1$*; (III) If exists, a steady state with a non-binding bank reserve constraint, i.e.,* $\lambda^B = 0$*, is unique. Provided that* $\lambda^B = 0$*, we have* $R^d = R^g = R^m = R^b$*,* $R^c = 1$*, and* $\pi = \beta^1 R^b$*. Moreover, monetary policy* $\bar{R}$ *has no effect, while* R^b *has no real effects on the economy.*

3.2.1 Property I - General Steady-State Features

Theorem 1 is crucial for understanding the steady state of money and banking. The first property provides some general features that apply to both types of steady states. That is, Property I holds whether the reserve constraint binds or not.

Borrowers and savers. First, from equation (A.4) in Appendix A, Property I shows that impatient households find it optimal not to make bank deposits, hold bonds, or invest in capital. Furthermore, later, Property II establishes that in a steady state with constrained lending, patient households do not borrow from the bank ($m_1 = 0$). As a standard result in models with patient and impatient households, patient households are savers, and impatient households are borrowers.

Nominal and real interest rates. Property I also shows the relation between lending rates R^m and R^c by equations (A.7) and (A.8). R^m is the nominal rate for a one-period loan, while R^c is an intra-period, real loan rate. Thus, to convert R^c to R^m, one must apply the inflation rate and the discount factor of the patient households, as they own banks. Moreover, all else equal both the reserve constraint multiplier λ^B and the real lending rate R^c strictly increase with the inflation rate π. As is explained in the text following equation (A.45), the higher the inflation rate, the tighter the binding reserve constraint, and thus the greater the real lending rate. Finally, the nominal deposit rate is the one-period saving rate, taking into account inflation and discounting by patient households, which are the savers in this economy.

3.2.2 Property II - Steady State with Constrained Lending

This property in the theorem characterizes the steady state in which banks are constrained in lending due to the reserve requirement.

Inflation and labor market conditions. An important condition for the existence and uniqueness of constrained lending steady state is that the inflation rate can be uniquely solved from the labor-market-clearing condition. This result provides a lens through which one can better understand the connection between inflation and labor market conditions. The left-hand side (LHS) of (56) is the aggregate labor demand, and the right-hand side (RHS) is the aggregate labor supply. Figure 8 illustrates how the steady-state inflation is determined. A blue line is $LHS\left(\pi\right)$ and a red line is $RHS\left(\pi\right)$ for a given policy level. Solid and dotted lines represent the impact of different policy levels, which will be discussed in the next section.

Labor demand $LHS\left(\pi\right)$ is decreasing (except for very high inflation rates), and labor supply $RHS\left(\pi\right)$ is an increasing function. Intuitively, higher inflation leads to higher loan rates, which tends to harm investment and housing demand and thus suppress labor demand by the production and housing sectors. Conversely, higher loan rates raise the interest burdens of borrowers and thus drive them to work more and, hence, create a greater labor supply. The crossing of the two curves pins down π_{ss}. All else equal, a policy change that increases (decreases) labor demand, that is, shifting the LHS curve upward (downward), is inflationary (deflationary). Conversely, a change in policy that increases (decreases) labor supply, that is, shifting the RHS curve upward (downward), is deflationary (inflationary).

Monetary policy transmission. Property II shows that monetary policy has a wide impact on all aspects of the steady state with constrained lending. Any change in the reserve ratio or the reserve rate affects bank decisions and, thus, the tightness of the binding reserve constraint. The latter directly affects all interest rates, which subsequently influence asset prices and individual decisions of households, entrepreneurs, and construction and production firms. Such transmission mechanism is demonstrated by the analytical algorithm in Appendix A.4 for solving the steady state with $\lambda^B > 0$. The key to the transmission mechanism is reflected by equations (A.44) and (A.45). The real loan rate R^c is a function of the constraint multiplier λ^B that is directly affected by policy instruments $\bar{R}$ and R^b. Through R^c, any impact to $\bar{R}$ and R^b is then transmitted to the asset prices and optimal decisions.

Housing collateral only. As shown by equation (A.18), the finding of $k_2 = 0$ is interesting because it shows borrowers' bank debt is collateralized by housing only ($h_2 > 0$), which is a result unique to our framework. In models with patient and impatient households where only capital can be used as collateral, such as Cordoba and Ripoll (2004), impatient households collateralize their debt by capital holdings. In contrast, our model allows households to use both capital and housing as collateral, yet borrowers optimally choose to use housing solely. Unlike capital, housing provides a direct utility benefit in addition to its value as collateral. Accordingly, impatient households are strictly better off investing in only housing to take advantage of the direct utility benefit, the asset value, and the collateral value of housing.

3.2.3 Property III - Steady State with Unconstrained Lending

This property is about the steady state in which the reserve requirement does not bind for banks. If exists, monetary policy has no real effect in this type of steady state. As explained previously, there is no spread between the nominal interest rates when the bank reserve constraint does not bind. Moreover, the net real loan rate is zero, as indicated by $R^c = 1$. Appendix A.6 provides an analytical algorithm for solving the steady state with $\lambda^B = 0$. This knife-edge case of a steady state is not the focus of our study. Therefore, our quantitative studies in the next section only consider levels of policy rates such that $\lambda^B > 0$.

4 Quantitative Studies

4.1 Parameterization

In our parameterization, one period is equivalent to a quarter. The discount factors for patient and impatient households adhere to the approach outlined in Iacoviello (2005). In particular, we use 0.99 for patient households and 0.95 for impatient households. The monitoring cost is set at 0.25, which is consistent with Carlstrom and Fuerst (1997). We set the value of the measure of entrepreneurs to 0.1 following Carlstrom and Fuerst (1997), and the measure of patient households to 0.64 following Iacoviello (2005). The depreciation rate of capital is 0.025, and the share of labor in production is 0.69, as is commonly used in the literature. In accordance with Greenwood et al. (1997), the housing depreciation rate is 0.0125. For the realization of the project outcome, ω_t, we assume a mean of one, with a normal distribution and a variance of σ. Similar to Carlstrom and Fuerst (1997), we use the entrepreneurs' discount factor β^e and σ to match (i) a quarterly bankruptcy rate of 0.974 percent and (ii) an average spread between the prime rate and the three-month commercial paper rate of 187 basis points per annum.

The steady-state value of the loan-to-value ratio is set to 0.765 using data from the Federal Housing Agency. For the steady-state value of housing demand, we follow Liu et al. (2013). We set the reserve requirement to 10% as it was the last reserve rate implemented. However, data obtained from the Federal Reserve Board and FDIC show that total reserves over deposits in the last 20 years have also been 10%. This observation suggests that banks do not lend all of their deposits, instead they prefer to keep them as reserves even in the absence of a

Table 1: Calibration

(Model Period: Quarter)	Parameters	Value	Source
Patient HH discount factor	β^1	0.99	Iacoviello (2005)
Impatient HH discount factor	β^2	0.95	Iacoviello (2005)
Entrepreneur's discount factor	β^e	0.97	SS target
Monitoring cost	μ	0.25	Carlstrom and Fuerst (1997)
Measure of entrepreneurs	ϱ	0.1	Carlstrom and Fuerst (1997)
Measure of patient HH	α	0.64	Iacoviello (2005)
Depreciation rate of capital	δ^k	0.025	literature
Depreciation rate of housing	δ^h	0.0125	Greenwood et al. (1997)
Share of labor in production	ν	0.69	literature
Standard deviation of $\Phi(\omega)$	σ	0.365	SS target
Steady State Values			
LTV	$\bar{\xi}$	0.765	Federal Housing Agency
Housing demand	$\bar{\varphi}$	0.0457	Liu et al. (2013)
Reserve requirement	$\bar{R}$	0.1	FED & FDIC
Reserve rate	R^b	1.005	Federal Reserve Board
Stochastic Processes			
Persistence of housing demand shock	ρ_φ	0.99	Liu et al. (2013)
Persistence of LTV shock	ρ_ξ	0.98	Liu et al. (2013)
Persistence of TFP shock	ρ_A	0.95	Iacoviello and Neri (2010)
Persistence of construction shock	ρ_{A^h}	0.997	Iacoviello and Neri (2010)
Persistence of labor supply shock	ρ_γ	0.92	Higgins and Sapci (2022)
Persistence of reserve requirement shock	$\rho_{\bar{R}}$	0.99	Carrera et al. (2012)
Persistence of reserve rate shock	ρ_{R^b}	0.92	Carrera et al. (2012)

reserve requirement. Using the data obtained from the Federal Reserve Board on interest rates on reserves, we set R^b to 2% annually, which is close to the long-run average. For stochastic processes, we follow the literature where similar shocks have been used as outlined in Table 1. Our next step is to perform quantitative exercises to analyze the effects of policies in both the long run and short run using this parameterization of the model.

4.2 Long-Run Policy Effects

For the long-run policy effects, we conduct comparative statics on the benchmark model with two sets of policies: the monetary policy of managing bank reserves $\left(\bar{R}, R^b\right)$ and the macroprudential policy of controlling the loan-to-value ratio $\left(\bar{\xi}\right)$. These analyses aim to understand the effects of a change in the policy on bank activities, real activities, and welfare defined as[6]

$$\mathcal{W} = (1-\varrho)\left[\alpha\frac{\ln(c_1)+\varphi\ln(h_1)-\gamma l_1}{1-\beta^1}+(1-\alpha)\frac{\ln(c_2)+\varphi\ln(h_2)-\gamma l_2}{1-\beta^2}\right]+\varrho\frac{C^e}{1-\beta^e}.$$

[6]Unless otherwise stated, all variables mentioned in this section refer to the steady-state levels.

Moreover, we define *commercial loan leverage* as the average entrepreneurial debt relative to the average entrepreneurial equity in the project, *i.e.*,

$$\frac{I-N}{N}=\frac{1}{\frac{R^c}{q^k g(\bar{\omega})}-1}.$$

A fall in the commercial loan leverage means de-leveraging of such loans. Finally, we provide a measure of the aggregate risk in the financial sector, the *Financial Risk Index* (FRI) defined as

$$FRI_t=\frac{\Phi\left(\bar{\omega}_t\right)C_t}{CA_t+C_t}=\frac{\Phi\left(\bar{\omega}_t\right)}{1+\frac{CA_t}{C_t}}. \tag{57}$$

FRI measures the total amount of defaulted commercial debt as a proportion of total debt (collateralized and commercial combined). As will be seen in what follows, macroprudential and monetary policy alters the risk structure of the financial sector through affecting (i) the riskiness of commercial debt contract as indicated by $\bar{\omega}_t$; and (ii) bank choices over the amount of collateralized (CA) vs. commercial debt (C), and thus the resulting CA/C ratio.

4.2.1 Monetary Policy

In this analysis, we examine how monetary policy affects the macroeconomy over the long run. Specifically, we focus on the impact of tightening monetary policy, which can be achieved through an increase in either the required reserve ratio ($\bar{R}$) or the interest rate paid on bank reserves (R^b). A higher required reserve ratio directly limits bank lending, resulting in a more restrictive monetary policy. The effect of a higher reserve rate is similar but through a different channel: An increase in R^b widens the gap between the deposit rate and the reserve rate and makes banks incur a strict loss on reserves kept at the central bank. To see this, for each unit of reserves, a bank earns a profit of $R^b-R^d=-\frac{\pi}{\beta^1}\lambda^B(1-\bar{R})<0$ by (55). Moreover, both π and λ^B rise with R^b (Figure 1), which means the marginal loss on reserves also rises with R^b. Therefore, a greater reserve rate essentially increases the lending cost and ends up discouraging bank lending. Figures 1-4 illustrate the long-term effects of a tighter monetary policy on the economy, with separate analyses for changes in $\bar{R}$ and R^b, respectively.

Financial risks. As can be seen in Figures 1 and 3, when monetary policy tightens in the long run, bank lending is further constrained. Therefore, both collateralized household loans and commercial loans decrease, and the real lending rate (R^c) increases. Additionally, bank lending displays a *flight to safety* as reflected by the rise in the ratio of collateralized loans relative to commercial loans. Reduced commercial lending also leads to deleveraging (*i.e.*, lower commercial leverage) and less default (*i.e.*, lower $\bar{\omega}$). As shown by (57), a rise in CA/C and a fall in $\bar{\omega}$ indicates that tighter monetary policy can help mitigate overall financial risk, which can be seen in the decrease of FRI.

Crowding-out. A tighter monetary policy leads to relative crowding out of commercial loans by collateralized loans in the sense that there is a rise in CA/C while the volumes of both types of loans are reduced. Capital investment falls due to not only high interest rates but also the reduction in C and the rise in CA/C (Figures 2 and 4). Overall, the crowding-out effect on the financial side makes the goods production worse off than it would have been without this spillover.

Figures 7a and 7b show the *output/housing elasticity*, which is defined as the percentage change in output relative to that in housing construction given a marginal change, respectively, in $\bar{R}$ and R^b. The figures demonstrate that the elasticity is always above one for the values of $\bar{R}$ and R^b under investigation. This means that due to the crowding-out effect of the loan distribution, the goods production sector suffers more compared to the housing market from the tightening of monetary policy in the long run. Nevertheless, the elasticity decreases with a rise in the policy parameters, which indicates that contractionary monetary policy decreases the gap in the impacts on the two sectors, as it has a relatively greater negative impact on the housing sector compared to the goods production sector, even though the latter still experiences larger declines than the former.

Inflation. Figures 1 and 3 illustrate the difference in the effect of the required reserve ratio and the reserve rate on long-term inflation. In particular, inflation falls in $\bar{R}$ but rises in R^b. Figure 8a displays the determination of the steady-state inflation using the labor-market-clearing condition (56). The blue lines in the figure represent the aggregate labor demand, which is the left-hand side (LHS) of the condition, and the red lines represent the aggregate labor supply, which is the right-hand side (RHS) of the condition. The solid curves show the condition when $\bar{R} = 0.1$, and the dotted curves represent the case when $\bar{R} = 0.01$. The intersection of each pair of curves determines the corresponding steady-state gross inflation rate, π_{ss}. The figure shows that decreasing the reserve requirement from 10 percent to 1 percent flattens both demand and supply curves to the effect that labor demand is decreased while labor supply is increased for the most part of the respective curves. This is because the reduction in $\bar{R}$ eases the reserve requirement and therefore ends up decreasing the real lending rate R^c. With less costly loans, both production and construction are boosted, raising labor demand from both sectors. In the meanwhile, household borrowers benefit from the lower loan rate and thus choose to work less, driving the overall labor supply down. The impact of $\bar{R}$ on the two curves makes them cross at a higher level of inflation. Similarly, Figure 8b illustrates the effects of a change in R^b on π_{ss}. A rise in the reserve rate relieves banks' lending constraint and thus stimulates labor demand (for lower inflation rates) while lowering labor supply. Thus, the long-run inflation rises.

Welfare. Changes in $\bar{R}$ and R^b also have qualitatively similar effects on the steady-state welfare. Figures 9 and 10 respectively report the welfare effects to changes in $\bar{R}$ and R^b. A tighter monetary policy improves overall welfare for the range of policy measures considered. However, the policy's impact varies across different types of agents. Both entrepreneurs and household savers benefit from tighter monetary policy. In particular, entrepreneurs are better off with tighter monetary policy because reduced capital investment leads to higher capital prices and, thus, more entrepreneurial income. Savers are better off because of higher earnings from asset holdings (capital, deposits, and bonds), enabling them to enjoy more leisure. In contrast, household borrowers are adversely affected as the policy tightens loan supply, making mortgages more costly.

Figure 12 provides a detailed representation of how variations in $\bar{R}$ and R^b respectively affect inflation and welfare. In all three panels, $\bar{R}$ takes values between 1 and 25 percent, which is the maximum value the steady state equilibrium supports and also the maximum value we observe in the last two decades.[7] R^b takes values between 1.001 and 1.1 (providing an annual rate of 0.4 percent to 40 percent). Figure 12a shows that inflation increases with R^b but decreases with $\bar{R}$. Figure 12b shows that welfare increases with both $\bar{R}$ and R^b as they represent policy tightening.

In Figure 12c, we plot the corresponding pairs of inflation and welfare for all values of $\left(\bar{R}, R^b\right)$ depicted in Figures 12a and 12b. Figure 12c shows a cascade of lines, each containing the same number of dots. Each dot represents a pair of inflation and welfare levels for a particular point on the grid of $\left(\bar{R}, R^b\right)$ depicted in the first two panels. The position of the dots changes monotonically with respective changes in $\bar{R}$ and R^b. According to the legend in Figure 12c, an increase in $\bar{R}$ alone moves a dot to a line above but does not change the position of the dot in the line. In contrast, an increase in R^b moves the dot along the same line to a position on the right. The main takeaway from Figure 12c is that higher long-run inflation does not necessarily mean lower welfare.[8]

4.2.2 Macroprudential Policy

We also consider the long-run effects of a macroprudential policy that involves adjusting the loan-to-value (LTV) requirement, $\bar{\xi}$. If $\bar{\xi}$ increases, it indicates a loosening of the macroprudential policy since it allows households to borrow a larger portion of their collateral value.

Financial risks. Figure 5 demonstrates that when the macroprudential policy is relaxed, bank lending increases and raises both types of loans in the long run. CA loans react more

[7] Data obtained from the Federal Reserve Board.

[8] Some search-theoretic monetary models, *e.g.*, Sun and Zhou (2018), *etc.*, can also generate the result that maximized welfare is achieved at positive mild inflation. Such a result requires an endogenous extensive margin for goods transactions in the search environment.

to macroprudential policy than C loans. In particular, when $\overline{\xi}$ increases, CA increases at an increasing rate, whereas C increases at a decreasing rate. In the end, CA/C takes the form of CA loans as the increase in collateral loans is larger than that of commercial loans. Moreover, more relaxed credit conditions also increase the default and commercial leverage, making commercial loans riskier. Despite the increase in $\bar{\omega}$ and commercial leverage, the increase in CA/C overcomes and decreases FRI. If we were to exclusively analyze a model involving a single loan type within the financial accelerator framework, our conclusion would inaccurately suggest an increase in FRI. These results highlight that considering the distributional effects of bank loans is critical in measuring the true financial risk.

Crowding-out. Relaxing macroprudential policy leads to a relative crowding out of commercial loans by collateral loans as the CA/C ratio increases. Strengthened commercial lending raises capital investment and goods production, as can be seen in Figure 6. However, this increase is not as high as it would have been if commercial loans were the only type of loans. Therefore, the choice of bank loan distribution affects the real economy through this relative crowding out as well. As Figure 7c shows, the relaxed credit conditions for collateralized borrowing favor the housing market over the goods market, as indicated by the value of the elasticity being below one. That is, the percentage expansion of construction is greater than that of production. Moreover, the gap between changes in the two sectors widens as relaxed credit conditions stimulate collateral lending more than commercial lending.

Inflation. Figure 8c illustrates that an increase in $\overline{\xi}$ causes both labor demand and supply curves to shift upward. Intuitively, a higher $\overline{\xi}$ relaxes the collateral constraint and thus raises the collateral value of housing. Labor demand increases due to strengthened demand for housing, resulting in a positive spillover to the goods sector. Additionally, borrowers tend to work more to afford more housing, stimulating the labor supply. The change in labor supply compensates for the change in labor demand, which creates little to no impact on inflation.

Welfare. Figure 11 shows that overall welfare is improved when the macroprudential policy is relaxed. Higher LTVs benefit both savers and borrowers, while entrepreneurial welfare decreases. Borrower welfare is hump-shaped in $\bar{\xi}$ changes, peaking at $\bar{\xi}$ close to 85%. Savers enjoy higher consumption and lower labor supply, while entrepreneurs suffer from lower capital prices, which reduces their consumption. Household borrowers also benefit from increased consumption and housing, but as $\bar{\xi}$ increases, they supply more labor, negatively impacting their welfare. Eventually, the negative impact on welfare from increased labor supply becomes dominant and causes the observed hump shape.

4.3 Short-Run Dynamics

Figures 13 and 14 display the responses of macroeconomic variables to an unexpected 1% change in the innovations of TFP (A_t), housing supply (A_t^h), housing demand (φ_t), LTV (ξ_t), and labor supply (γ_t) shocks in the benchmark model, as well as newly created reserve rate (R_t^b) and reserve requirement ($\bar{R}_t$) shocks, all of which create *adverse* shocks in the economy. In particular, the monetary policy tools ($\bar{R}_t$ and R_t^b) are modified as AR(1) processes:

$$\ln \bar{R}_t = (1 - \rho_{\bar{R}}) \ln \bar{R} + \rho_{\bar{R}} \ln \bar{R}_{t-1} + \varepsilon_{\bar{R},t} \tag{58}$$

$$\ln R_t^b = (1 - \rho_{R^b}) \ln R^b + \rho_{R^b} \ln R_{t-1}^b + \varepsilon_{R^b,t}, \tag{59}$$

where $\rho_{\bar{R}}$ and $\rho_{R^b} \in (-1, 1)$ are the persistence parameters, and $\varepsilon_{\bar{R},t}$ and $\varepsilon_{R^b,t}$ are *i.i.d.* standard normal processes. $\bar{R}$ and R^b can be thought of as the long-run monetary policy rates that the central bank controls.

Similar to the long-run effects, Figure 13 shows that a tighter monetary policy mitigates financial risks (Financial Risk Index, FRI) by lowering commercial loan defaults ($\bar{\omega}$) and by increasing collateral loans to commercial loans ratio (CA/C). Following a reserve rate shock (R^b), FRI initially increases as commercial loans are affected more than collateral loans due to an initial decrease in CA/C. When the interest rate increases, savers are encouraged to deposit more, which in turn improves the availability of loans. However, as the interest rate gradually decreases (while still remaining above the steady-state level) over time, deposits decrease, negatively affecting the loan supply. However, FRI soon decreases, behaving similarly to a reserve requirement shock ($\bar{R}$). After this initial increase with the reserve rate shock, both loan types respond negatively to adverse monetary policy shocks. Collateralized loans, however, tend to crowd commercial loans out, leading to a relative crowding out and contributing to the reduction of FRI. These findings suggest that monetary policy can be an effective tool for mitigating financial risks.

Unlike monetary policy, a restrictive macroprudential policy shock (a decrease in LTV) decreases the CA/C ratio. When borrowers cannot borrow a high percentage of their house value, collateral loan demand decreases, which also decreases the housing demand. Lower demand reduces house prices (Figure 14). As seen in Figure 14, while the housing sector gets a direct hit through the sharp reduction in collateral loans, goods production benefits from it. Banks supply more commercial loans and less collateralized loans to meet the demand, creating an absolute crowding out. Crowding out effects of the macroprudential policy are slightly different as long-run suggests a relative, but short-run suggests an absolute crowding out. The difference comes from the fact that long-run investment also decreases due to high capital prices as a result of a decrease in LTV. However, in the short run, capital prices barely change (if anything decreases), which allows entrepreneurs to invest in capital and thus slightly increase the demand for commercial loans. Overall, the strong decrease in CA/C, combined with the increase in defaults due to high commercial leverage, leads to an increase in FRI. It is worth noting that a decrease in housing demand (φ) produces similar results to the decrease in LTV, as the latter affects the housing demand indirectly while the former has a direct effect.

Adverse shocks on TFP (A) and labor supply (γ) both favor the housing market over goods production, leading to an increase in CA/C. The increase in CA/C, combined with a decrease in default risk, results in a lower FRI. In contrast, a construction shock has the opposite effect as it favors goods production over construction, resulting in a decrease in the CA/C ratio and an increase in FRI. Another difference between these shocks is that while TFP and labor supply shocks cause an absolute crowding out, an adverse construction shock creates a relative crowding out. In particular, TFP and labor supply shocks increase collateral loans but decrease commercial loans, whereas an adverse construction shock decreases both types of loans. These findings have empirical implications as they suggest a closer look at the ratio of bank loan volumes across types to find clues of financial crowding out, especially when the volumes appear to move in the same direction. This is because while a shock in the production sector stays isolated, a shock in the housing sector has a negative spillover to the real economy through the collateral channel.

Figure 14 shows the responses of real sector variables to the different shocks. Negative shocks cause an overall decrease in GDP but have differing effects in the goods and housing sectors. For instance, the shocks that directly affect the housing market, such as the adverse housing demand and LTV shocks, favor goods production while simultaneously hurting construction. In contrast, adverse TFP shock favors the housing market over goods production. Whereas a shock like labor supply that affects both construction and production leads to a decrease in both sectors. Moreover, the housing market responds expectedly to all shocks. In particular, house prices tend to decrease due to adverse TFP, housing demand, and LTV shocks. Meanwhile, house prices increase due to adverse monetary policy, construction, and labor supply shocks. The direction of the changes in house prices to the adverse shocks is not surprising, except potentially for the adverse labor supply shock. The reason is that the shock negatively impacts both goods production and construction, leading to a decrease in housing supply. Therefore, the low supply creates upward pressure on house prices.

5 Empirical Support

5.1 Riskiness of Loan Types

In our theoretical framework, there are inherited risks associated with commercial loans that are not present in collateralized loans. For instance, borrowers can default on their commercial loans if the outcome of their investment falls below the threshold $\bar{\omega}$, making commercial loans riskier. While, in reality, people can also default on their mortgages, widespread collateralization of mortgages makes it a safer bet for banks. As shown in Table 2, Commercial and Industrial loans have been twice as volatile as real estate loans over the past two decades. This gap widens further when it comes to residential loans specifically.

Table 2: Coefficient of Variation of C&I Loans Against Real Estate Loans

2004:6-2023:3	**Coefficient of variation**
Commercial and Industrial Loans	0.321
Real Estate Loans	0.166
Commercial Real Estate Loans	0.250
Residential Real Estate Loans	0.106

Source: Board of Governors of the Federal Reserve System (US).

Although volatility is important for the overall financial system, individual banks may be more concerned with the losses they may incur from each type of loan. Therefore, Table 3 shows the charge-off rates of each type, which are defined as the value of loans and leases that have been removed from the books and charged against loss reserves. Charge-off rates are determined by dividing the flow of a bank's net charge-offs (*i.e.*, gross charge-offs minus recoveries) of a bank during a quarter by the average amount of its outstanding loans throughout that quarter.

Table 3: Charge-off Rates of C&I Loans Against Real Estate Loans

Charge-off Rate (percent)	**All Commercial Banks**	**Top 100**	**Non-top 100**
All loans	0.883	0.982	0.627
Commercial and industrial loans	0.779	0.731	0.87
Loans secured by real estate	0.438	0.522	0.319
Single-family residential mortgages	0.389	0.433	0.197
Commercial real estate loans	0.487	0.614	0.395

Note: The ratios are multiplied by 400 to express them in annual percentages. Data are obtained from the Federal Reserve Board.

5.2 VAR Study on Relative Crowding-Out

After showing that commercial and industrial loans are riskier than real estate loans, this section takes our analysis a step further and searches for clues as to whether monetary policy really displays the behavior we observe in our model. In particular, we theoretically showed that while banks decrease both types of loans when there is a contractionary monetary policy, they display "*flight to safety*" behavior and prefer relatively safer loans. This distribution change in loans increases the collateralized-to-commercial loan ratio, creating a relative crowding out in the economy.

Empirically measuring the effects of monetary policy on loans is not simple due to endogeneity concerns. In particular, while monetary policy can affect loans, the amount of loans can affect monetary policy as well. For instance, in a time when there are not enough loans, the central bank could decide to engage in expansionary monetary policy to relieve the pressure in the financial sector. Therefore, we conduct the following five-variable VAR to better comprehend the effects of a tighter monetary policy on loans while addressing macroeconomic endogeneity:

$$\Delta Y_t = \alpha + A(L)\Delta Y_{t-1} + e_t,$$

where Y is the vector of variables that include (1) the ratio of real estate loans to commercial and industrial loans to capture the loan distribution of banks, (2) the federal funds effective rate to capture the monetary policy, (3) the industrial production index to account for the business cycle changes, (4) house prices, specifically Median Sales Price for New Houses Sold in the United States, to capture the housing market dynamics, and (5) Chicago Fed National Financial Conditions Index to account for the risk in the financial sector. $A(L)$ is a matrix of lagged coefficients, and e is the robust error term.[9] Δ indicates the first difference of the logged data to ensure the series are stationary. The data period is 1963:1-2023:3 and is monthly.[10]

Figure 15 illustrates the impact of a one standard deviation increase in the federal funds effective rate on each loan type, commercial and industrial (C&I) and real estate. As expected, Figures 15a and 15b confirm that tighter monetary policy decreases both loans. However, Figure 15c shows that while both loans decrease, there is a significant increase in the ratio of real estate to commercial and industrial loans, which corresponds to the CA/C ratio in our model. Overall, banks favor real estate loans relative to commercial and industrial loans

[9]The most conservative order is used in estimation, and the optimal lags are chosen by using both Akaike's information criterion and Schwarz's Bayesian information criterion.

[10]The data are retrieved from FRED, Federal Reserve Bank of St. Louis.

when there is a contractionary monetary policy. These findings are consistent with our model predictions on the relative crowding out of tighter monetary policy.

6 Conclusion

We have constructed a tractable general-equilibrium model of money, banking and finance that allows banks to make a portfolio choice over risky commercial and collateralized household loans. Our key findings are the following: First, monetary policy is transmitted through an interest-rate channel and a bond-supply channel. Fractional reserve banking can be welfare-improving. Second, inflation is influenced, but not directly controlled by monetary authority. Instead, it is determined by the interaction of labor demand and supply. Third, short-run disturbances to the economy and long-run policy changes alter the bank loan distribution, which results in crowding-out effects between construction and production and has direct implications for overall financial risk. Finally, contractionary monetary policy reduces loan volumes, pivots bank lending toward household mortgage loans, and mitigates financial risk, and *vice versa* for expansionary policy.

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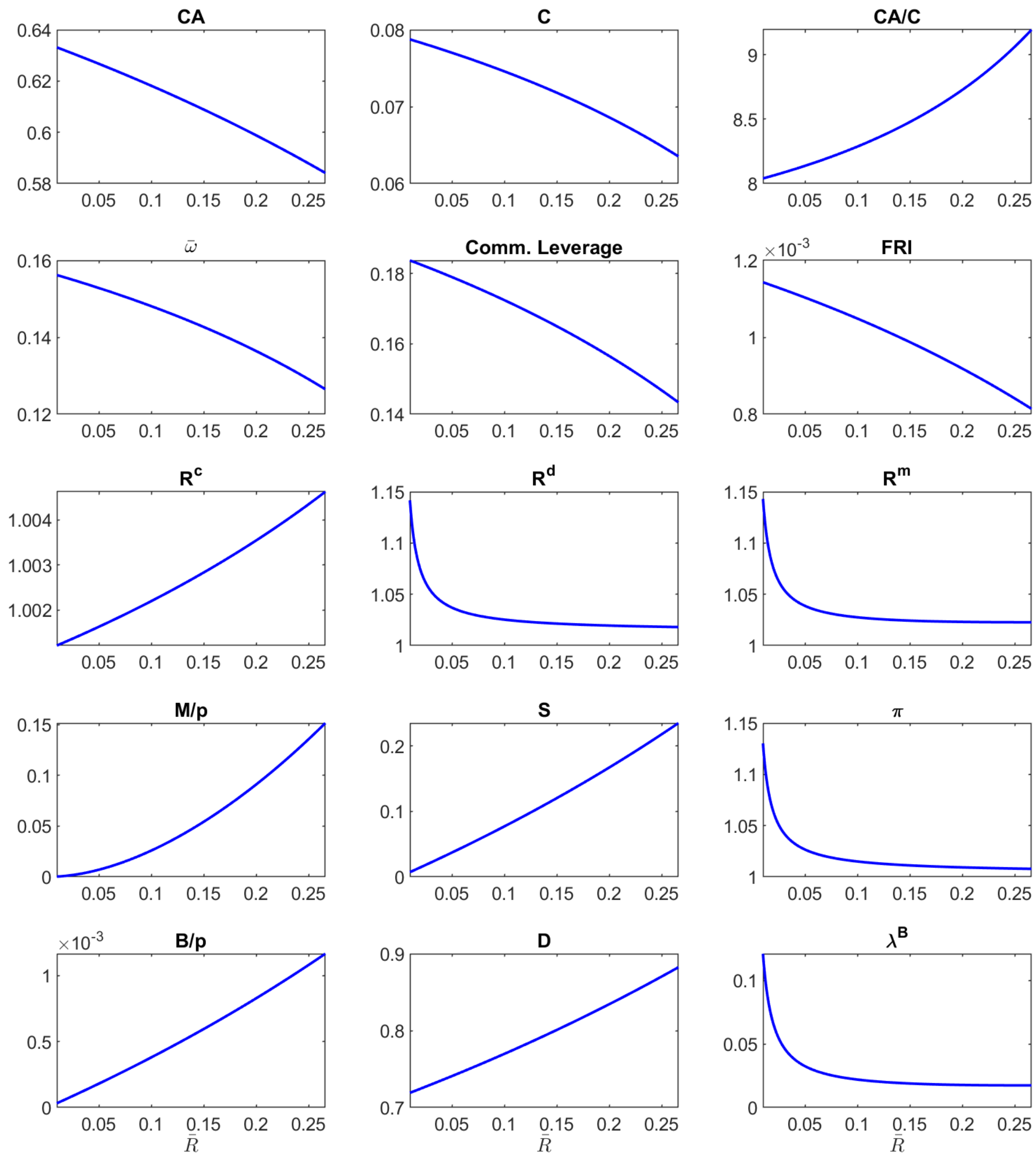
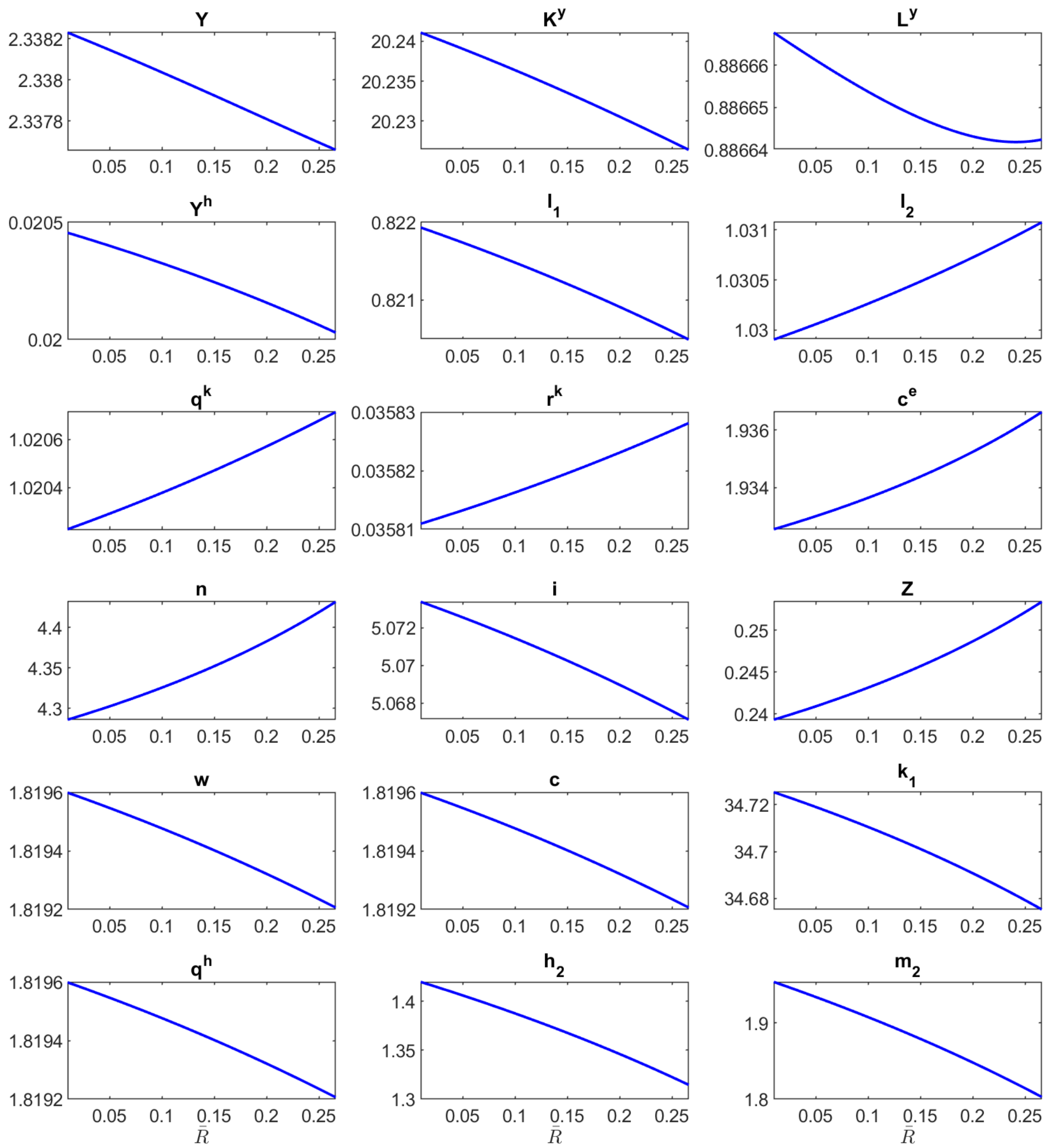
Figure 1: Long-Run Effects of $\bar{R}$ on the Banking Sector Variables



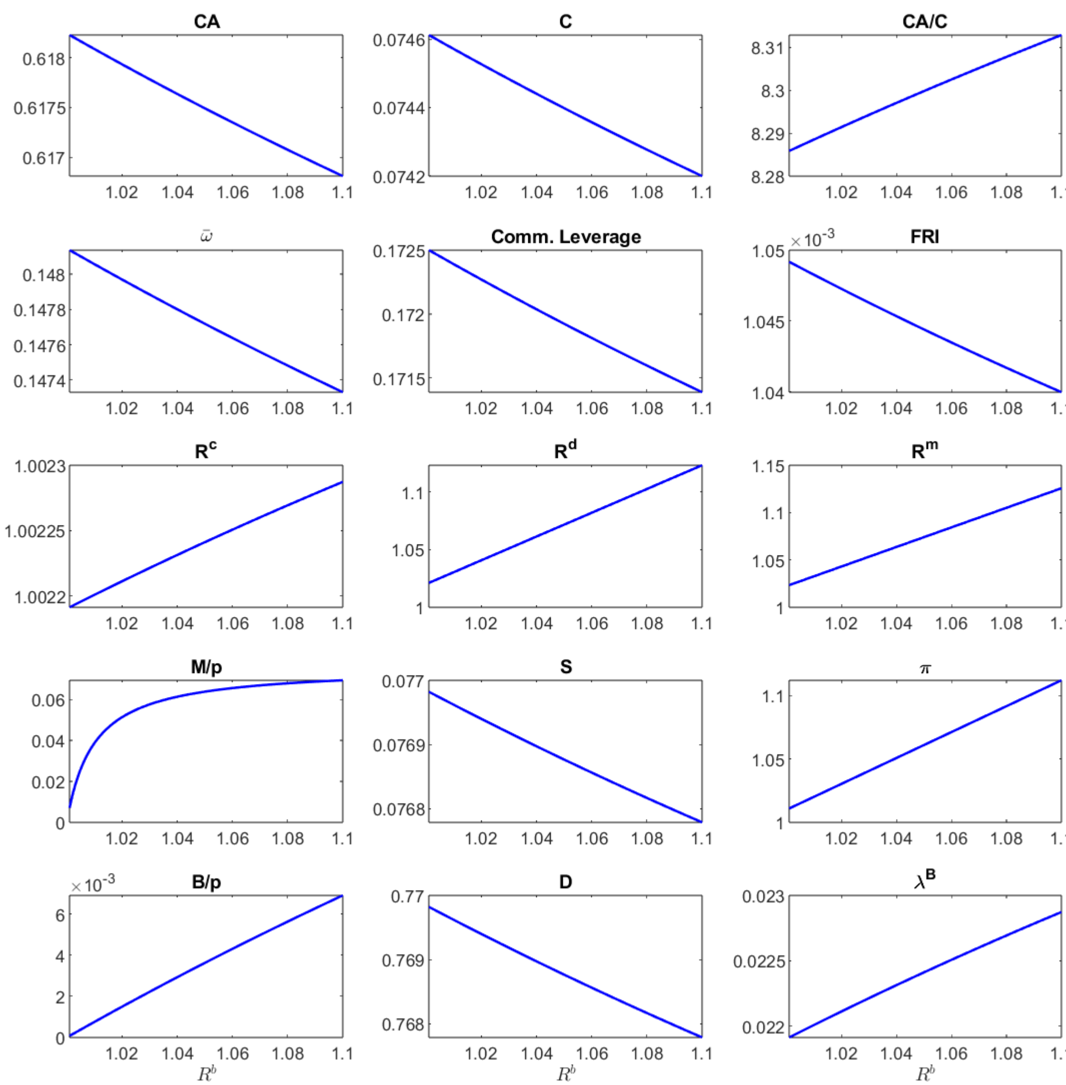
Note: The figure plots the steady-state values of variables associated with the banking sector under different levels of $\bar{R}$.

Figure 2: Long-Run Effects of $\bar{R}$ on the Real Sector Variables



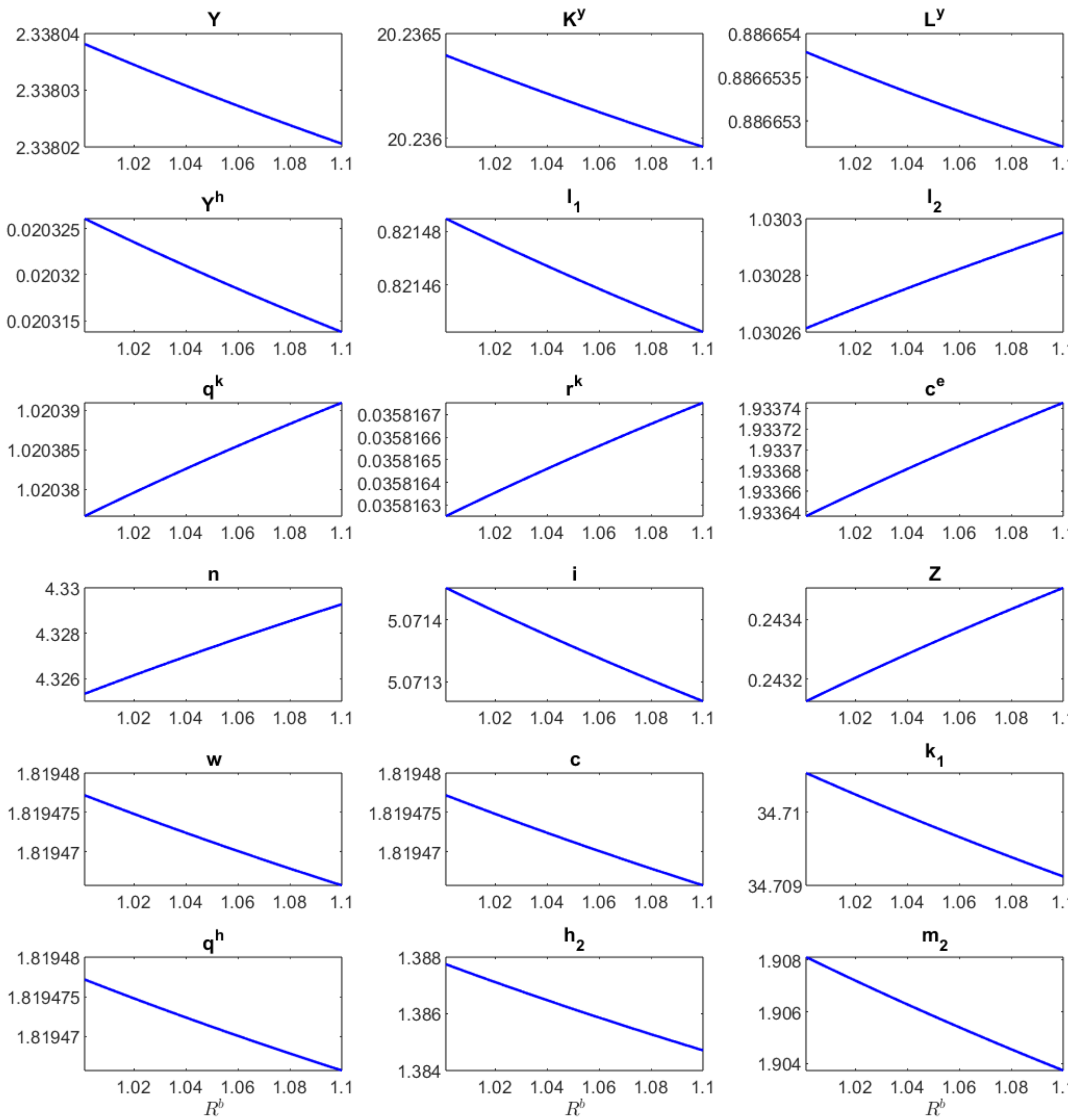
Note: The figure plots the steady-state values of variables associated with the real sector under different levels of $\bar{R}$.

Figure 3: Long-Run Effects of R^b on the Banking Sector Variables



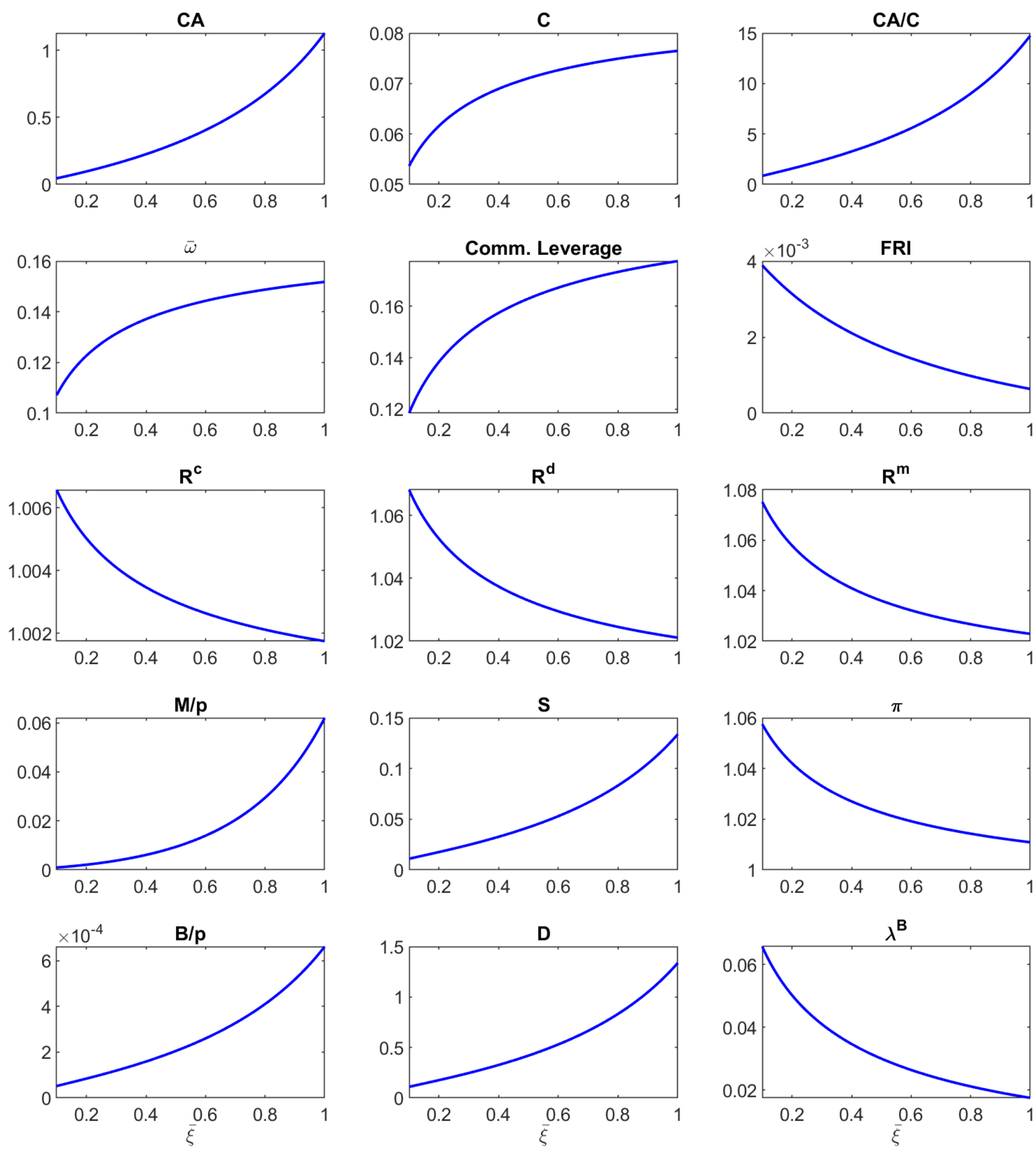
Note: The figure plots the steady-state values of variables associated with the banking sector under different levels of R^b.

Figure 4: Long-Run Effects of R^b on the Real Sector Variables



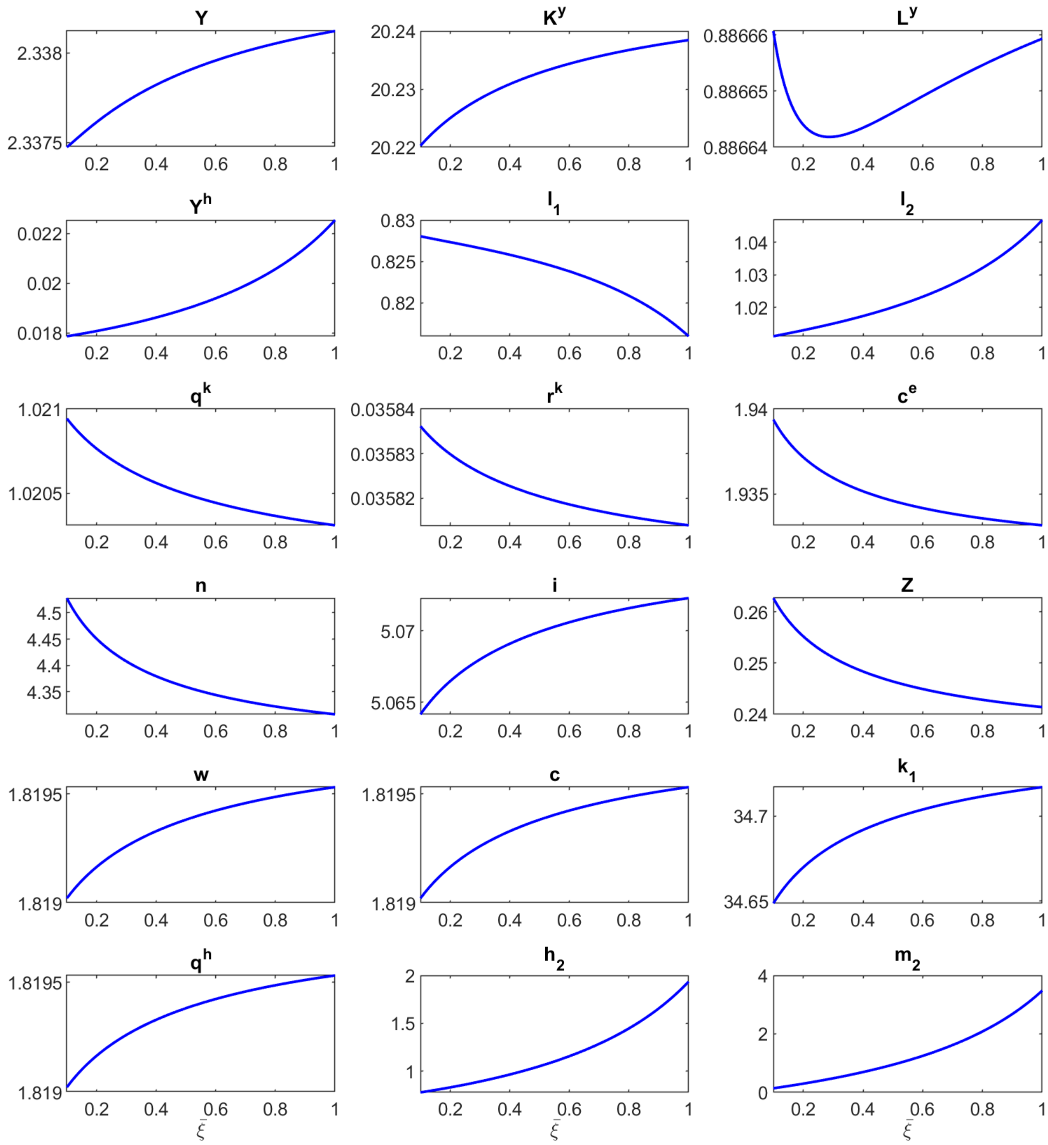
Note: The figure plots the steady-state values of variables associated with the real sector under different levels of R^b.

Figure 5: Long-Run Effects of $\bar{\xi}$ on the Banking Sector Variables



Note: The figure plots the steady-state values of variables associated with the banking sector under different levels of $\bar{\xi}$.

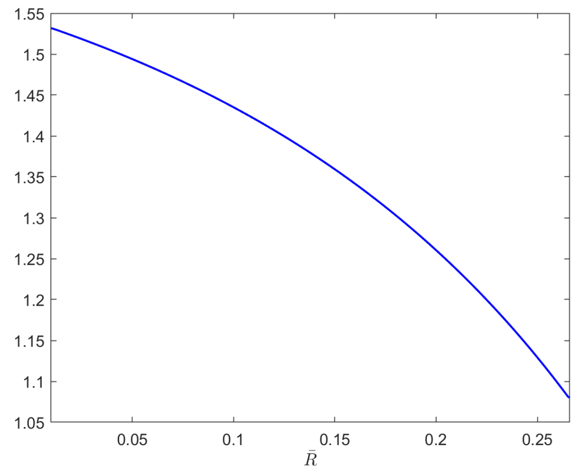
Figure 6: Long-Run Effects of $\bar{\xi}$ on the Real Sector Variables



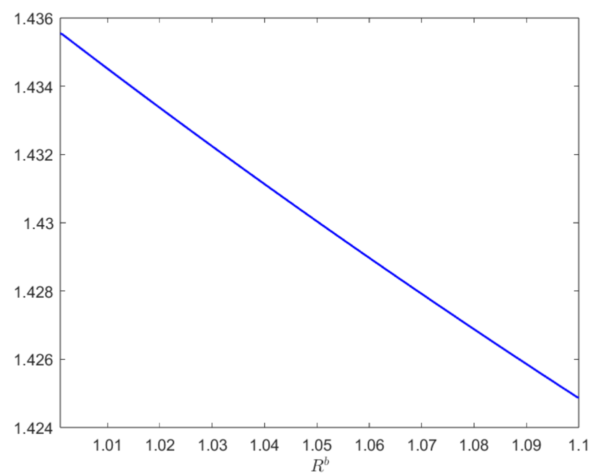
Note: The figure plots the steady-state values of variables associated with the real sector under different levels of $\bar{\xi}$.

Figure 7: Output/Housing Elasticity

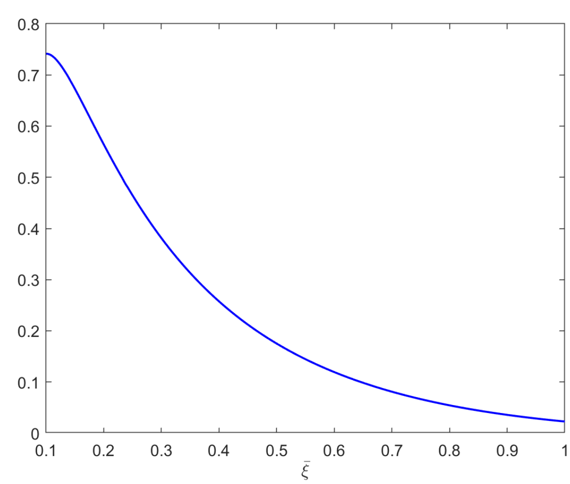
(a) For Changes in $\bar{R}$



(b) For Changes in R^b



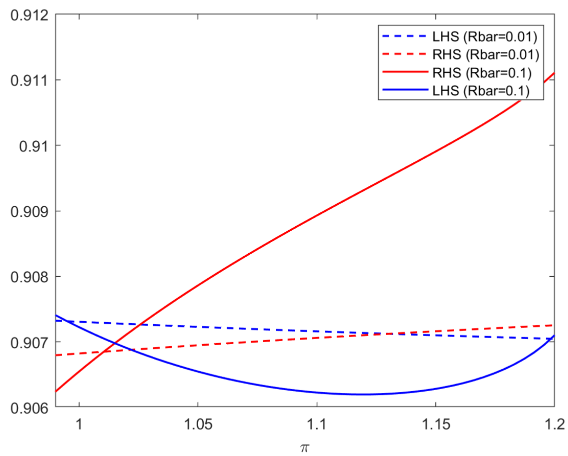
(c) For Changes in $\bar{\xi}$



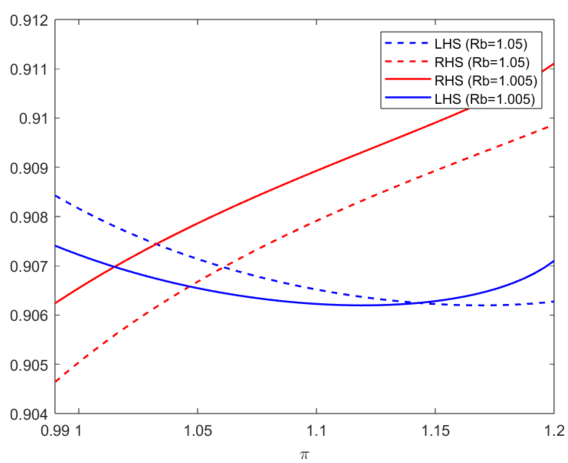
Note: The figure plots the gradient of Y (goods production) versus Y^h (house construction) as $\bar{R}$, R^b, and $\bar{\xi}$ change, respectively.

Figure 8: Policy Effects on Steady-State Inflation

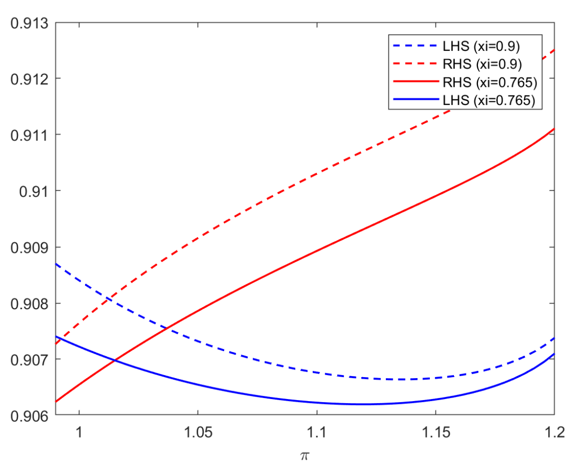
(a) Effects of $\bar{R}$



(b) Effects of R^b

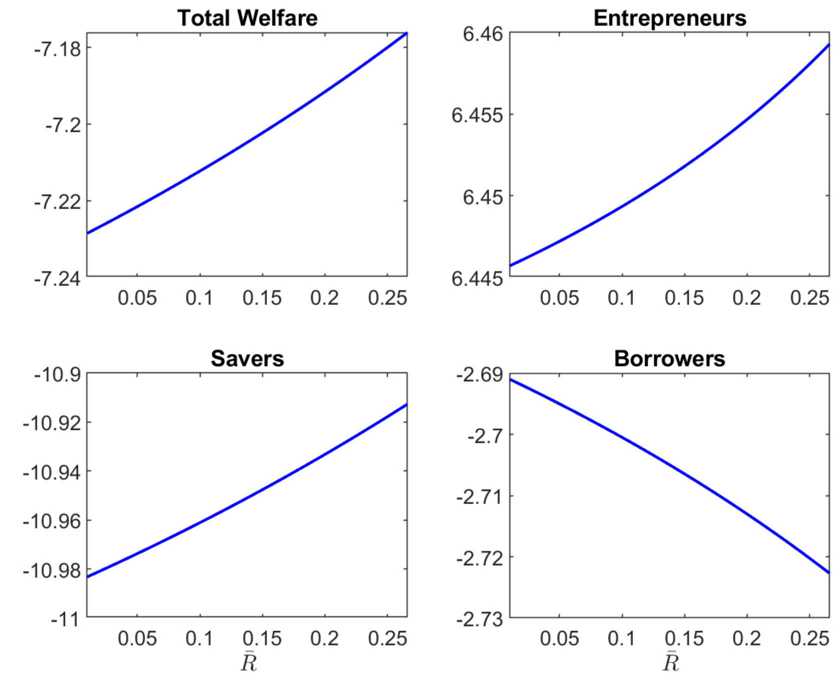


(c) Effects of $\bar{\xi}$



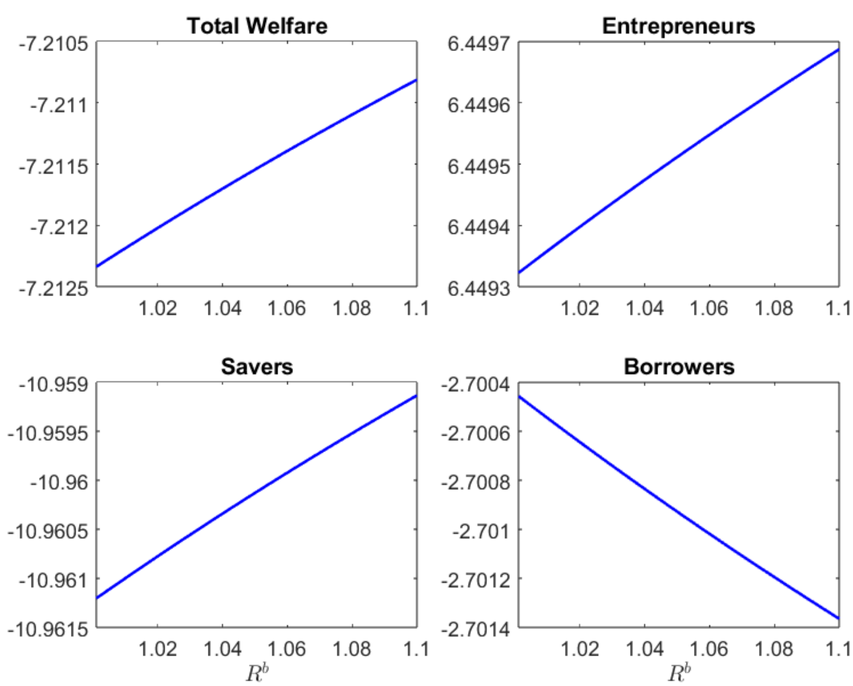
Note: The figure plots the demand (LHS) and the supply (RHS) of the labor market clearing condition, which helps determine the steady-state inflation. The figure depicts LHS and RHS given respective levels of $\bar{R}$, R^b, and $\bar{\xi}$.

Figure 9: Effects of $\bar{R}$ on Welfare



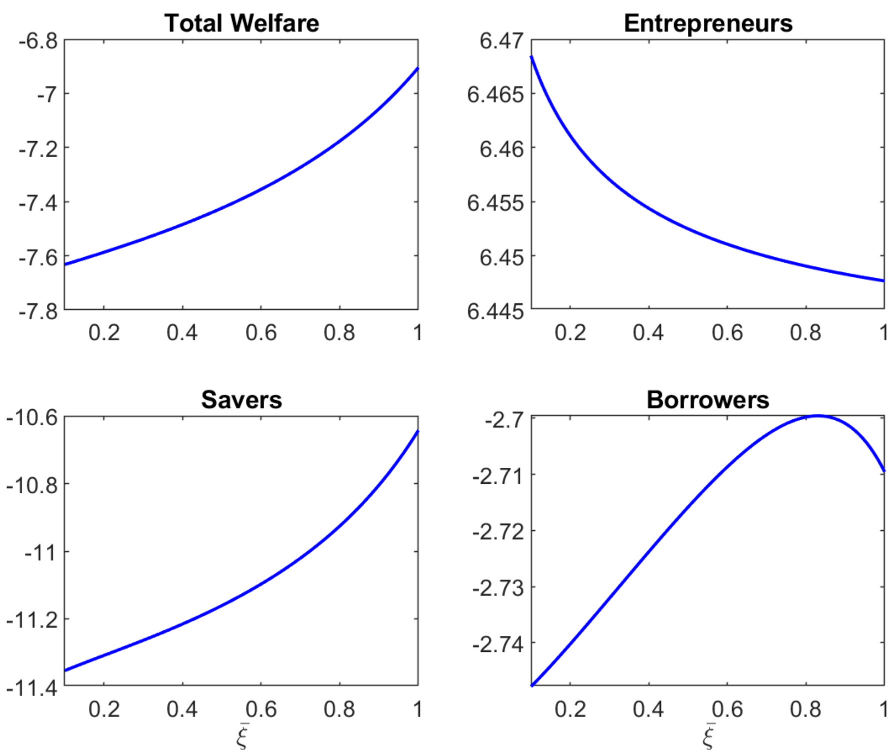
Note: The figure plots the welfare measures for aggregate economy, savers, borrowers, and entrepreneurs for different levels of $\bar{R}$.

Figure 10: Effects of R^b on Welfare



Note: The figure plots the welfare measures for aggregate economy, savers, borrowers, and entrepreneurs for different levels of R^b.

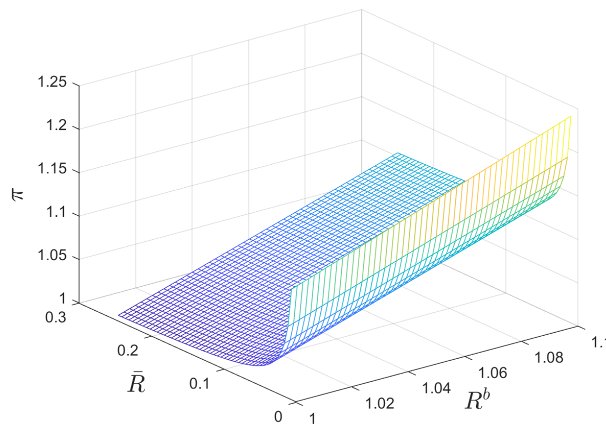
Figure 11: Effects of $\bar{\xi}$ on Welfare



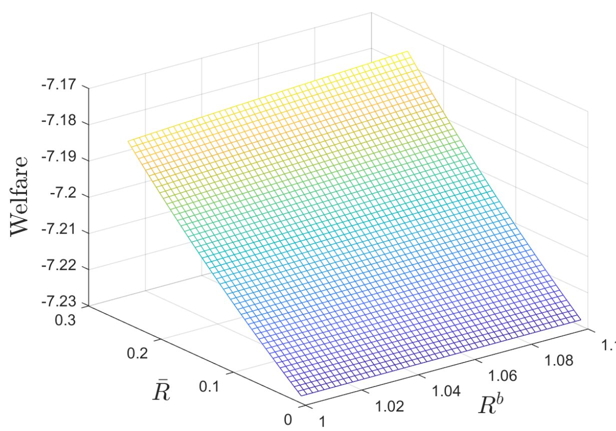
Note: The figure plots the welfare measures for aggregate economy, savers, borrowers, and entrepreneurs for different levels of $\bar{\xi}$.

Figure 12: Effects of Inflation on Total Welfare

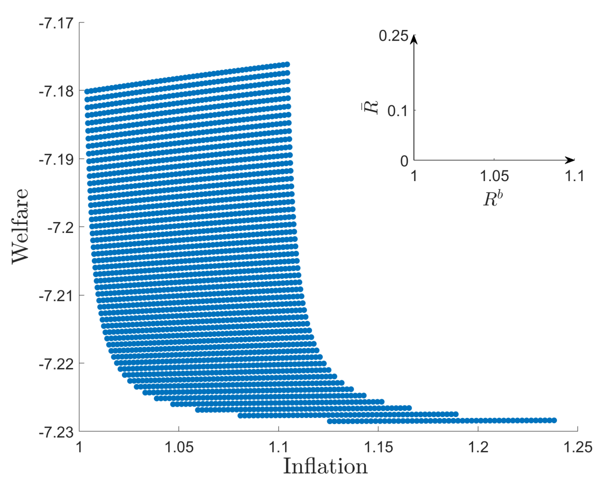
(a) Inflation vs. $\bar{R}$ and R^b



(b) Welfare vs. $\bar{R}$ and R^b

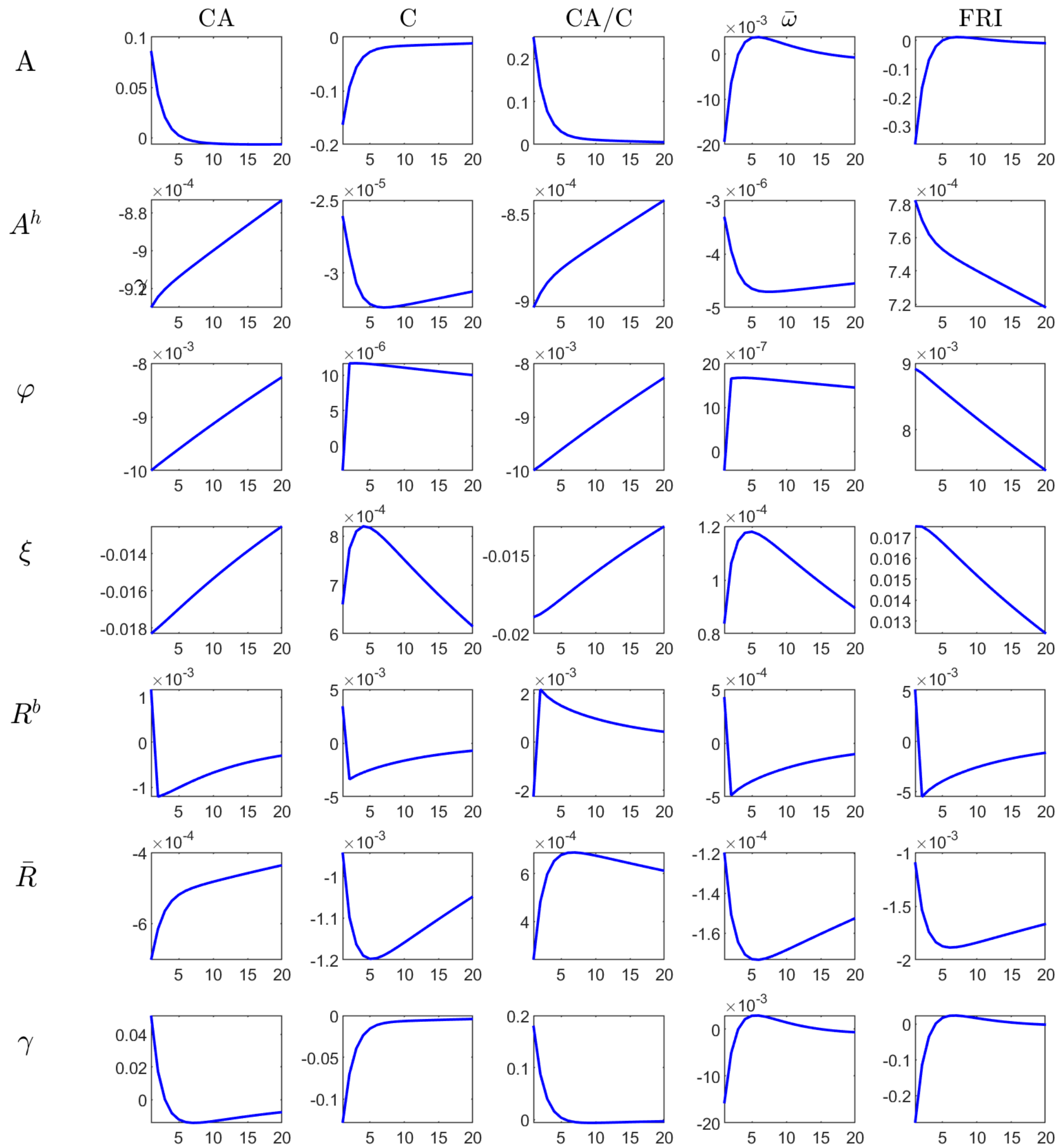


(c) Welfare vs. Inflation



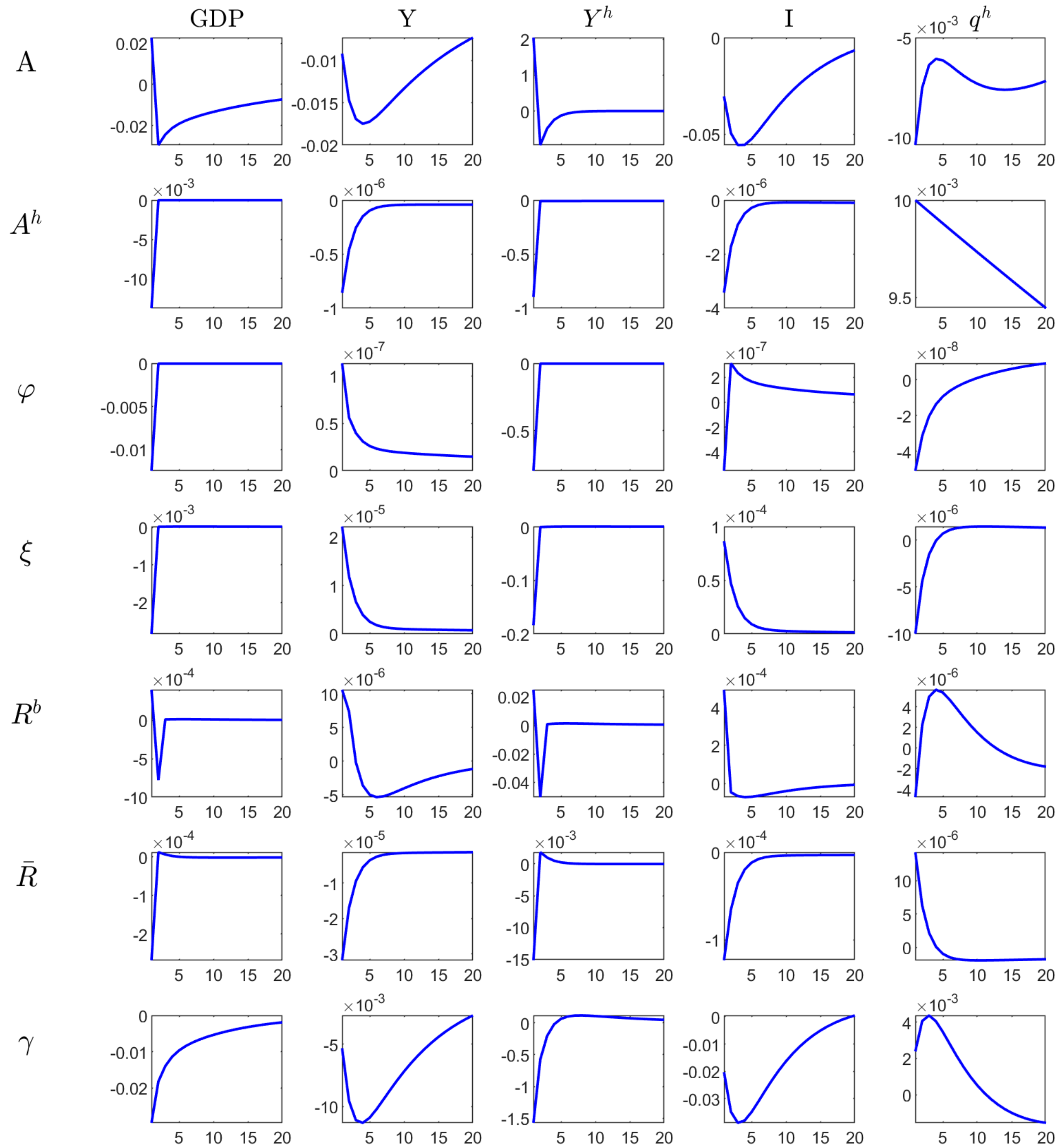
Note: The figure plots the changes in inflation and welfare against policy parameters ($\bar{R}$, R^b) as well as welfare against inflation. Each dot in Panel (c) represents the welfare and inflation values given a pair of $\bar{R}$ and R^b.

Figure 13: Impulse Responses of Banking Sector Variables to Adverse Shocks



Note: The figure plots the responses of variables associated with the banking sector to all adverse shocks in the economy. In particular, we initiate a 1% increase to the innovation of the reserve requirement shock ($\varepsilon_t^{\bar{R}}$) and reserve rate shock ($\varepsilon_t^{R^b}$), creating a contractionary monetary policy. We also initiate a 1% decrease in TFP (ε_t^{A}), construction ($\varepsilon_t^{A^h}$), LTV (ε_t^{ξ}), housing demand (ε_t^{φ}), and labor supply (ε_t^{γ}) shocks. All responses are normalized so that the units of the vertical axes represent percentage deviations from the steady state.

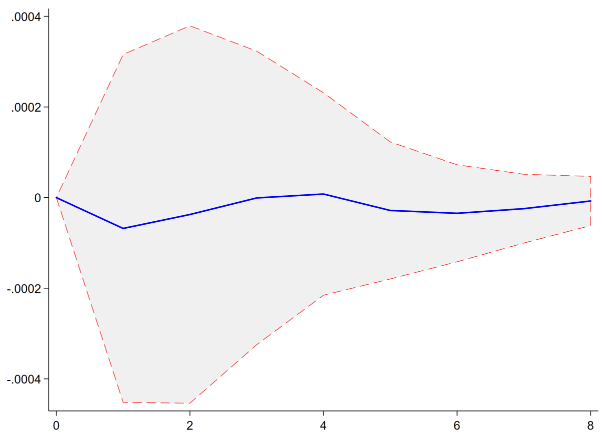
Figure 14: Impulse Responses of Real Sector Variables to Adverse Shocks



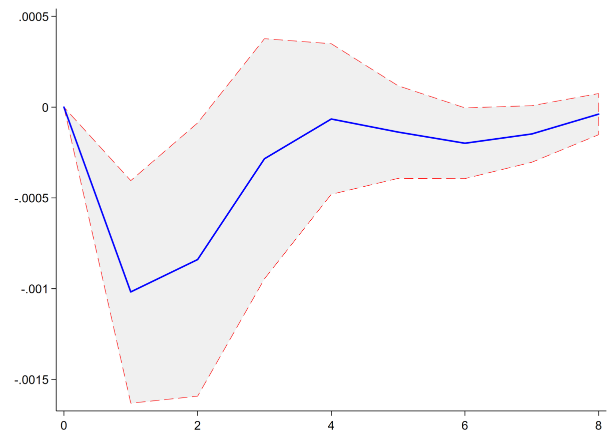
Note: The figure plots the responses of variables associated with the real sector to all adverse shocks in the economy. In particular, we initiate a 1% increase to the innovation of the reserve requirement shock ($\varepsilon_t^{\bar{R}}$) and reserve rate shock ($\varepsilon_t^{R^b}$), creating a contractionary monetary policy. We also initiate a 1% decrease in TFP (ε_t^{A}), construction ($\varepsilon_t^{A^h}$), LTV (ε_t^{ξ}), housing demand (ε_t^{φ}), and labor supply (ε_t^{γ}) shocks. All responses are normalized so that the units of the vertical axes represent percentage deviations from the steady state.

Figure 15: Responses of Real Estate, C&I Loans, and Real Estate/C&I to an Increase in Federal Funds Effective Rate

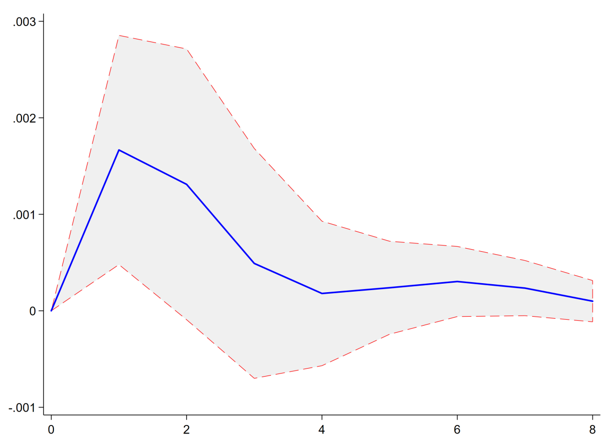
(a) Real Estate Loans



(b) Commercial and Industrial Loans



(c) Real Estate/C&I



Note: The figure plots the real estate loans, commercial and industrial loans, and the collateral to commercial loan ratio responses to a 1% increase in the federal funds effective rate. Shaded areas indicate the 95% confidence intervals.

A Appendix: Proof of Theorem 1

This appendix contains a detailed proof of Theorem 1. First, recall that the representative bank's optimal decision in the steady state implies (55). Based on that, we continue to analyze the rest of the steady-state conditions.

A.1 SS household decisions

All households, banks, and firms take policy parameters $\left(\bar{R}, R^b\right)$ as given when making decisions. Conditions (12) to (14) in steady state become

$$1 \geq \beta^j \frac{R^d}{\pi}, \quad d_j \geq 0 \tag{A.1}$$

$$1 \leq R^m \left[\beta^j \frac{1}{\pi} + \lambda_j c_j\right], \quad m_j \geq 0 \tag{A.2}$$

$$1 \geq \beta^j \frac{R^g}{\pi}, \quad b_j \geq 0, \tag{A.3}$$

where all pairs in the above hold with complementary slackness. Given $\beta^2 < \beta^1$, condition (A.1) implies that in the steady state,

$$1 = \beta^1 \frac{R^d}{\pi} = \beta^1 \frac{R^g}{\pi} > \beta^2 \frac{R^d}{\pi} = \beta^2 \frac{R^g}{\pi}. \tag{A.4}$$

The inequality in the above implies $d_2 = b_2 = 0$ and the equality yields $R^d = R^g = \pi/\beta^1$. Hence, we have Properties I of Theorem 1.

Condition (55) implies $R^c = 1$ if $\lambda^B = 0$ and $R^c > 1$ if $\lambda^B > 0$, which together with $R^d = \pi/\beta^1$ yields

$$R^b = \frac{\pi}{\beta^1}\left(1 - \lambda^B + \lambda^B \bar{R}\right). \tag{A.5}$$

Take π as given and solve for

$$\lambda^B = \frac{1}{1-\bar{R}}\left(1 - \frac{\beta^1 R^b}{\pi}\right). \tag{A.6}$$

Given (A.6), condition (55) yields

$$R^c = \frac{\beta^1}{\pi} R^b + \lambda^B = \frac{1}{1-\bar{R}} \left(1 - \frac{\beta^1}{\pi} R^b \bar{R}\right) \tag{A.7}$$

$$R^m = R^b + \frac{\pi}{\beta^1} \lambda^B. \tag{A.8}$$

Note that the real lending rate, R^c, is a function of the SS inflation rate, π. Later, we will show that this is the key channel through which monetary policy makes long-run real impacts on the economy given $\lambda^B > 0$. Recall that $R^m > R^d$ if $\lambda^B > 0$. Thus given $\lambda^B > 0$ and $R^d = \pi/\beta^1$, we have

$$R^m > R^d = \frac{1}{\frac{1}{R^d}} \geq \frac{1}{\frac{1}{R^d} + \lambda_1 c_1} = \frac{1}{\frac{\beta^1}{\pi} + \lambda_1 c_1},$$

where $\lambda_1 \geq 0$ is the multiplier of the household collateral constraint. Therefore, the first inequality in condition (A.3) for $j = 1$ is strict, which implies $m_1 = 0$. Given the collateral constraint (15), $m_1 = 0$ implies $\lambda_1 = 0$ for any strictly positive holdings of capital and housing by the patient households. Thus, we have $m_1 = \lambda_1 = 0$ in the steady state if $\lambda^B > 0$. Given $m_1 = 0$, an equilibrium with collateralized debt must have $m_2 > 0$. Then condition (A.3) for $j = 2$ requires

$$1 = R^m \left[\beta^2 \frac{1}{\pi} + \lambda_2 c_2\right], \tag{A.9}$$

which given $\frac{\pi}{\beta^1} R^c = R^m$ yields

$$\lambda_2 = \frac{1}{\pi c_2} \left(\frac{\beta^1}{R^c} - \beta^2\right). \tag{A.10}$$

Moreover, condition (15) solves for

$$m_2 = \frac{1}{R^c} \beta^1 \xi \left(q^k k_2 + q^h h_2\right). \tag{A.11}$$

Assuming interior solutions, optimality conditions, (9) - (11), imply that in the steady state,

$$\begin{aligned}
w &= \gamma c_1 = \gamma c_2 \equiv \gamma c && \text{(A.12)}\\
q^k &= \beta^1 \left(r^k + \left(1-\delta^k\right) q^k\right) && \text{(A.13)}\\
q^k &= \beta^2 \left(r^k + \left(1-\delta^k\right) q^k\right) + \pi c \lambda_2 \xi q^k && \text{(A.14)}\\
q^h &= \varphi \frac{c}{h_1} + \beta^1 \left(1-\delta^h\right) q^h && \text{(A.15)}\\
q^h &= \varphi \frac{c}{h_2} + \beta^2 \left(1-\delta^h\right) q^h + \pi c \lambda_2 \xi q^h. && \text{(A.16)}
\end{aligned}$$

Equation (A.13) implies

$$r^k = q^k \left[\frac{1}{\beta^1} - \left(1-\delta^k\right)\right]. \tag{A.17}$$

Then substituting (A.10) and (A.13) into (A.14) yield

$$q^k = \frac{r^k}{\frac{\frac{\beta^1}{R_c}-\beta^2}{\beta^1-\beta^2}\xi - (1-\delta^k)}.$$

The above becomes $\frac{\beta^1-\beta^2}{\frac{\beta^1}{R_c}-\beta^2} = \xi\beta^1$ by (A.17). Given $R^c \geq 1$ if $\lambda^B \geq 0$, we have $\frac{\beta^1-\beta^2}{\frac{\beta^1}{R_c}-\beta^2} \geq 1$ and it is not possible to have $\frac{\beta^1-\beta^2}{\frac{\beta^1}{R_c}-\beta^2} = \xi\beta^1$ given $\xi, \beta^1 < 1$. It follows that condition (A.14) cannot hold with equality. Therefore,

$$q^k > \beta^2 \left(r^k + \left(1-\delta^k\right) q^k\right) + \pi c \lambda_2 \xi q^k, \tag{A.18}$$

and thus $k_2 = 0$. Then (A.11) yields

$$m_2 = \frac{1}{R^c} \beta^1 \xi q^h h_2. \tag{A.19}$$

Finally, budget constraints of the patient and impatient households simplify to:

$$c + q^h \delta^h h_1 = w l_1 + \left(r^k - q^k \delta^k\right) k_1 + \left(\frac{1}{\beta^1} - 1\right)(d_1 + b_1) \tag{A.20}$$

$$c+q^h\delta^h h_2 \;\; = \;\; wl_2+\left(1-\frac{R^c}{\beta^1}\right)m_2, \tag{A.21}$$

where we have incorporated $c_1=c_2=c$, $d_2=k_2=m_1=0$, $R^d=\pi/\beta^1$, $R^m=\pi R^c/\beta^1$, and that dividends are zero in equilibrium.

A.2 Steady state entrepreneur decisions

First, the steady state version of conditions (32) and (33) are given by

$$\begin{gathered}\frac{R^c}{q^k}=g\left(\bar{\omega}\right)+\frac{\beta^e}{\beta^1}f\left(\bar{\omega}\right)R^c\\ \frac{R^c}{q^k}=1-\mu\Phi\left(\bar{\omega}\right)-\frac{\mu\phi\left(\bar{\omega}\right)f\left(\bar{\omega}\right)}{1-\Phi\left(\bar{\omega}\right)},\end{gathered} \tag{A.22}$$

which imply

$$g\left(\bar{\omega}\right)+\frac{\beta^e}{\beta^1}f\left(\bar{\omega}\right)R^c=1-\mu\Phi\left(\bar{\omega}\right)-\frac{\mu\phi\left(\bar{\omega}\right)f\left(\bar{\omega}\right)}{1-\Phi\left(\bar{\omega}\right)}.$$

The steady state version of condition (20) is

$$g\left(\bar{\omega}\right)=1-\mu\Phi\left(\bar{\omega}\right)-f\left(\bar{\omega}\right). \tag{A.23}$$

The above two equations together imply

$$\frac{\mu\phi\left(\bar{\omega}\right)}{1-\Phi\left(\bar{\omega}\right)}=1-\frac{\beta^e}{\beta^1}R^c. \tag{A.24}$$

Define the left-hand side of the above as $\Omega\left(\bar{\omega}\right)$. It follows that

$$\bar{\omega}=\Omega^{-1}\left(1-\frac{\beta^e}{\beta^1}R^c\right). \tag{A.25}$$

Next we solve for the capital price q^k from combining (A.22) and (A.23):

$$q^k=\frac{R^c}{1-\mu\Phi\left(\bar{\omega}\right)+\left(\frac{\beta^e}{\beta^1}R^c-1\right)f\left(\bar{\omega}\right)}, \tag{A.26}$$

where according to (18), we have

$$f(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega d\Phi(\omega) - \bar{\omega}\left[1 - \Phi(\bar{\omega})\right]. \tag{A.27}$$

Then (A.17) solves for

$$r^k = q^k \left[\frac{1}{\beta^1} - \left(1 - \delta^k\right)\right]. \tag{A.28}$$

A.3 Other steady state conditions

The steady state version of the optimality conditions by productive and construction firms are

$$w = \nu A (L^y)^{\nu-1} (K^y)^{1-\nu} \tag{A.29}$$

$$r^k = (1-\nu) A (L^y)^{\nu} (K^y)^{-\nu} \tag{A.30}$$

$$w = q^h A^h. \tag{A.31}$$

Next, the laws of motion for capital and housing stocks are given by

$$\delta^k K^y = \varrho I \left[1 - \mu\Phi(\bar{\omega})\right] \tag{A.32}$$

$$\delta^h H = A^h L^h. \tag{A.33}$$

Finally, the steady state version of the market-clearing conditions (46) to (52) and the binding bank-balance-sheet condition (34) imply

$$L^y + L^h = (1-\varrho)\left[\alpha l_1 + (1-\alpha) l_2\right] + \varrho \tag{A.34}$$

$$K^y = (1-\varrho)\left[\alpha k_1 + (1-\alpha) k_2\right] + Z \tag{A.35}$$

$$H = (1-\varrho)\left[\alpha h_1 + (1-\alpha) h_2\right] \tag{A.36}$$

$$A (L^y)^{\nu} (K^y)^{1-\nu} = (1-\varrho) c + \varrho C^e + \varrho I \tag{A.37}$$

$$\left(1 - \bar{R}\right)(1-\varrho)\alpha d_1 = \varrho (I - N) + (1-\varrho)(1-\alpha) m_2 \tag{A.38}$$

Condition (A.38) is essentially the credit-market-clearing condition. The aggregate supply of credit is on the left-hand side. We have maintained the assumption that $\lambda^B > 0$ and thus the aggregate supply of credit is given by $D - S = \left(1 - \bar{R}\right) D = \left(1 - \bar{R}\right)(1 - \varrho)\,\alpha d_1$ in (A.38), where $(1 - \varrho)\,\alpha$ is the measure of patient households. The aggregate demand for credit is on the right-hand side of the equation and is the sum of commercial loan demand and collateralized loan demand. In particular, the aggregate demand for commercial loans is given by $\varrho\,(I - N)$, where ϱ is the measure of entrepreneurs and $(I - N)$ is the average entrepreneurial debt. According to (43) - (45) and (A.13), the steady state levels of (Z, I, N) are given by

$$Z = \varrho\left[f(\bar{\omega})\,I - \frac{C^e}{q^k}\right] \tag{A.39}$$

$$I = \frac{N}{1 - \frac{q^k g(\bar{\omega})}{R^c}} \tag{A.40}$$

$$N = w + \frac{q^k Z}{\beta^1 \varrho}. \tag{A.41}$$

A.4 Algorithm for solving the steady state with $\lambda^B > 0$

We now provide an analytical algorithm for obtaining a full set of solutions to the steady-state variables given $\lambda^B > 0$. The algorithm takes three steps:

Step 1. Recall that given $\lambda^B > 0$, we have

$$m_1^{ss} = \lambda_1^{ss} = d_2^{ss} = k_2^{ss} = 0.\text{[11]} \tag{A.42}$$

Step 2. This is a complex step in which we take the inflation rate π as given and solve for the rest of the steady state variables. Later in Step 3, we derive the equation for solving the steady state level of inflation, π_{ss}. First, recall that

$$R^d_{ss} = R^g_{ss} = \frac{\pi}{\beta^1} \tag{A.43}$$

[11] Throughout the paper, we use a subscript/superscript of "ss" to denote the analytical solution of a steady-state variable.

and that equations (A.6), (A.7), (A.8), (A.25), (A.26), (A.27), and (A.28) give

$$
\begin{aligned}
\lambda_{ss}^{B} &= \frac{1}{1-\bar{R}}\left(1-\frac{\beta^{1}R^{b}}{\pi}\right) && \text{(A.44)}\\
R_{ss}^{c} &= \frac{\beta^{1}}{\pi}R^{b}+\lambda_{ss}^{B}=\frac{1}{1-\bar{R}}\left(1-\frac{\beta^{1}}{\pi}R^{b}\bar{R}\right) && \text{(A.45)}\\
R_{ss}^{m} &= R^{b}+\frac{\pi}{\beta^{1}}\lambda_{ss}^{B} && \text{(A.46)}\\
\bar{\omega}_{ss} &= \Omega^{-1}\left(1-\frac{\beta^{e}}{\beta^{1}}R_{ss}^{c}\right) && \text{(A.47)}\\
f\left(\bar{\omega}_{ss}\right) &= \int_{\bar{\omega}_{ss}}^{\infty}\omega d\Phi\left(\omega\right)-\bar{\omega}_{ss}\left[1-\Phi\left(\bar{\omega}_{ss}\right)\right] && \text{(A.48)}\\
q_{ss}^{k} &= \frac{R_{ss}^{c}}{1-\mu\Phi\left(\bar{\omega}_{ss}\right)+\left(\frac{\beta^{e}}{\beta^{1}}R_{ss}^{c}-1\right)f\left(\bar{\omega}_{ss}\right)} && \text{(A.49)}\\
r_{ss}^{k} &= q_{ss}^{k}\left[\frac{1}{\beta^{1}}-\left(1-\delta^{k}\right)\right]. && \text{(A.50)}
\end{aligned}
$$

Next, given r_{ss}^{k}, equation (A.30) yields

$$\left(\frac{K^{y}}{L^{y}}\right)_{ss}=\left[\frac{\left(1-\nu\right)A}{r_{ss}^{k}}\right]^{1/\nu}.$$

Given the above, equations (A.12) and (A.29) solve for (c_{ss}, w_{ss}):

$$c_{ss}=\frac{w_{ss}}{\gamma}=\frac{\nu}{\gamma}A\left(\frac{K^{y}}{L^{y}}\right)_{ss}^{1-\nu}. \tag{A.51}$$

Then equation (A.31) yields

$$q_{ss}^{h}=\frac{w_{ss}}{A^{h}}. \tag{A.52}$$

Given $(\pi, c_{ss}, R_{ss}^{c})$, equation (A.10) yields

$$\lambda_{2}^{ss}=\frac{1}{\pi c_{ss}}\left(\frac{\beta^{1}}{R_{ss}^{c}}-\beta^{2}\right). \tag{A.53}$$

Then given $\left(c_{ss}, q_{ss}^{h}\right)$, equation (A.15) solves for

$$h_1^{ss} = \frac{\varphi c_{ss}}{q_{ss}^h \left[1 - \beta^1 \left(1 - \delta^h\right)\right]} = \frac{\varphi A^h}{\gamma \left[1 - \beta^1 \left(1 - \delta^h\right)\right]} \tag{A.54}$$

and equations (A.16) and (A.53) together yield

$$h_2^{ss} = \frac{\varphi A^h}{\gamma \left[1 - \beta^2 \left(1 - \delta^h\right) - \xi \left(\frac{\beta^1}{R_{ss}^c} - \beta^2\right)\right]}. \tag{A.55}$$

Note that h_1^{ss} is independent of R_{ss}^c and thus monetary policy. Equations (A.36) and (A.33) imply

$$H_{ss} = (1 - \varrho) \left[\alpha h_1^{ss} + (1 - \alpha) h_2^{ss}\right] \tag{A.56}$$

$$L_{ss}^h = \frac{\delta^h}{A^h} H_{ss}. \tag{A.57}$$

Next, we use equations (A.37), (A.39), (A.40), and (A.41) to solve for

$$C_{ss}^e = \frac{\left(\frac{A}{\delta^k}\left[1 - \mu\Phi\left(\bar{\omega}_{ss}\right)\right]\left(\frac{K^y}{L^y}\right)_{ss}^{-\nu} - 1\right) w_{ss} - \left(\frac{1}{\varrho} - 1\right) c_{ss} \left(1 - \frac{q_{ss}^k g(\bar{\omega}_{ss})}{R_{ss}^c} - \frac{q_{ss}^k f(\bar{\omega}_{ss})}{\beta^1}\right)}{1 - \frac{q_{ss}^k g(\bar{\omega}_{ss})}{R_{ss}^c} - \frac{q_{ss}^k f(\bar{\omega}_{ss})}{\beta^1} + \frac{1}{\beta^1}\left(\frac{A}{\delta^k}\left[1 - \mu\Phi\left(\bar{\omega}_{ss}\right)\right]\left(\frac{K^y}{L^y}\right)_{ss}^{-\nu} - 1\right)} \tag{A.58}$$

Given C_{ss}^e, equations (A.32), (A.39), (A.40) and (A.41) together solve for

$$N_{ss} = \frac{\beta^1 w_{ss} - C_{ss}^e}{\beta^1 - \frac{f(\bar{\omega}_{ss})}{\frac{1}{q_{ss}^k} - \frac{g(\bar{\omega}_{ss})}{R_{ss}^c}}} \tag{A.59}$$

$$I_{ss} = \frac{N_{ss}}{1 - \frac{q_{ss}^k g(\bar{\omega}_{ss})}{R_{ss}^c}} \tag{A.60}$$

$$Z_{ss} = \varrho \left[f\left(\bar{\omega}_{ss}\right) I_{ss} - \frac{C_{ss}^e}{q_{ss}^k}\right] \tag{A.61}$$

$$K_{ss}^y = \frac{\varrho}{\delta^k} I_{ss} \left[1 - \mu\Phi\left(\bar{\omega}_{ss}\right)\right]. \tag{A.62}$$

Then, given $\left(\left(K^y/L^y\right)_{ss}, K_{ss}^y\right)$, we have

$$L_{ss}^y = K_{ss}^y / \left(\frac{K^y}{L^y}\right)_{ss}. \tag{A.63}$$

Given $k_2^{ss} = 0$, equation (A.35) implies

$$k_1^{ss} = \frac{K_{ss}^y - Z_{ss}}{(1-\varrho)\,\alpha}. \tag{A.64}$$

Equations (A.19) and (A.38) yield

$$\begin{aligned} m_2^{ss} &= \xi q_{ss}^h h_2^{ss} \frac{\beta^1}{R_{ss}^c} && \text{(A.65)} \\ d_1^{ss} &= \frac{\varrho\,(I_{ss} - N_{ss}) + (1-\varrho)\,(1-\alpha)\,m_2^{ss}}{\left(1-\bar{R}\right)(1-\varrho)\,\alpha}. && \text{(A.66)} \end{aligned}$$

It is straightforward to obtain the following from market-clearing conditions and the binding reserve constraint:

$$\begin{aligned} Y_{ss} &= A\,(L_{ss}^y)^{\nu}\,(K_{ss}^y)^{1-\nu} && \text{(A.67)} \\ D_{ss} &= (1-\varrho)\,\alpha d_1^{ss} && \text{(A.68)} \\ C_{ss} &= \varrho\,(I_{ss} - N_{ss}) && \text{(A.69)} \\ CA_{ss} &= (1-\varrho)\,(1-\alpha)\,m_2^{ss} && \text{(A.70)} \\ S_{ss} &= \bar{R} D_{ss}. && \text{(A.71)} \end{aligned}$$

Given (A.71), equation (1) implies

$$\left(\frac{B}{p}\right)_{ss} = \frac{R^b - 1}{\pi} S_{ss} \tag{A.72}$$

and equation (2) yields

$$\left(\frac{M}{p}\right)_{ss} = \frac{R_{ss}^g}{\pi - 1}\left(\frac{B}{p}\right)_{ss} = \frac{\pi}{\beta^1\,(\pi-1)}\left(\frac{B}{p}\right)_{ss}. \tag{A.73}$$

Thus,

$$b_1^{ss} \quad = \quad \frac{\left(\frac{B}{p}\right)_{ss}}{(1-\varrho)\,\alpha}. \tag{A.74}$$

Note that $\left(\frac{B}{p}\right)_{ss} \geq 0$ requires

$$R^b \geq 1. \tag{A.75}$$

Then the households' budget constraints (A.20) and (A.21) can be used to solve for

$$l_1^{ss} \quad = \quad \frac{1}{w_{ss}}\left[c_{ss} + q_{ss}^h \delta^h h_1^{ss} - \left(r_{ss}^k - q_{ss}^k \delta^k\right) k_1^{ss} - \left(\frac{1}{\beta^1} - 1\right)\left(d_1^{ss} + b_1^{ss}\right)\right] \tag{A.76}$$

$$l_2^{ss} \quad = \quad \frac{1}{w_{ss}}\left[c_{ss} + q_{ss}^h \delta^h h_2^{ss} - \left(1 - \frac{R_{ss}^c}{\beta^1}\right) m_2^{ss}\right]. \tag{A.77}$$

Step 3. All of the steady state variables on the left-hand sides of equations (A.43) to (A.77) are functions of π, the inflation rate. Therefore, the steady-state system boils down to one equation with one unknown, which allows us to solve for π_{ss} from the labor-market-clearing equation (56). Therefore, there exists a unique steady state of money and banking with a binding reserve constraint if and only if there exists a unique solution to equation (56) that also satisfies the existence conditions specified in the next subsection.

A.5 Existence conditions for a steady state with $\lambda^B > 0$

Equations (A.42) to (A.77) give a list of solutions to all steady state variables except for π_{ss}. For the existence of a steady state of money and banking with $\lambda^B > 0$, the solution to equation (56), π_{ss}, must also be such that $R_{ss}^d \geq 1$ and

$$\lambda_{ss}^B, \Omega\left(\bar{\omega}_{ss}\right), f\left(\bar{\omega}_{ss}\right), q_{ss}^k, r_{ss}^k, \lambda_2^{ss}, h_2^{ss}, K_{ss}^y, C_{ss}^e, N_{ss}, I_{ss}, Z_{ss}, k_1^{ss}, C_{ss}, l_1^{ss}, l_2^{ss}, \left(\frac{M}{p}\right)_{ss} > 0. \tag{A.78}$$

First, $R_{ss}^d \geq 1$ requires

$$\pi_{ss} \geq \beta^1. \tag{A.79}$$

Next, $\lambda_{ss}^B > 0$ requires

$$\pi_{ss} > \beta^1 R^b. \tag{A.80}$$

Given that R_{ss}^c is a function of π as expressed by (A.45), $\Omega\left(\bar{\omega}_{ss}\right) > 0$ and $q_{ss}^k > 0$ together require that π_{ss} satisfy

$$1 - \frac{1 - \mu\Phi\left(\bar{\omega}_{ss}\right)}{f\left(\bar{\omega}_{ss}\right)} < \frac{\beta^e}{\beta^1} R_{ss}^c\left(\pi_{ss}\right) < 1. \tag{A.81}$$

Then $\lambda_2^{ss} > 0$ and $h_2^{ss} > 0$ together require π_{ss} satisfy

$$0 < \frac{\beta^1}{R_{ss}^c\left(\pi_{ss}\right)} - \beta^2 < \frac{1}{\xi}\left[1 - \beta^2\left(1 - \delta^h\right)\right]. \tag{A.82}$$

Given (A.47), $f\left(\bar{\omega}_{ss}\right) > 0$ and $r_{ss}^k > 0$ together require

$$\int_{\bar{\omega}_{ss}(\pi_{ss})}^{\infty} \omega d\Phi\left(\omega\right) - \bar{\omega}_{ss}\left(\pi_{ss}\right)\left[1 - \Phi\left(\bar{\omega}_{ss}\left(\pi_{ss}\right)\right)\right] > 0 \tag{A.83}$$

$$\frac{1}{\beta^1} - \left(1 - \delta^k\right) > 0. \tag{A.84}$$

Furthermore, $K_{ss}^y > 0$ always holds given $\mu \in (0,1)$ and $\Phi(\cdot)$ is a CDF. In addition, π_{ss} must also be such that

$$C_{ss}^e\left(\pi_{ss}\right), N_{ss}\left(\pi_{ss}\right), I_{ss}\left(\pi_{ss}\right), Z_{ss}\left(\pi_{ss}\right), k_1^{ss}\left(\pi_{ss}\right), C_{ss}\left(\pi_{ss}\right), l_1^{ss}\left(\pi_{ss}\right), l_2^{ss}\left(\pi_{ss}\right) > 0. \tag{A.85}$$

Hence, we have Properties II of Theorem 1.

A.6 Algorithm for solving the steady state with $\lambda^B = 0$

Similar to the previous one, this section describes an analytical algorithm for fully characterizing the steady-state variables given $\lambda_{ss}^B = 0$. First, given $\lambda_{ss}^B = 0$ equation (55) implies $\frac{\pi}{\beta^1} R^c = R^m = R^d = R^g = R^b$. Moreover, (A.5) yields $R_{ss}^d = \frac{\pi}{\beta^1}$. These two sets of results together imply

$R^c_{ss} = 1$, $R^d_{ss} = R^g_{ss} = R^m_{ss} = R^b$, and $\pi_{ss} = \beta^1 R^b$. Thus we have

$$\left(\frac{B}{p}\right)_{ss} = \frac{R^b - 1}{\beta^1 R^b} S_{ss} \tag{A.86}$$

$$\left(\frac{M}{p}\right)_{ss} = \frac{R^b}{\beta^1 R^b - 1} \left(\frac{B}{p}\right)_{ss}. \tag{A.87}$$

Moreover, recall that both conditions (A.4) and (A.18) are derived for all $\lambda^B \geq 0$. Thus we still have $d_2^{ss} = b_2^{ss} = k_2^{ss} = 0$ in this case of $\lambda^B = 0$. Hence, we have Property I of Theorem 1.

Now suppose $\lambda_1 > 0$ given $\lambda^B = 0$. Then (A.3) implies that $1 < 1 + \lambda_1 c_1 \pi / \beta_1$ and that $m_1 = 0$. However, $\lambda_1 > 0$ and $m_1 = 0$ together contradict the collateral constraint (15). Therefore, it must be the case that $\lambda_1 = 0$, which then implies $m_1 > 0$ is given by (A.90) according to (15). Given $R^c_{ss} = 1$, it is straightforward to check that the following steady-state variables can still be solved by the exact same equations as those listed in Section A.4:

$$\{\bar{\omega}_{ss}, q^k_{ss}, r^k_{ss}, \left(\frac{K^y}{L^y}\right)_{ss}, c_{ss}, w_{ss}, q^h_{ss}, \lambda_2^{ss}, h_1^{ss}, h_2^{ss}, H_{ss}, \tag{A.88}$$

$$L^h_{ss}, K^y_{ss}, L^y_{ss}, C^e_{ss}, N_{ss}, I_{ss}, Z_{ss}, m_2^{ss}, k_1^{ss}, b_1^{ss}, Y_{ss}, C_{ss}, l_1^{ss}, l_2^{ss}\}. \tag{A.89}$$

Moreover, the solutions to the following variables revise to:

$$m_1^{ss} = \beta^1 \xi \left(q^k k_1^{ss} + q^h h_1^{ss}\right) \tag{A.90}$$

$$CA_{ss} = (1 - \varrho) \left[\alpha m_1^{ss} + (1 - \alpha) m_2^{ss}\right] \tag{A.91}$$

$$d_1^{ss} = \frac{\varrho (I_{ss} - N_{ss}) + CA_{ss} + S}{(1 - \varrho) \alpha} \tag{A.92}$$

$$l_1^{ss} = \frac{1}{w_{ss}} \left[c_{ss} + q^h_{ss} \delta^h h_1^{ss} - \left(r^k_{ss} - q^k_{ss} \delta^k\right) k_1^{ss} - \left(\frac{1}{\beta^1} - 1\right) \left(d_1^{ss} + b_1^{ss} - m_1^{ss}\right)\right] \tag{A.93}$$

$$l_2^{ss} = \frac{1}{w_{ss}} \left[c_{ss} + q^h_{ss} \delta^h h_2^{ss} - \left(1 - \frac{1}{\beta^1}\right) m_2^{ss}\right]. \tag{A.94}$$

Finally, solving the steady state with $\lambda^B = 0$ boils down to solving the bank reserve, S_{ss}, from the labor-market-clearing condition:

$$L^y_{ss} + L^h_{ss} = (1-\varrho)\left[\alpha l_1^{ss}(S) + (1-\alpha) l_2^{ss}\right] + \varrho,$$

where l_1^{ss} is a function of S because d_1^{ss} and b_1^{ss} are through (A.74) and (A.92). The existence of this steady state requires $S_{ss} > \bar{R} D_{ss}$. Moreover, since $l_1^{ss}(S)$ is a linear function, the solution S_{ss} exists and is unique. Thus the steady state with $\lambda^B = 0$ exists and is unique, provided that $S_{ss} > \bar{R} D_{ss}$ and all steady-state variables listed in (A.88) to (A.89) are strictly positive.

Note that in this steady state with $\lambda^B = 0$, reserve requirement $\bar{R}$ is ineffective because the reserve constraint does not bind. Moreover, the reserve rate R^b, in this case, only affects the nominal interest rates (R^d, R^m) and the real values of reserves, money and nominal bonds, *i.e.*, S_{ss}, $\left(\frac{B}{p}\right)_{ss}$ and $\left(\frac{M}{p}\right)_{ss}$. Note that none of $\left(S_{ss}, \left(\frac{B}{p}\right)_{ss}, \left(\frac{M}{p}\right)_{ss}\right)$ affects the rest of the real economy. Therefore, R^b has no meaningful real effect. Hence we have the Property III of Theorem 1.