A proposal to solve some puzzles in charmed semileptonic $B$ decays


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I. Introduction: Summary of the exp. and theo. situation
   a Recap of incl. and excl. measurements
   b Recap of the '1/2' vs '3/2' problem
II. Discovery of potential 2S charmed state(s) by BABAR
III. Our Proposal and its Viability
IV. Prediction of $\Gamma(B \rightarrow D'(\ast) \ell \bar{\nu}_\ell)$ using light-cone sum rules
V. Summary
I.a Experimental situation for $B \to X_c \ell \bar{\nu}_\ell$

- $BABAR$ and $Belle$: 1.1 ab$^{-1}$ at $\Upsilon(4S)$
- $\approx 25\%$ of all $B$ decay semileptonic

- Most abundant $b \to c$: $\mathcal{B}(B^+ \to X_c \ell^+ \nu_\ell) = (10.92 \pm 0.16)\%$
  
  $X_c$: charmed system; isospin averaged value from [HFAG]

- Major focus of experimental attention from $B$ factories
- Inclusive $X_c$ mass spectrum (not unfolded; $p_i^* > 0.8$) [PRD81:032003]
- Presence of charm decays up to $\approx 3$ GeV (resolution 0.36 GeV)
  
- $D^{(*)}, D^{**}, D'(*) \leftrightarrow 1S, 1P, 2S$
I.a Experimental situation for $B \to X_c \ell \bar{\nu}_\ell$

<table>
<thead>
<tr>
<th>Charm state $X_c$</th>
<th>$B(B^+ \to X_c \ell^+ \nu)$</th>
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<tbody>
<tr>
<td>$D$</td>
<td>$(2.31 \pm 0.09)$ %</td>
</tr>
<tr>
<td>$D^*$</td>
<td>$(5.63 \pm 0.18)$ %</td>
</tr>
<tr>
<td>$\sum D^{(*)}$</td>
<td>$(7.94 \pm 0.20)$ %</td>
</tr>
</tbody>
</table>

$D_0^* \to D \pi$ | $(0.41 \pm 0.08)$ %         |
$D_1^* \to D^* \pi$ | $(0.45 \pm 0.09)$ %         |
$D_1 \to D^* \pi$ | $(0.43 \pm 0.03)$ %         |
$D_2^* \to D^{(*)} \pi$ | $(0.41 \pm 0.03)$ %         |
$\sum D^{**} \to D^* \pi$ | $(1.70 \pm 0.12)$ %         |

$D \pi$ | $(0.66 \pm 0.08)$ %         |
$D^* \pi$ | $(0.87 \pm 0.10)$ %         |
$\sum D^* \pi$ | $(1.53 \pm 0.13)$ %         |

$\sum D^{(*)} + \sum D^* \pi$ | $(9.47 \pm 0.24)$ %         |
$\sum D^{(*)} + \sum D^{**} \to D^{(*)} \pi$ | $(9.64 \pm 0.23)$ %         |

Inclusive $X_c$ | $(10.92 \pm 0.16)$ %

All values from [HFAG 2010]. For the values of $B \to D \pi \ell \bar{\nu}_\ell$ and $B \to D^* \pi \ell \bar{\nu}_\ell$ an uncertainty weighted average of both isospin modes was calculated assuming a 100% correlation between both values.

⇒ 'Gap' of $(1.45 \pm 0.29)$ % emerges which is not accounted for

Uses semi-inclusive $D^{(*)} \pi$ branching fractions; with measured $1P ~ D^{**} \to D^{(*)} \pi \Rightarrow (1.28 \pm 0.29)$ %
I.b Theoretical situation of $B \rightarrow X_c \ell \bar{\nu}_\ell$

- Comparable rates for the narrow and broad $D^{**}$ states problematic:

<table>
<thead>
<tr>
<th>State</th>
<th>Branching Fraction</th>
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<tbody>
<tr>
<td>$D_0^* \rightarrow D \pi$</td>
<td>$(0.41 \pm 0.08)$%</td>
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</tr>
</tbody>
</table>

\{ broad states $(0.86 \pm 0.12)$% \}
\{ narrow states $(0.84 \pm 0.04)$% \}

- Uraltsev’s sum rule + covariant quark model estimate from [EPJ:C52975]

$$\mathcal{B}(B^+ \rightarrow D^{**}_{1/2=\text{broad}} \ell^+ \nu) / \mathcal{B}(B^+ \rightarrow D^{**}_{3/2=\text{narrow}} \ell^+ \nu) \sim 0.1 - 0.2$$

i.e. clear dominance of narrow over broad.

- Experimental violation known as '1/2' vs '3/2' puzzle.

- Persistent $\sim 2 - 3\sigma$ difference between $|V_{cb}|$ from inclusive vs exclusive

$$\left( 41.9 \pm 0.4_{\text{exp.}} \pm 0.6_{\text{theo.}} \right) \times 10^{-3} \quad \text{vs} \quad \left( 38.7 \pm 0.6_{\text{exp.}} \pm 0.5_{\text{theo.}} \right) \times 10^{-3}$$

[ARNPS:201161119]

- Any connections?

\begin{center}
\begin{tikzpicture}
\node (vcb_inclusive) at (0,0) {$|V_{cb}|$ inclusive};
\node (vcb_exclusive) at (3,0) {$|V_{cb}|$ exclusive};
\node (gap_inclusive) at (1.5,1.5) {gap inclusive vs exclusive};
\node (1P_states) at (0.5,0.5) {measured 1P states};
\node (1/2_vs_3/2) at (1.5,0) {'1/2' vs '3/2' Problem};
\end{tikzpicture}
\end{center}
II. Discovery of new charmed states at $\text{BABAR}$

- $\text{BABAR}$ observed four new charmed states [PRD82:111101]:

<table>
<thead>
<tr>
<th>Notation</th>
<th>$m$ (GeV)</th>
<th>$\Gamma$ (GeV)</th>
<th>Sig.</th>
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<tbody>
<tr>
<td>$D(2550)^0$</td>
<td>$2.54 \pm 0.01$</td>
<td>$0.130 \pm 0.018$</td>
<td>$3\sigma$</td>
</tr>
<tr>
<td>$D(2600)^0$</td>
<td>$2.61 \pm 0.01$</td>
<td>$0.093 \pm 0.014$</td>
<td>$7\sigma$</td>
</tr>
<tr>
<td>$D(2750)^0$</td>
<td>$2.75 \pm 0.01$</td>
<td>$0.071 \pm 0.013$</td>
<td>$4\sigma$</td>
</tr>
<tr>
<td>$D(2760)^0$</td>
<td>$2.76 \pm 0.01$</td>
<td>$0.061 \pm 0.006$</td>
<td>$9\sigma$</td>
</tr>
</tbody>
</table>

The $m_{D^*\pi}$ mass distribution for $D(2550)$, $D(2600)$, $D(2750)$ is shown. $D(2760)$ reconstructed in $D\pi$ channel.

- Helicity angles of $D(2550)$ and $D(2600)$ helicity consistent with $2S$:

Helicity angle from $D^* \rightarrow D\pi$ is defined as the angle between the primary pion and $\pi_{\text{slow}}$ from $D^* \rightarrow D\pi_{\text{slow}}$ (in the $D^*$ rest frame)

- $D(2750)$ candidate for $1D$ (Likely no relevant for semileptonic decays $\rightarrow$ cf. Backup)
II. Strong decays of $2S$ states $D'$ & $D'^*$

- Strong $D'$ and $D'^*$ decays:

$$2S \rightarrow 1S$$

or

$$2S \rightarrow 1P \rightarrow 1S$$

E.g. $p$-wave $+ \pi \rightarrow 1S$

$s$-wave $+ \pi \rightarrow 1P_{\text{broad}} \ (\rightarrow 1S)$

Mom. of the emitted pion $p_\pi \sim 0.01 - 0.5 \text{ GeV}$

$s$-wave $+ \pi\pi \rightarrow 1S$

$d$-wave $+ \pi \rightarrow 1P_{\text{narrow}} \ (\rightarrow 1S)$

Signature $s$-wave: $D'(\ast) \rightarrow D(\ast)\pi\pi$

Signature $p$-wave: $D'(\ast) \rightarrow D(\ast)\pi$

More decays involving $\rho$ and $\eta$ in principle allowed

- Significant $2S \rightarrow 1P_{\text{broad}}$ cross

feed plausible [PRD:83014009]
III. Our Proposal and its Viability

Proposal  
Explore possibility that the sum of $D'(\ast)$ rate is substantial,
\[
\mathcal{B}(B^+ \to D'(\ast) \ell^+ \nu_\ell) \sim \mathcal{O}(1\%) 
\]
and show that this can help resolve the problems mentioned earlier without giving rise to new ones.

1. This is a big enough contribution to the sum over exclusive states to close the gap between inclusive and exclusive without e.g. introducing non-resonant $B^+ \to D(\ast) \pi \ell^+ \nu_\ell$ contributions. A large non-resonant rate at high $D^* \pi$ invariant mass would disagree with the inclusive lepton spectrum and the measured semi-exclusive $B^+ \to D(\ast) \pi \ell^+ \nu_\ell$ rate.

2. The $D'(\ast)$ states can decay with one pion in an $s$-wave to members of the $s_\pi^I = \frac{1}{2}^+$ states, and could thus enhance the observed decay rate to the $\frac{1}{2}^+$, and thus give rise to the '1/2' vs '3/2' puzzle.

3. With the relatively low mass of the $D'(\ast)$ the lepton spectrum can stay quite hard, in agreement with the observations.

4. The $\mathcal{B}(B^+ \to D^* \pi \ell^+ \nu_\ell)$ semi inclusive measurement is not in conflict with our hypothesis, since the decay of the $D'(\ast)$ would yield two or more pions most of the time.

IV. Prediction for \( \Gamma(B^+ \rightarrow D'(\ast) \ell^+ \nu_\ell) \)

- \( D'(\ast) \) and \( D(\ast) \): identical quantum numbers
  i.e. same formulae for decay rate and definitions of form factors

\[
\frac{d\Gamma_{D'\ast}}{dw} = \frac{G_F^2 |V_{cb}|^2 m_B^5}{48\pi^3} r^3 (1 - r)^2 \sqrt{w^2 - 1} (w + 1)^2 \times \left[ 1 + \frac{4w}{w + 1} \frac{1 - 2w + r^2}{(1 - r)^2} \right] [F(w)]^2,
\]

where \( r = m_{D'(\ast)}/m_B \) and \( w = v \cdot v' \) denotes the recoil parameter, where \( v \) denotes the velocity of the \( B \) meson, and \( v' \) of the \( D'(\ast) \).

- In \( m_{b,c} \gg \Lambda_{QCD} \) limit: 6 form factors \( \rightarrow \) single universal Isgur-Wise function \( \zeta_2(w) \)
  i.e. \( F(w) = G(w) = \zeta_2(w) \)

- Heavy quark symmetry: \( \zeta_2(w = 1) = 0 \)
  \( \rightarrow \) Non-zero rate at zero recoil entirely due to \( \Lambda_{QCD}/m_{b,c} \) corrections

- For \( w > 1 \) no power suppression, but low kinematic range of \( 1 < w < 1.3 \)
  role of \( \Lambda_{QCD}/m_{b,c} \) corrections can be very large.

- Naive expectation: \( \frac{d\zeta_2}{dw} \bigg|_{w=1} > 0 \) In quark model main effect of wave function of the brown muck is to increase the expectation value of the distance from the heavy quark of a spherically symmetric wave function. Overlap of initial and final state wave functions should increase as \( w \) increases.
IV. The $B^+ \rightarrow D'(\ast) \ell^+ \nu_\ell$ form factors

Not easy to calculate the $B^+ \rightarrow D'(\ast) \ell^+ \nu_\ell$ form factors:

a. Quark model [PRD:62:014032] hoped to be trustable near $w = 1$,

b. Modify QCD light-cone sum rule calculation [PJC:60603] hoped to be reasonable near max. recoil

But Both models were developed, tuned, and tested for states that are the lightest within a given set of quantum numbers, thus take prediction with truck load of salt. But even rough estimates can be helpful!

Quark Model form factors at $w = 1$ and linear extrapolation to $w = 1.05$:

![Diagram showing ansatz for the $B \rightarrow D'\ell\nu_\ell$ Isgur-Wise function with linear extrapolation to $G(w=1.05)$ and $F(w=1.05)$ from [PRD:62:014032].]

- Quark model predictions
- QCD Sum rule prediction
- Quadratic Isgur-Wise function
- Linear Isgur-Wise function

Linear extrapolation to $G(w=1.05)$ from [PRD:62:014032]

Linear extrapolation to $F(w=1.05)$ from [PRD:62:014032]
IV. The $B^+ \rightarrow D'(\ast) \ell^+ \nu_\ell$ form factors

Modify QCD light-cone sum rules so that the $2S$ state can be projected out

e.g. schematically for the decay constant

$$\begin{align*}
\frac{m_D^4 f_D^2}{m_c^2 (m_D^2 - q^2)} &+ \frac{m_D'^4 f_D'^2}{m_c^2 (m_D'^2 - q^2)} + \int_{s_D'}^\infty ds \frac{\rho(s)}{s - q^2}.
\end{align*}$$

where $\rho$ is the spectral density function, and $f_D$ and $f_D'$ denote the $1S$ and $2S$ decay constant, respectively.


Form factors sensitive fo chosen decay constants, Borel, and duality parameters

Effect of variation on duality and Borel parameters in calculation
IV. Prediction for $\Gamma(B^+ \to D'(\ast) \ell^+ \nu_\ell)$

Parametrize $F(w)$ and $G(w)$ which determine the $D'(\ast)$ as quad. polynom. i.e.

\[
\begin{align*}
F(w) &= \beta_0^* + (w - 1)\beta_1^* + (w - 1)^2 \beta_2^*, \\
G(w) &= \beta_0 + (w - 1)\beta_1 + (w - 1)^2 \beta_2.
\end{align*}
\]

$\Rightarrow$ Rough estimate for sum of two semileptonic $B^+ \to D'(\ast) \ell^+ \nu_\ell$ decays:

\[
\mathcal{B}(B^+ \to D'(\ast) \ell^+ \nu_\ell) \sim (0.3 - 0.7) \%
\]

Earlier quark models without accounting for $\Lambda_{\text{QCD}}/m_{b,c}$ effects obtained smaller rates, c.f. [PRD:39799],[PTP:91757]. Including $\Lambda_{\text{QCD}}/m_{b,c}$ effects a value of 0.4% was obtained by [PRD:62:014032].

With a linear parametrization and the quark model result only:

\[
\mathcal{B}(B^+ \to D'(\ast) \ell^+ \nu_\ell) \sim 1.4 \%
\]

We take this as an indication that a large radial contribution is plausible, and that $B^+ \to D'(\ast) \ell^+ \nu_\ell$ may account for a substantial part of the observed 'Gap' between inclusive and exclusive decays.
- **Indication** that hypothesis plausible and that $B \rightarrow D'(\ast) \ell \bar{\nu}_\ell$ may account for a substantial part of the observed 'gap'.

- **Interesting measurement for LHCb (or B-factories):** $B \rightarrow D'(\ast) \pi = [D(\ast) \pi^+ \pi^-] \pi^-$

  Factorization [PRL:87201806] implies relation between these channels and semileptonic decay rate at $w_{\text{max}}$:

  $$
  \Gamma(B \rightarrow D'(\ast) \pi) = \frac{3\pi^2 C^2 |V_{ud}|^2 f^2}{m_B m_{D'(\ast)}} \left. \frac{d\Gamma(B \rightarrow D'(\ast) \ell \bar{\nu}_\ell)}{dw} \right|_{w=w_{\text{max}}}
  $$

  $C$ combination of Wilson coefficients with $C |V_{ud}| \approx 1$, and $w_{\text{max}}$ corresponds to $q^2 = 0 \leq m_{\pi}^2$

- **If future measurement find a $B \rightarrow D'(\ast) \ell \bar{\nu}_\ell$ decay rate ...**

  the precise determination of the branching fraction and form factors would impact other measurements and the theory of semileptonic decays, e.g. it may yield a better understanding . . .

  i. . . . of $b \rightarrow c$ backgrounds and improve $|V_{ub}|$ and $|V_{cb}|$

  ii. . . . missing exclusive contributions to **inclusive** $B \rightarrow X_c \ell \bar{\nu}_\ell$

  iii. . . . of the measured $B \rightarrow D(\ast) \tau \bar{\nu}_\tau$ and its tension with the SM

Further

  iv. Help improve the measurements of the semileptonic branching fractions of the $s_i^{\pi} = \frac{1}{2}$ and $\frac{3}{2}$ states, thus maybe help resolving the '1/2' vs '3/2' puzzle

  v. Help improve the sum rule bound on the $B \rightarrow D^* \ell \bar{\nu}_\ell$ form factor.

**Thank you for your attention!**
Backup
A. Prediction for $1D$ from QCD sum rules

- QCD sum rule result of [PRD:79034025] suggests that $1D$ contributions to the inclusive semileptonic decay rate are small

<table>
<thead>
<tr>
<th>Decay</th>
<th>PRD:79034025</th>
<th>PLB:478408</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \rightarrow D_{1}^* \ell \bar{\nu}_\ell$</td>
<td>$6 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow D_{2}' \ell \bar{\nu}_\ell$</td>
<td>$6 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow D_{2} \ell \bar{\nu}_\ell$</td>
<td>$1.5 \times 10^{-4}$</td>
<td>$1 \times 10^{-5}$</td>
</tr>
<tr>
<td>$B \rightarrow D_{3}^* \ell \bar{\nu}_\ell$</td>
<td>$2.1 \times 10^{-4}$</td>
<td>$1 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

The branching fractions for the four $1D$ states are quoted. Note that the $D_{1}^*$ is not identical with the $1P$ state with the same name (which is sometimes denoted as $D_{1}'$ to avoid this confusion)

The Isgur-Wise functions for the $\frac{3}{2}$ and the $\frac{5}{2}$ $1D$ doublets as a function of the recoil param. $y (= w)$ are shown.