Phys 410 – Topics in Mathematical Physics I

Course Outline: Sept-Dec 2022

Instructor: Dr. Mark Laidlaw
Office: Elliott 106
Phone: 721-7701
Email: laidlaw@uvic.ca

Lectures: 8:30 – 9:30 TWF, ELL 160
Office Hours: TBD, they will be posted on the Bright-Spaces site. Predominantly via Zoom. If my door is open, feel free to knock. If I'm busy/hiding, it will be closed.

In this course I will discuss mathematical techniques used to solve physical problems with you. The goal of this course is to help you make connections between some of your preparatory math work and your current and future physics courses.

I assume that you have taken the following math courses:

- MATH 211 – Matrix Algebra (and perhaps MATH 312 Linear Algebra)
- MATH 342 and Math 346 – Differential Equations
- MATH 301 – Calculus of complex variables

The mathematical techniques covered in this course will be particularly relevant to the following physics courses:

- PHYS 321A, 321B – Classical Mechanics
- PHYS 323, 423 – Quantum Mechanics
- PHYS 326, 422 – Electromagnetism
- PHYS 415 – General Relativity

This course will cover a good deal on the theory and applications of linear algebra. We will discuss transformations of coordinate systems. This will lead us to explore tensors and, more generally, representations of groups. The example of curvilinear coordinate systems will force us to consider how differential operators transform. From there we will consider how to solve differential equations. A differential equation of extraordinary interest will be the Sturm-Liouville equation, and we will discuss how to solve it and show that a broad, useful class of functions can be seen as a vector. This insight is the basis for orthogonal expansions of functions such as Fourier Analysis. We will develop a number of techniques including variational calculus.

Throughout the course physical examples and problems will be interposed; I will try and draw them broadly, however my interests will skew my choices of examples.

References:

There is no required text for this course. Content will be drawn heavily from
Arfken & Weber, *Mathematical Methods for Physicists* 6th ed. Elsevier.
Lea, *Mathematics for Physicists* Thompson Brooks Cole.
Courant & Hilbert, *Methods of Mathematical Physics*
Other useful references include:
Butkov, *Mathematical Physics* Addison Wesley.
Dennery & Krzwicki, *Mathematics for Physicists* Dover.
Landau & Lifshitz, *The Classical Theory of Fields,* Butterworth Heinemann.
Misner, Thorne & Wheeler, *Gravitation*, Freeman.
Jones, *Groups, Representations and Physics,* Institute of Physics.
Gasiorowicz, *Quantum Physics,* Wiley.
Griffiths, *Introduction to Quantum Mechanics,* Pearson.
Ahlfors, *Complex Analysis,* McGraw-Hill.

Marking Scheme:

There will be four assessed components of the course:

1. Final exam in the December exam period
2. Written project and associated presentation
3. Midterm exam tentatively scheduled for Oct 21
4. Approximately weekly assignments

The final exam and the written project and presentation are essential course elements.
If you do not write the final exam, or if you do not both submit the project and do the presentation you will be assigned the failing grade "N".

My grading philosophy:

The university has narrative descriptions of the meanings of various letter grades. It can be found in the calendar. A-range and B-range grades are earned by students who are fully engaged with the course; participating, doing the assigned work well, and demonstrating competence with or mastery of the new material covered in class. A's are distinguished from B's by the consistent quality of their work. Passing grades are earned by either full engagement with the course but marginal quality work, or by inconsistent engagement with adequate quality. Failing grades are earned by inadequate quality work, inadequate engagement, or both. The description that follows is an attempt to codify this animating idea.

Individual questions are marked on a 5-step scale:

4 – indicates a question that is fully, correctly, and elegantly solved.

3 – indicates a question that is correctly solved, with at most minor defects in the mathematical exposition, and may be poorly explained.

2 – indicates a question where a substantial and discipline-appropriate attempt was made to solve the question. Mathematical errors do not undermine the marker's assessment that the student knows what to do. Physics errors are subtle.

1 – indicates a question which has significant mathematical or conceptual errors.

0 – indicates a question which was not submitted or which did not receive a meaningful attempt.

In each of the assessed components of the course you will be assigned a mark A*, A, B, P, F. These marks are ordered, with A* the best, and F the worst. If your minimum mark is A or higher you will be assigned an A-range grade, if your minimum mark is B you will be assigned a B-range grade, if your minimum mark is P you will be assigned a C or D-range grade, and if your minimum mark is F you will be assigned an F.

Final exam: Details of the exact assessment method for the final exam will be available prior to the exam. It is a Physics exam so answering more questions better will result in a higher assessed grade.

Midterm exam: Details of the exact assessment method for the midterm will be available prior to the exam. If your final exam grade is better than your midterm grade or you do not write the midterm the final exam grade will replace it with the exception that an A* on the final exam will result in a maximum of A on the midterm.

Assignments: A requirement for the grade B is that essentially all questions are assessed at the level 2 or better with many at the level 3. Better performance results in higher grades. A requirement for P is a minimum of 60% of the questions assessed at level 2 or better.

Project: In this course you are going to choose a mathematical physics topic to research and learn about. You are going to write a summary which explains why the topic is interesting, the key mathematical pieces of the derivation, and develop some assignment and exam-style questions to supplement the material. You will also give an approximately 15-20 min lecture-style presentation.
Key timelines:

- Week of Sept 9 – initial question formulation
- Week of Sept 23 – sources due
- Week of Oct 7 – initial summary and question refinement exercise
- Week of Oct 28 – draft due
- Week of Nov 11 – final paper due
- Nov 15 – 30 – presentations

Detailed rubrics will be available as the milestones come up.

Tentative Lecture Schedule:

Vectors and rigid coordinate transformations. Tensors. Representations, tensor rank, and addition of angular momentum. Curvilinear coordinates. Differential operators and connections. Series solutions of differential equations and Sturm-Liouville problem. Approximation techniques.