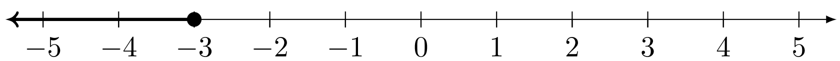
1.5 Graphing intervals on the real line

We can represent intervals of real numbers graphically on the real line by shading in the relevant portions. A **filled-in circle** indicates that an endpoint is included, an **empty circle** indicates that the endpoint is not included, and an **arrow** indicates that the interval extends forever in that direction. When an interval is expressed in the form $\{x : x > a\}$, $\{x : x < a\}$, $\{x : x \geq a\}$, or $\{x : x \leq a\}$, this is straightforward.

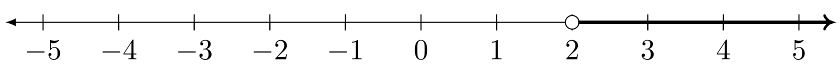
Example 4

The interval $(-\infty, -3] = \{x : x \leq -3\}$, is represented by the following graph.



The filled-in circle at the point $x = -3$ indicates that -3 is included in the interval. The arrow indicates that all real numbers less than -3 are also included.

The interval $\{x : x > 2\}$ is represented by the following graph.



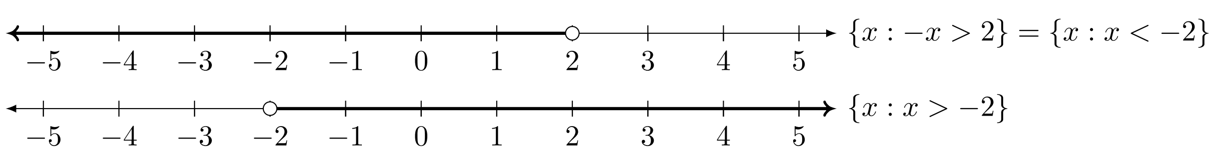
The empty circle at the point $x = 2$ indicates that 2 is not included in the interval, and the arrow indicates that all real numbers greater than 2 are included.

Dividing by a negative number. You may recall that when dividing both sides of an inequality by a negative number, the inequality switches direction. This is because dividing by a negative number corresponds to subtraction, as for example in:

$$\begin{array}{rcl} -x & > & 2 \\ -x + x & > & 2 + x \\ 0 & > & 2 + x \\ 0 - 2 & > & -2 + 2 + x \\ -2 & > & x. \end{array}$$

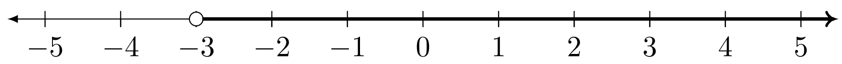
Thus, $-x > 2$ is equivalent to $x < -2$.

Graphically, the interval $\{x : -x > 2\}$ is the mirror image of the interval $\{x : x > -2\}$ on the number line.



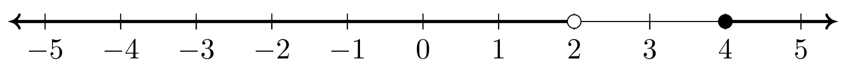
Example 5 *Represent* $\{x : -3x < 9\}$ *graphically.*

In this example, the inequality is not yet expressed in the form $x < a$ or $x > a$, and so we need first to rewrite the inequality. Dividing both sides by -3, remembering that this switches the direction of the inequality, gives us $\{x : x > -3\}$:

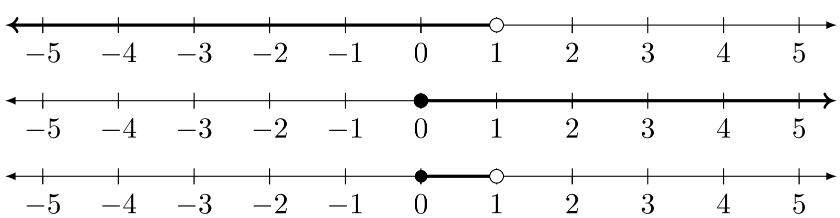


Example 6 *Representing unions and intersections graphically.*

The set $(-\infty, 2) \cup [4, \infty)$ is the union of two intervals, both of which we can represent graphically. The union is just represented by drawing both:



The set $(-\infty, 1) \cap [0, \infty)$ is the intersection of two intervals. To represent this set graphically, we draw only the portion that is common to both:



The interval $(-\infty, 1)$

The interval $[0, \infty)$

The set $(-\infty, 1) \cap [0, \infty)$

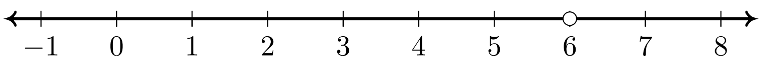
Notice that it is easy to determine from the graph that $(-\infty, 1) \cap [0, \infty) = [0, 1)$.

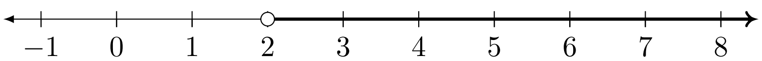
1.5.1 Practice Problems

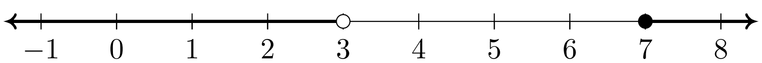
Represent each of the following sets graphically on a number line:

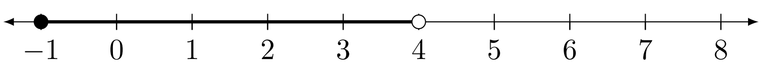
1. $x \neq 6$
2. $-5x<10$
3. $(-\infty, 3) \cup [7, \infty)$
4. $(-\infty, 4) \cap [-1, \infty)$
5. $(-\infty, 1) \cap [5, \infty)$

1.5.2 Solutions

1. 

2. 

3. 

4. 

5.