1.4 Intervals on the real line

In what follows, we assume that x and y are real numbers.

Set builder notation is a way of describing sets of real numbers that satisfy some condition: $\{x$: "some condition" $\}$ describes the set of all real numbers for which the condition is true.

For example: $\{x : -2 \leq x \leq 5\}$ represents the set of all real numbers that are greater than or equal to -2 and less than or equal to 5. This set is often described using interval notation; see below.

Interval Notation. Intervals of numbers on the real line are denoted using brackets. Different types of brackets have different meanings.

- $\big[$ and $\big]$ are used to indicate that the endpoints of the interval are included. These correspond to $\leq$ and $\geq$ when the interval is described using inequalities.

  For example, $\big[a, b\big] = \{x : a \leq x \leq b\}$. Therefore $\big[0, 9\big] = \{x : 0 \leq x \leq 9\}$. This set contains numbers such as 0, 4, π , 8.3759838459, $\frac{1}{6}$, 9, etc. There are infinitely many members of this set.

- $\big($ and $\big)$ are used to indicate that the endpoints of the interval are *not* included. These correspond to $<$ and $>$ when the interval is described using inequalities.

  For example, $\big(a, b\big) = \{x : a < x < b\}$. Therefore, $\big(2, 5\big)$ = $\{x : 2 < x < 5\}$. This set contains numbers such as 2.0000000001, 4, π, $\frac{7}{3}$ etc. There are also infinitely many members of this set.

- The two types of brackets discussed above can be used together. For example, $\big[-3, 7\big)$ = $\{x : -3 \leq x < 7\}$. This set also has infinitely members, and in particular contains -3 but not 7.

- **Note:** Since infinity is not a real number, we must always use an open bracket for infinity or negative infinity. ex $\big(-\infty, 4\big]$.

Warning: It is important to remember that not all real numbers are integers. For example, the set $\big[2, 3\big]$ is *not* equal to $\big(1, 4\big)$ – there are many numbers, such as 1.3, that are in the interval $(1, 4)$ but are not in the interval $[2, 3]$.

Combining intervals. The set $\{x : 2 < x < 5$ or $6 < x < 9\}$ is the set of all real numbers that are in the interval $(2, 5)$ *or* are in the interval $(6, 9)$. We use the $\cup$ symbol to signify the *union* of two sets. For example, $\{x : 2 < x < 5$ or $6 < x < 9\} = (2, 5) \cup (6, 10)$, and $\{x : x \neq -4\} = (-\infty, -4) \cup (-4, \infty)$.

More generally, if A and B are two sets of real numbers then their **union**, denoted $A \cup B$, is the set of all numbers that are in A, B, or both. Their **intersection**, denoted $A \cap B$, is the set of all numbers that are in both A and B.

For example, $(-\infty, 2) \cap [-1, 5]$ is the set of all real numbers x that are less than 2 *and* satisfy $-1 \leq x \leq 5$. That is, $(-\infty, 2) \cap [-1, 5] = [-1, 2)$. Another way to think about this set is as the solution set to the **system of inequalities** (or **compound inequality**) $x < 2, \;\; x \geq -1, \;\; x \leq 5$.

1.4.1 Practice Problems

Write the following expressions in set builder notation or interval notation:

1. $\big[-1, 10\big]$
2. $\{x : -7 \leq x < 5\}$
3. $\big[7, 28\big)$
4. $\{x : 2 < x < 7\}$
5. $\big(-8, 5\big)$
6. $\{x : 0 < x \leq 6\}$
7. $\{x : x > -3\}$
8. $(-\infty, 2) \cup (2, 8) \cup (8, \infty)$

1.4.2 Solutions

1. $\{x : -1 \leq x \leq 10\}$ 2. $\big[-7, 5\big)$ 3. $\{x : 7 \leq x < 28\}$ 4. $\big(2, 7\big)$ 5. $\{x : -8 < x < 5\}$
6. $\big(0, 6\big]$ 7. $(-3, \infty)$ 8. $\{x : x \neq 2$ and $x \neq 8\}$