1.2 Fractions

When **adding or subtracting fractions,** express them as proportions of a common amount (get a common denominator) and then combine them.

For fractions, $\frac{a}{b}$ and $\frac{c}{d}$, any common multiple of b and d can be used as a common denominator. For example, bd works.

Multiplying fractions corresponds to taking a portion of a portion of a number: multiply the numerators together and the denominators together.

When **dividing fractions**, remember that $\frac{1}{c/d} = \frac{d}{c}$, so that $\frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{c}{d}$ (i.e. multiply by the reciprocal of the divisor). So dividing by $\frac{c}{d}$ is the same as multiplying by $\frac{d}{c}$.

Example 1

- $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$
- $\frac{1}{2} + \frac{4}{5} = \frac{1 \cdot 5}{2 \cdot 5} + \frac{4 \cdot 2}{5 \cdot 2} = \frac{5}{10} + \frac{8}{10} = \frac{13}{10}$
- $\frac{5}{6} - \frac{3}{4} = \frac{5 \cdot 4}{6 \cdot 4} - \frac{3 \cdot 6}{4 \cdot 6} = \frac{20}{24} - \frac{18}{24} = \frac{2}{24} = \frac{1}{12}$
- $\frac{6}{5} \cdot \frac{3}{7} = \frac{6 \cdot 3}{5 \cdot 7} = \frac{18}{35}$
- $\frac{10}{3} \div \frac{7}{8} = \frac{10/3}{7/8} = \frac{10}{3} \cdot \frac{8}{7} = \frac{10 \cdot 8}{3 \cdot 7} = \frac{80}{21}$
- $\frac{8}{15} \div \frac{4}{3} = \frac{8}{15} \cdot \frac{3}{4} = \frac{8 \cdot 3}{15 \cdot 4} = \frac{24}{60} = \frac{2}{5}$

Remember that $\frac{a+b}{c} = (a+b) \cdot \frac{1}{c} = a \cdot \frac{1}{c} + b \cdot \frac{1}{c} = \frac{a}{c} + \frac{b}{c}$. For example: $\frac{4+\pi}{4} = \frac{4}{4} + \frac{\pi}{4} = 1 + \frac{\pi}{4}$. This is just the distributive rule, which sometimes leads (as in that example) to some cancellation of factors common to both the numerator and denominator. On the other hand, no cancellation is possible with $\frac{4}{4+\pi}$ because the denominator cannot be factored.

Example 2 *(a) Without using a calculator, show that* $\frac{4}{4+\pi} = \frac{1}{1+\frac{\pi}{4}}$. *(b) Use a calculator to verify that* $\frac{4}{4+\pi} \neq 1 + \cfrac{1}{\frac{\pi}{4}}$ *and that* $\frac{4}{4+\pi} \neq \frac{1}{1+\pi}$.

(a) We can factor a 4 out of the denominator: $\frac{4}{4+\pi} = \frac{4}{4\left(1+\frac{\pi}{4}\right)} = \frac{4}{4} \cdot \frac{1}{1+\frac{\pi}{4}} = \frac{1}{1+\frac{\pi}{4}}$.

(b) $\frac{4}{4+\pi} \approx 0.56$, and $1 + \frac{1}{\frac{\pi}{4}} \approx 2.27$, and $\frac{1}{1+\pi} \approx 0.24$.

Example 2(b) emphasizes that only factors common to *all terms* in both the numerator and the denominator can be cancelled.

1.2.1 Practice Problems

Evaluate and put the following expressions into the lowest terms:

1. $\frac{2}{7} + \frac{3}{7}$
2. $\frac{7}{9} + \frac{5}{6}$
3. $\frac{1}{4} - \frac{3}{5}$
4. $\frac{3}{2} \cdot \left(-\frac{5}{15}\right)$
5. $\frac{27}{16} \div \frac{3}{4}$
6. $\frac{\frac{11}{4} - \frac{5}{6}}{\frac{3}{2}}$
7. Without using a calculator, show that $\frac{3}{6+\pi} = \frac{1}{2+\frac{\pi}{3}}$.

1.2.2 Solutions

1. $\frac{5}{7}$ 2. $\frac{29}{18}$ 3. $-\frac{7}{20}$ 4. $-\frac{1}{2}$ 5. $\frac{9}{4}$ 6. $\frac{23}{18}$