

4.2 Inequalities Involving Rational Functions

In this section we discuss graphing the solution to inequalities like $y > \frac{x+1}{x-1}$ or, more generally, $y > f(x)$, where $f(x)$ is a rational function. The graph of $y = f(x)$, together with the vertical lines at the points where f is undefined, partition the plane into several regions. As before, in each region either all points satisfy the inequality or none do. Thus our method is:

- Find the domain of f .
- Sketch the graph of $y = f(x)$. Use a solid line if the points on the graph belong to the solution set, and a dashed line if not.
- Pick a test point in each region and determine the regions for which every point belongs to the solution set, then shade these regions on the graph.

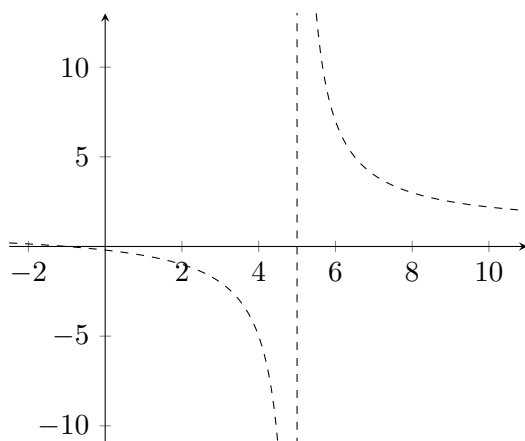
Example 45 Graph the solution set to $y < \frac{x+1}{x-5}$.

The solution set is $\{(x, y) : y < \frac{x+1}{x-5}\}$.

The domain of $f(x) = \frac{x+1}{x-5}$ is the set of all real numbers except 5.

By plotting a few well-chosen points and being careful what happens as x gets large, small, or near the vertical line $x = 5$, we arrive a sketch like the one shown below. The graph is drawn with a dashed line because no points on the graph belong to the solution set.

The vertical line $x = 5$ is included in the picture because it is involved in partitioning the plane into regions. It is drawn as a dashed line because $\frac{x+1}{x-5}$ is undefined at $x = 5$, hence $x = 5$ can not belong to the solution set.



The plane is now partitioned into 4 regions: above and below each curve, and on each side of the line $x = 5$.

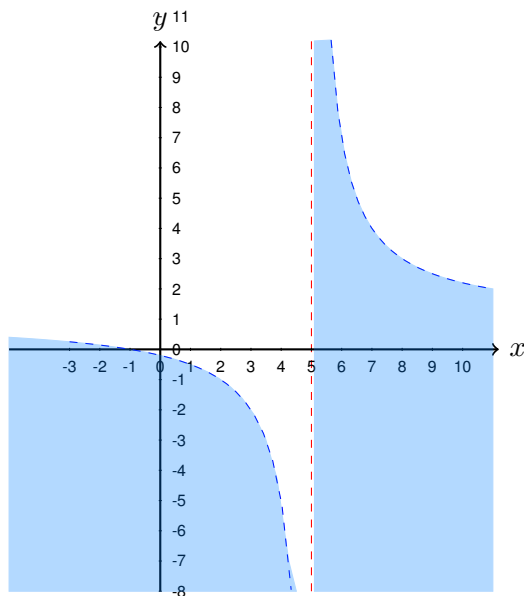
For the region below the leftmost curve we choose $(-3, 0)$ as the test point. Since $0 < \frac{-2}{-8}$, all points in this region belong to the solution set.

For the region above the leftmost curve and left of the line $x = 5$, we choose $(0, 0)$ as the test point. Since $0 > \frac{1}{-5}$, no points in this region belong to the solution set.

For the region below the rightmost curve and right of the line $x = 5$, we choose $(6, 0)$ as the test point. Since $0 < \frac{7}{1}$, all points in this region belong to the solution set.

For the region above the rightmost curve and right of the line $x = 5$, we choose $(8, 8)$ as the test point. Since $8 < \frac{9}{3}$, no points in this region belong to the solution set.

Finally, we shade the regions for which all points belong to the solution set.



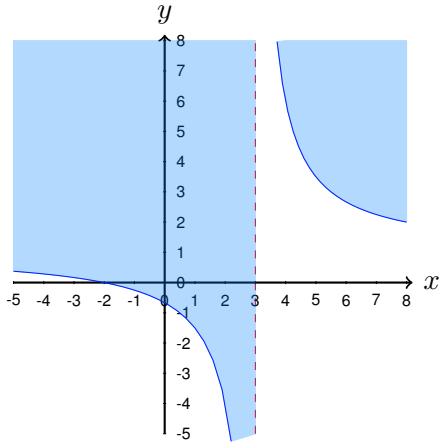
4.2.1 Practice Problems

Graph the solution set to the following:

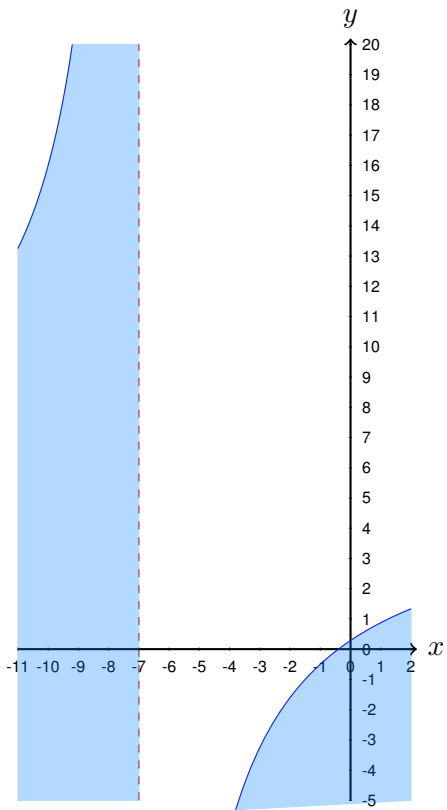
1. $y \geq \frac{x+2}{x-3}$
2. $y \leq \frac{5x+2}{x+7}$
3. $y < \frac{1}{x} + 1$
4. $y < \frac{x-2}{x+3}$

4.2.2 Solutions

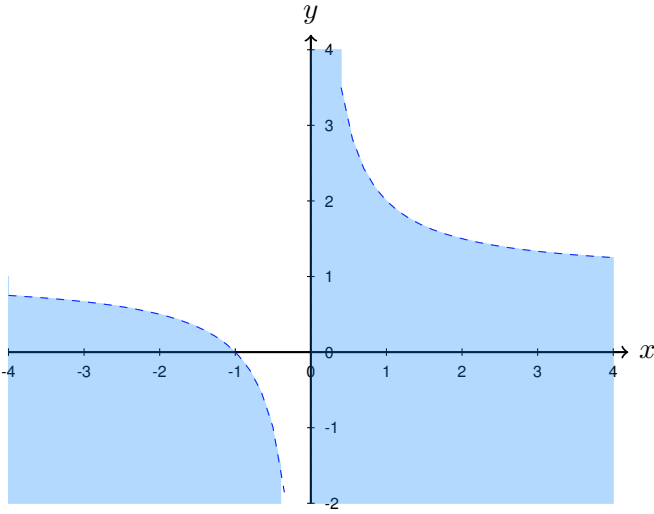
1. Solution set:



2. Solution set:



3. Solution set:



4. Solution set:

