3.4 Inequalities in $\mathbb{R}^2$

In this section we discuss graphing the solution set to an inequality like $y \leq f(x)$, or $y \geq f(x)$, where f is a function. The solution set is the set of all points for which the inequality is true. The material in this section is very similar to our previous work on graphing the solution set to inequalities involving rational expressions.

The graph of a function $f(x)$ is the set of all points (x, y) such that $y = f(x)$. It partitions the plane into two regions. The same inequality holds for all points in the same region. Hence what we need to do is sketch the graph of $y = f(x)$, pick a test point in each region and check which inequality holds, and then be careful about whether the points on the graph need to be included.

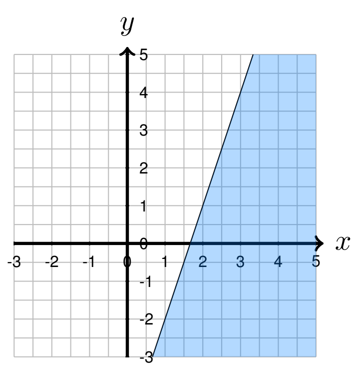
Example 38 *Graph the solution set of* $y \leq 3x - 5$*.*

The solution set is $\{(x, y) : y \leq 3x - 5\}$.

The first step is to sketch the graph of (the line) $y = 3x - 5$. All points on the line belong to the solution set, so we draw a solid line (otherwise, we would draw a dashed line).

The next step is to pick a (convenient) test point in each region. In the leftmost region, we pick $(0, 0)$ as the test point. Since $0 > 3 \cdot 0 - 5$, no points in this region belong to the solution set. In the rightmost region, we pick $(4, 0)$ as the test point. Since $0 < 3 \cdot 4 - 5$, all points in this region belong to the solution set.

A graph of the solution set is shown below. The collection of points in the solution set and not on the line is shaded (blue). The line is drawn solid to indicate that the points on the line are in the solution set (as indicated above).



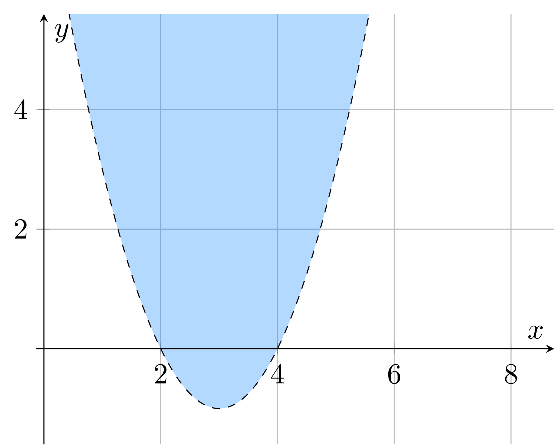
Example 39 *Graph the solution set of* $y > x^2 - 6x + 8$*.*

The solution set is $\left\{(x, y) : y > x^2 - 6x + 8\right\}$.

The first step is to sketch the graph of $y = x^2 - 6x + 8$. We know it is a parabola, and since $x^2 - 6x + 8 = (x - 2)(x - 6)$, it crosses the x axis at 2 and 4, and has its vertex at $x = 3$. By computation, the y-coordinate of the vertex is -1. Since $y = x^2 - 6x + 8$ for every point on the graph, no points on the parabola belong to the solution set and so we sketch the parabola with a dashed line.

The next step is to pick a test point in each region: inside the parabola and outside of it. For the region outside the parabola we pick $(0, 0)$ as the test point. Since $0 < 8$, no points in this region belong to the solution set. For the region inside the parabola, we pick $(3, 0)$ as the test point. Since $0 > -1$, all points in this region are in the solution set.

A graph of the solution set is shown below.



3.4.1 Exercises

1. Graph $\{(x, y) : y < 4x - 9\}$.

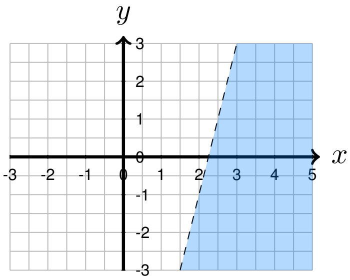
2. Graph the solution set to $y > \frac{1}{3}x + 4$.

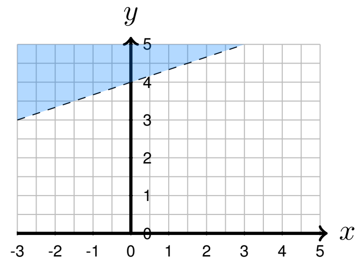
3. Graph $\{(x, y) : y \geq 8x\}$.

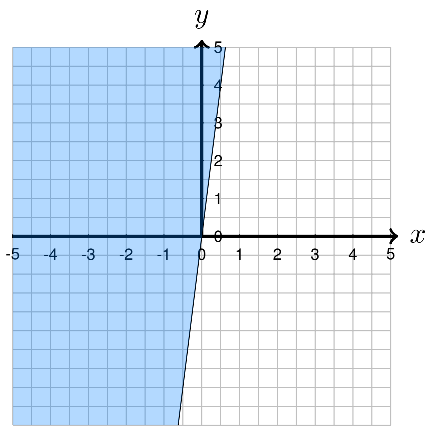
4. Graph the solution set to $y > x^2 + 2x + 5$.

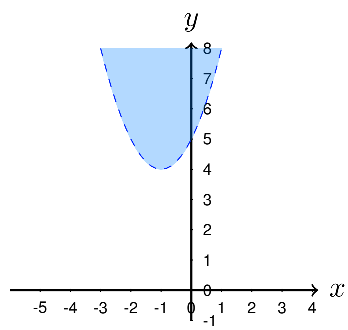
5. Graph the solution set to $y < \frac{x^7+2x^6+x^5}{x^5}$.

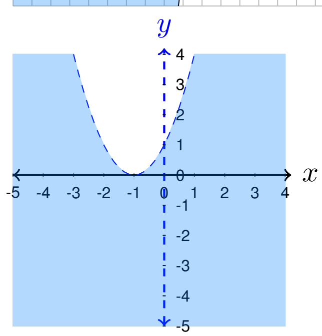
3.4.2 Solutions

1. 

2. 

3. 

4. 

5. 

Note: Because $\frac{x^7+2x^6+x^5}{x^5}$ is undefined for $x = 0$, we have excluded the line $x = 0$ as well as the curve $y = x^2 + 2x + 1$.