University of Victoria | Graduate Studies

Notice of the Final Oral Examination for the Degree of Master of Science

of

ASHNA WRIGHT

BSc (University of Victoria, 2022)

"Counting X-free sets"

Department of Mathematics and Statistics

Thursday, June 13, 2024
2:30 P.M.
David Strong Building
Room C108

Supervisory Committee:
Dr. Natasha Morrison, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Jonathan Noel, Department of Mathematics and Statistics, UVic (Co-Supervisor)
Dr. Anthony Quas, Department of Mathematics and Statistics, UVic (Member)

External Examiner:
Dr. Adam Zsolt Wagner, Department of Mathematical Sciences, Worcester Polytechnic Institute

Chair of Oral Examination:
Dr. Panajotis Agathoklis, Department of Electrical and Computer Engineering, UVic

Dr. Robin G. Hicks, Dean, Faculty of Graduate Studies

Abstract

Let X be a finite subset of $\mathbb{Z}^d$. A set $A \subseteq [n]^d$ is X-*free* if it does not contain *a copy of* X, that is subset of the form $\boldsymbol{b} + r \cdot X$ for any $r > 0$ and $\boldsymbol{b} \in \mathbb{R}^d$. Let $r_X(n)$ denote the cardinality of the largest X-free subset of $[n]^d$. In this thesis we explore X-free sets in three ways. Firstly, we give an exposition of a standard multidimensional extension of Behrend's construction that gives a lower bound on $r_X(n)$ for all $|X| \geq 3$. Next, using this lower bound on $r_X(n)$, we lower bound the number of copies of X guaranteed in subsets with cardinality larger than $r_X(n)$, a *supersaturation* result. Finally, using our supersaturation result, we show that for infinitely many values of n the number of X-free subsets is $2^{O(r_X(n))}$. This result is obtained using the powerful hypergraph container method. Further, it generalizes previous work of Balogh, Liu, and Sharifzadeh and Kim.

This thesis includes joint work with Natalie Behague, Joseph Hyde, Natasha Morrison, and Jonathan Noel.