Brendan Steed

- BSc Hons. (University of Victoria, 2021)

Notice of the Final Oral Examination for the Degree of Master of Science

Topic

The Heisenberg Spectral Triple and Associated Zeta Functions

Department of Mathematics and Statistics

Date & location

- Monday, November 27, 2023
- 10:00 A.M.
- David Turpin Building, Room A203

Examining Committee

Supervisory Committee

- Dr. Heath Emerson, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)
- Dr. Ian Putnam, Department of Mathematics and Statistics, UVic (Co-Supervisor)

External Examiner

- Dr. Rodrigo Trevino, Department of Mathematics, University of Maryland

Chair of Oral Examination

- Dr. Catherine Stevens, School of Earth and Ocean Sciences, UVic

Abstract

The construction of Butler, Emerson, and Schultz [2] produced a certain spectral triple, which they called the Heisenberg cycle, by way of the quantum mechanical annihilation and creation operators, $d/dx \pm x$, along with their relationships to the harmonic oscillator, $-d^2/dx^2 + x^2$; Where all of these operators are defined (initially) to act on smooth functions over R. In particular, their Heisenberg cycle was over a crossed-product generated by the natural translation action on the (commutative) C^*-algebra of uniformly continuous, bounded, functions on $\mathbb{R}$.

In this thesis, we generalize the Heisenberg cycle of Butler, Emerson, and Schultz to allow for the construction of a spectral triple over a crossed-product generated by the natural translation action on the $C*$-algebra of uniformly continuous, bounded, functions on a Euclidean space, V, of arbitrary finite dimension n. For such a generalization, the annihilation and creation operators are replaced using the exterior derivative and codifferential, exterior and interior multiplication by a certain differential 1-form, and the relationship these four operators have to the n-dimensional harmonic oscillator acting on differential forms. Similarly to [2], we will show that our generalized Heisenberg cycle provides a new way of producing spectral triples over crossed-products of the form $C(M) \rtimes_\alpha \Gamma$, where Γ is a discrete subgroup of V and $\alpha : V \times M \to M$ is a smooth V-action on a compact manifold M.

In Chapter 1, we introduce the problem and briefly discuss some historical background behind Alain Connes program of noncommutative geometry, as well as touch on some elementary constructions in multi-linear algebra. Chapter 2 is where we define the classes of differential forms which appear most frequently in this thesis. Therein, we also rigorously define the operators mentioned in the paragraph above, and use them to produce the so-called Dirac-Heisenberg which will be associated to our generalization of the Heisenberg cycle. For the first half of Chapter 3, we discuss some basic C^*-algebra theory and introduce the crossed-product native to the Heisenberg cycle. In the latter half of that chapter, we verify that our Heisenberg cycle satisfies the conditions of a spectral triple, compute an integral formula for the resulting ζ-functions, and show how one uses the Heisenberg cycle to produce spectral triples over crossed-products generated by smooth actions of V on compact manifolds.