University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

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BSc (University of Victoria, 2021)

"The Edwards–Sokal Coupling for the Potts Higher Lattice Gauge Theory on $\mathbb{Z}^d$"

Department of Mathematics and Statistics

Wednesday, August 9, 2023
9:30 A.M.
David Strong Building
Room C108

Supervisory Committee:
Dr. Anthony Quas, Department of Mathematics and Statistics, UVic (Co-Supervisor)
Dr. Gourab Ray, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)

External Examiner:
Dr. Matan Harel, Department of Mathematics, Northeastern University

Chair of Oral Examination:
Dr. Denise Cloutier, Department of Geography, UVic

Dr. Robin G. Hicks, Dean, Faculty of Graduate Studies

Abstract

The Edwards–Sokal coupling of the standard Potts model with the FK–Potts (random-cluster) bond percolation model can be generalized to arbitrary-dimension cells. In particular, the Potts lattice gauge theory on $\mathbb{Z}^d$ has a graphical representation as a plaquette percolation measure. We systematically develop these previously-known results, using the frameworks of cubical (simplicial) homology and discrete Fourier analysis.

We show that, in the finite-volume setting, the Wilson loop expectation of a higher cycle γ is equal to the probability that γ is a homological boundary in the higher FK–Potts model. We also prove the strong FKG property of the higher FK–Potts model. These results culminate in a simple proof for the existence of infinite-volume limits in the higher Potts model and, in certain cases, of their invariance under translations and other symmetries. Additionally, we thoroughly examine the behavior of boundary conditions as they relate to the Edwards–Sokal coupling, for the purpose of understanding the higher Potts Gibbs states. In particular, we discuss spatial Markov properties and conditioning in the higher FK–Potts model, and generalize to more general boundary conditions the FKG property, the aforementioned identity forWilson loop expectations, and a result about monotonicity in the coupling strength parameter. Also, we prove a theorem regarding the sharpness of thresholds of increasing symmetric events for the higher FK–Potts model with periodic boundary conditions.

In the final section, we describe some matrix-based sampling algorithms. Lastly, we prove a new characterization of the ground states of the random-cluster model, motivated by the problem of understanding the ground states in the higher FK–Potts model.