University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

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BSc Hons. (University of Southampton, UK, 2019)

"Topological Data Analysis:
Persistent Homology of Uniformly Distributed Points"

Department of Mathematics and Statistics

Wednesday, July 5, 2023
9:00 A.M.
David Strong Building
Room C126

Supervisory Committee:
Dr. Ryan Budney, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Farouk Nathoo, Department of Mathematics and Statistics, UVic (Member)
Dr. Alan Mehlenbacher, Department of Economics, UVic (Outside Member)

External Examiner:
Dr. Chad Giusti, Department of Mathematics, Oregon State University

Chair of Oral Examination:
Dr. JJ Kavelaars, Department of Physics and Astronomy, UVic

Dr. Robin G. Hicks, Dean, Faculty of Graduate Studies

Abstract

Topological Data Analysis (TDA) is a branch of computational topology that provides methods to extract qualitative information from high-dimensional, noisy, and incomplete data. TDA combines techniques from various fields, such as algebraic topology, computational geometry, algorithms, statistics, and graph theory. Persistent Homology (PH), based on homology theory from algebraic topology, is the principal tool used in TDA; PH tracks the evolution of topological features of the data across multiple scales through persistent homological bars, which represent the creation (birth) and disappearance (death) of these features. These bars are graphically depicted through persistence diagrams and persistence barcodes. The challenge in using PH for the analysis of noisy real-world data is to separate the bars generated by noise from the bars that provide meaningful topological information of the underlying geometric object from which the data is sampled; this problem remains unresolved despite various proposed techniques. A limited number of papers analyzed the PH of noise by considering points in $\mathbb{R}^d$ generated using probability distributions. This thesis introduces persistent homology concentrating on the computational side, and it examines the birth and death times of persistent homology bars generated by Vietoris-Rips complexes of uniformly distributed points in three spaces: a unit interval, a unit square, and a unit cube. Through numerical simulations, it is identified that the birth and death times of the persistent homology bars adhere to distinct statistical distributions, whose precise nature varies according to the space from which the points are sampled and the homological dimension of the persistent homology bars; the research examines the behaviour of their parameters as the number of points increases, providing insights into the persistent homology of noise and laying the groundwork for further research.