



University
of Victoria

Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

TYLER SCHULZ

BSc (University of Victoria, 2021)

“Supercritical Phase Transitions from Number Theory”

Department of Mathematics and Statistics

Thursday, April 13, 2023

3:30 P.M.

David Strong Building

Room C128

Supervisory Committee:

Dr. Marcelo Laca, Department of Mathematics and Statistics, University of Victoria (Supervisor)

Dr. Stephen Scully, Department of Mathematics and Statistics, UVic (Member)

External Examiner:

Prof. Aidan Sims, School of Mathematics and Applied Statistics, University of Wollongong

Chair of Oral Examination:

Dr. Andrew Marton, Department of Pacific and Asian Studies, UVic

Dr. Robin G. Hicks, Dean, Faculty of Graduate Studies

Abstract

We classify the KMS_β states of the right $ax + b$ C^* -dynamical system of \mathbb{N} in the supercritical range $\beta \in (0,1]$, thus completing the classification of KMS_β states initiated in [13]. We show that the simplex of KMS_β states is affinely isomorphic to the simplex of subconformal measures on the circle. We then provide explicit formulas for the extremal subconformal measures and corresponding KMS_β states in terms of classical arithmetic functions. For $\beta \in (0,1]$, our measures are parameterized by the compact space $\mathbb{N}^\times \cup \{\infty\}$, and in particular, demonstrate phase transition at each value of β , a novel feature among C^* -dynamical systems related to number theory.

Another new feature of the right $ax + b$ system is the existence of equivariant quotients, corresponding to the quotient rings $\mathbb{Z}/m\mathbb{Z}$ for $m \in \mathbb{N}^\times$. We provide a classification of the KMS_β states of the quotient C^* -dynamical systems, and show that the quotient systems exhibit spontaneous symmetry-breaking with respect to the group of units $(\mathbb{Z}/m\mathbb{Z})^*$. We then use this action to compute the type of the high-temperature KMS_β states with parameter belonging to $\mathbb{N}^\times \subseteq \mathbb{N}^\times \cup \{\infty\}$.