University of Victoria | Graduate Studies

Notice of the Final Oral Examination for the Degree of Doctor of Philosophy

of

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MSc (University of Victoria, 2012)
BSc (Thompson Rivers University, 2010)

“The i-Graph and Other Variations of the γ-Graph”

Department of Mathematics and Statistics

Wednesday, December 14, 2022
9:00 A.M.
Virtual Defence

Supervisory Committee:
Dr. Christina Mynhardt, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)
Dr. Richard Brewster, Department of Mathematics and Statistics, UVic (Co-Supervisor)
Dr. William Bird, Department of Computer Science, UVic (Outside Member)

External Examiner:
Dr. M.E. Messinger, Department of Mathematics & Computer Science, Mount Allison University

Chair of Oral Examination:
Dr. Erica Woodin, Department of Psychology, UVic

Dr. Robin Hicks, Dean, Faculty of Graduate Studies

Abstract

In graph theory, reconfiguration is concerned with relationships among solutions to a given problem for a specific problem. For a graph *G*, the *γ-graph* of *G*, *G*(*γ*), is the graph whose vertices correspond to the minimum dominating sets of *G*, and where two vertices of *G*(*γ*) are adjacent if and only if their corresponding dominating sets in *G* differ by exactly two adjacent vertices. We present several variations of the *γ*-graph including those using identifying codes, locating-domination, total-domination, paired-domination, and the upper domination number. For each, we show that for any graph *H*, there exist infinitely many graphs whose *γ*-graph variant is isomorphic to *H*.
The independent domination number *i*(*G*) is the minimum cardinality of a maximal independent set of *G*. The i-graph of *G*, denoted $\mathscr{I}$(*G*), is the graph whose vertices correspond to the *i*-sets of *G*, and where two *i*-sets are adjacent if and only and if the they differ by two adjacent vertices. In contrast to the parameters mentioned above, we show that not all graphs are *i*-graph realizable. We build a series of tools to show that known i-graphs can be used to construct new *i*-graphs and applied these results to build other classes of *i*-graphs, such as block graphs, hypercubes, forests, and unicyclic graphs. We determine the structure of the *i*-graphs of paths and cycles, and in the case of cycles, discuss the Hamiltonicity of their *i*-graphs. We also construct the *i*-graph seeds for certain classes of line graphs, a class of graphs known as theta graphs, and maximal planar graphs. In doing so, we characterize the line graphs and theta graphs that are *i*-graphs.